Group Analysis

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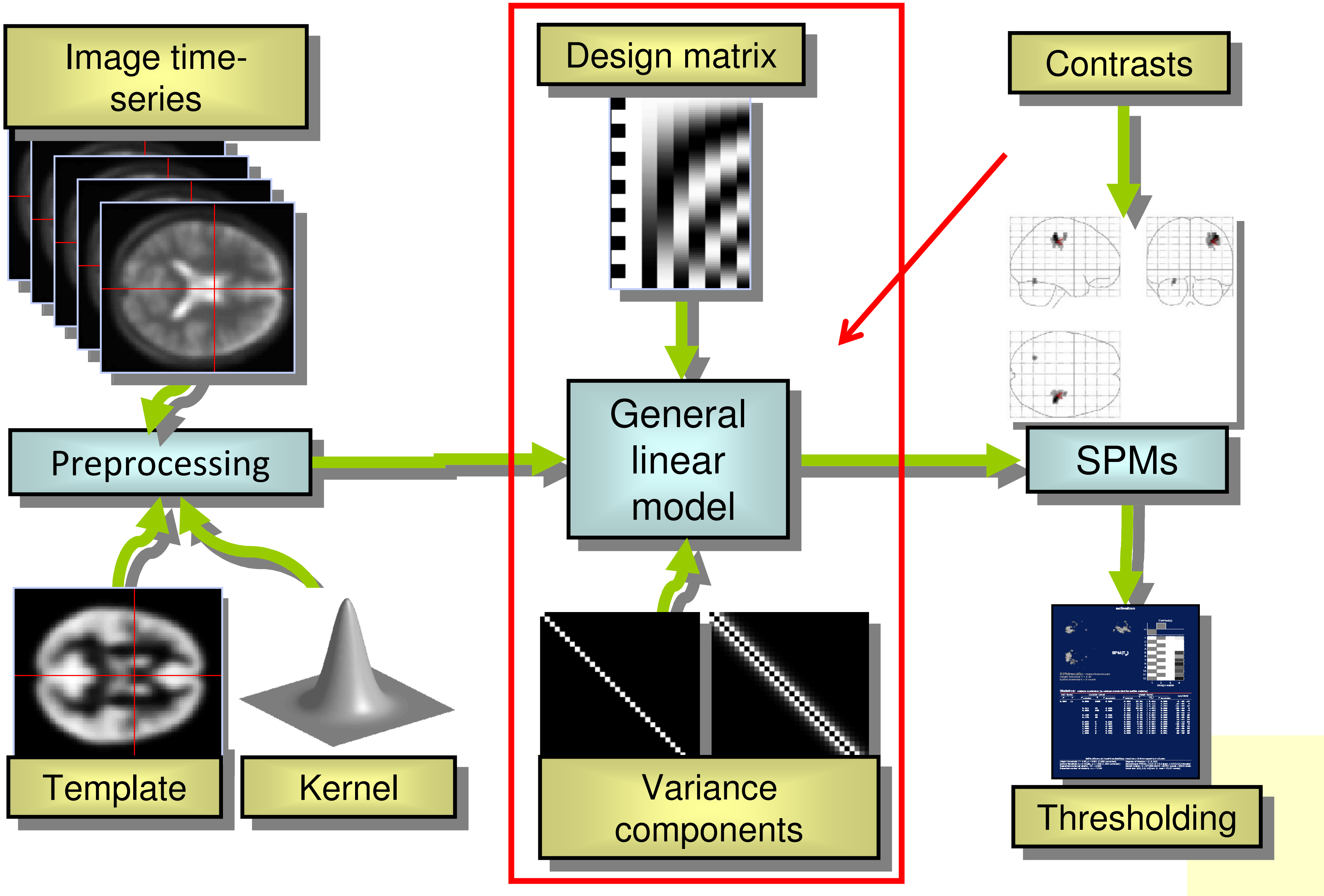
Edinburgh SPM course, 2013

Thanks to Jesper Anderson, Tom Nichols, Jean Daunizeau, Stephan Kiebel & other SPM authors for slides

Overview of SPM



Overview

Making the group inferences we want

- Optimising the GLM
- The two-stage GLM
- Two methods of RFX inference

2-stage GLM

Single subject

Each subject's scans are modelled separately
Single subject parameter estimates

1^{st} level

Single subject **contrasts of parameter estimates** taken to 2^{nd} level as (spm_con*.img) **'con images'**

Group/s of subjects

For a given effect, the whole group is modelled
Parameter estimates apply to group effect/s

2^{nd} level

Group level **contrasts of 2^{nd} level parameter estimates** are used to form statistics

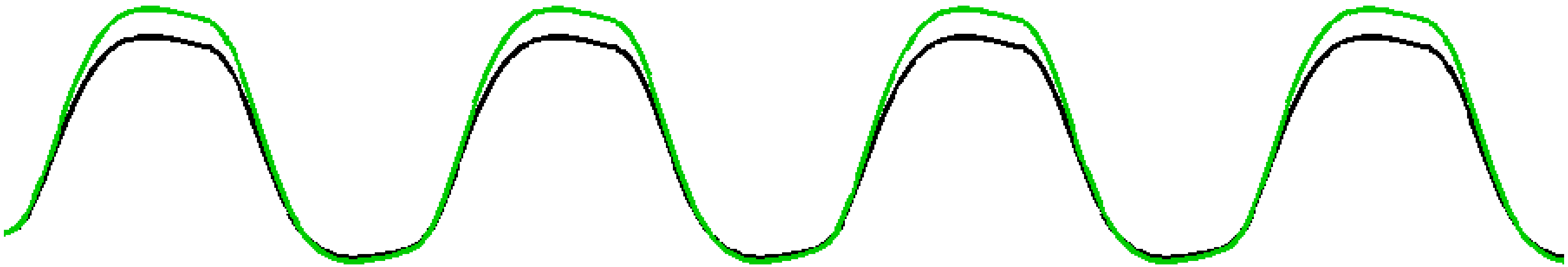
Overview

- Hierarchical models
- Mixed-effects models
- Random effects (RFX) models
- Variance components

... All the same

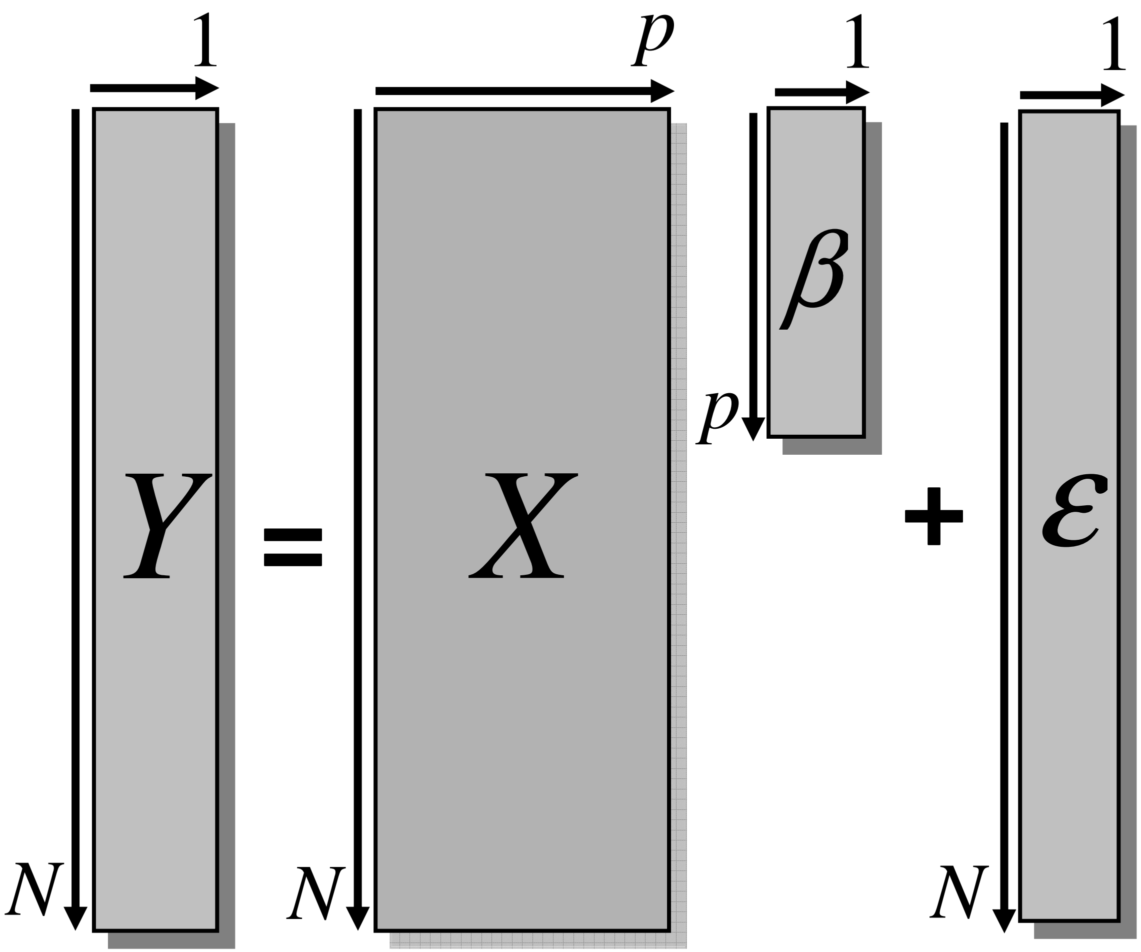
Refer to dealing with multiple sources of variation and making the inferences we want, i.e. generalising to a population

Why isn't this easy?



