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Within and between subject contrasts

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What are contrasts?

A **contrast** is a method of **asking a question** about your model in **mathematical terms**

The GLM **partitions** the data into a **model prediction** and **error**

$$\mathbf{Y} = \boxed{\mathbf{X}\boldsymbol{\beta}} + \boxed{\boldsymbol{\epsilon}}$$

The **model prediction** is the **linear combination** of **predictor variables** and **parameters**

The **parameter values** are **unknown** and must be **estimated** from the data

The **estimated values** provide a summary of the **magnitude** and **direction** of the **experimental effect** — the part of the GLM that we want to ask questions about, using **contrasts**

The General Linear Hypothesis

All hypothesis testing in the GLM is based on the following

$$\mathscr{H}_0 : \mathbf{L}\boldsymbol{\beta} = \mathbf{0}$$

L is the *m* x *k* **matrix** of **weights**

$\boldsymbol{\beta}$ is the *k* x 1 **vector** of **model parameters**

0 is an *m* x 1 **vector** of **zeros**

The hypothesis is that some **linear combination** of the (population) **parameters** equals **zero**

This is just standard **null hypothesis significance testing** in a more general framework

The General Linear Hypothesis

All hypothesis testing in the GLM is based on the following

$$\mathscr{H}_0 : \mathbf{L}\boldsymbol{\beta} = \mathbf{0}$$

This is a **very flexible** system e.g. for **two parameters**

$\mathbf{L} = [1 \quad 0]$ β_1 is equal to 0

$\mathbf{L} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ β_1 **OR** β_2 is equal to 0

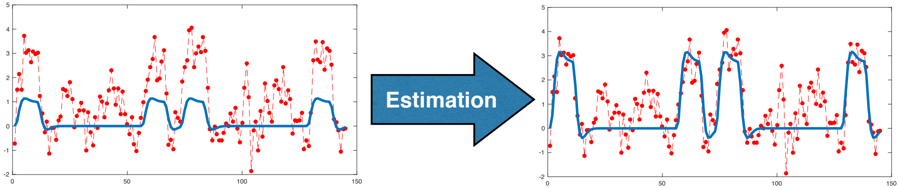
$\mathbf{L} = [1 \quad -1]$ the **difference** between β_1 and β_2 is 0

$\mathbf{L} = [0.5 \quad 0.5]$ the **average** of β_1 and β_2 is 0

Parameters and interpretation

As **contrasts** are **linear combinations** of parameters, **interpretation** depends on understanding the parameters

1st-level



The parameters **scale** the **predicted shape of the response** to **best fit the data**

Their values tell us about the **magnitude** and **direction** of the **average change from baseline** in each experimental condition

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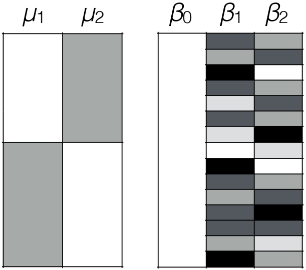
Parameters and interpretation

As **contrasts** are **linear combinations** of parameters, **interpretation** depends on understanding the parameters

2nd-level



Indicator variables

- Contain only 1 or 0 to model a **factor**
- Parameters are **cell means**

Continuous covariates

- Contain any value **within a range**
- Parameters are **regression slopes**

Comparing **cell means** is sensible, but comparing **regression slopes** only makes sense if the predictors are **scaled identically**

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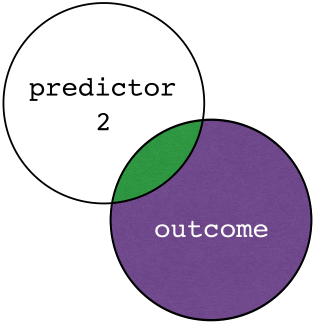
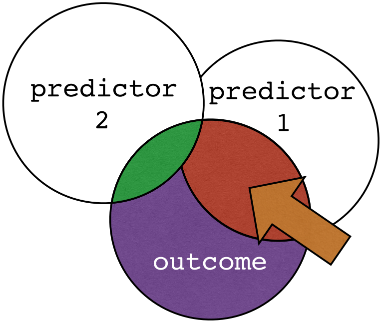
Parameters and interpretation

For **multiple** predictor variables, tests for each **parameter** are interpreted as testing the **unique variability** explained **after adjusting for all other variables**

