Bayesian inference

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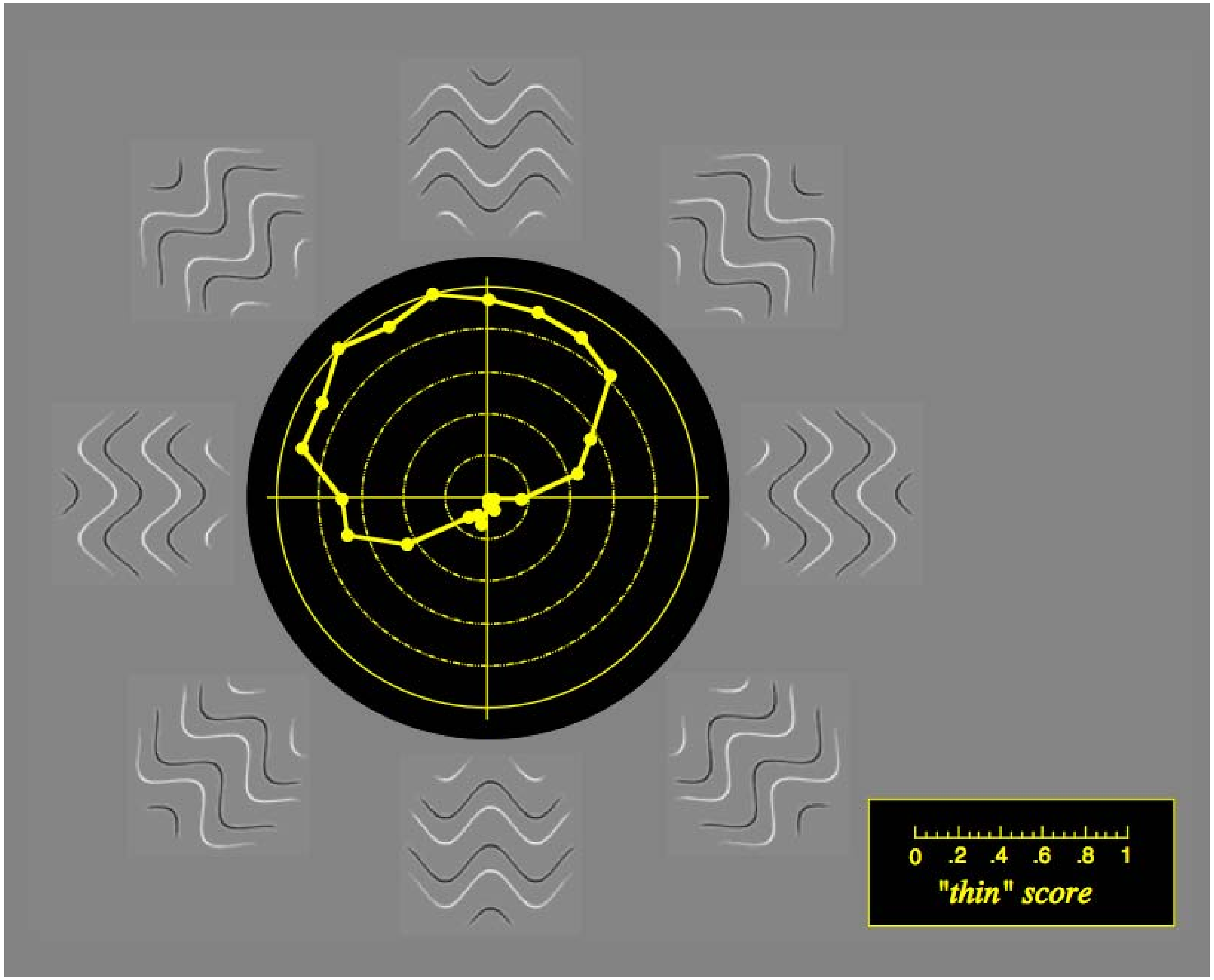

Prior knowledge on the illumination position
Mamassian & Goutcher 2001.

Overview of the talk

1 Probabilistic modelling and representation of uncertainty

1.1 Bayesian paradigm

1.2 Hierarchical models

1.3 Frequentist versus Bayesian inference

2 Numerical Bayesian inference methods

2.1 Sampling methods

2.2 Variational methods (EM, VB)

3 SPM applications

3.1 aMRI segmentation

3.2 fMRI time series analysis with spatial priors

3.3 Dynamic causal modelling

3.4 EEG source reconstruction

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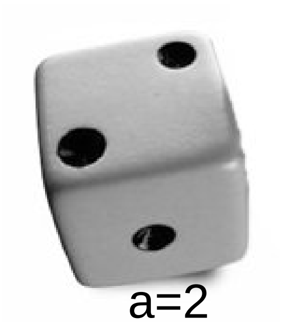
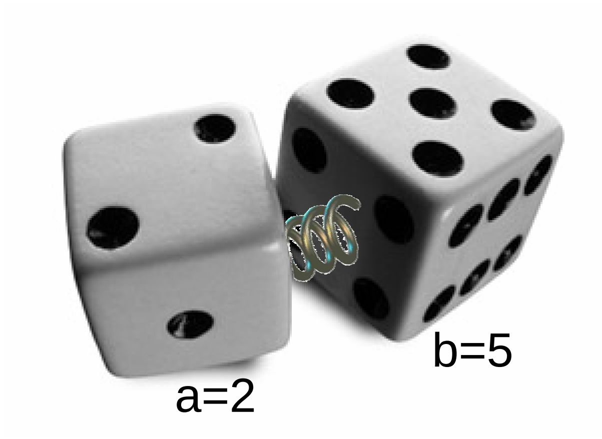
Bayesian paradigm

theory of probability

Degree of plausibility *desiderata*:
- should be represented using real numbers (D1)
- should conform with intuition (D2)
- should be consistent (D3)