- Multiple levels of model
- Multiple components of error in each

The GLM revisited



$$y = X\beta + \varepsilon$$

$$\varepsilon \sim N(0, \sigma^2 I)$$

Estimate with **Ordinary Least Squares** (OLS)

N: number of scans
p: number of regressors

Model is specified by

1. Design matrix *X*
2. **Assumptions about ε**

Ordinary Least Squares revisited

Find $\hat{\beta}$ that minimises $$\boxed{\left\| y - X\beta \right\|^2 = \varepsilon^T \varepsilon}$$

The Ordinary Least Squares parameter estimates are:

$$\boxed{\hat{\beta} = (X^T X)^{-1} X^T y}$$

Estimation is **direct** – multiply data by the (pseudo) inverse of X

This is only valid if errors are i.i.d. – if there is a single error covariance component, the variance s^2.

$$\boxed{\varepsilon \sim N(0, \sigma^2 I)}$$

Because covariance affects the statistics…

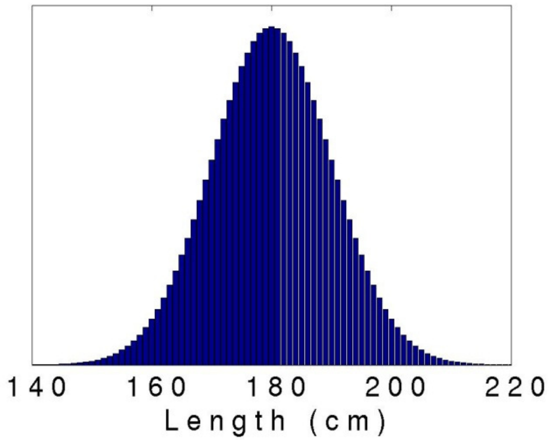
Covariance and non-sphericity

Classical inference is about what is **surprising**

- A statistic tests an effect's size relative to its expected behaviour under null hypothesis
- Degrees of freedom must reflect **how related** (correlated) different observations are
- If observations covary, there are fewer independent observations than we think, so significance of statistics can be overrated

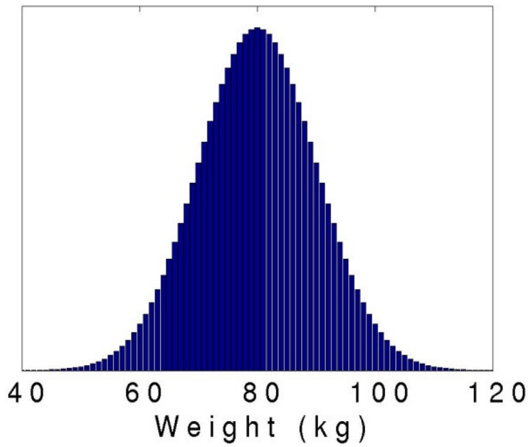
Variance

Length of men



μ=180cm, σ=14cm (σ^2=200)

Weight of men



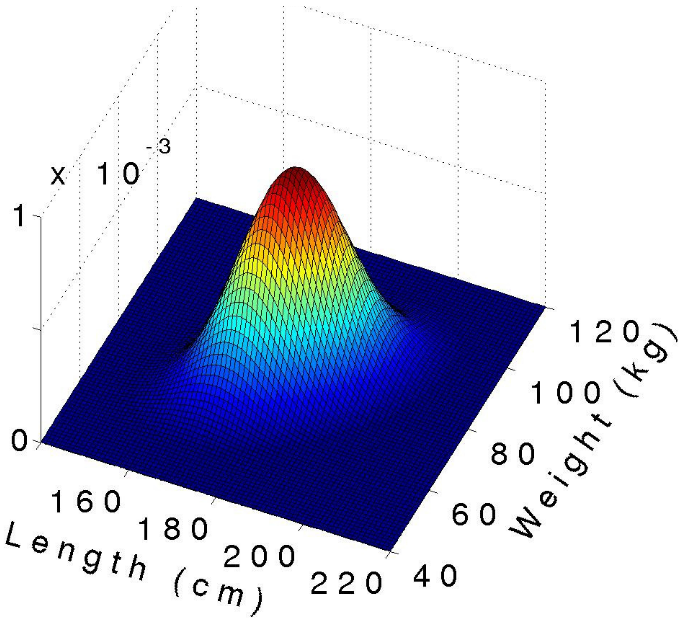
μ=80kg, σ=14kg (σ^2=200)

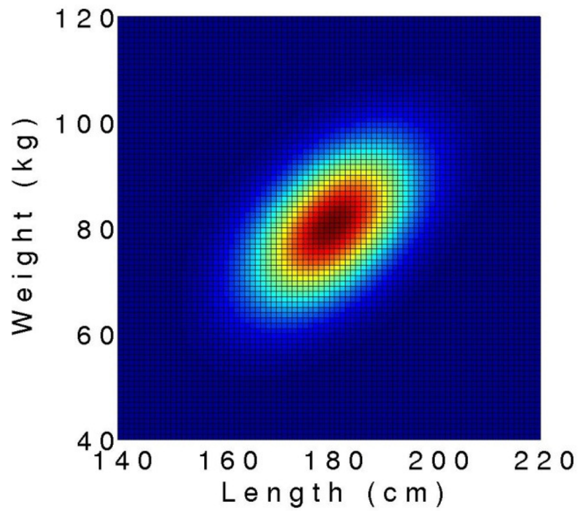
Each 1-dimensional variable is completely characterised by μ (mean) and σ^2 (variance)

i.e. can calculate $p(l|\mu,\sigma^2)$ for any l and $p(w|\mu,\sigma^2)$ for any w

Variance-covariance matrix

- Can also view length and weight as a 2-dimensional random variable (p(*l*,*w*)).





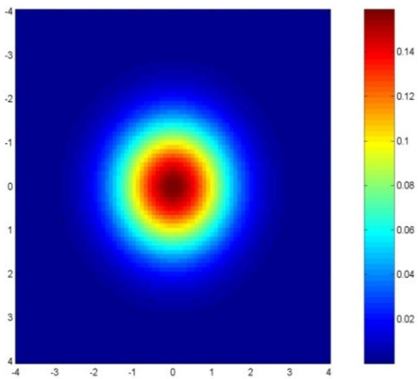
$p(l,w|\mu,\Sigma)$

$$\mu = \begin{bmatrix} 180 \\ 80 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 200 & 100 \\ 100 & 200 \end{bmatrix}$$

Length and weight are related – i.e., covary

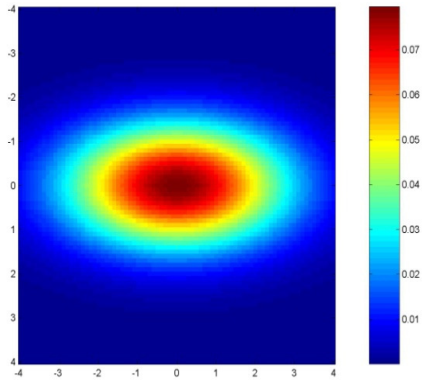
What is (and isn't) sphericity?

sphericity => i.i.d.
error covariance
It is a multiple of the identity matrix:
Cov(e) = σ^2I



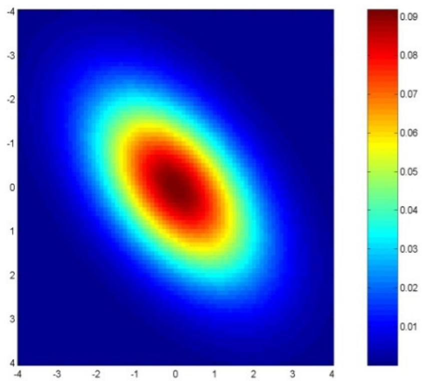
$$Cov(e) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Examples of non-sphericity:



$$Cov(e) = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$$

non-identity



$$Cov(e) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

non-independence

Covariance and statistics

$$T = \frac{c^T \hat{\beta}}{\sqrt{\mathrm{var}(c^T \hat{\beta})}} = \frac{c^T \hat{\beta}}{\sqrt{\hat{\sigma}^2 c^T \left(X^T X\right)^{-1} c}} \sim t_{N-p} = \frac{\text{contrast of estimated parameters}}{\sqrt{\text{variance estimate}}}$$

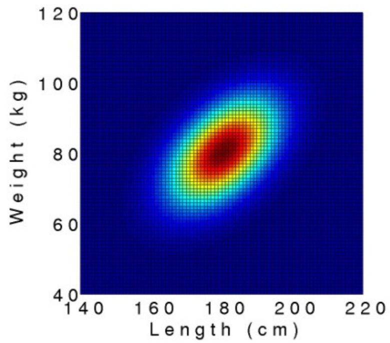
- How good an estimator (precise) is effect β?
- How much do we think the betas covary? – a precise (low) C_β maximises T

- The df are also a function of C_ε & design matrix X…

Covariance and degrees of freedom

- Measure departure from sphericity (episilon)
- Evaluate significance of sum of squares ratios using F with (approx) Greenhouse-Geisser df – i.e. fewer

Heights & weights



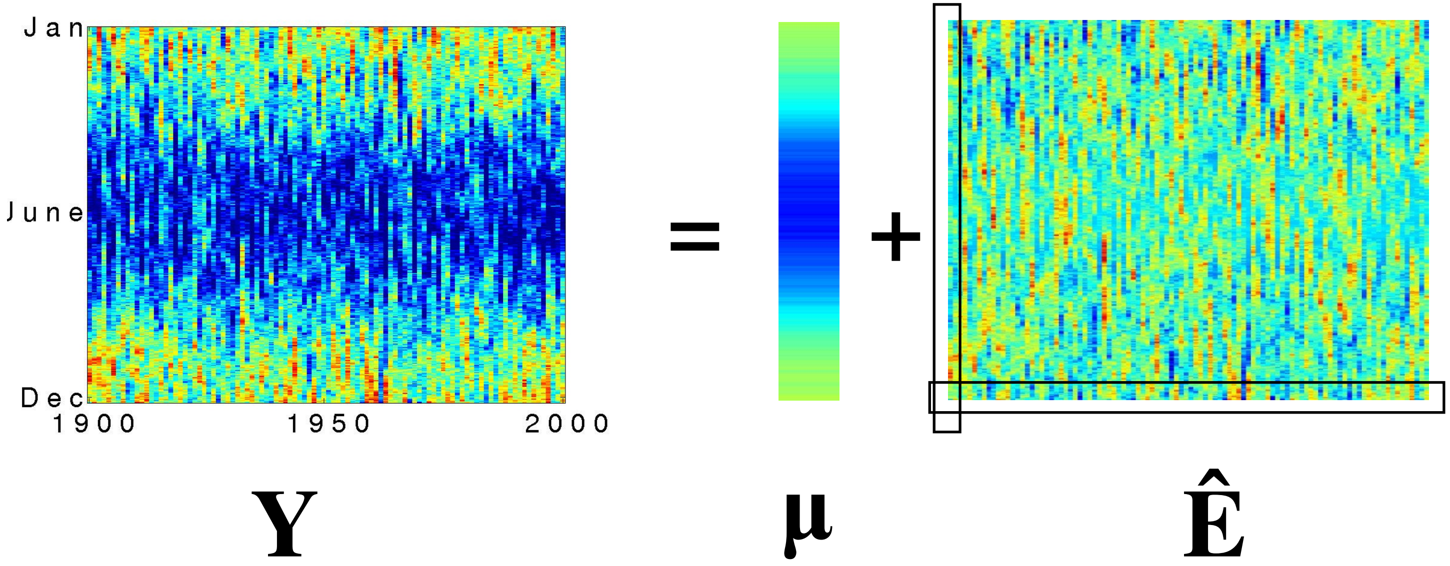
$$\Sigma = \begin{pmatrix} 200 & 100 \\ 100 & 200 \end{pmatrix} \quad \varepsilon = 0.8$$

= Satterthwaite correction (SPSS)

(in theory sl. liberal – but see Mumford & Nichols, 2009)

The rain in Bergen

12 months for 100 years



A simple GLM: model monthly rainfall using mean

Data from whole 20th century

The rain in Bergen

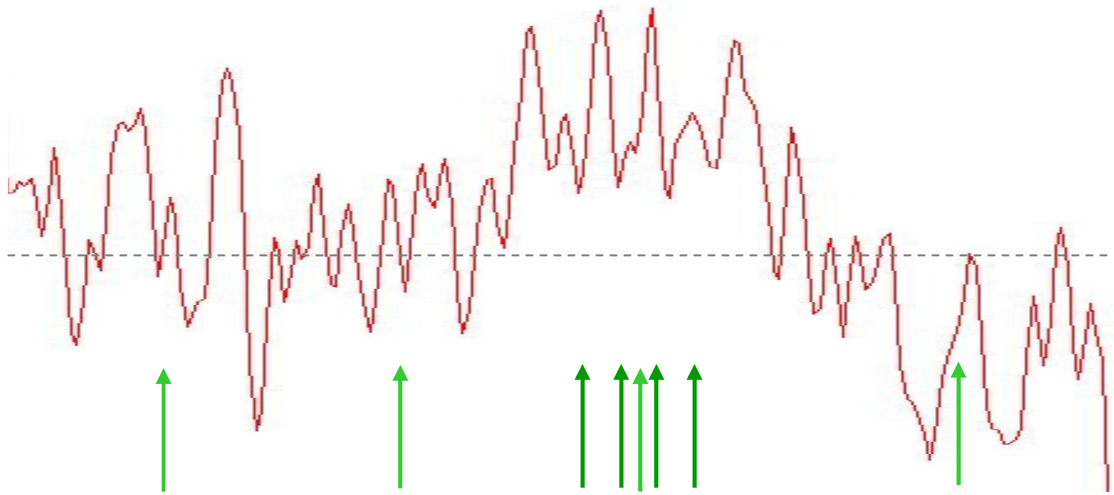
How much do the following observations tell us?