Effect of predictor 1



VS



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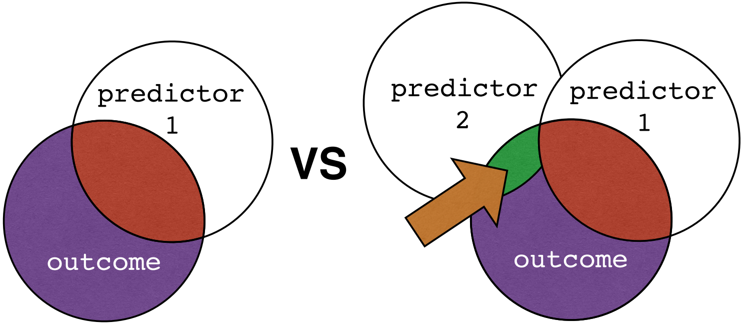
Parameters and interpretation

For **multiple** predictor variables, tests for each **parameter** are interpreted as testing the **unique variability** explained **after adjusting for all other variables**

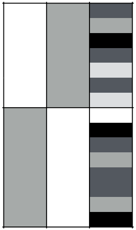
Effect of predictor 2



Parameters and interpretation

For **multiple** predictor variables, tests for each **parameter** are interpreted as testing the **unique variability** explained **after adjusting for all other variables**

For **contrasts**, this means that even if a parameter is given a **weight of 0**, the other parameters are **still adjusted** for its presence



$$\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

The individual **cell means** are still **adjusted** for the **covariate**

The value of the **parameter estimates** depends on **other variables in the model**

Contrasts ask questions about the model — they **do not alter the model**

Parameters and interpretation

Remember that the parameters need **estimating** — how this is done provides some **clues** for **interpretation** of the values:

$$\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$$

Notice that this **depends** on the **data** and the **design matrix**

Important consequences

1. The estimates depend on the **scaling** of **X**
2. If **X** contains **redundant columns** then there are an **infinite number of solutions** — **(X′X)**$^{-1}$ **does not exist**

SPM will allow for a **redundant parameterisation** of the design matrix — creates **complications** in **specifying** and **interpreting** contrasts

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Estimable functions

Allowing for a **redundant parameterisation** means we have to be quite careful with our **contrasts**

SPM uses a **pseudo-inverse** to solve for the parameters

```
beta = pinv(X'*X)*X'*Y
```

For a **redundant parameterisation** this produces parameters whose **individual values** are not very meaningful

- Find two values that sum to 5?
- We could get **2 and 3** or **4,322 and -4,317**

To make sure we don't specify **contrasts** that give us values that **depend** on the solution from the pseudo-inverse we need a **restriction**

These **restriction** are known as ***estimable functions***

Estimable functions

A contrast that defines an **estimable function** of the model parameters can be expressed as

$$\mathbf{L} = \mathbf{T}\mathbf{X}$$

A contrast is only **meaningful** if it is formed from a **linear combination** of the **rows** of the **design matrix**

$$\mathbf{L}\hat{\boldsymbol{\beta}} = \mathbf{T}\mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{T}\hat{\mathbf{Y}}$$

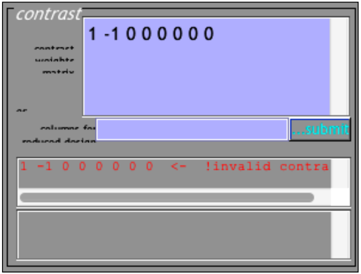
An **estimable contrast** is also one that can be expressed as a **linear combination** of the **model estimates**

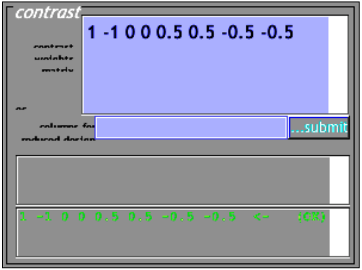
The **model estimates** dictate how **meaningful** values are formed from combining the **predictors** and the **parameters**

A question is only meaningful if it **respects this combination**

Estimable functions

To make sure that the contrast we are testing is **estimable**, SPM will perform an **estimability test**