• normalization: $$\sum_a P(a) = 1$$

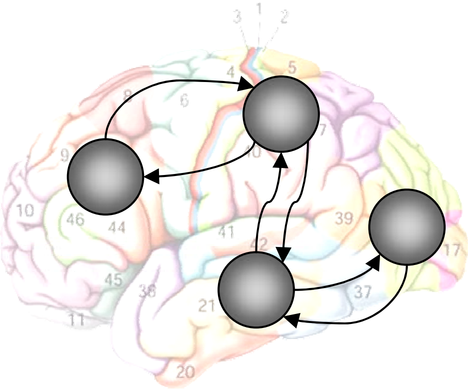


• marginalization: $$P(b) = \sum_a P(a,b)$$

• conditioning : (*Bayes rule*) $$P(a,b) = P(a|b)\,P(b) = P(b|a)\,P(a)$$

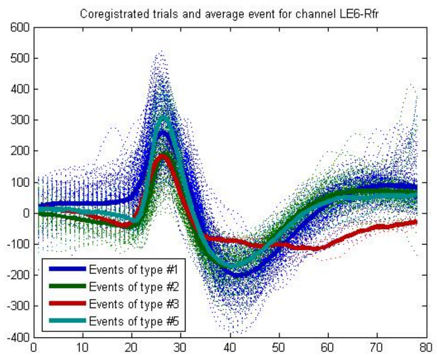
Bayesian paradigm

Likelihood and priors



generative model m

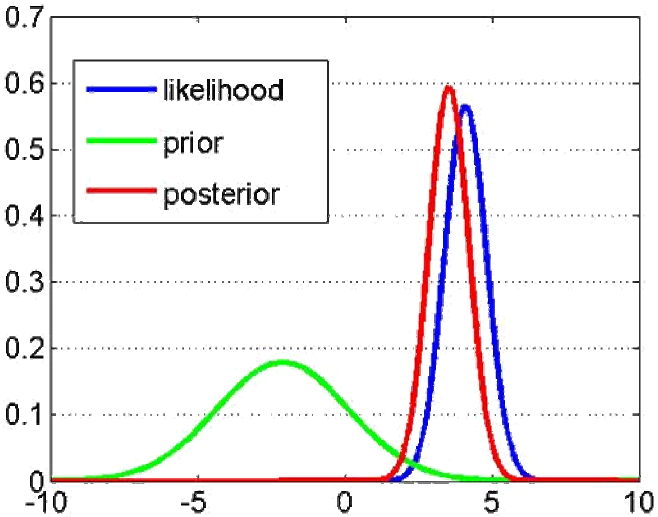
y



Likelihood: $p(y|\theta,m)$

Prior: $p(\theta|m)$

Bayes rule: $$p(\theta|y,m)=\frac{p(y|\theta,m)\,p(\theta|m)}{p(y|m)}$$



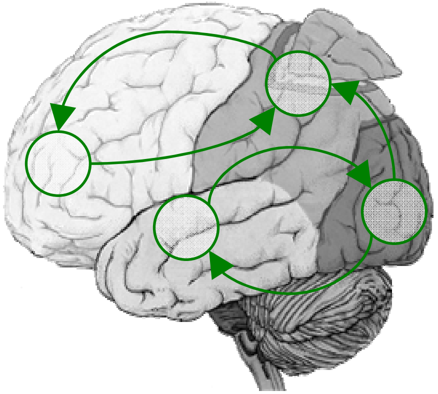
Bayesian paradigm

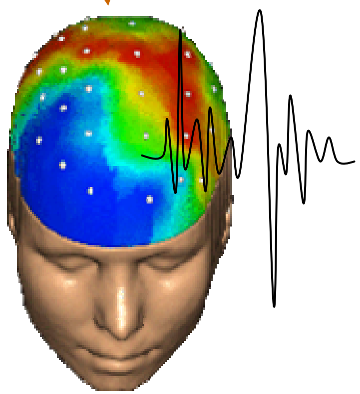
forward and inverse problems

forward problem

$$p\left(y\middle|\vartheta,m\right)$$

likelihood





posterior distribution

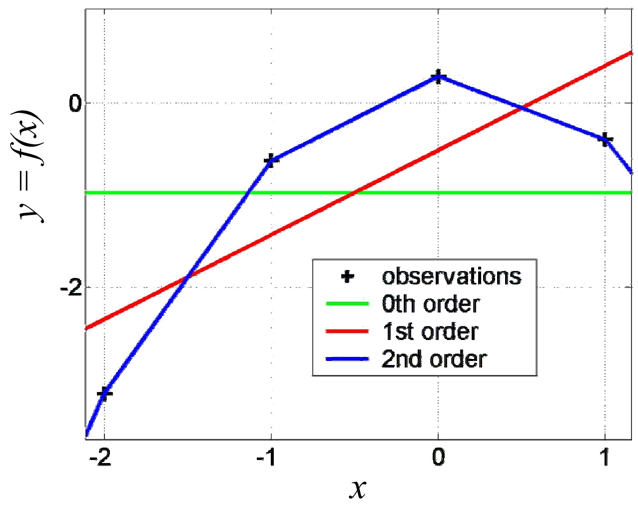
$$p\left(\vartheta\middle|y,m\right)$$

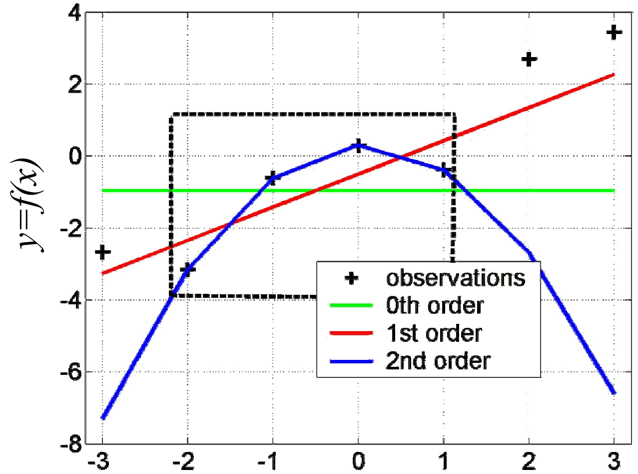
inverse problem

Bayesian paradigm

Model comparison

Principle of parsimony :
« plurality should not be assumed without necessity »

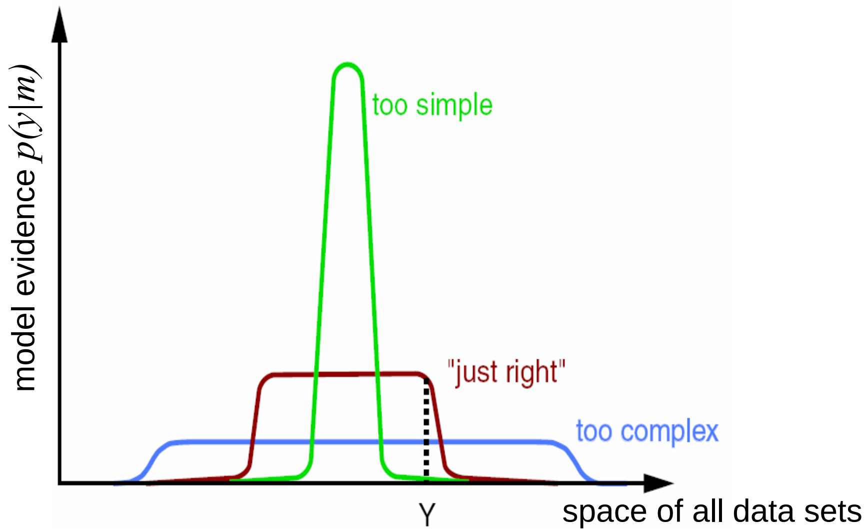




Model evidence:

$$p(y|m) = \int p(y|\theta, m) p(\theta|m) d\theta$$

"Occam's razor" :

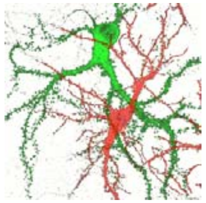


Hierarchical models

principle

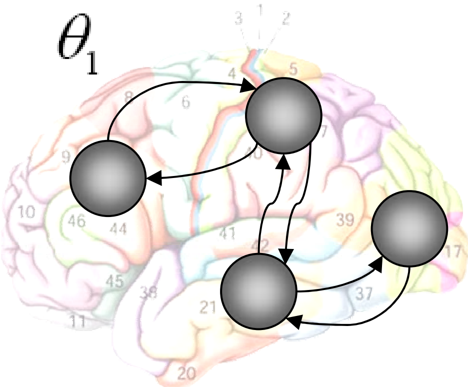
$\vdots$

θ_2

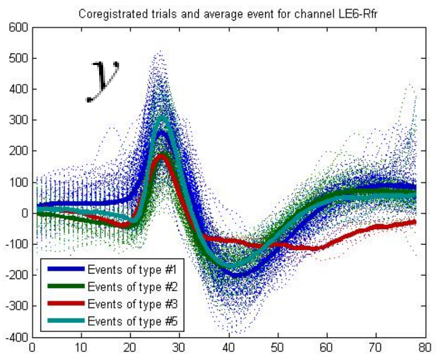


$$p(\theta_2|\theta_3, m)$$

θ_1



$$p(\theta_1|\theta_2, m)$$



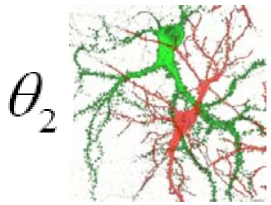
$$p(y|\theta_1, m)$$

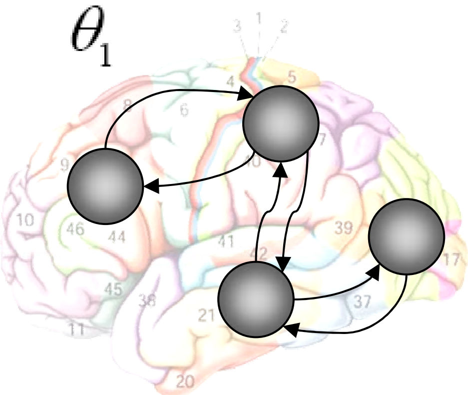
hierarchy

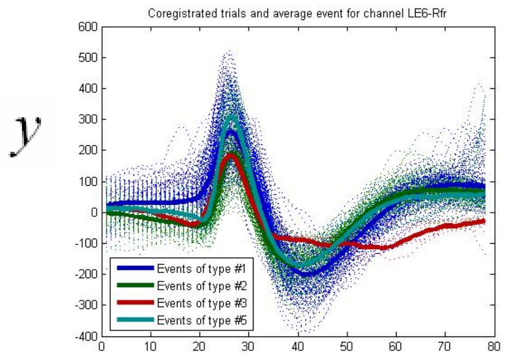
causality

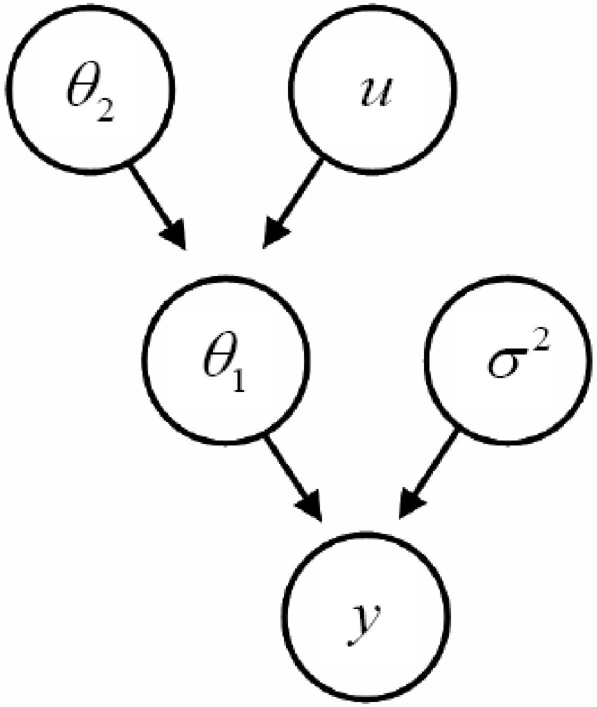
Hierarchical models

directed acyclic graphs (DAGs)









