

Online Supplemental Materials for “Accounting for correlated  
observations in an age-based state-space stock assessment model”

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## Simulation study

A small simulation study is performed to ensure correct implementation of the model and the ability to differentiate between datasets simulated with independent and correlated observations using AICc. The simulations are set up to resemble the North Sea whiting case study by using the same number of age-groups, years, and fleets. The  $F$ -process is modelled with an AR(1)-structure using  $\rho = 0.5$ , which results in time-varying selectivity. This was done in order to test the model's ability to differentiate selectivity changes from correlations in the observation error. In each run two datasets are generated, one with independent observations corresponding to Model 1, and one with correlated observations corresponding to Model 5. The covariance matrices for the observation errors is set equal to those estimated in the whiting case study. For each dataset we fit both Models 1 and 5 giving a total of 4 estimations per pair of datasets, and with a total of 20 runs we obtain 40 simulated datasets and 80 estimations in total. The number of times each combination of model and simulation is selected by AICc is recorded in a cross-classification table. The efficiency of the model to reconstruct the true  $N$ s and  $F$ s is also cross-tabulated. We choose a simple measure of model efficiency, namely the root mean squared error of state estimates, i.e.  $\sqrt{\sum_y \sum_a \left( \log N_{y,a} - \log \hat{N}_{y,a} \right)^2 / N_1} + \sqrt{\sum_y \sum_a \left( \log F_{y,a} - \log \hat{F}_{y,a} \right)^2 / N_2}$ , where  $N_1$  and  $N_2$  is the total number of  $N$ -states and  $F$ -states respectively.

	Est 1	Est 5
Sim 1	20	0
Sim 5	0	20

Table 1: Cross-classification table from the simulation study (number of times each combination of simulation and estimation model was selected by the AICc criterion). The AICc criterion selected the correct model in every simulation.

	Est 1	Est 5
Sim 1	6.467	6.830
Sim 5	6.656	5.869

Table 2: Root mean squared error of the estimated state-vector. Rows indicate the true covariance structure in the simulation model (1=independent, 5=unstructured), whereas columns indicate the assumed covariance structure of the estimation model.