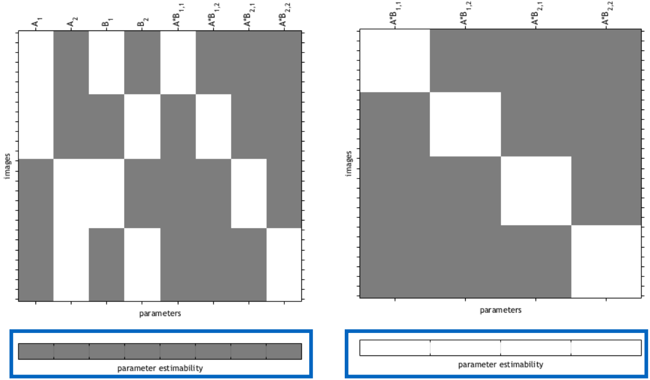




If you ever see `!invalid contrast` it means you are trying to use a **non-estimable** contrast in an **overparameterised design**

See McFarquhar (2016) for more on this

Estimable functions



Non-estimable parameters are indicated by a **grey box**

These parameters will not have **unique values** and contrasts that involve them need to **specified carefully**

Contrast interpretation

Most of the time we work with **well parameterised** models and so do not have to worry about **estimability** — always check the grey boxes!

Despite this, a contrast can be **estimable** but **misinterpreted**

This largely comes down to

1. Understanding **what** the parameters mean
2. Making sure that the contrast is testing **what you think it is**

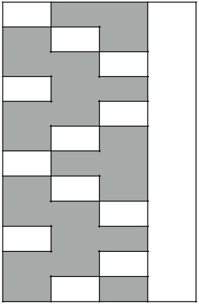
You should always ensure that you are **clear** how to **interpret** the **parameters** and **contrasts** from your model

How can you know **what your results mean** of you don't understand **what the model is telling you**, or **what your questions are**?

Contrast interpretation

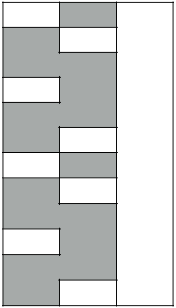
1st-level explicit vs implicit baselines

Model 1
(A, B, rest, const.)



$$\mathbf{L}_{\mathrm{A-B}} = \begin{bmatrix} 1 & -1 & 0 & 0 \end{bmatrix}$$
$$\mathbf{L}_{\mathrm{A-R}} = \begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix}$$
$$\mathbf{L}_{\mathrm{B-R}} = \begin{bmatrix} 0 & 1 & -1 & 0 \end{bmatrix}$$

Model 2
(A, B, const.)



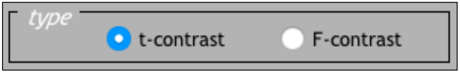
$$\mathbf{L}_{\mathrm{A-B}} = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}$$
$$\mathbf{L}_{\mathrm{A-R}} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$
$$\mathbf{L}_{\mathrm{B-R}} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

An **explicit** baseline over an **implicit** baseline alters the **interpretation of the parameters** and the **form of the contrasts**

Contrast interpretation

Assuming our **contrast** is **estimable** and we are clear on **what it means** we can test the **estimated value** by forming a **test statistic**

In **SPM** we have **two** options for how to test our **contrast value** — as a ***t*-contrast** or an ***F*-contrast**



Each type is used in **different contexts** and it is important to understand their **differences**, and **similarities**, to ensure you use the **most appropriate method** for your questions

t-contrasts

In **SPM** a *t*-contrast is defined by

- An **L** matrix with a **single row**
- Hypothesis testing using a *t*-statistic
- **One-tailed** *p*-values

$$t = \frac{\mathbf{L}\hat{\beta}}{\hat{\sigma}\sqrt{\mathbf{L}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{L}'}}$$

Notice that **L** appears in the **numerator** and **denominator** — scaling of **L** does not matter

All lead to the same *t*-statistic:

$$\mathbf{L} = \begin{bmatrix} 1 & -1 \end{bmatrix} \qquad \mathbf{L} = \begin{bmatrix} 2 & -2 \end{bmatrix} \qquad \mathbf{L} = \begin{bmatrix} 1000 & -1000 \end{bmatrix}$$

t-contrasts

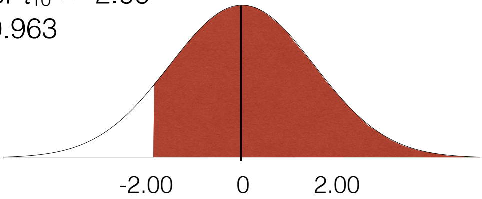
In **SPM** a t-contrast is defined by

- An **L** matrix with a **single row**
- Hypothesis testing using a t-statistic
- **One-tailed** p-values