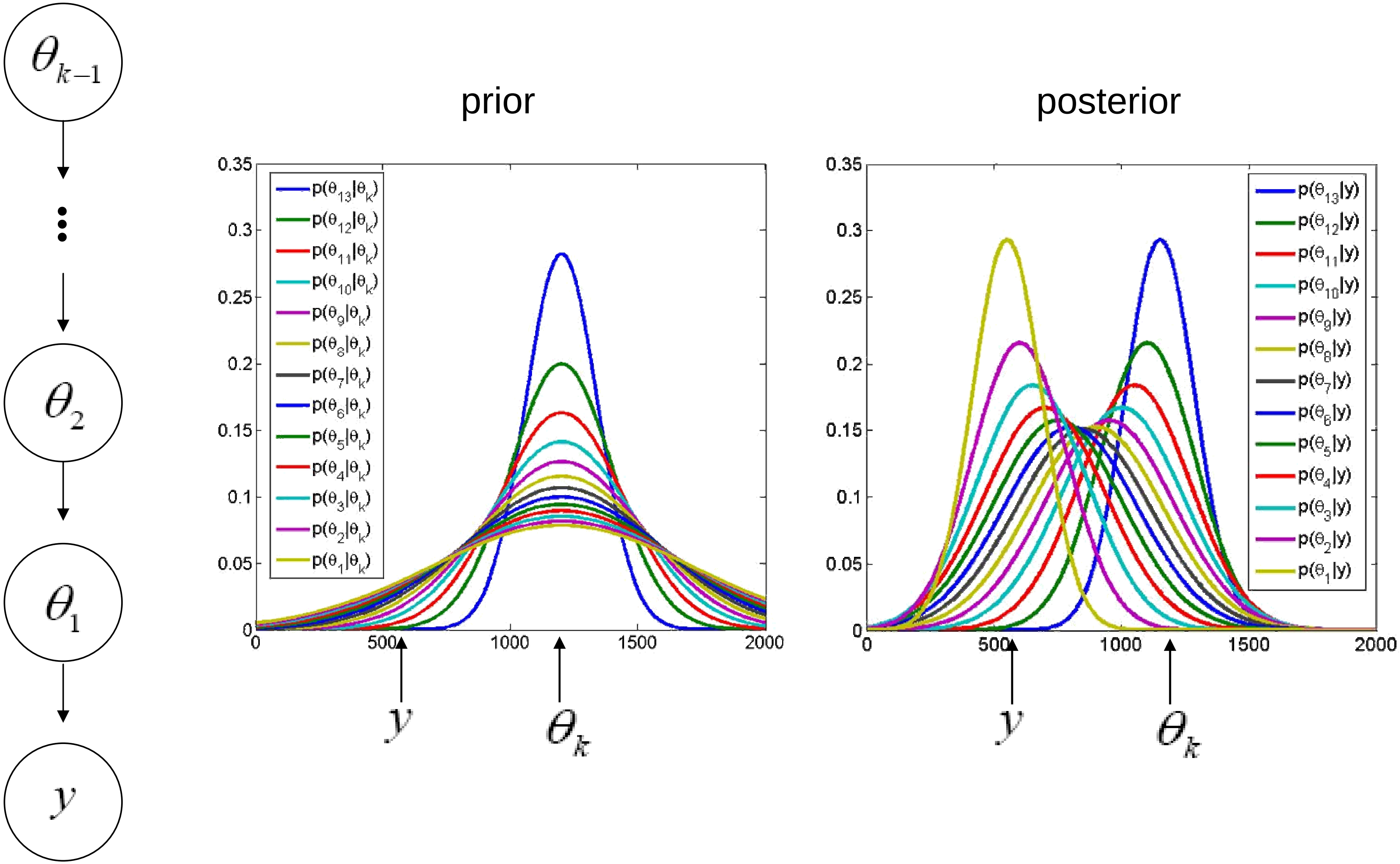
$$p(\theta_1 | \theta_2, u, m)$$

$$p(y | \theta_1, \sigma^2, m)$$

$$p(\theta | m) = \prod_j p(\theta_j | par(\theta_j), m)$$

Hierarchical models

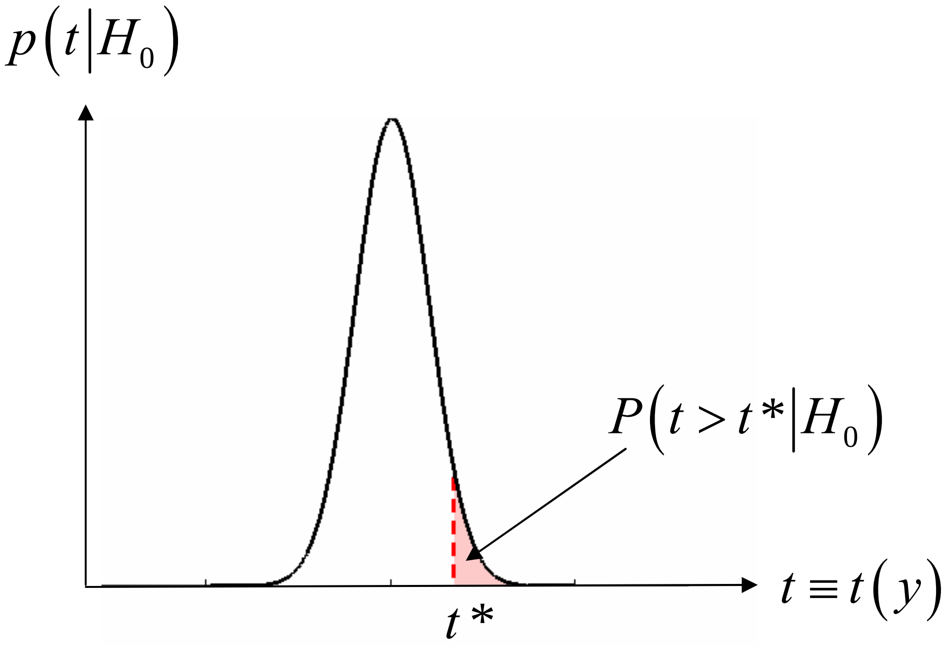
univariate linear hierarchical model



Frequentist versus Bayesian inference

a (quick) note on hypothesis testing

- define the null, e.g.: $H_0 : \theta = 0$

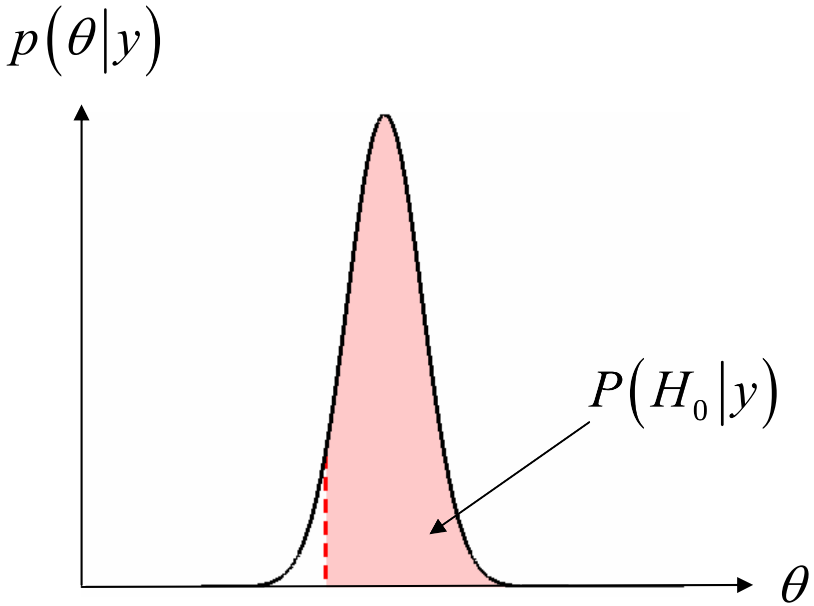


- estimate parameters (obtain test stat.)
- apply decision rule, i.e.:

if $P\left(t > t^* \middle| H_0\right) \leq \alpha$ then reject H0

classical SPM

- invert model (obtain posterior pdf)