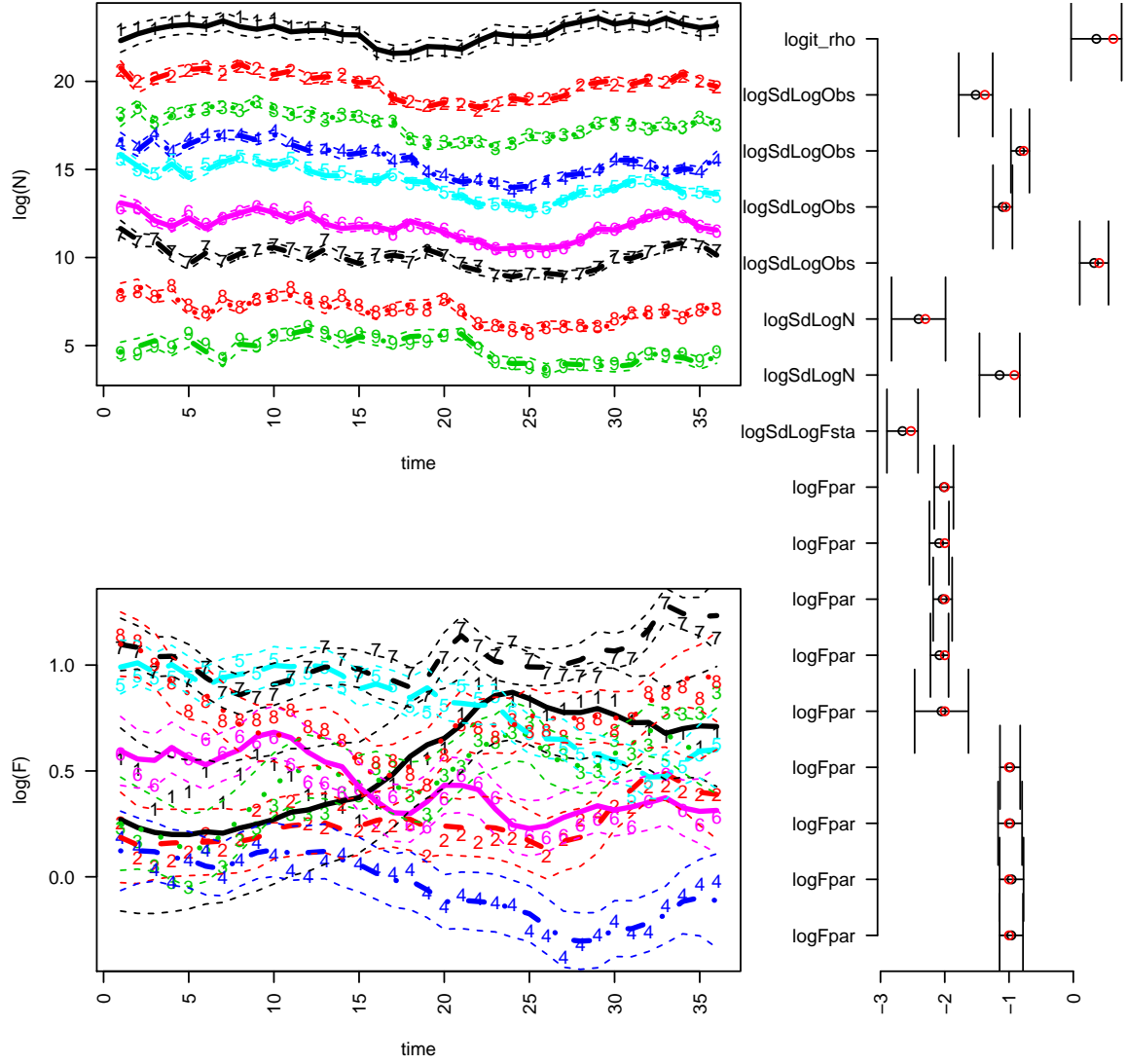


Figure 1: Example from simulation study of true parameters (excluding observation correlation parameters) and simulated states along with their estimated values and 95% confidence intervals (lines). Estimated log-numbers-at-age (top), log-fishing-mortalities, (bottom), and parameters (right).

## Supplemental Tables

This section contains the specification of the covariance structure for the observations for all the models. In all tables the rows represent fleets and the columns represent age groups, the first row applies to the total catches, while the second and third rows apply to the surveys. A “-” in a table cell means that there are no observations for this combination of fleet and ages. A zero in a table cell means that it is assumed to be independent. Positive integers specify coupling of distances between lattice nodes in the correlation structure, that is, repeated integers specify that these distances are assumed to be equal. Note, that since table cells correspond to *distances* between lattice nodes, the number of columns is one less than the number of age groups. “UC” in a row specify that this fleet should use the unconstrained correlation structure. The specifications for Models 1 and 5 have been omitted since these tables consist purely of zeroes and “UC”.

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1
2	-	2	2	2	2	-	-	-	-
3	3	3	3	3	3	-	-	-	-

Table 3: Herring: observation covariance structure configuration for model 2

	1	2	3	4	5	6	7	8	9
1	1	2	3	3	3	3	3	3	3
2	-	4	5	6	6	-	-	-	-
3	7	4	5	6	6	-	-	-	-

Table 4: Herring: observation covariance structure configuration for model 3

	1	2	3	4	5	6	7	8	9
1	UC	UC	UC	UC	UC	UC	UC	UC	UC
2	-	1	2	2	2	-	-	-	-
3	3	2	4	4	4	-	-	-	-

Table 5: Herring: observation covariance structure configuration for model 4

	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	-	2	2	2	2	-	-	-
3	2	2	2	2	2	-	-	-

Table 6: Whiting: observation covariance structure configuration for model 2

	1	2	3	4	5	6	7	8
1	1	1	1	2	2	2	2	2
2	-	1	2	2	2	-	-	-
3	1	1	2	2	2	-	-	-

Table 7: Whiting: observation covariance structure configuration for model 3

	1	2	3	4	5	6	7	8
1	UC	UC	UC	UC	UC	UC	UC	UC
2	-	1	2	1	1	-	-	-
3	3	4	4	4	3	-	-	-

Table 8: Whiting: observation covariance structure configuration for model 4

	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	0	0
2	-	1	1	1	1	-	-	-
3	2	2	2	2	2	-	-	-