The p-values are **upper-tail** values — you will only see results for **positive t-statistics**

Upper-tail for $t_{10} = -2.00$

$p = 0.963$



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t-contrasts

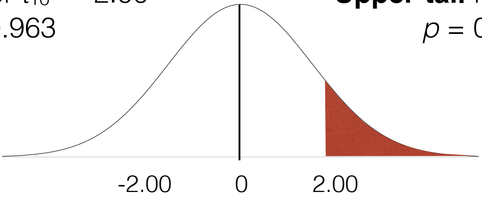
In **SPM** a *t*-contrast is defined by

- An **L** matrix with a **single row**
- Hypothesis testing using a *t*-statistic
- **One-tailed** *p*-values

The *p*-values are **upper-tail** values — you will only see results for **positive *t*-statistics**

Upper-tail for $t_{10} = -2.00$
$p = 0.963$

Upper-tail for $t_{10} = 2.00$
$p = 0.037$



t-contrasts

In **SPM** a *t*-contrast is defined by

- An **L** matrix with a **single row**
- Hypothesis testing using a *t*-statistic
- **One-tailed** *p*-values

The *p*-values are **upper-tail** values — you will only see results for **positive *t*-statistics**

A **positive** *t*-statistic occurs when the **direction of the effect** matches the **direction of the contrast**

$\mathbf{L} = \begin{bmatrix} 1 & 0 \end{bmatrix}$ **Positive** effects $\mathbf{L} = \begin{bmatrix} -1 & 0 \end{bmatrix}$ **Negative** effects

One-tailed *p*-values only suitable for **strong directional hypotheses** — overused in imaging (see Chen *et al.*, 2018)

F-contrasts

In **SPM** an *F*-contrast is defined by

- An **L** matrix with (possibly) **multiple rows**
- Hypothesis testing using a *F*-statistic
- **Two-tailed** *p*-values

$$F = \frac{(\mathbf{L}\hat{\boldsymbol{\beta}})'(\mathbf{L}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{L}')^{-1}(\mathbf{L}\hat{\boldsymbol{\beta}})}{r\hat{\sigma}^2}$$

The **numerator** forms a **sum-of-squares** — divided by *r* to form a **mean square**

Multiple rows can be thought of as an **OR** question

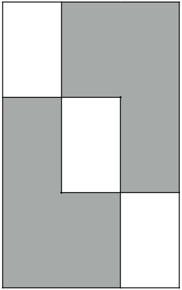
An *F*-contrast with a **single row** is the same as t^2 — in SPM this allows for a **two-tailed** alternative to a *t*-contrast

F-contrasts

Simple example: 1-way ANOVA with a 3-level factor

The **weights** for an F-contrast testing the **main effect** of the factor are:

$$\mathbf{L} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$



Does this seem odd?

Why is it not?

$$\mathbf{L} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

F-contrasts

Simple example: 1-way ANOVA with a 3-level factor

Why is it not?

$$\mathbf{L} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \boxed{1 \quad 0 \quad -1} \end{bmatrix}$$

This row is redundant

This last row is the **sum** of the other rows — the value from this row is **not independent of the other rows**

$$\boldsymbol{\beta} = \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix} \quad \mathbf{L}\boldsymbol{\beta} = \begin{bmatrix} -5 \\ 3 \\ -2 \end{bmatrix} \Rightarrow (-5) + 3 = -2$$

The last row **provides no more information** — connection with **numerator degrees of freedom**

Do contrast weights have to sum to zero?

A potential **source of confusion** relates to wether the **weights** of the contrast **must sum to zero**

Distinction between a **linear combination** and a **contrast**:

- A **contrast** is specifically about **comparing parameters**
- A **linear combination** is more general e.g. average, sum etc.

In **SPM** the term **contrast** is used generically for either

In the statistics literature a **contrast** has the **additional requirement** that the weights sum to zero — not necessary for a **linear combination**

It is more important that you **understand the question** you are asking, rather than enforcing that the weights **must sum to zero**

Do contrast weights have to sum to zero?