- define the null, e.g.: $H_0 : \theta > 0$
- apply decision rule, i.e.:

if $P\left(H_0 \middle| y\right) \geq \alpha$ then accept H0

Bayesian PPM

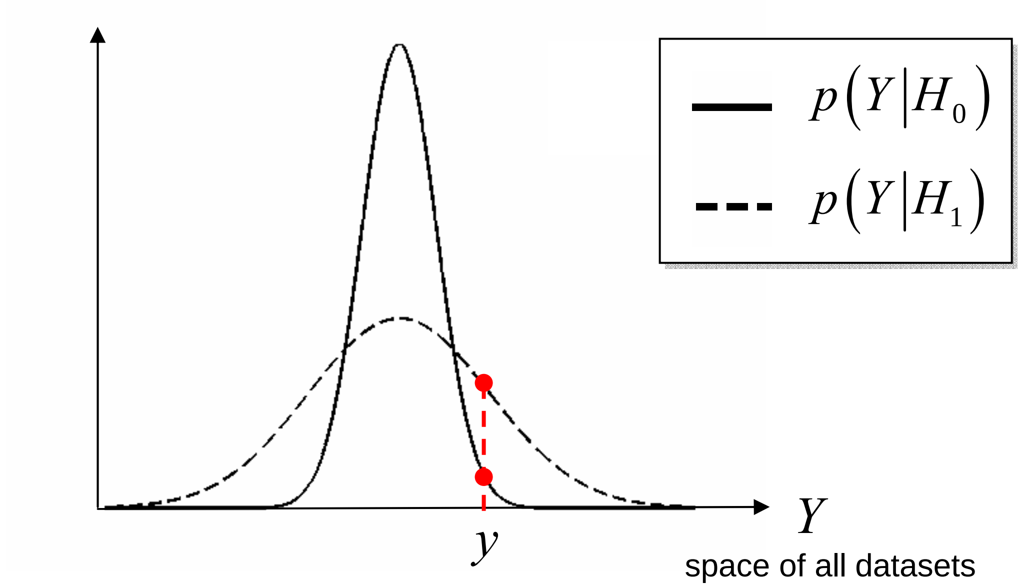
Frequentist versus Bayesian inference

Bayesian inference: what about point hypothesis testing?

- define the null and the alternative hypothesis *in terms of priors*, e.g.:

$$H_0 : p(\theta | H_0) = \begin{cases} 1 & \text{if } \theta = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$H_1 : p(\theta | H_1) = N(0, \Sigma)$$



space of all datasets

- invert both generative models (obtain both model evidences)
- apply decision rule, i.e.:

$$\text{if } \quad \frac{P(H_0|y)}{P(H_1|y)} \leq 1 \quad \text{then reject H0}$$

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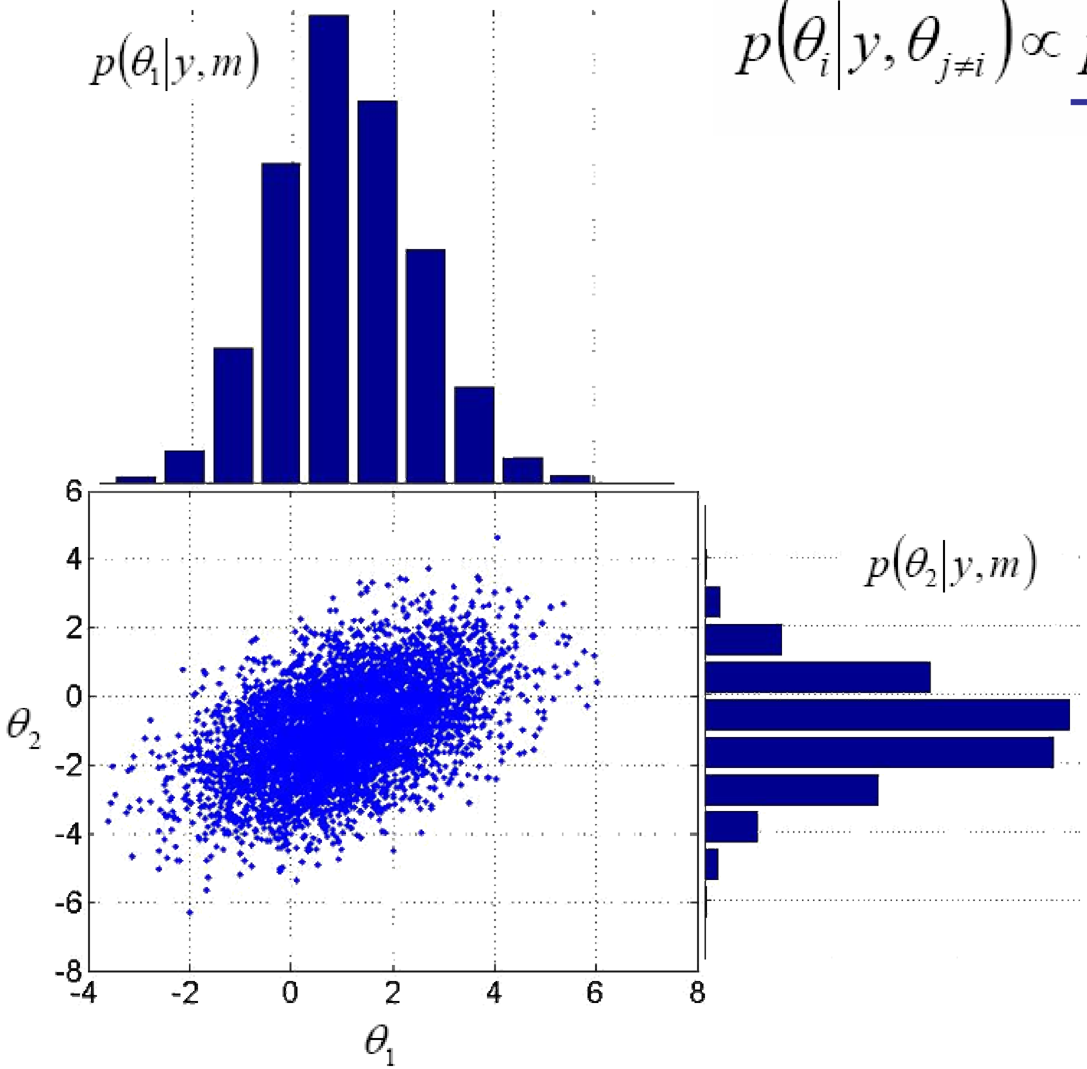
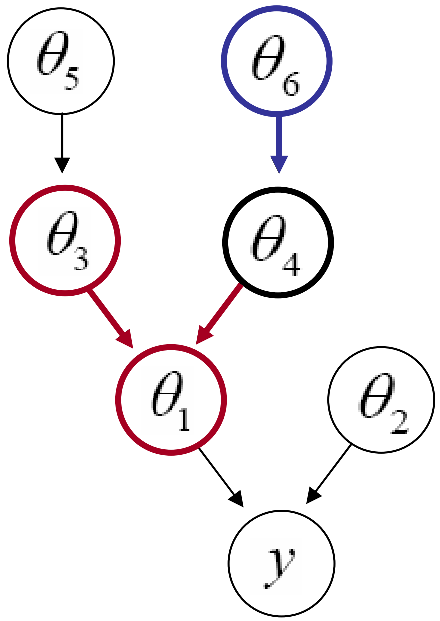
Sampling methods

MCMC example: Gibbs sampling

$$p\left(\theta_i \middle| y, \theta_{j\neq i}\right) \propto \underline{p\left(\theta_i \middle| par(\theta_i)\right)} \underline{\prod_{j=ch(i)} p\left(\theta_j \middle| par(\theta_j)\right)}$$