Rain on 4 consecutive days in June

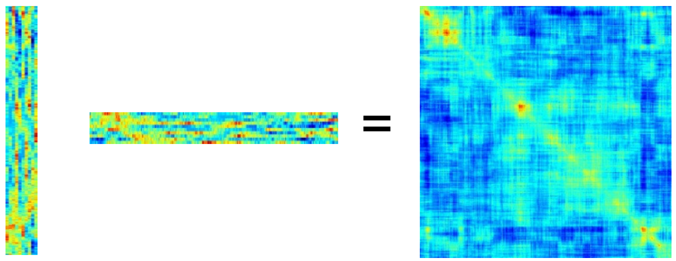
Rain on the same day in May, June, July and August

…which is more likely to indicate a wet summer?

Can we determine the patterns of correlation?

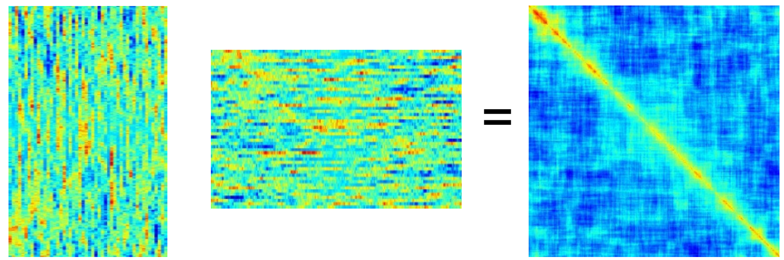


The rain in Bergen



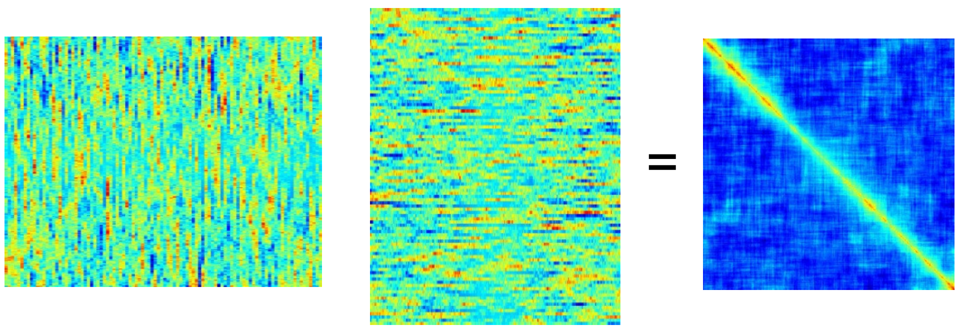
$\hat{\mathbf{E}}$ $\hat{\mathbf{E}}^T$ $\mathbf{S}$

Estimate based on 10 years



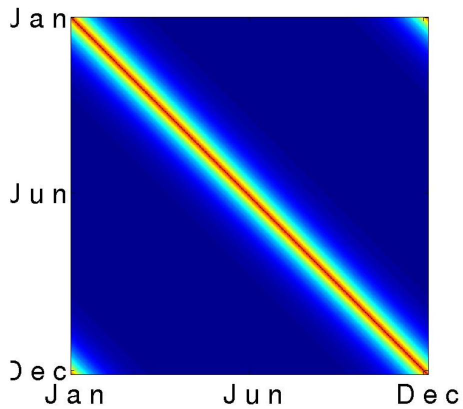
$\hat{\mathbf{E}}$ $\hat{\mathbf{E}}^T$ $\mathbf{S}$

Estimate based on 50 years



$\hat{\mathbf{E}}$ $\hat{\mathbf{E}}^T$ $\mathbf{S}$

Estimate based on 100 years

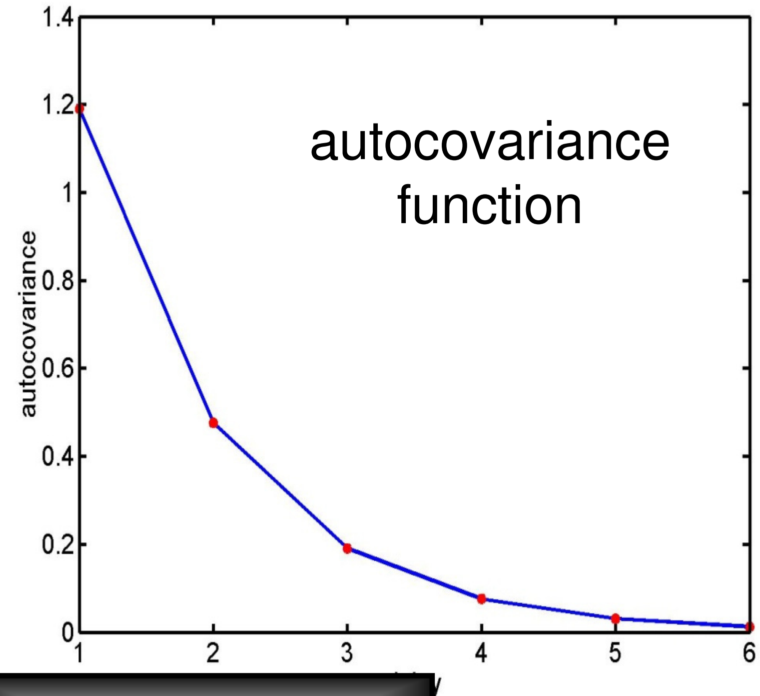


True $\boldsymbol{\Sigma}$ – as if there were not 100*365=36500 data points, but 2516!

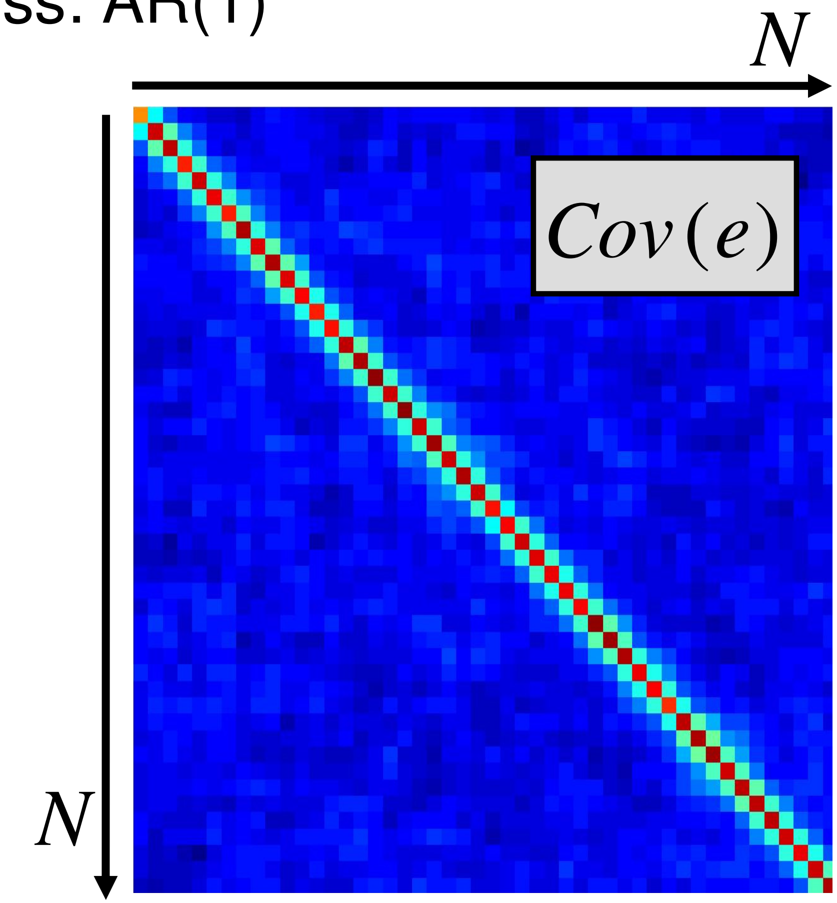
Serial correlations in fMRI

$$e_t = ae_{t-1} + \varepsilon_t \text{ with } \varepsilon_t \sim N(0, \sigma^2)$$

1st order autoregressive process: AR(1)



Also: high-pass filtering



Serial correlations in fMRI

Pre-whitening

- Use an enhanced noise model with multiple error covariance components
- Estimate components AR (1) + white noise
- Specify a filter matrix *W* to whiten the data – *'undoing'* the serial correlations

$$\boxed{Wy = WX\beta + We}$$

$$\boxed{We \sim N(0, \sigma^2 W^2 V)}$$

Serial correlations in fMRI

SPM prewhitening model: AR(1) + white noise

- AR(1) cannot be estimated precisely at each voxel
- But precision is critical, or estimates are worse than OLS – biased AND imprecise
- Use spatial regularisation – pool estimation over active voxels, defined using 1st pass OLS estimate (*P* < .001)
- + White noise – voxel-specific variance s^2

$$e_t = ae_{t-1} + \varepsilon_t \text{ with } \varepsilon_t \sim N(0, \sigma^2)$$

Serial correlations in fMRI

Once data are 'pre-whitened', estimation can proceed using Ordinary Least Squares

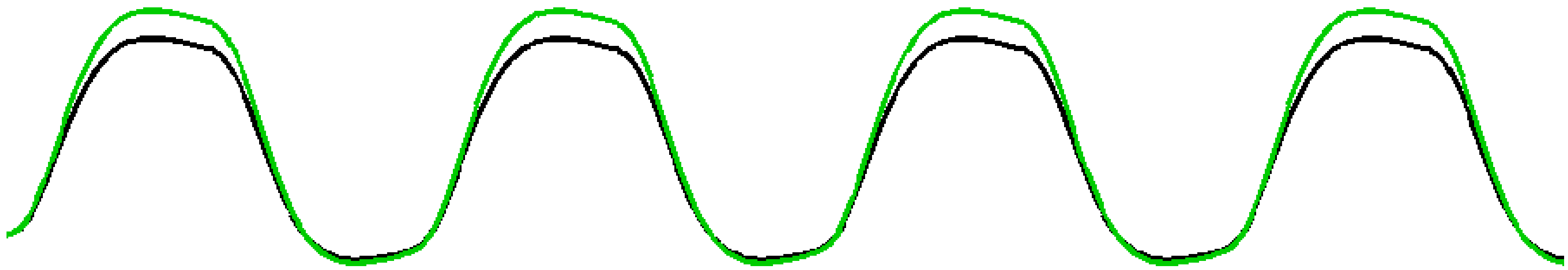
- The parameter estimates are again optimal – unbiased and minimum variance
- The df are also correct, if we want to do our statistical inference at the first level

Take-home message (1)

- If error structure is complex with multiple components of covariance – not just i.i.d. – our inference depends on modelling the error structure

- What does this have to do with 2-level models?