$$\mathcal{H}_0 : \beta_1 - \beta_2 = 0$$

$$\mathbf{L} = \begin{bmatrix} 1 & -1 \end{bmatrix} \quad \sum l_i = 0 \ ✔$$
$$\mathbf{L} = \begin{bmatrix} 2 & -2 \end{bmatrix} \quad \sum l_i = 0 \ ✔$$
$$\mathbf{L} = \begin{bmatrix} 1 & -2 \end{bmatrix} \quad \sum l_i = -1 \ ✘$$

$$\mathcal{H}_0 : (\beta_1 + \beta_2)/2 - \beta_3 = 0$$

$$\mathbf{L} = \begin{bmatrix} 0.5 & 0.5 & -1 \end{bmatrix} \quad \sum l_i = 0 \ ✔$$
$$\mathbf{L} = \begin{bmatrix} 1 & 1 & -2 \end{bmatrix} \quad \sum l_i = 0 \ ✔$$
$$\mathbf{L} = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \quad \sum l_i = 1 \ ✘$$

In both cases we are looking at **differences** of parameters — these are **true contrasts**

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Do contrast weights have to sum to zero?

The **alternative** is when we look at **individual parameters** or **averages of parameters**

$$\mathcal{H}_0 : \beta_1 = 0$$

$$\mathbf{L} = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \sum l_i = 1 \quad ✔$$

$$\mathcal{H}_0 : (\beta_1 + \beta_2)/2 = 0$$

$$\mathbf{L} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \sum l_i = 1 \quad ✔ \qquad \mathbf{L} = \begin{bmatrix} 1 & 1 \end{bmatrix} \sum l_i = 2 \quad ✔$$

If the parameters are **estimable** then these are all valid

Interpretation must be done with **care** — depends on how the model is parameterised (e.g. **implicit** vs **explicit** baselines)

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Contrasts in ANOVA models

Often, it is the **group-level** where our **hypotheses** are focussed

Typically, data collected from **factorial designs** will be analysed using a **ANOVA** models

For designs with >2 factors, the ANOVA has a number of effects that we may be interested in — **main effects**, **interactions**, **simple effects**

Important to understand how these are tested using **contrasts**

SPM **defaults** to a **cell means** ANOVA rather than an **overparameterised** ANOVA — no need to worry about **estimability**, but the effects of interest have to be constructed from the contrasts

Contrasts in ANOVA models

2 x 2 ANOVA

Cell means are the **means** from the **intersection** of the factors and the **marginal means** are the **means** from **across** a factor

		Factor A		
		1	**2**	
Factor B	**1**	μ_{11}	μ_{21}	$\mu_{.1}$
	2	μ_{12}	μ_{22}	$\mu_{.2}$
		$\mu_{1.}$	$\mu_{2.}$	

Cell means

Marginal means for Factor A

Marginal means for Factor B

The **dot** notation indicates a **subscript averaged over**

Contrasts in ANOVA models

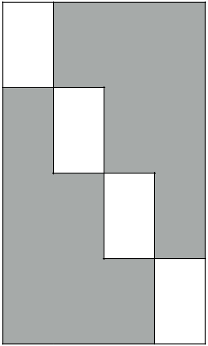
2 x 2 ANOVA

The simplest specification of 2 x 2 ANOVA is the **cell means** approach

$$y_{ijk} = \mu_{jk} + \epsilon_{ijk}$$

The **GLM parameters** are simply the **cell means**

The **ANOVA effects** are then formed using **contrasts of the cell means**



Main effect of A $\mathbf{L} = [1 \quad 1 \quad -1 \quad -1]$

Main effect of B $\mathbf{L} = [1 \quad -1 \quad 1 \quad -1]$

A x B interaction $\mathbf{L} = [1 \quad -1 \quad -1 \quad 1]$

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Contrasts in ANOVA models

2 x 2 ANOVA

The **interaction** contrast

We are looking for a **difference** of **two differences**

$$A \times B = \underbrace{\boxed{(A1B1 - A2B1)}}_{\text{Effect of A at the first level of B}} - \underbrace{\boxed{(A1B2 - A2B2)}}_{\text{Effect of A at the second level of B}}$$