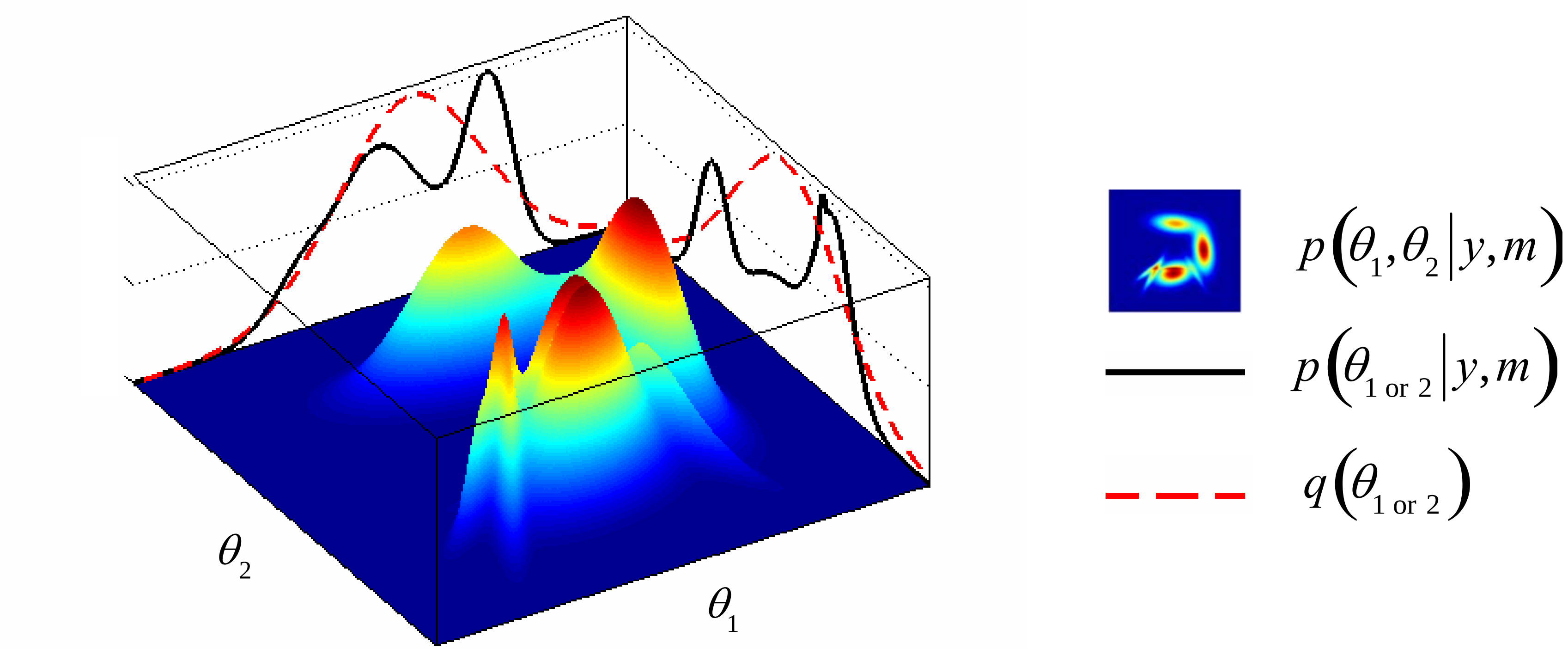


$$\frac{1}{N}\sum_{n=1}^{N} p\left(y \middle| \theta^{(n)}, m\right) \approx p\left(y \middle| m\right)$$

Variational methods

$$\ln p(y|m) = \underbrace{\langle \ln p(\theta, y|m) \rangle_q + S(q)}_{\text{free energy } F(q)} + D_{KL}\left(q(\theta); p(\theta|y,m)\right)$$

VB/EM/ReML: find (iteratively) the “variational” posterior $q(\theta)$ which maximizes the free energy $F(q)$ under some approximation