Table 9: Haddock: observation covariance structure configuration for model 2

	1	2	3	4	5	6	7	8
1	1	2	3	4	4	4	4	4
2	-	2	3	4	4	-	-	-
3	1	2	3	4	4	-	-	-

Table 10: Haddock: observation covariance structure configuration for model 3

	1	2	3	4	5	6	7	8
1	UC	UC	UC	UC	UC	UC	UC	UC
2	-	1	2	4	1	-	-	-
3	1	2	2	4	3	-	-	-

Table 11: Haddock: observation covariance structure configuration for model 4

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	-	-	-
3	3	3	3	3	3	3	-	-	-

Table 12: Turbot: observation covariance structure configuration for model 2

	1	2	3	4	5	6	7	8	9
1	1	1	2	2	2	2	2	2	2
2	3	4	4	4	4	4	-	-	-
3	5	5	5	5	5	5	-	-	-

Table 13: Turbot: observation covariance structure configuration for model 3

	1	2	3	4	5	6	7	8	9
1	UC	UC	UC	UC	UC	UC	UC	UC	UC
2	1	2	2	3	2	1	-	-	-
3	4	3	3	1	2	4	-	-	-

Table 14: Turbot: observation covariance structure configuration for model 4

## Supplemental Figures

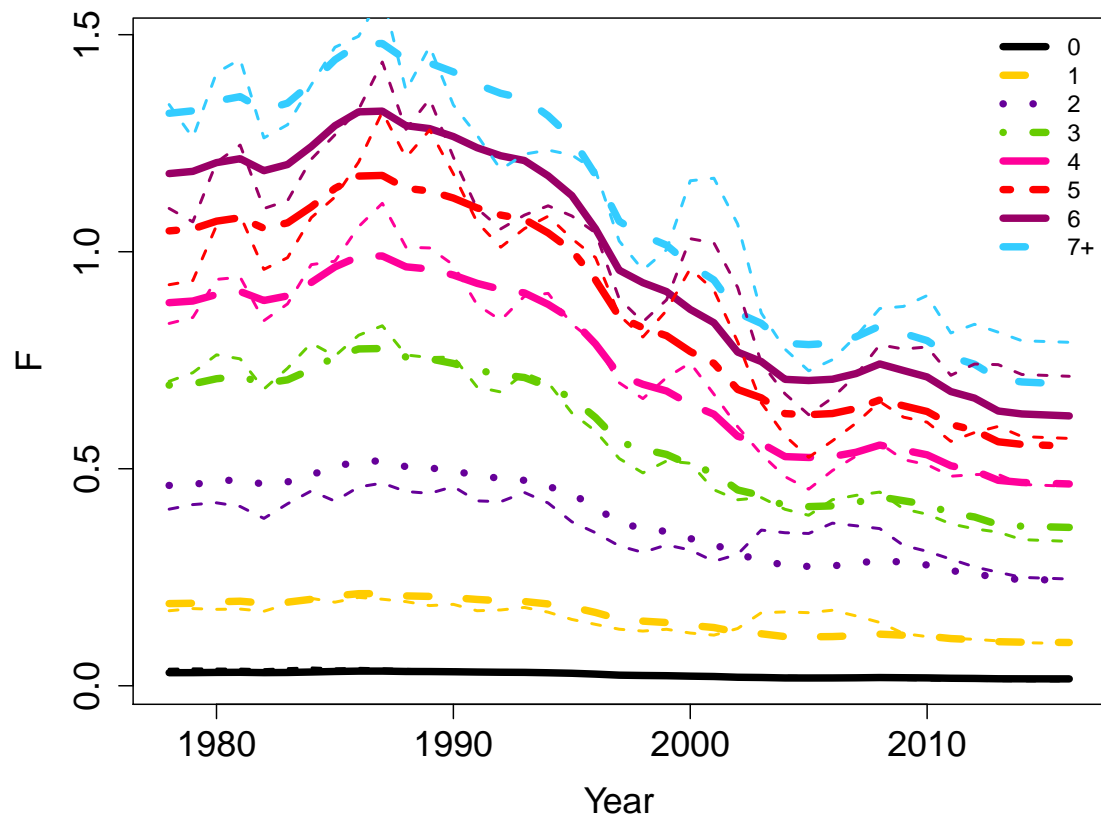


Figure 2: Whiting: Estimated fishing mortalities by age. Thick lines are estimates from Model 4, while thin lines are estimates from Model 1 (independent observation errors).



