Group Analysis

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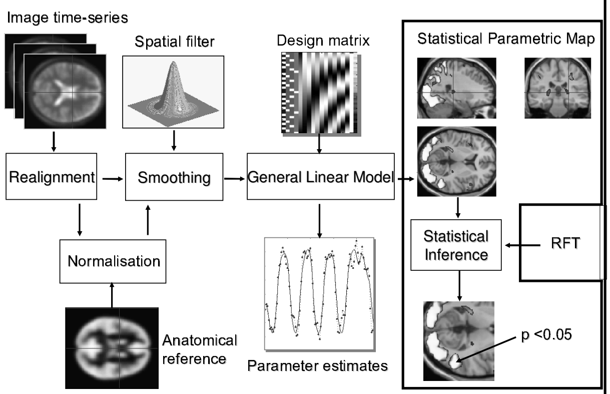
SPM Course
Edinburgh, April 2011



Lexicon

- ❑ Hierarchical models
- ❑ Mixed effect models
- ❑ Random effect (RFX) models
- ❑ Components of variance

… all the same
… all alluding to multiple sources of variation
(in contrast to fixed effects)

Hierarchical models: definition

Hierarchical model

$$y = X^{(1)}\theta^{(1)} + \varepsilon^{(1)}$$
$$\theta^{(1)} = X^{(2)}\theta^{(2)} + \varepsilon^{(2)}$$
$$\vdots$$
$$\theta^{(n-1)} = X^{(n)}\theta^{(n)} + \varepsilon^{(n)}$$

Multiple variance components at each level

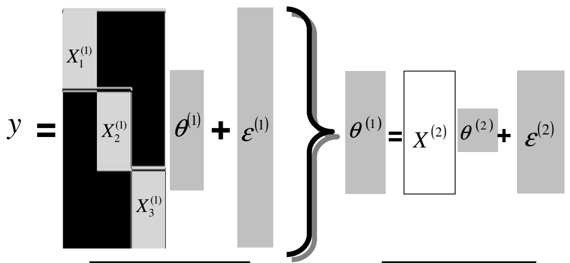
$$C_\varepsilon^{(i)} = \sum_k \lambda_k^{(i)} Q_k^{(i)}$$

At each level, changes in any parameter is driven by changes at the level above.

What we don't know: GLM parameters (theta) and variance hyperparameters (lambda).

Hierarchical models: fMRI group analysis

$$y = X^{(1)}\theta^{(1)} + \varepsilon^{(1)}$$
$$\theta^{(1)} = X^{(2)}\theta^{(2)} + \varepsilon^{(2)}$$



First level

Second level

Overview

- Group analysis: fixed versus random effects
- Two RFX methods:
  - *Holmes & Friston (HF) approach*
  - *non-sphericity modelling*
- Examples
- Power and efficiency: summary

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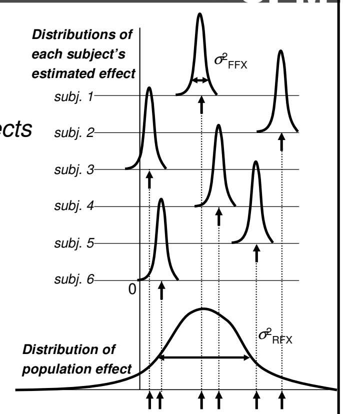
FFX versus RFX

❑ Fixed effects:
Intra-subjects variation suggests *all these subjects different from zero*

❑ Random effects:
Inter-subjects variation suggests *population not different from zero*



Fixed effect

$$y = X^{(1)}\theta^{(1)} + \varepsilon^{(1)}$$

- ❑ Only source of variation (over sessions) is measurement error
- ❑ True response magnitude is *fixed*

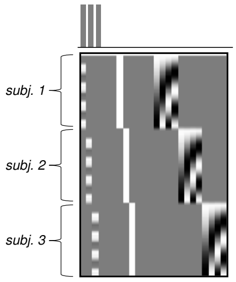
Random effects

$$y = X^{(1)}\theta^{(1)} + \varepsilon^{(1)}$$
$$\theta^{(1)} = X^{(2)}\theta^{(2)} + \varepsilon^{(2)}$$

- Two sources of variation
  - measurement errors
  - response magnitude (over subjects)
- Response magnitude is *random*
  - each subject/session has random magnitude
  - but note, population mean magnitude is *fixed*