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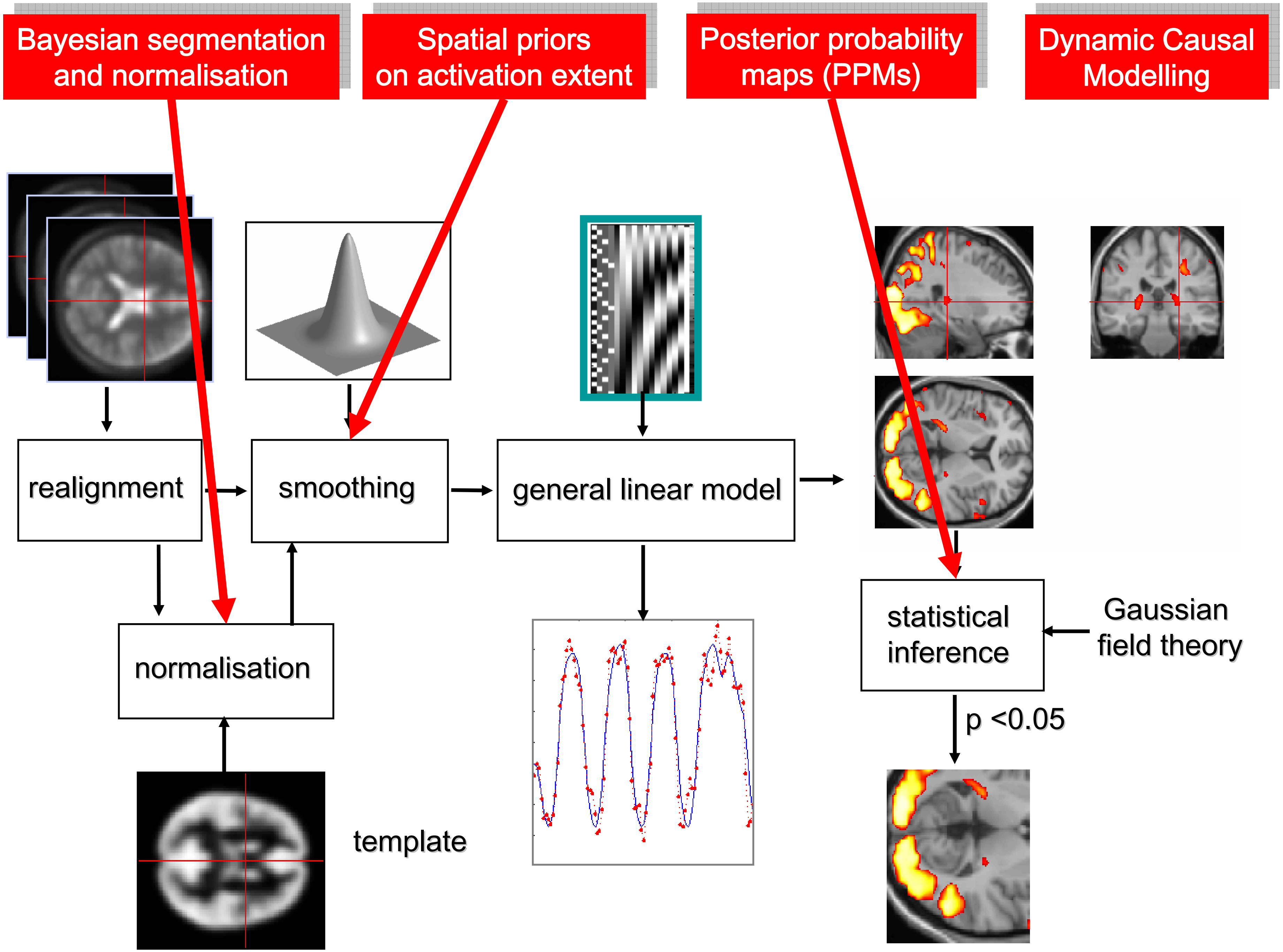
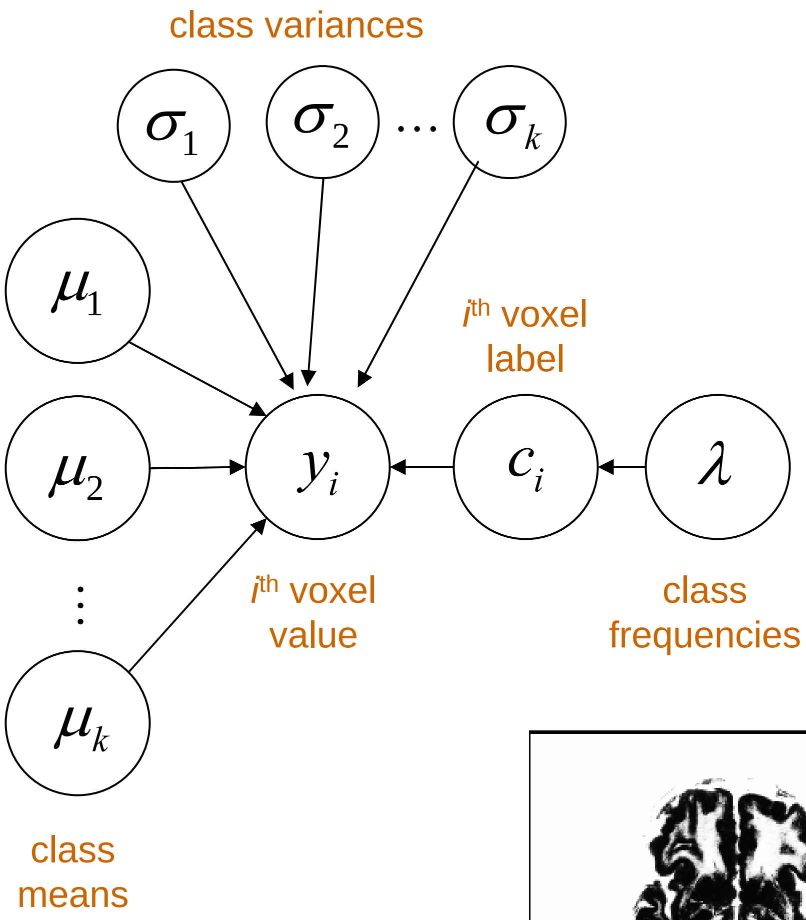
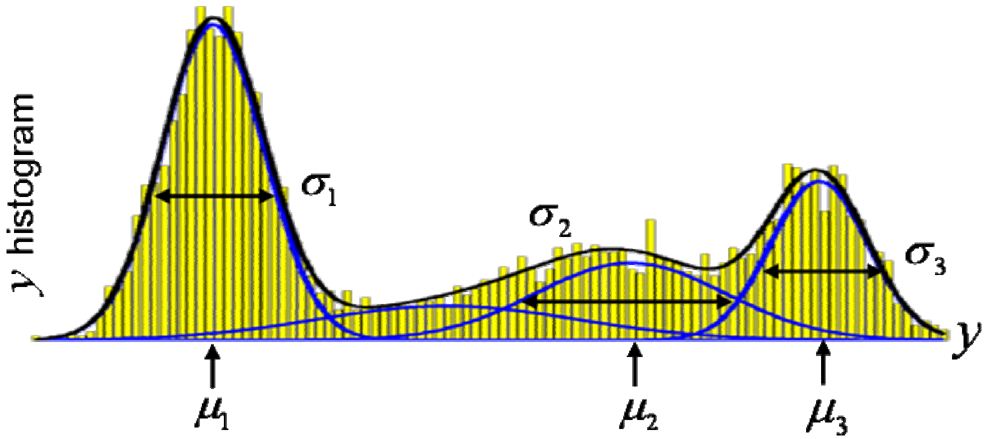
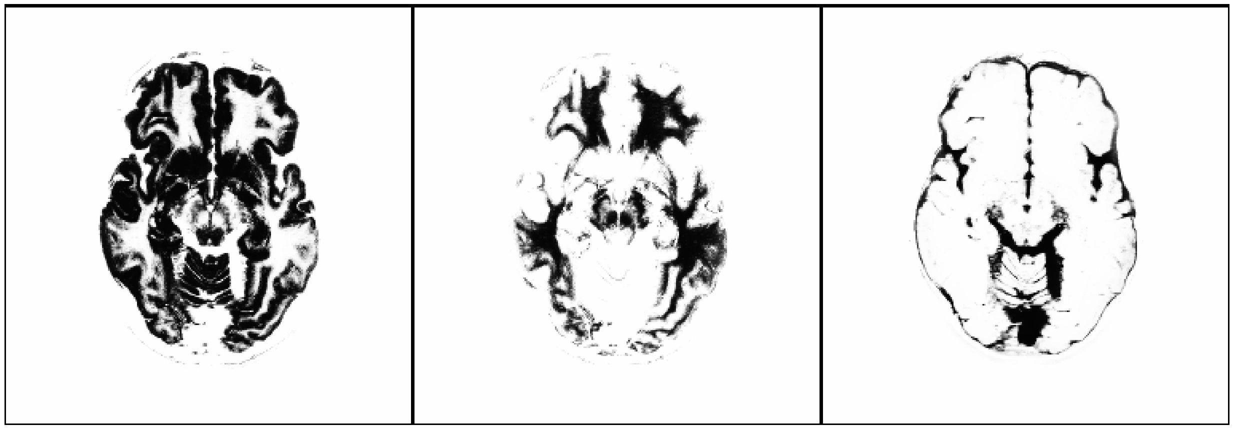


Bayesian segmentation and normalisation
Spatial priors on activation extent
Posterior probability maps (PPMs)
Dynamic Causal Modelling
realignment
smoothing
general linear model
normalisation
template
statistical inference
Gaussian field theory
p <0.05

aMRI segmentation





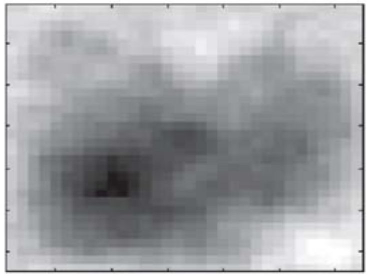


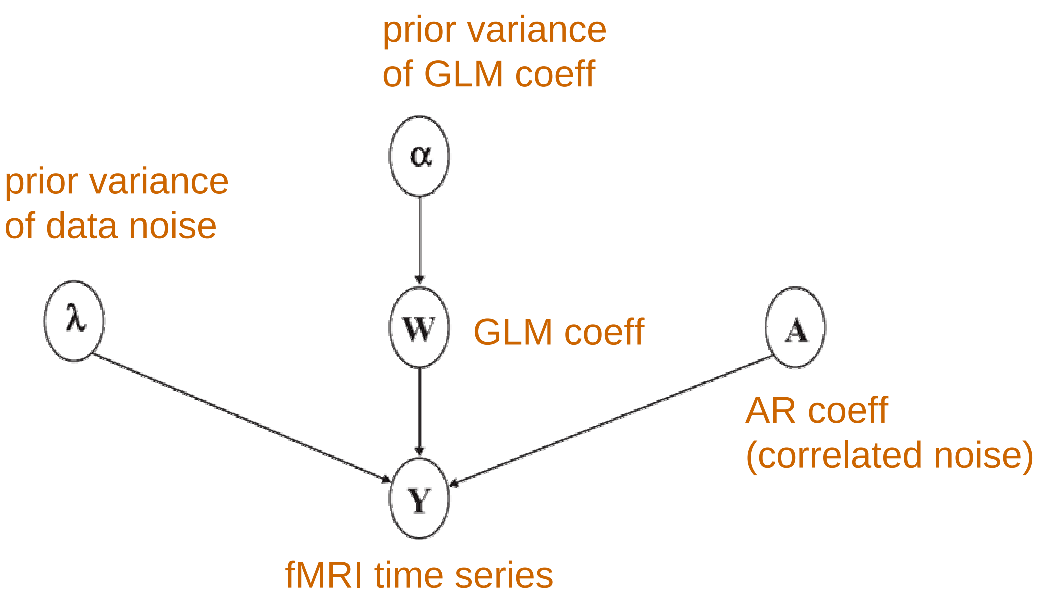
grey matter | white matter | CSF

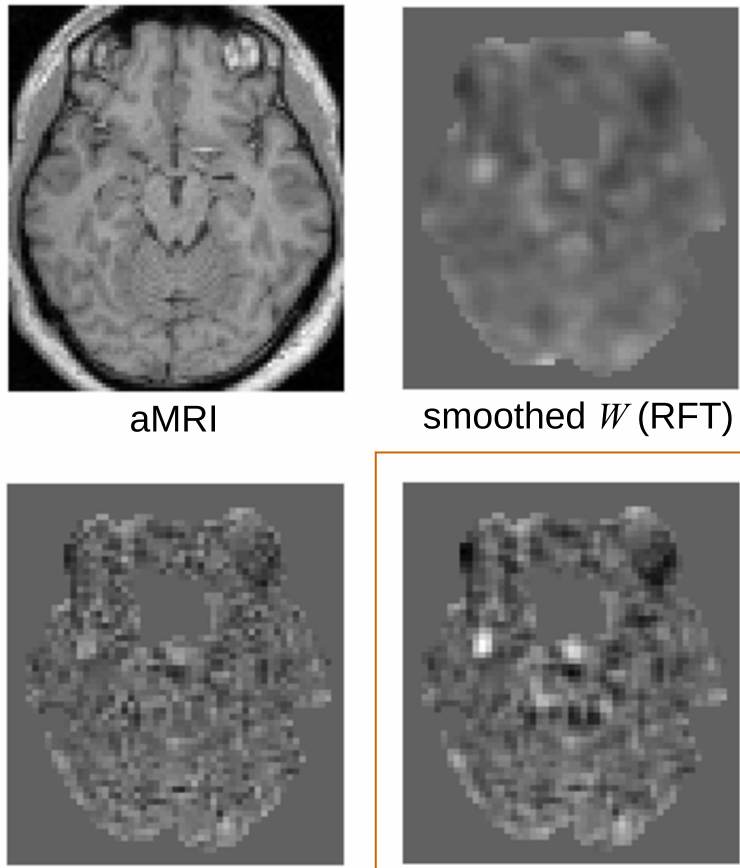
[Ashburner et al., Human Brain Function, 2003]