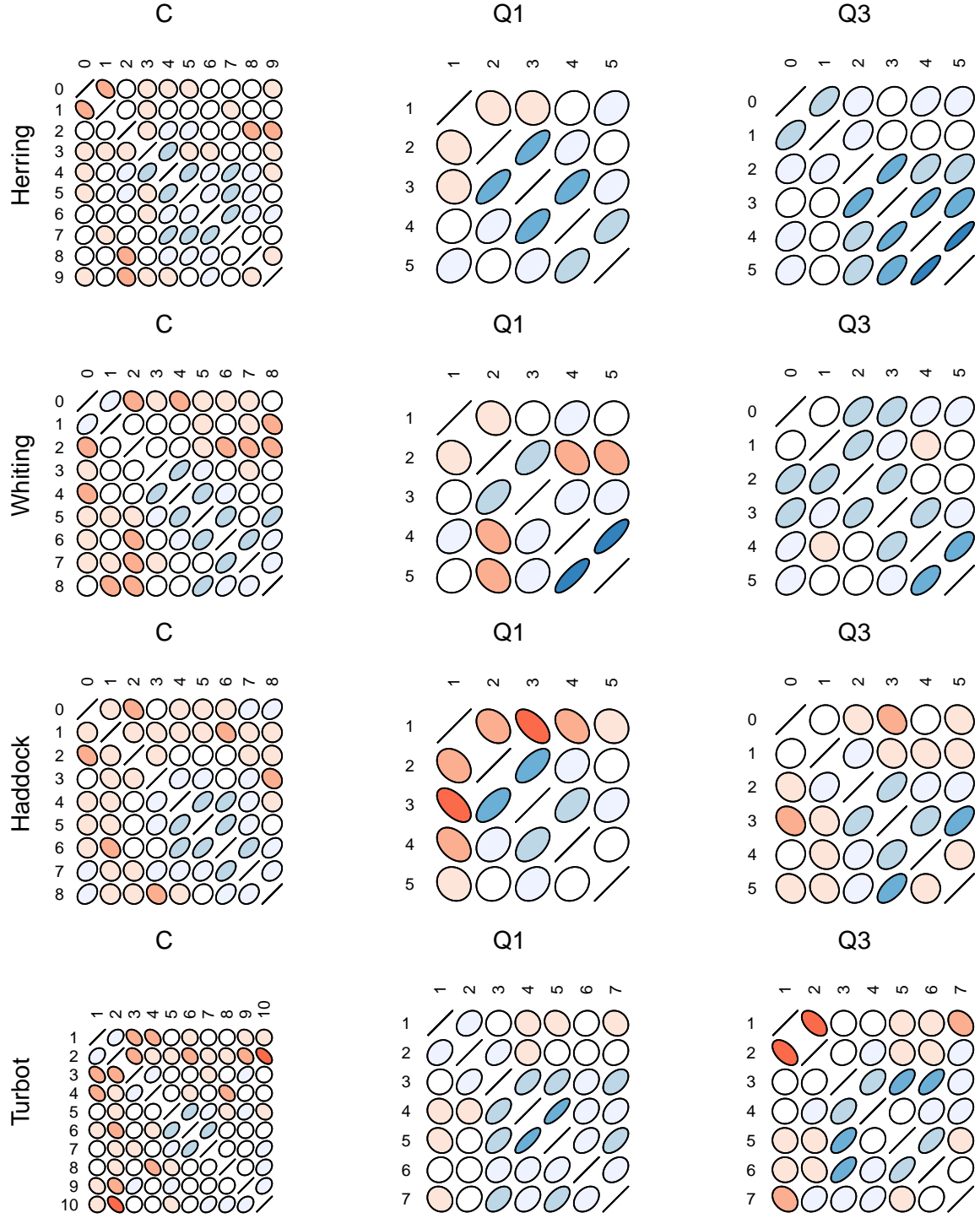


Figure 3: Model 1: correlation between ages of OSA residuals for herring, whiting, haddock, and turbot (from the top to bottom). The columns represent from left to right: commercial catches, survey 1, and survey 2.