Effect of A at the first level of B

Effect of A at the second level of B

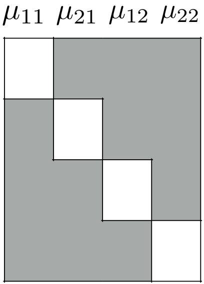
Contrasts in ANOVA models

2 x 2 ANOVA

The **interaction** contrast

We are looking for a **difference** of **two differences**

$$A \times B = (\mu_{11} - \mu_{21}) - (\mu_{12} - \mu_{22})$$



Contrasts in ANOVA models

2 x 2 ANOVA

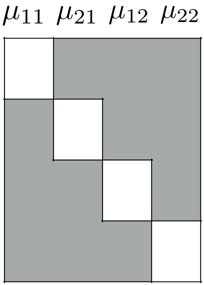
The **interaction** contrast

We are looking for a **difference** of **two differences**

$$\mathrm{A} \times \mathrm{B} = \boxed{(\mu_{11} - \mu_{21})} - \boxed{(\mu_{12} - \mu_{22})}$$

$$\begin{array}{cccc} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}$$

$$\mathbf{L} = \begin{bmatrix} 1 & -1 & -1 & 1 \end{bmatrix}$$



The **weights** for the **interaction** come from **subtracting** the weights for the **simple effects**

Contrasts in ANOVA models

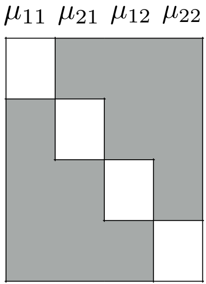
2 x 2 ANOVA

The **interaction** contrast

We are looking for a **difference** of **two differences**

$$A \times B = (\mu_{11} - \mu_{21}) - (\mu_{12} - \mu_{22})$$
$$= \mu_{11} - \mu_{21} - \mu_{12} + \mu_{22}$$

$$\mathbf{L} = \begin{bmatrix} 1 & -1 & -1 & 1 \end{bmatrix}$$



Alternatively, we can **remove the brackets** from the expression for the interaction

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Contrasts in ANOVA models

2 x 2 ANOVA

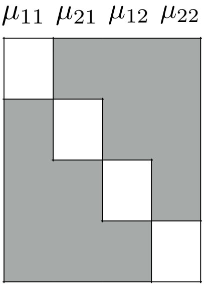
The **interaction** contrast

We are looking for a **difference** of **two differences**

$$\mathrm{A} \times \mathrm{B} = (\mu_{11} - \mu_{21}) - (\mu_{12} - \mu_{22})$$

$$= \boxed{\mu_{11}}\boxed{- \mu_{21}}\boxed{- \mu_{12}}\boxed{+ \mu_{22}}$$

$$\mathbf{L} = \begin{bmatrix} 1 & -1 & -1 & 1 \end{bmatrix}$$

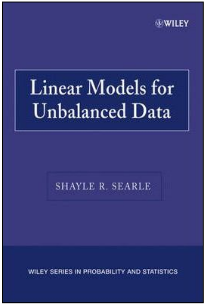


Alternatively, we can **remove the brackets** from the expression for the interaction

Unbalanced designs

ANOVA models with **unequal numbers of subjects** in each cell used to be a source of hot debate in statistics

Whole books written on the topic (e.g. Searle, 1987)



The debate centres on **which sums-of-squares** to use when decomposing the ANOVA effects — choice of **Type I**, **Type II** and **Type III**

Recent resurgence of the debate with more people using **R**, which defaults to **Type I**

Type III tend to be the default in most statistical packages — debate about how **sensible** this is vs **Type II** (e.g. Venables, 1998; Langsrud, 2003; Fox, 2008; Fox and Weisberg, 2011)

Unbalanced designs

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$$

Type I

Each effect is tested **after** adding it to the model **in order**

Effect of α_i

$$y_{ijk} = \mu + \epsilon_{ijk}$$

$$y_{ijk} = \mu + \alpha_i + \epsilon_{ijk}$$

Type II

Main effects are tested **assuming any interaction is 0**

$$y_{ijk} = \mu + \beta_j + \epsilon_{ijk}$$

$$y_{ijk} = \mu + \alpha_i + \beta_j + \epsilon_{ijk}$$

Type III

Main effects are tested **after correcting for interactions**

$$y_{ijk} = \mu + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$$

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$$

Unbalanced designs

Implementation using contrasts

Both **Type I** and **Type II** are tricky to calculate **weights** for

$$\mathbf{L}_{\mathsf{I}} = [0.5 \quad 0.5 \quad -0.667 \quad -0.333]$$
$$\mathbf{L}_{\mathsf{II}} = [0.6 \quad 0.4 \quad -0.6 \quad -0.4]$$
$$\mathbf{L}_{\mathsf{III}} = [0.5 \quad 0.5 \quad -0.5 \quad -0.5]$$

Not particularly **intuitive** — both **Type I** and **Type II** weights depend on the **number of subjects in each cell**

Type III weights correspond to the usual contrasts used in SPM — unweighted means that **do not depend** on the number of subjects