Why isn’t this easy?

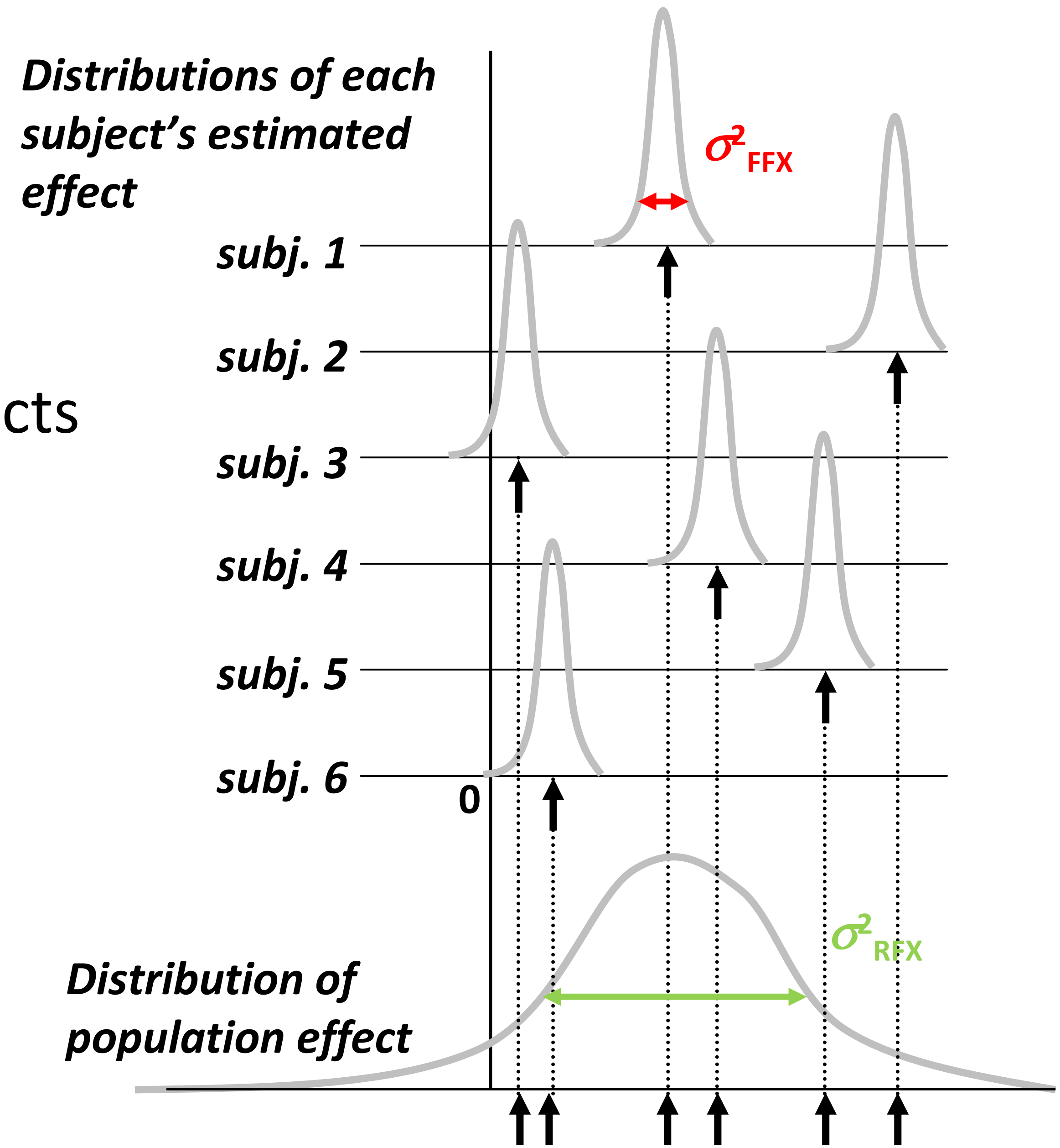


- Multiple levels of model
- Multiple components of error in each

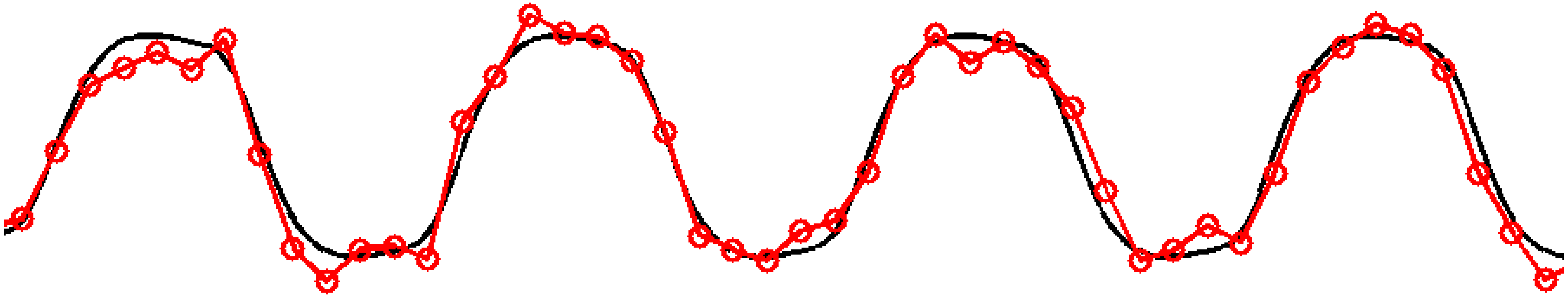
Fixed vs. random effects

- **Fixed effects:**
  Intra-subjects variation suggests all these subjects different from zero

- **Random effects:**
  Inter-subjects variation suggests population not different from zero



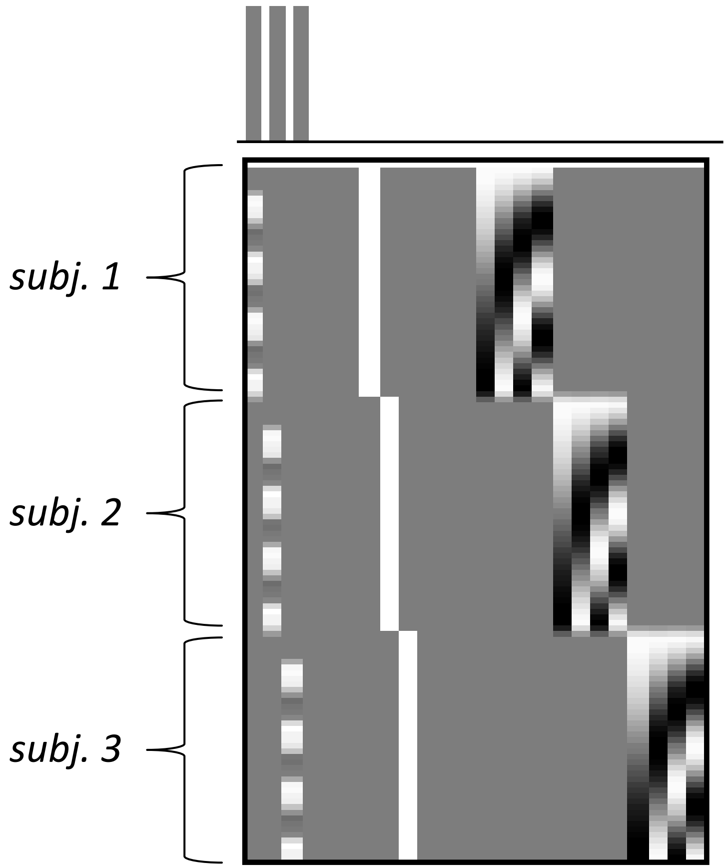
Fixed effects



- Only source of variation (over sessions) is measurement error
- True response magnitude is *fixed*

Fixed effect modelling in SPM

- Grand GLM (single level) approach (model all subjects at once)
- Good:
  - *max df*
  - *simple model*
- Bad:
  - *assumes common variance over subjects at each voxel*



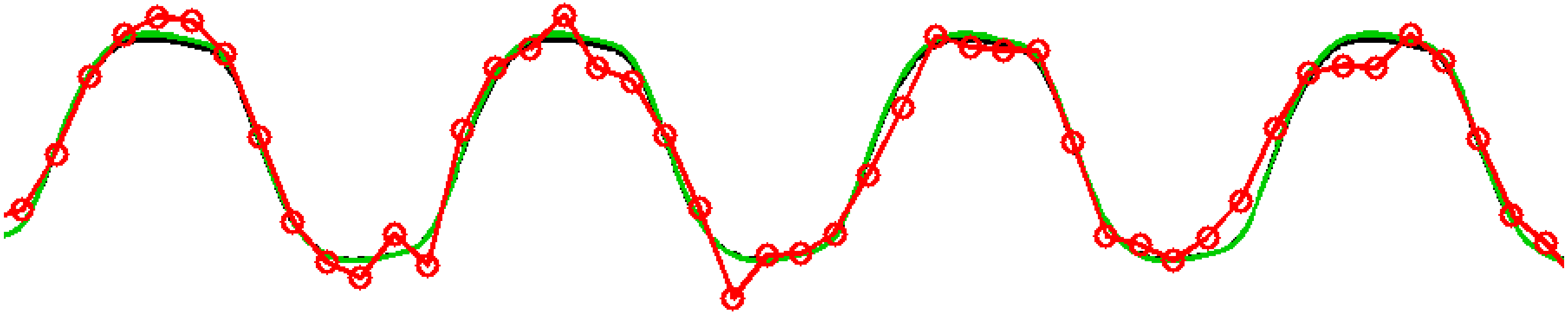
Fixed vs. random effects

Summary

- Fixed effect inference: *“I can see this effect in this cohort”*
- Random effect inference: *“If I were to sample a new cohort from the same population I would get the same result”*

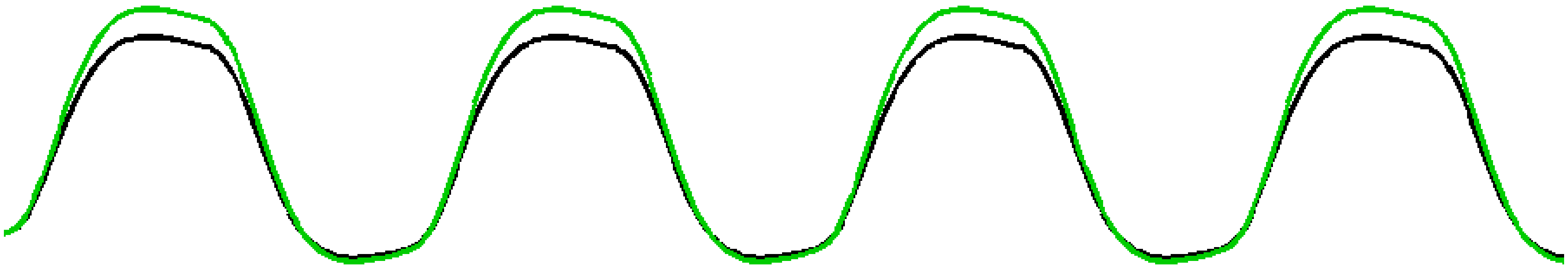
- Fixed isn’t ‘wrong’, but is not usually of interest