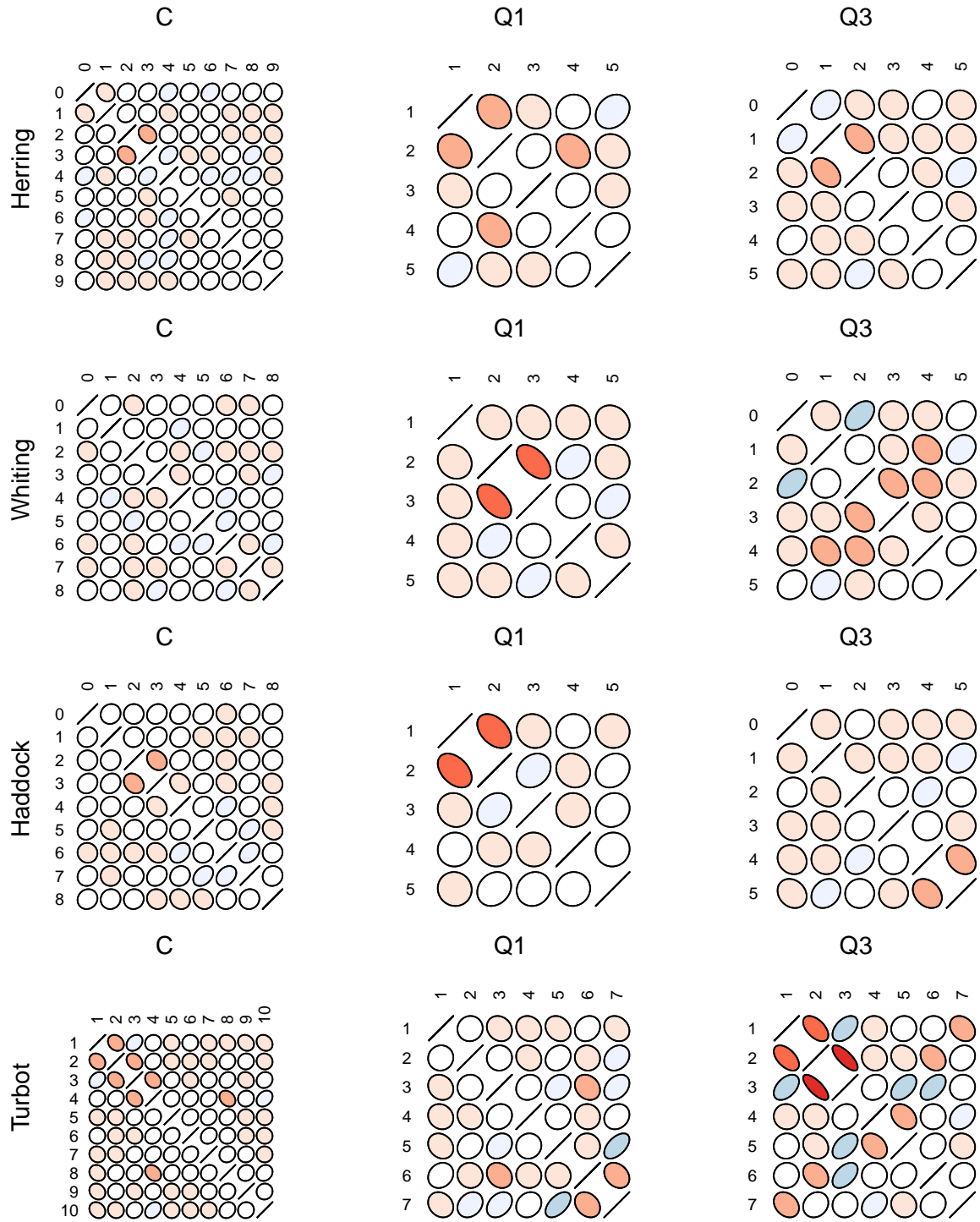


Figure 4: Final model: correlation between ages of OSA residuals for herring, whiting, haddock, and turbot (from the top to bottom). The columns represent from left to right: commercial catches, survey 1, and survey 2.