fMRI time series analysis with spatial priors

		1		
	2	-8	2	
1	-8	20	-8	1
	2	-8	2	
		1		







aMRI

smoothed W (RFT)

ML estimate of W

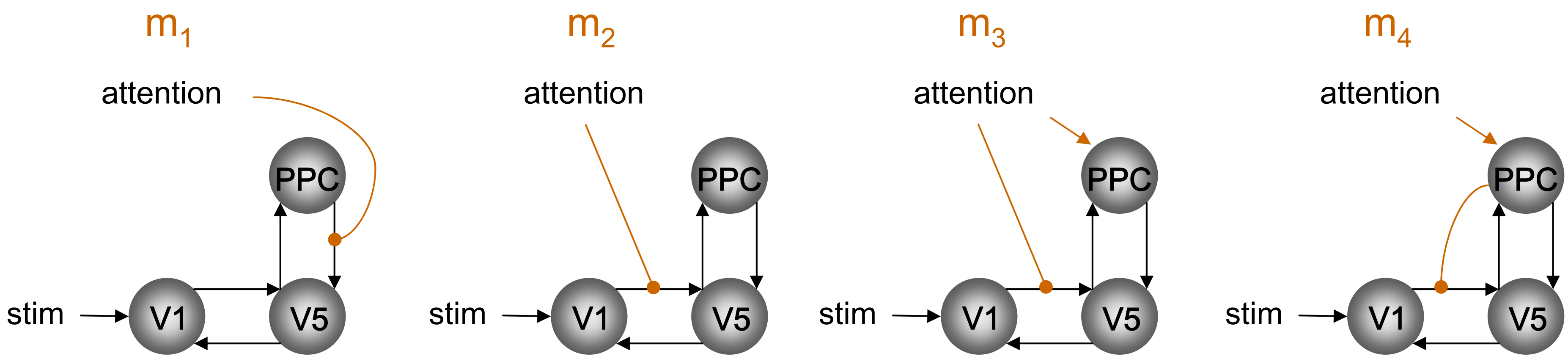
VB estimate of W

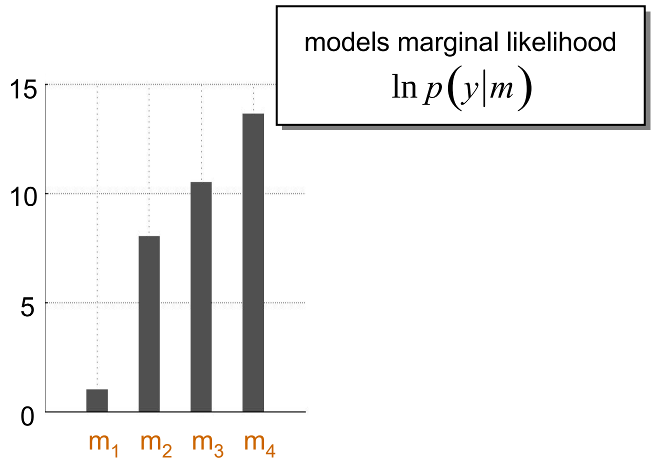
PPM

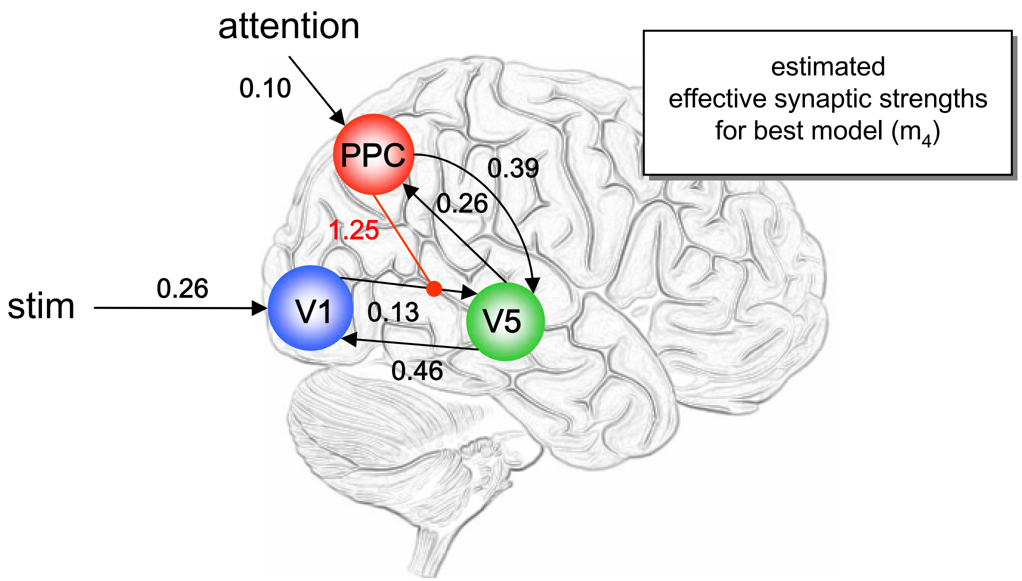
[Penny et al., Neuroimage, 2004]

Dynamic causal modelling

model comparison on network structures





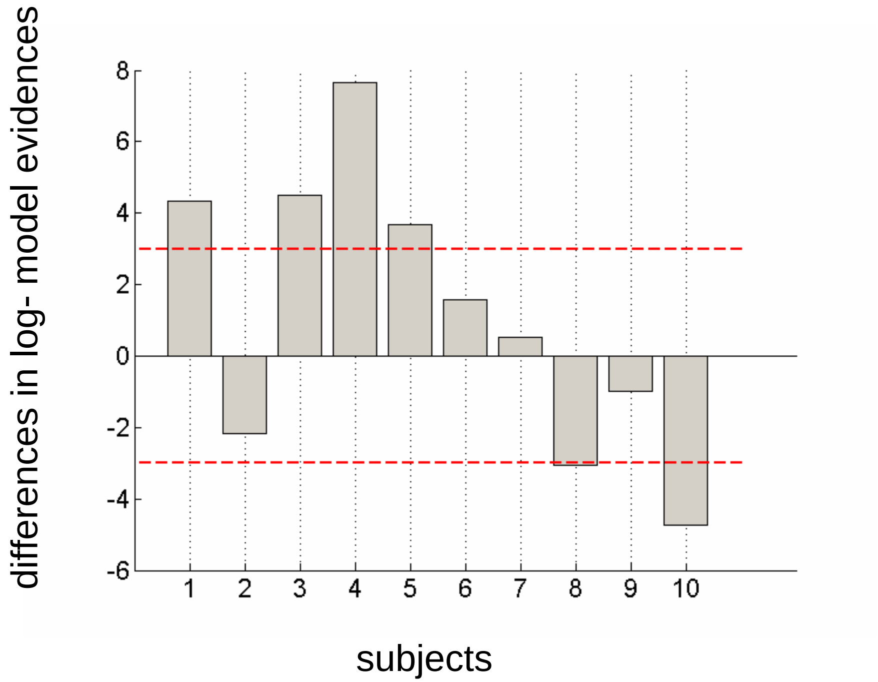


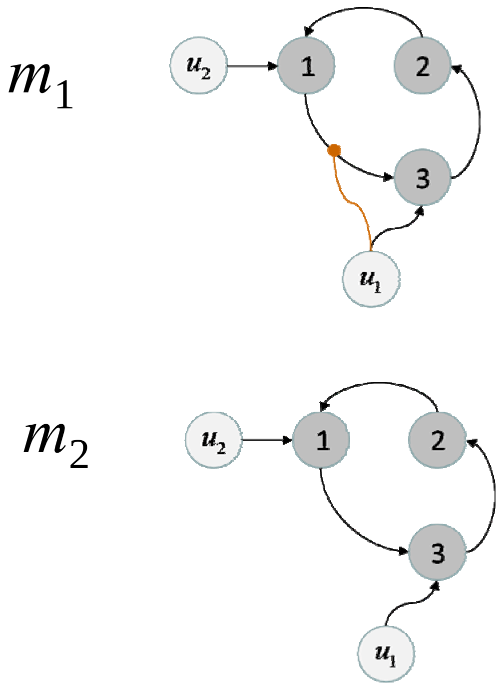
[Stephan et al., Neuroimage, 2008]