Random effects



- Two sources of variation
  - measurement errors
  - response magnitude (over subjects)
- Response magnitude is *random*
  - each subject/session has random magnitude

Random effects

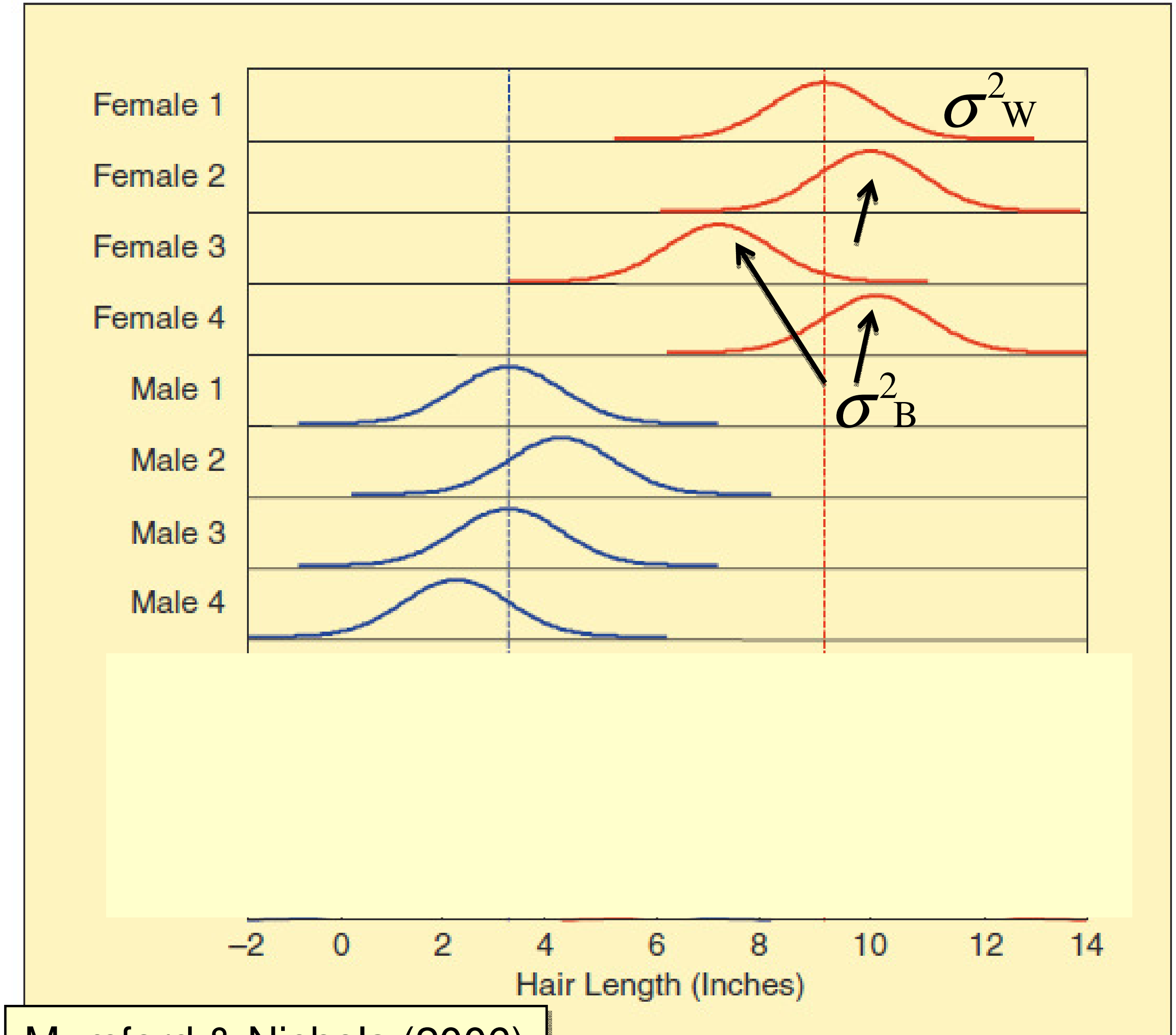


- Two sources of variation
  - measurement errors
  - response magnitude (over subjects)
- Response magnitude is *random*
  - each subject/session has random magnitude
  - but note, population mean magnitude is *fixed*

Why bother with two stages?

- We want to make an inference to the population, not a single subject, so why do we care?
- Why can't we just do group stats on data for each voxel, as in SPSS?
- We could, if data Y were simple values per voxel – precisely known.
- Instead, we have estimates of individual subjects' effects – so more than 1 covariance component

Hierarchical models



Mumford & Nichols (2006)

Does hair length differ by gender?

2 sources of variability

Within-subject: σ^2_W

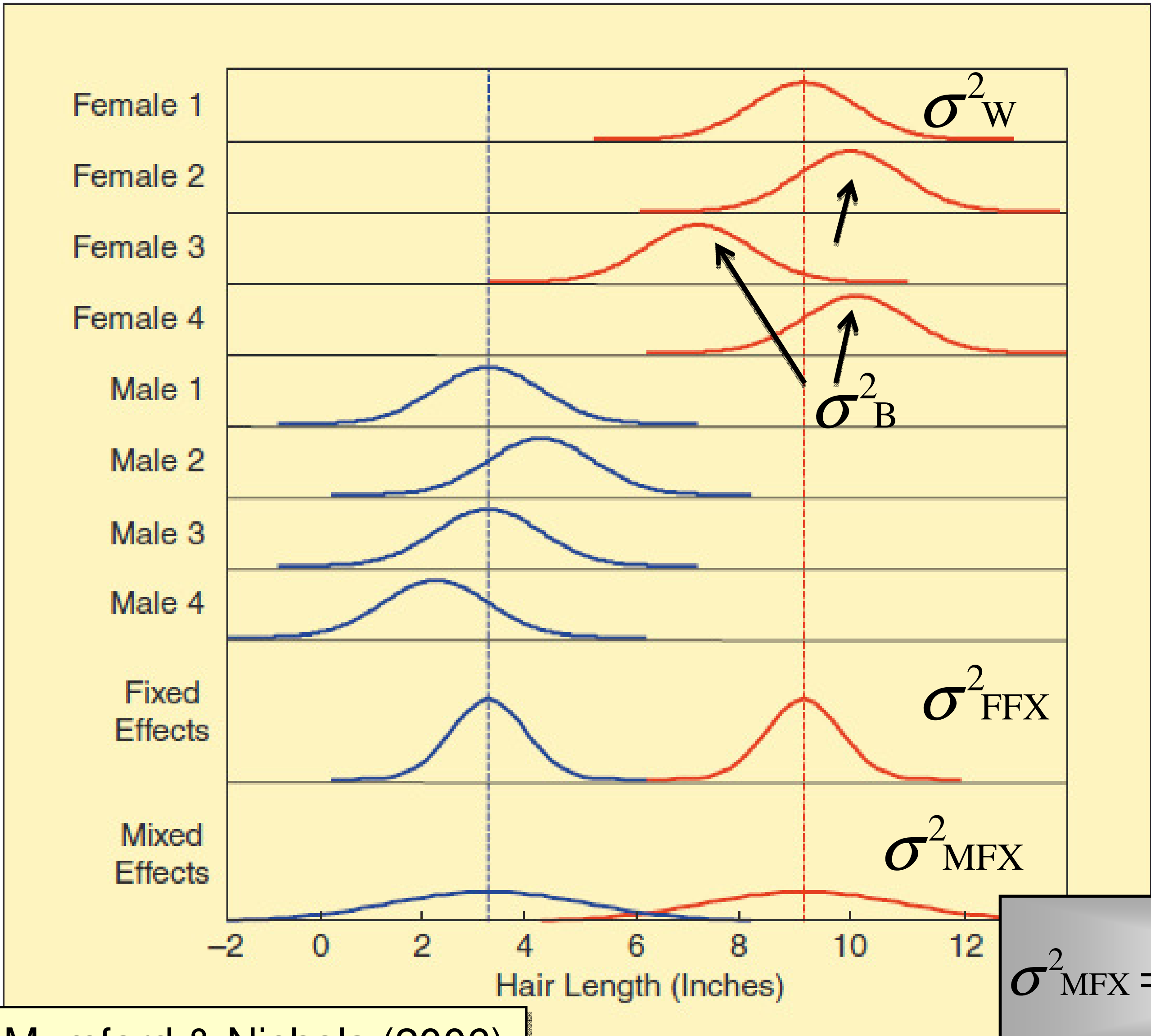
Between-subjects: σ^2_B

To generalise across this sample, combine data from hairs measured in all subjects, get σ^2_{FFX}

To generalise to population, use estimates of hair length for each subject, get σ^2_{MFX}

MIX of between/ within variability

Hierarchical models



Mumford & Nichols (2006)

Does hair length differ by gender?

2 sources of variability

Within-subject (1)

Between-subjects (49)

To generalise across this sample if p = 25 hairs per subject

$$\sigma^2_{FFX} = \frac{1}{4} * \frac{\sigma^2_W}{25} = 0.01$$

To generalise to population, given N = 4 subjects per group

$$\sigma^2_{MFX} = \frac{1}{4} * \frac{\sigma^2_W}{25} + \boxed{\frac{1}{4}\sigma^2_B} = 12.26$$