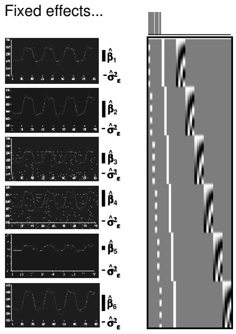
FFX modelling in SPM: basics

- Grand GLM approach
  (model all subjects at once)
- Good:
  - *max dof*
  - *simple model*
- Bad:
  - *assumes common variance over subjects at each voxel*

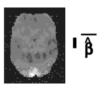


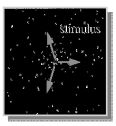
FFX modelling in SPM: summary

Fixed effects...



estimated mean activation image…

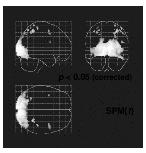




…to be compared with residuals variance:

- $\sigma^2_\varepsilon / nw$

n – subjects
w – error dof



FFX versus RFX

- FFX isn't "wrong", just usually isn't of interest
- Summary:
  - **FFX inference**:
    "I can see this effect in this cohort"
  - **RFX inference**:
    "If I were to sample a new cohort from the same population I would get the same result"

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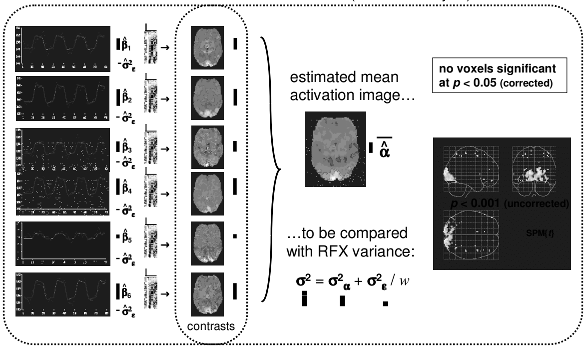
HF approach: basics

- "Summary statistics" approach
- 1- or 2- sample *t* test on contrast image
  - intra-subject variance not used
- Procedure:
  - Fit GLM for each subject i
    and compute contrast estimate $c\hat{\beta}_i$ (first level)
  - Analyze $\left\{c\hat{\beta}_i\right\}_{i=1,\ldots,n}$ (second level)

HF approach: summary



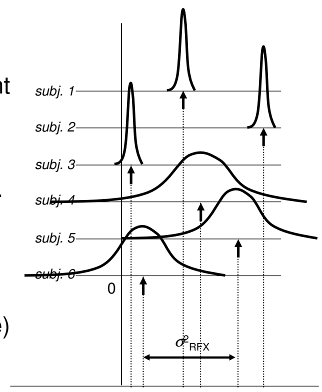
HF approach: assumptions

- Distribution
  - Normality
  - Independent subjects
- Homogeneous variance:
  - Residual error the same for all subjects
  - Balanced designs

HF approach: efficiency & validity

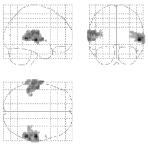
- If assumptions true
  - Optimal, fully efficient
  - Exact p-values
- If σ^2_{FFX} differs btw subj.
  - Reduced efficiency
  - Biased σ^2_{RFX}
  - Liberal dof

  (here 3 subj. dominate)

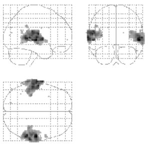


HF approach: robustness (I)

Summary statistics



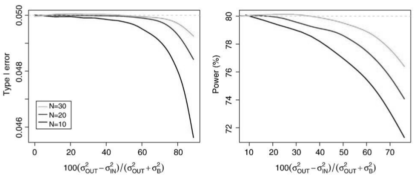
Full Bayesian Hierarchical Modelling



Friston et al. (2004) Mixed effects and fMRI studies, Neuroimage

HF approach: robustness (II)

- In practice, validity and efficiency are excellent
  - For 1-sample case, HF impossible to break



Mumford & Nichols. Simple group fMRI modeling and inference. *Neuroimage*, 47(4):1469--1475, 2009.

