SDS321HW23Sol

1. A student, wishing to estimate the average time it takes to load blackboard on her Galaxy-4®, does a study. As data, she will randomly select three times of day and will count the time it takes to load (in seconds). Suppose that the time it takes to load the webpage is exponentially distributed for which the value of the parameter, Θ, is unknown.

   a. Give the likelihood function assuming that the waiting times are exponentially distributed with parameter Θ.

      $L(\Theta)=\Theta^n e^{-\theta\Sigma xi}$ x>0

   b. Derive the maximum likelihood estimator for Θ.

      $LL(\Theta)=n\log\Theta+(-\Theta\Sigma x_i)$

      So solve $n/\Theta-\Sigma x_i=0$

      So $\Theta^*_{MLE}=n/(\Sigma x_i)$

   c. Now the student gathers her data. The results are $y_1=22$, $y_2=18$, $y_3=35$. Find the maximum likelihood estimate for Θ. 3/75=.04

   d. The student decides to estimate the waiting time using a Bayesian approach. Based on her subjective opinion about the waiting time, she uses the following gamma prior for Θ:

      $p(\Theta)=\Theta e^{-2\Theta}, 0<\Theta.$

      To conduct a Bayesian analysis, she will need to determine the posterior distribution for Θ. Using the same three data points, write an expression that is proportional to the posterior, that is, p(Θ ly).

      $p(\Theta\, ly)\propto \Theta^n e^{-\theta\Sigma xi}\ \Theta e^{-2\Theta}=\Theta^4 e^{-77\Theta}$

   e. Recognizing your answer to the previous question as a particular parametric distribution, name it and its parameters.

      Gamma(5,77)

   f. What are the posterior mean and variance of Θ? (numeric answers)

      $E[\Theta]=5/77$, $Var[\Theta]=5/77^2$

   g. If you use a mean square loss function, what is a Bayesian estimate for Θ?

5/77

h. Find the MAP estimate for Θ. 4/77

i. Suppose that the 90% credible set produced in the previous question is (0.03, 0.12). The Bayesian interpretation is (circle one):

   i. There is 90% probability that the average time to upload in a randomly sampled task is between 0.03 and 0.12.

   ii. Circle this one! Assuming this prior, there is 90% probability that the true parameter for the time it takes to upload is between 0.03 and 0.12.

   iii. We have used a procedure that, for 90% of samples, will produce an interval that contains the true parameter value.