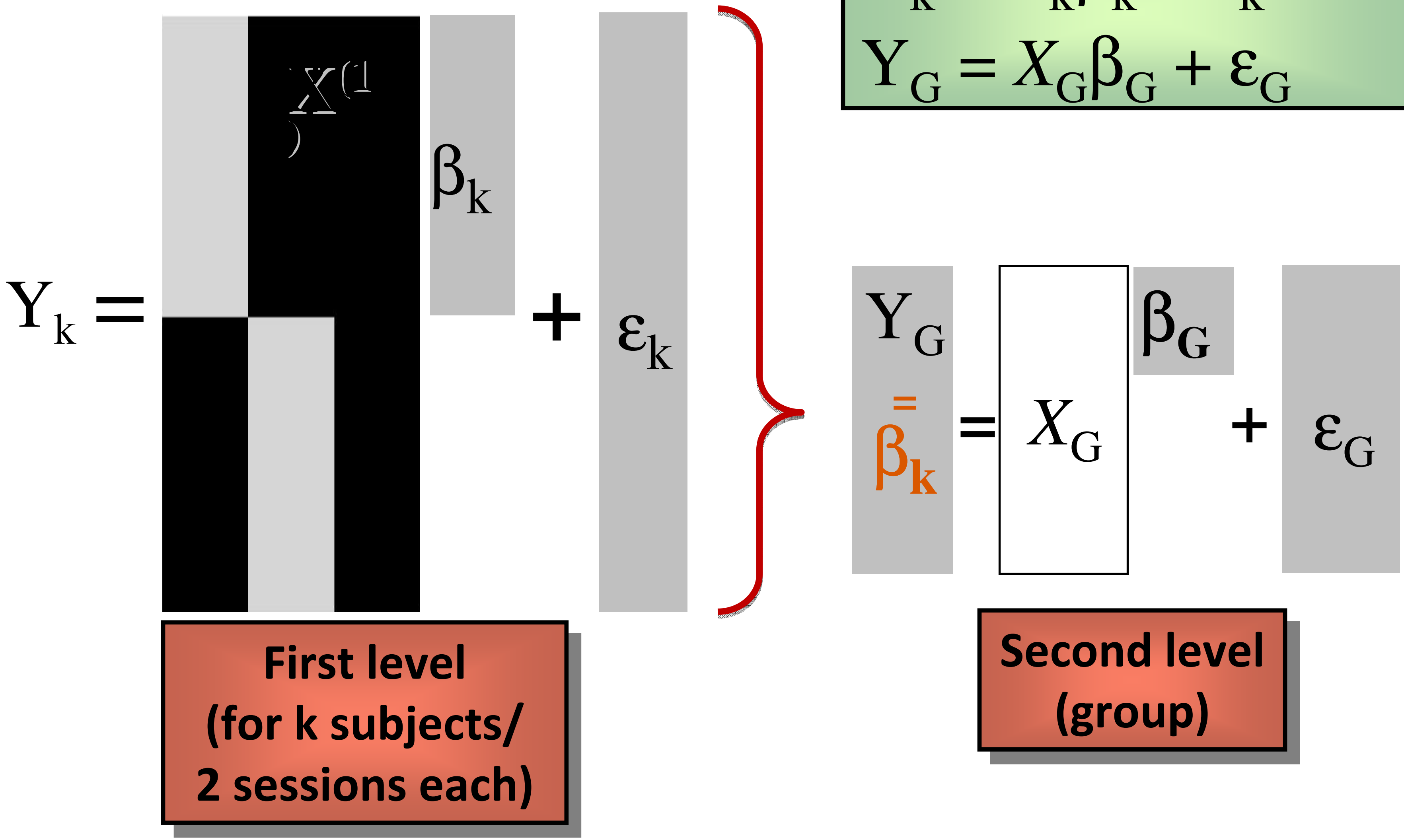
Why bother with two stages?

Why can’t we just do group stats on the data from each voxel?

- ...that could be valid but would not be optimal
- Hierarchical models deal with mixed sources of variance, not just between-subject variance
- Model both scan-to-scan and subject-to-subject variability
- More than 1 variance component (nonsphericity) at the group level

Hierarchical models

$$Y_k = X_k \beta_k + \varepsilon_k$$
$$Y_G = X_G \beta_G + \varepsilon_G$$



Hierarchical models

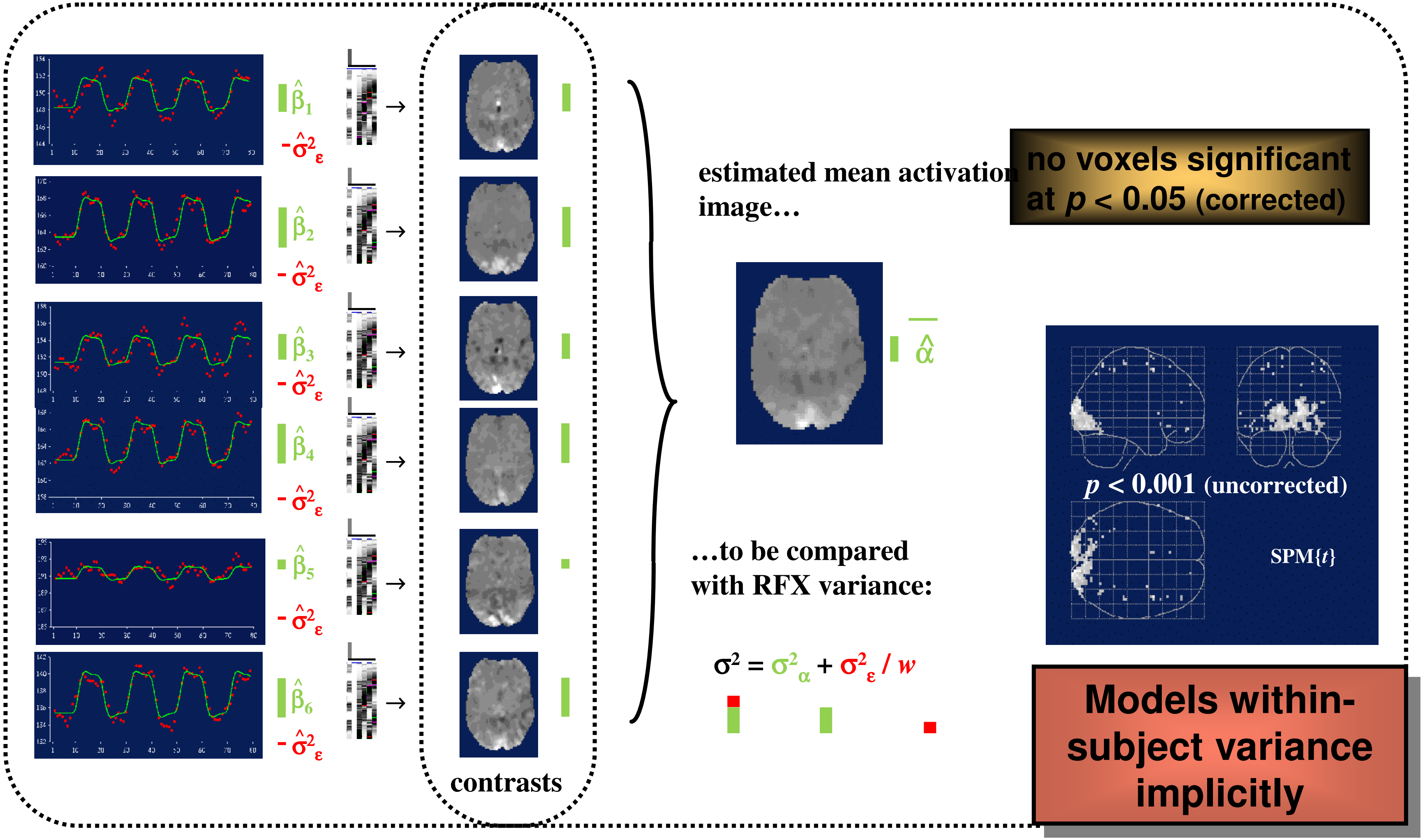
Two approaches in SPM

1. Simple summary statistic – Holmes & Friston
2. Non-sphericity modelling at group level

- Pros and cons – assumptions vs. flexibility
  - Subject variances equivalent
  - Subject design matrices equivalent
  - (2) enables a wide range of 2nd level models

Simple summary statistic approach (‘HF’)

1^{st} level (within subjects) **2^{nd} level** (between-subject)



Simple summary statistic approach (‘HF’)

Assumptions

- Distribution normal, independent subjects
- Homogeneous variance
  - Subjects’ residual errors same
  - Subjects’ design matrices same

  - 2 covariance components
  - Collapse into 1 if these elements of the group level covariance are homogenous over subjects

Simple summary statistic approach ('HF')

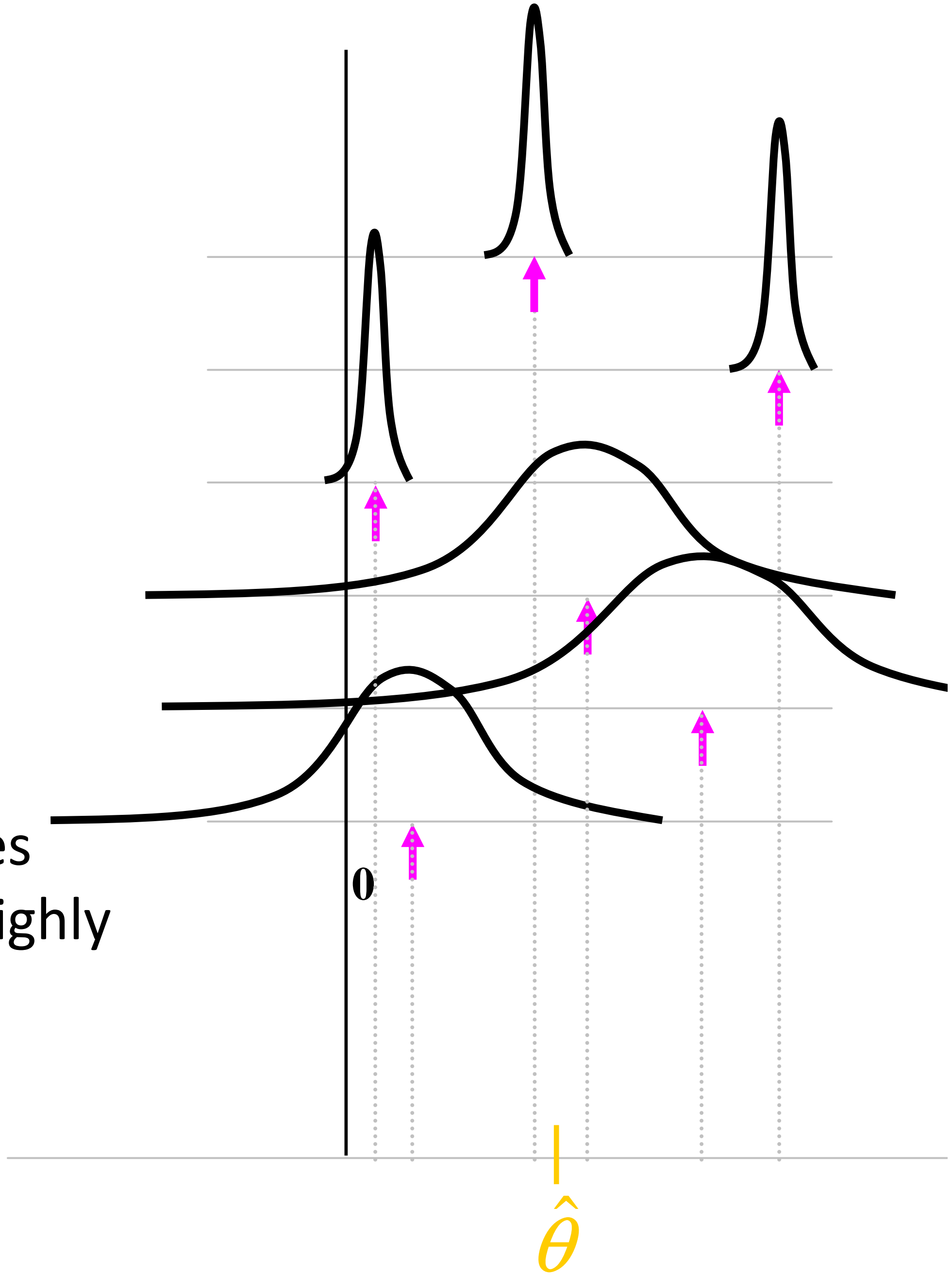
Use only a single image per subject

- Limited to 1- or 2-sample t-tests at the 2nd level
- Balanced designs

- Limitation = strength
    - No 2nd level sphericity assumption
    - 'Partitioned' error term @ 2nd level