See McFarquhar (2016) for **MATLAB code** and **more detailed discussion** on these approaches

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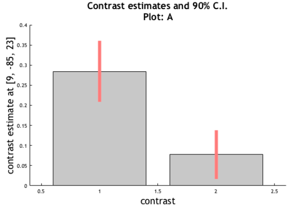
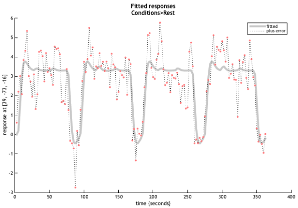
Other uses of contrasts in SPM

Contrasts are **generic methods** of selecting **linear combinations** of the **model parameters**

Contrasts are also used in SPM for

- Selecting parameters to **plot** in **bar charts**
- Partitioning the model to **plot** only **effects of interest**





Understanding how this works is **crucial** to getting the most out of the SPM plotting facilities

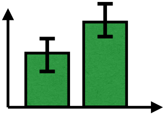
Making plots using contrasts

Rules for making plots using contrasts

1. Each **row** will be a **separate bar** in the plot
2. **Height** of the bar is the **combination** of parameters **in that row**

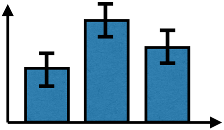
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}$$

Bar 1: value of parameter 1
Bar 2: average of parameters 2 and 3



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

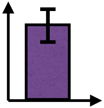
Bar 1: value of parameter 1
Bar 2: value of parameter 2
Bar 3: value of parameter 3



```
>> eye(3)
ans =
    1  0  0
    0  1  0
    0  0  1
```

$$\begin{bmatrix} 1 & -1 & 0 \end{bmatrix}$$

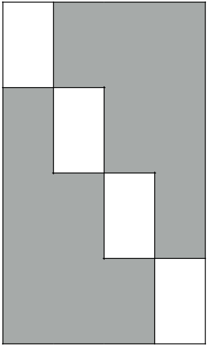
Bar 1: difference between parameters 1 and 2



Making plots using contrasts

2 x 2 ANOVA

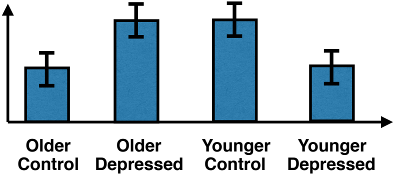
Age (older/younger) x **Diagnosis** (control/depressed)



To plot the **interaction** we want to select **each cell mean**

$$\text{Plot } \mathrm{A} \times \mathrm{B} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4 bars for the **cell means**



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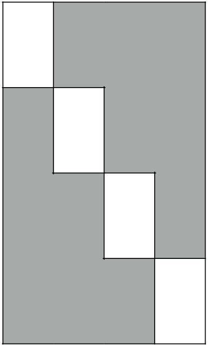
Making plots using contrasts

2 x 2 ANOVA

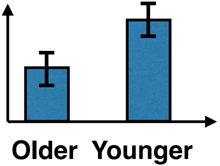
Age (older/younger) x **Diagnosis** (control/depressed)



To plot the **main effects** we average over the cells of the **other factor** to form the **marginal means**

$$\text{Plot Age} = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \end{bmatrix}$$

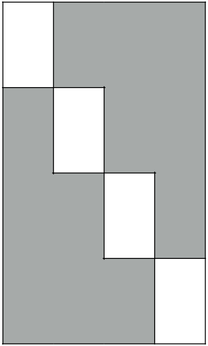
2 bars each **averaged across Diagnosis**



Making plots using contrasts

2 x 2 ANOVA

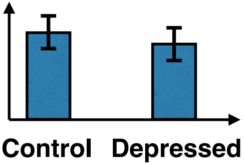
Age (older/younger) x **Diagnosis** (control/depressed)



To plot the **main effects** we average over the cells of the **other factor** to form the **marginal means**

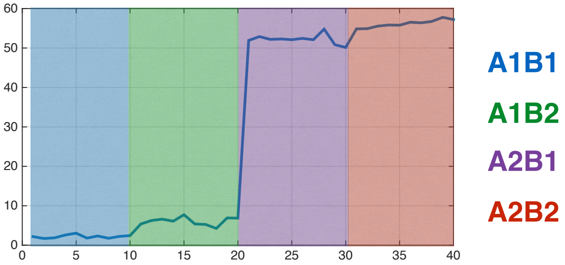
$$\text{Plot Diagnosis} = \begin{bmatrix} 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.5 \end{bmatrix}$$

2 bars each **averaged across Age**



Partitioning effects using contrasts

When attempting to **visualise** effects in our model, sometimes it can be difficult to see what is going on because the **effect of interest** may be **obscured** by another **effect/confound**



Clear **main effect of A** — is there a **main effect of B**?