- 2-sample and correlation might give trouble
  - Dramatic imbalance and/or heteroscedasticity

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Non sphericity modelling: basics

- 1 effect per subject
  - Use Holmes & Friston approach

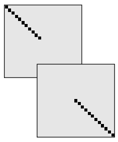
- >1 effects per subject
  - Can't use HF, must use non sphericity modelling
  - Covariance components and ReML

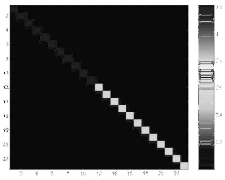
  (c.f. "Bayesian inference" talk)

Multiple covariance components (I)

- E.g., 2-sample t-test
  - Errors are independent but not identical.
  - 2 covariance components

Q_k's:



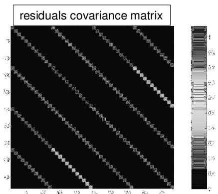


residuals covariance matrix

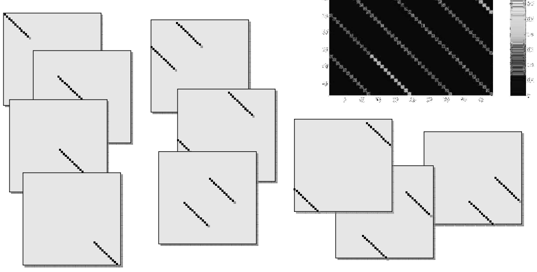
Multiple covariance components (II)

- Errors are not independent and not identical

residuals covariance matrix



Q_k's:



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Example 1: data

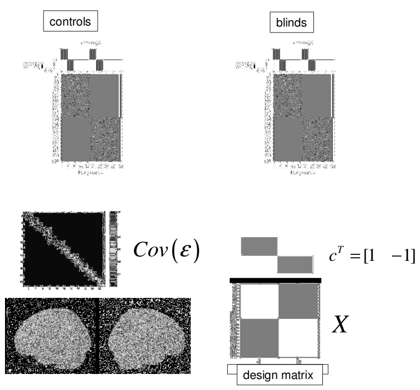
SPM

- Stimuli:
  - Auditory presentation (SOA = 4 sec)
  - 250 scans per subject, block design
  - Words, e.g. “book”
  - Words spoken backwards, e.g. “koob”
- Subjects:
  - 12 controls
  - 11 blind people

Example 1: population differences

SPM

- 1^{st} level
- 2^{nd} level



Example 2: data

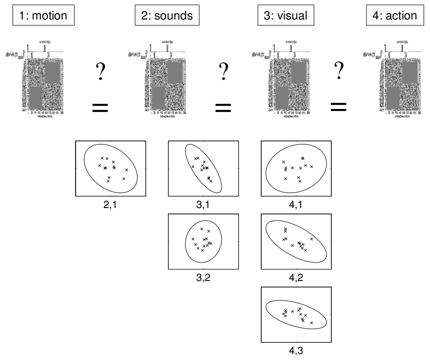
- Stimuli:
  - Auditory presentation (SOA = 4 sec)
  - 250 scans per subject, block design
  - Words:

Motion	Sound	Visual	Action
“jump”	“click”	“pink”	“turn”

- Subjects:
  - 12 controls
- Question:
  - What regions are affected by the semantic content of the words?

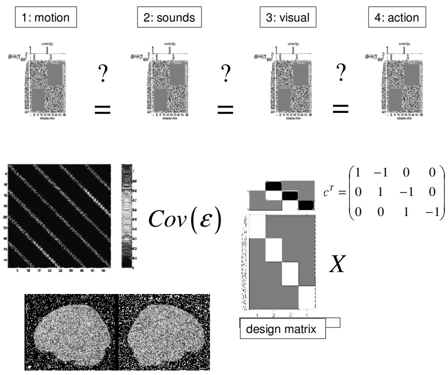
Example 2: repeated measures ANOVA

- 1st level
- 2nd level



Example 2: repeated measures ANOVA

- 1st level
- 2nd level



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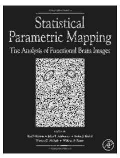
Power & efficiency: summary (II)

- Efficiency = 1/ [estimator variance]
  - goes up with *n* (number of subjects)
  - c.f. “experimental design” talk
- Power = chance of detecting an effect
  - goes up with degrees of freedom (*dof* = *n*-*p*).
  - I reject the null when *P<0.05*. Is my risk of false positive rate (FPR) controlled at 5%?

    Well, not exactly, but valid control: $FPR \leq \alpha$.

    This is potentially conservative.

Bibliography:



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With many thanks to G. Flandin, J.-B. Poline and Tom Nichols for slides.