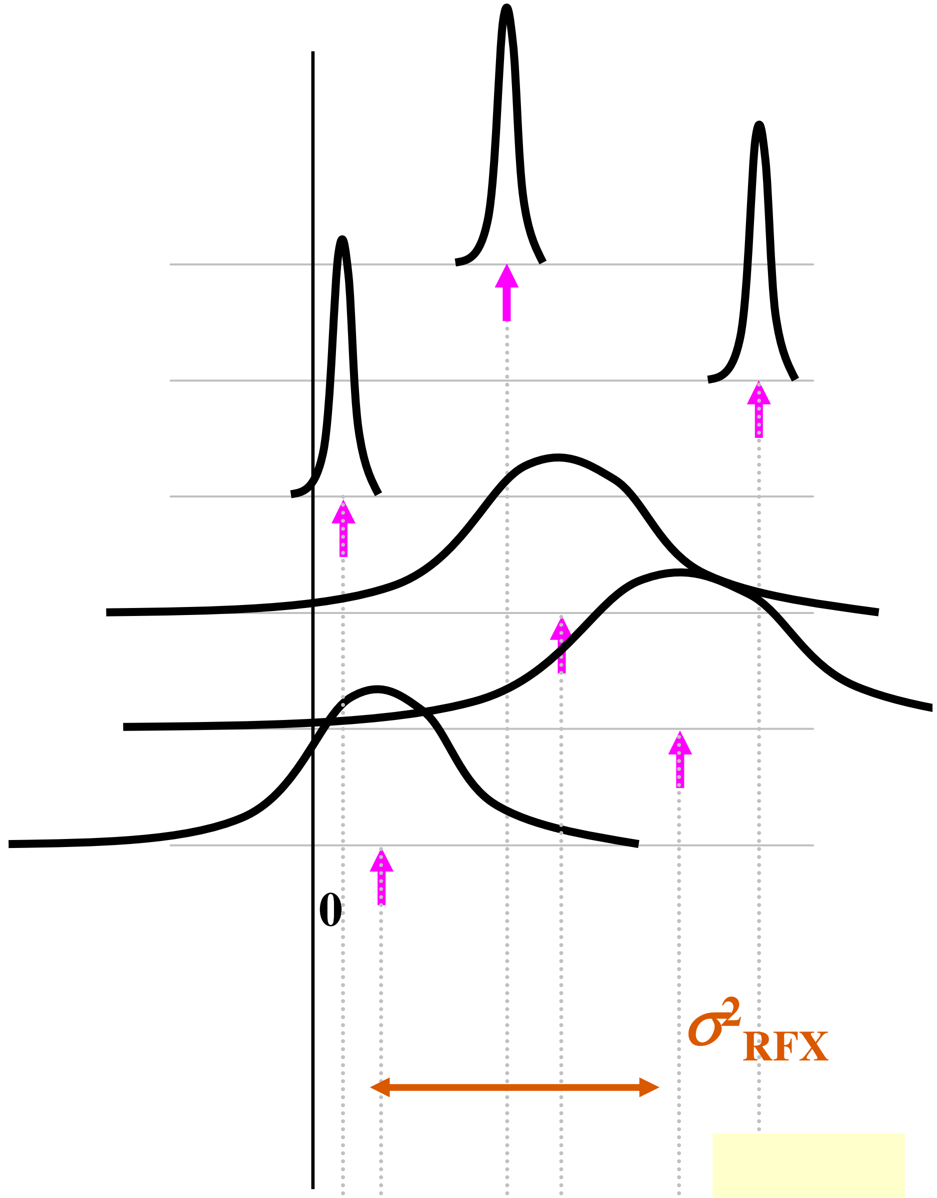
HF - efficiency

- If assumptions true
  - Optimal, fully efficient
- If σ^2_{FFX} differs between subjects
  - Reduced efficiency
  - Here, optimal $\hat{\theta}$ requires down-weighting the 3 highly variable subjects



HF - validity

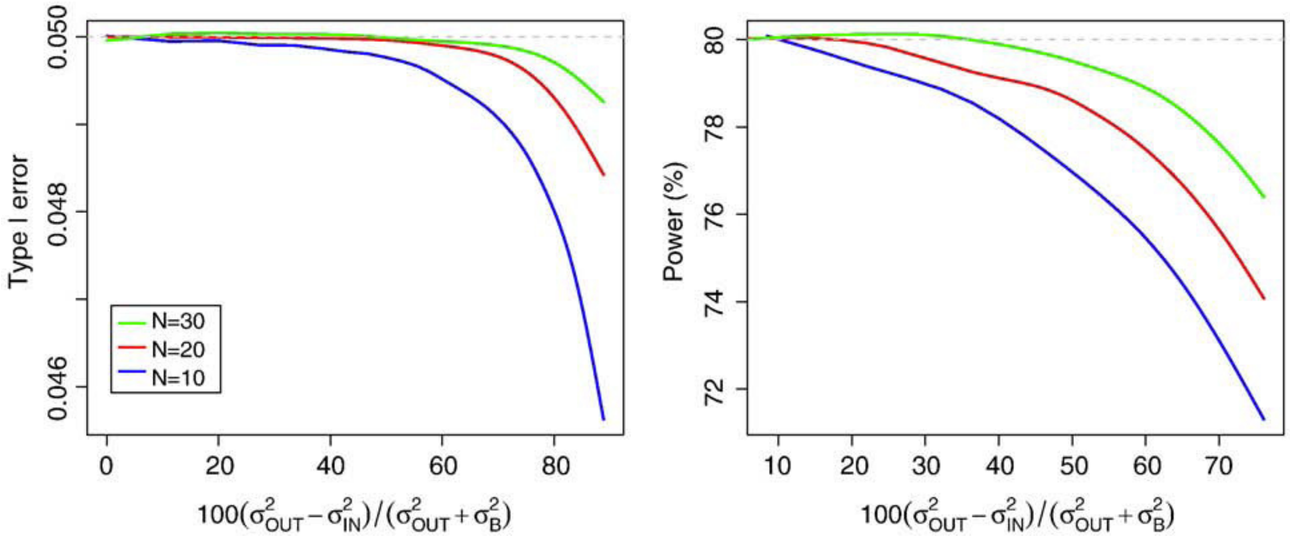
- If assumptions true
  - Exact *P*-values
- If σ^2_{FFX} differs btw subj.
  - Standard errors not OK
    - Est. of σ^2_{RFX} may be biased
  - df not OK
    - Here, 3 Ss dominate
    - DF < 5 = 6-1



HF – robustness

- In practice, validity & efficiency are excellent
  - For the one sample case, HF is very robust

False Positive Rate **Power Relative to Optimal**



(outlier severity) **(outlier severity)**

  - Potential concern with 2-sample or correlation if outliers/ large imbalance

Modelling 2nd level non-sphericity

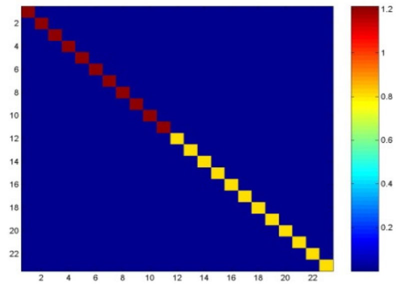
A more flexible summary statistic approach

- Model within-level just as at 1st level
- Represent different sources of covariance using linear combination of basis functions
- Multiple covariance components
- Same estimation using prewhitening approach, and cross-voxel ‘pooling’

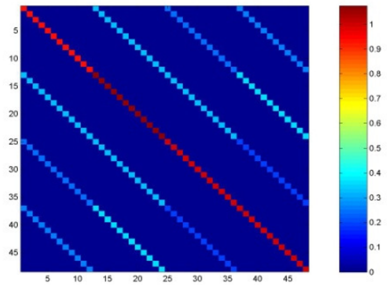
Modelling 2^{nd} level non-sphericity

Error Covariance

- Errors are independent
  but not identical



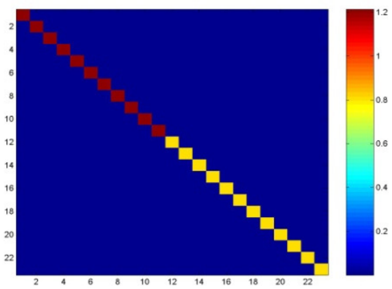
- Errors are not
  independent
  and not identical

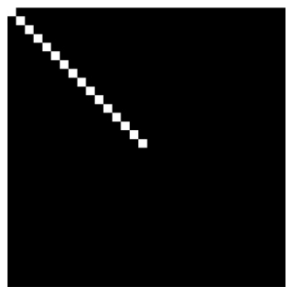


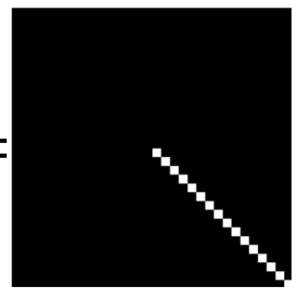
Modelling 2nd level non-sphericity

Errors can be Independent but Non-Identical when…

1) One parameter but from different groups – 2-sample t-test e.g. patients and control groups



$Q_1 =$ 

$Q_2 =$ 

Modelling 2nd level non-sphericity

Error can be Non-Independent and Non-Identical when…

1)Several contrasts per subject are taken to 2nd level

e.g. Repeated Measures ANOVA

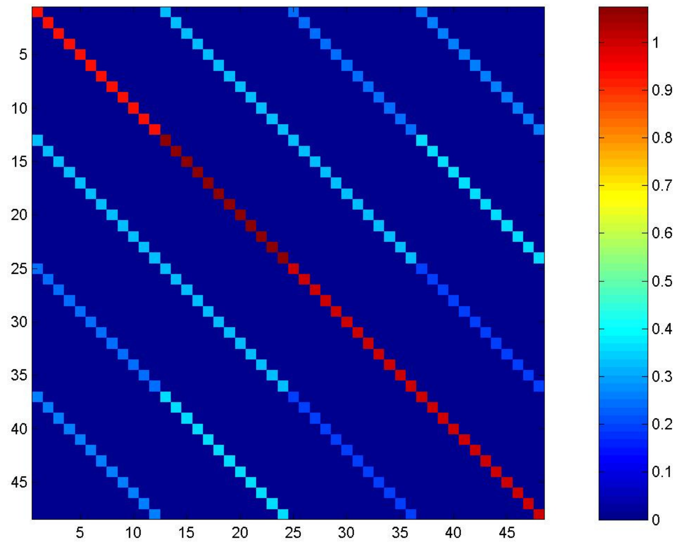
2) Omnibus test is needed across several basis functions characterising the hemodynamic response