Dynamic causal modelling

model comparison for group studies

$$\ln p(y|m_1) - \ln p(y|m_2)$$





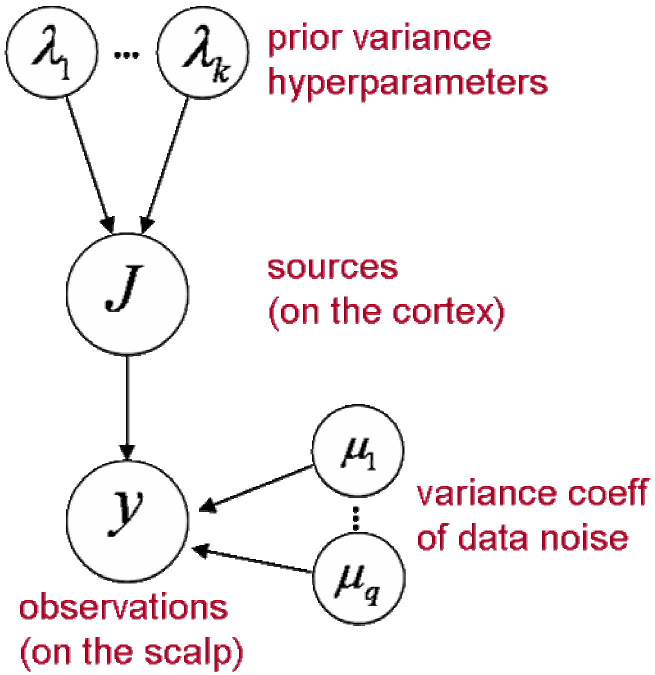
fixed effect — assume all subjects correspond to the same model

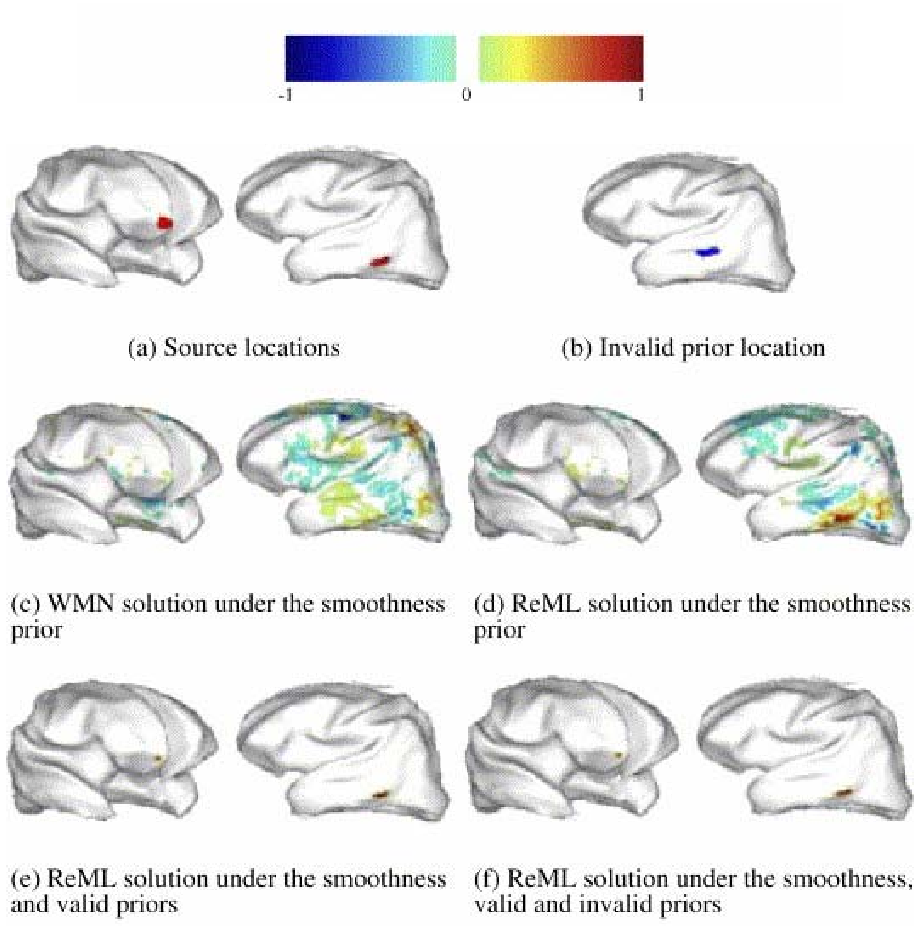
random effect — assume different subjects might correspond to different models

[Stephan et al., Neuroimage, 2009]

EEG source reconstruction

finessing an ill-posed inverse problem



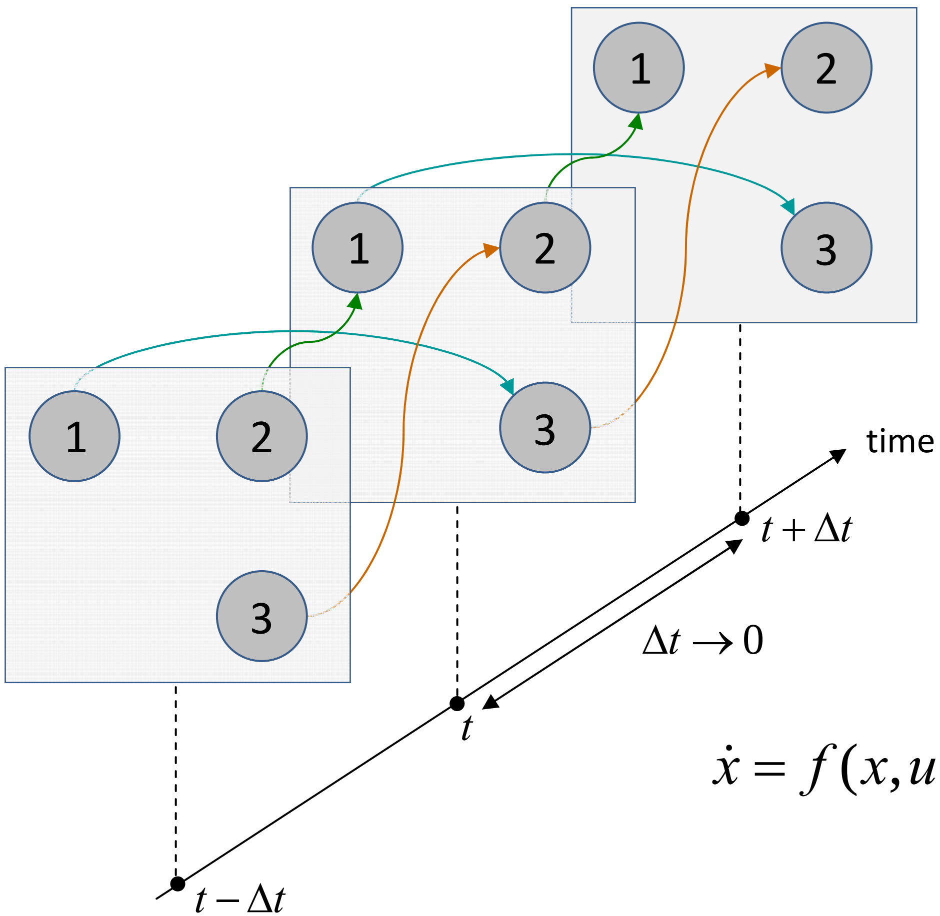


[Mattout et al., Neuroimage, 2006]

I thank you for your attention.

DCMs and DAGs

a note on causality



$$\dot{x} = f(x,u,\theta)$$