At present, any effect of B is **obscured** by the effect of A

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Partitioning effects using contrasts

We can use the **contrast** for the **effect of B** to **remove** from the data those effects that are **not related** to the **effect of B**

$$\mathbf{L} = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}$$

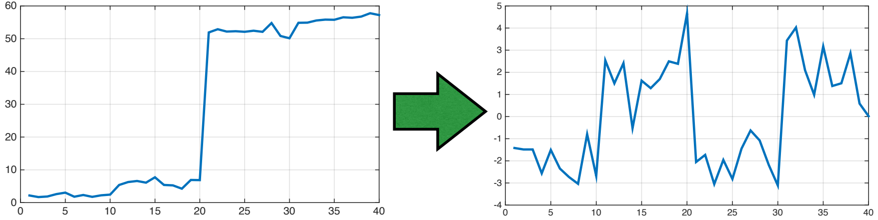
$$\mathbf{L}_0 = \mathbf{I}_4 - \mathbf{L}'(\mathbf{L}')^+$$

$$\mathbf{X}_0 = \mathbf{X}\mathbf{L}_0$$

$$\mathbf{R}_0 = \mathbf{I}_n - \mathbf{X}_0\mathbf{X}_0^+$$

$$\mathbf{Y}_{\text{adj}} = \mathbf{R}_0\mathbf{Y}$$

We use the **contrast** to **partition the model** into **effect of interest** and **effects of no interest** and then **remove** the **effects of no interest** from the data



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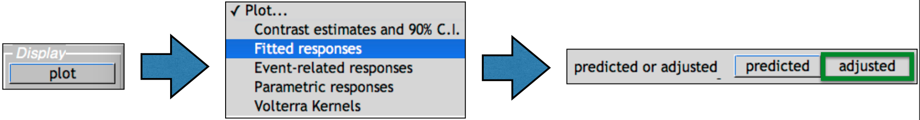
Partitioning effects using contrasts

We can use the **contrast** for the **effect of B** to **remove** from the data those effects that are **not related** to the **effect of B**

$$\mathbf{L} = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}$$
$$\mathbf{L}_0 = \mathbf{I}_4 - \mathbf{L}'(\mathbf{L}')^+$$
$$\mathbf{X}_0 = \mathbf{X}\mathbf{L}_0$$
$$\mathbf{R}_0 = \mathbf{I}_n - \mathbf{X}_0\mathbf{X}_0^+$$
$$\mathbf{Y}_{\mathrm{adj}} = \mathbf{R}_0\mathbf{Y}$$

We use the **contrast** to **partition the model** into **effect of interest** and **effects of no interest** and then **remove** the **effects of no interest** from the data

This is what **SPM** does when you plot **fitted responses** and select **adjusted**



Summary

Contrasts are an important topic in SPM as they are the means of **asking questions** about our models

These questions take the form of **hypothesis tests** about **linear combinations** of the **estimated model parameters**

An important element of this is **understanding what the model parameters mean**

For **overparameterised** models this is more difficult — contrasts must form **estimable functions** — even when models are **well conditioned**, contrasts can still be **misinterpreted**

In SPM contrasts are also used as means of **selecting parameters to plot** and **partitioning** the model into **effects of interests**

Take-home messages

If you are a **beginner** with SPM, the **contrast** framework can be **difficult** and **confusing**

Although I have talked about concepts such as **estimability**, **unbalanced designs** and **model partitioning**, these are **not that important** when starting out

As a beginner, you need to:

- Start **thinking in contrasts** — engage with your models and think about **what the model parameters mean**
- Try to **understand** what the contrast weights **mean** — especially with **auto-generated** weights
- Make sure you are clear on the **difference** between ***t*-** and ***F*-contrasts**
- **Don't** just use contrast weights you've **learned in the past** as they **may not be suitable** for your model

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McFarquhar, M. (2016) Testable Hypotheses for Unbalanced Neuroimaging Data. *Frontiers in Neuroscience: Brain Imaging Methods*, 10, 270.

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