e.g. F-test combining HRF, temporal derivative and dispersion regressors

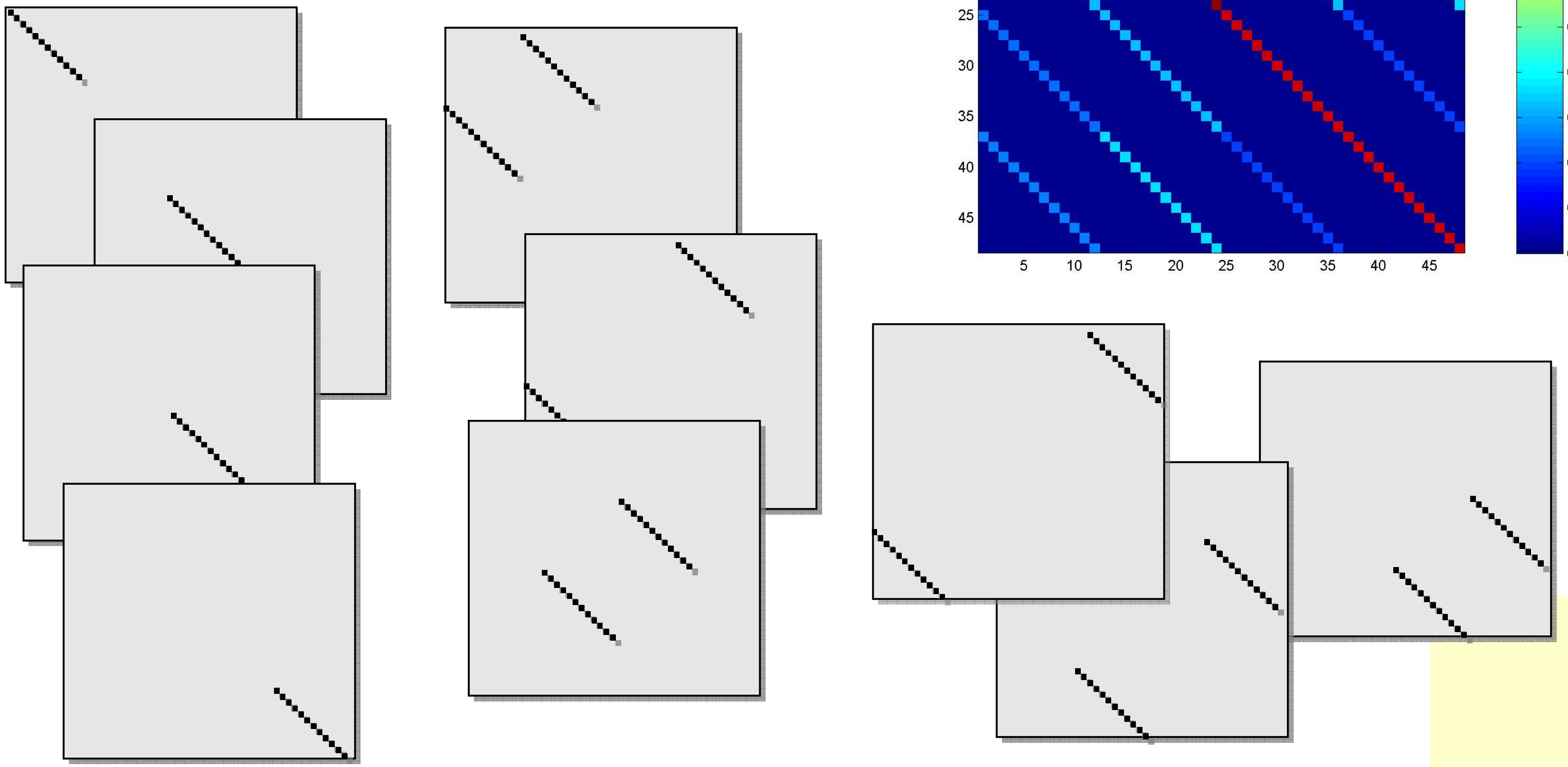
Modelling 2nd level non-sphericity

Errors are not independent
and not identical

residuals covariance matrix



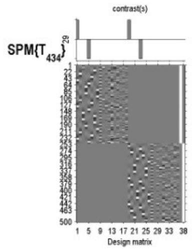
Q_k's:



Modelling 2^{nd} level non-sphericity

1: motion **2: sounds** **3: motion** **4: sounds**

1^{st} level



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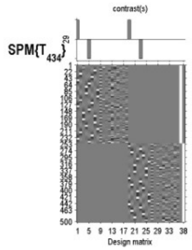

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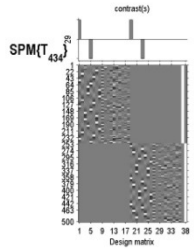

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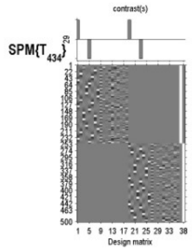
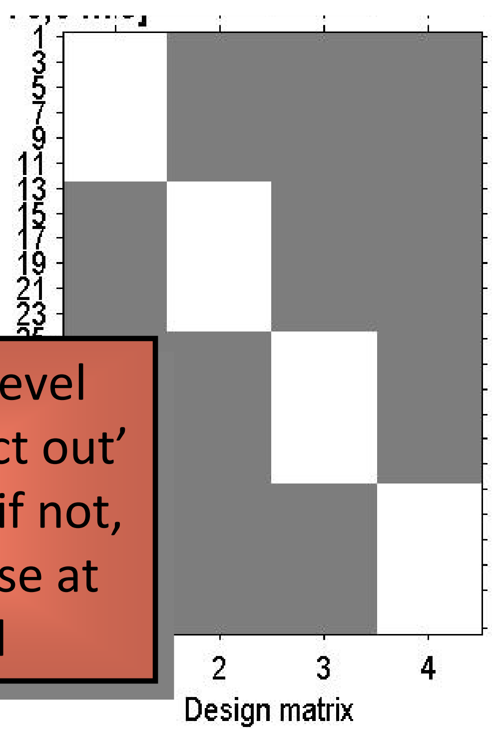
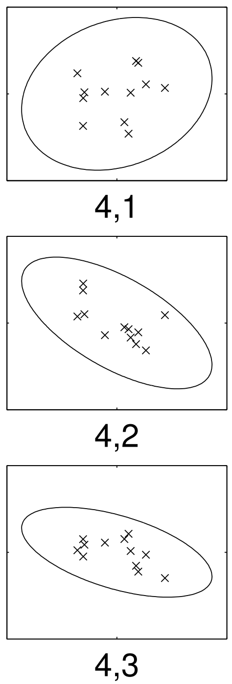


2^{nd} level



N.B. These 1st level contrasts 'subtract out' subject effects – if not, must model these at the 2^{nd} level



Block design study

Repeated measures ANOVA model

Which regions are sensitive to semantic content of words across 4 conditions?

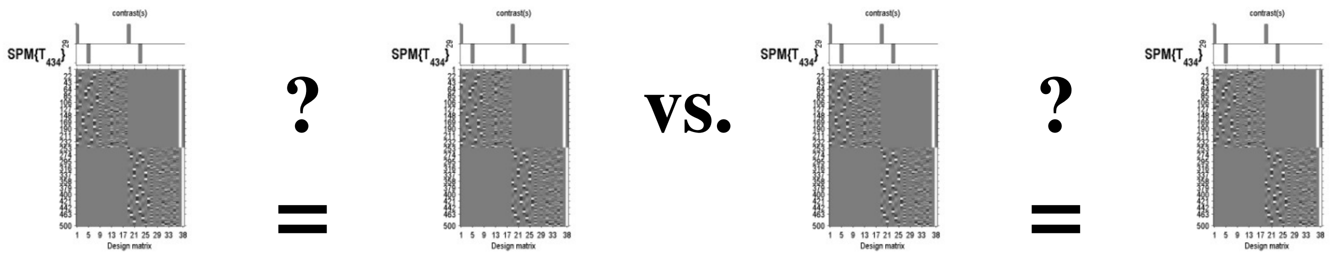
Noppeney et al.

Modelling 2nd level non-sphericity

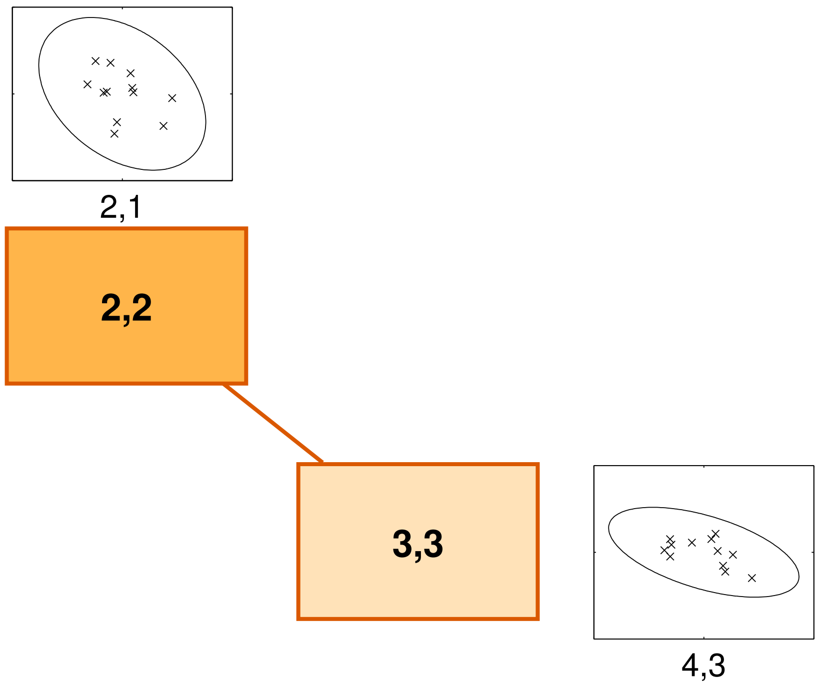
YOUNG ADULTS | **OLDER ADULTS**

1: motion | **2: sounds** | **3: motion** | **4: sounds**

1st level



2nd level



Mixed ANOVA model

2 x 1st level contrasts for each subject

Possible non-independence only on some off-diagonals

Also model non-identical variances by group on diagonals

Modelling 2^{nd} level non-sphericity

Assumptions

- Needed for cross-voxel pooling, homogenous across ‘active’ voxels
- Within subject covariance still homogenous
- **HF plus** pooled variance at 2^{nd} level

Advantages

- Fast relative to ‘full’ mixed-effects procedures
- Flexibility of possible 2^{nd} level models

Summary

fMRI models need to take account of

- Multiple sources of variability at each level
- Hierarchical nature of data

Estimation & correction for resulting nonsphericity

- Some assumptions
- If correct, optimise estimation & inference
- SPM enables very flexible 2^{nd} level models

2-stage GLM

Single subject

Each has an independently acquired set of data
These are modelled separately
Models account for **within subjects variability**
Parameter estimates apply to individual subjects

1st level

Single subject **contrasts of parameter estimates** taken forward to 2nd level as (spm_con*.img) **'con images'**

Group/s of subjects

To make population inferences, 2nd level models account for **between subjects variability**
Parameter estimates apply to group effect/s

2nd level

Statistics compare **contrasts of 2nd level parameter estimates to 2nd level error**

Bibliography

- *Statistical Parametric Mapping: The Analysis of Functional Brain Images*. Elsevier, 2007.
- *Generalisability, Random Effects & Population Inference*. Holmes & Friston, NeuroImage,1999.
- *Classical and Bayesian inference in neuroimaging: theory*. Friston et al., NeuroImage, 2002.
- *Classical and Bayesian inference in neuroimaging: variance component estimation in fMRI*. Friston et al., NeuroImage, 2002.
- Simple group fMRI modeling and inference.

  Mumford & Nichols, *Neuroimage*, 2009 [ALSO ON POWER]
- *Flexible factorial tutorial* by Glascher and Gitelman www.sbirc.ed.ac.uk/cyril/cp_fmri.html

SPM