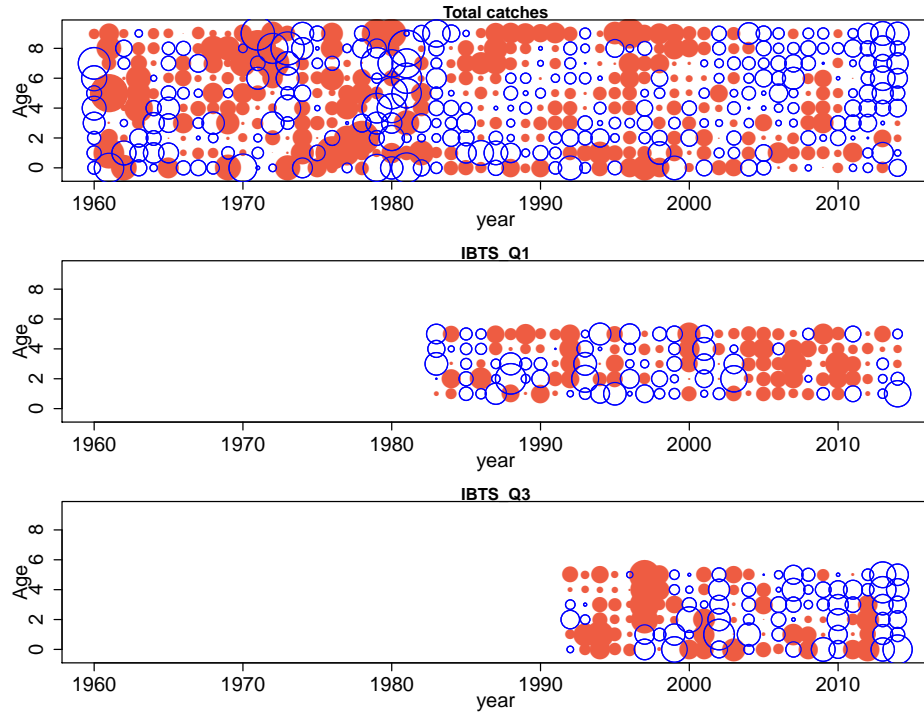


Figure 5: Herring: Normalized one-step ahead residuals for model 1 (independent observations).

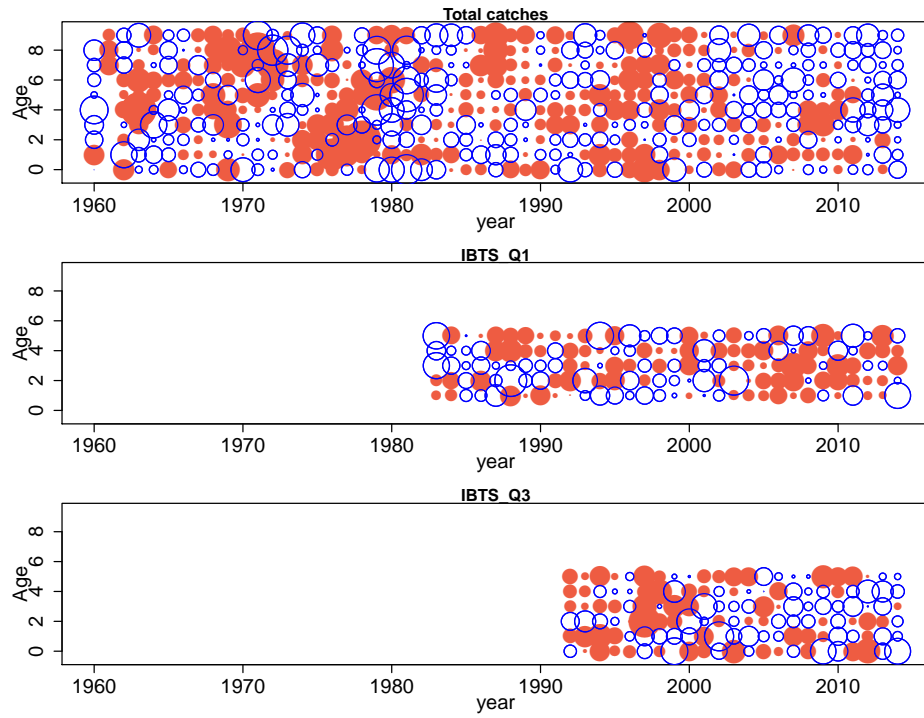


Figure 6: Herring: Normalized one-step ahead residuals for model 4 (final model).

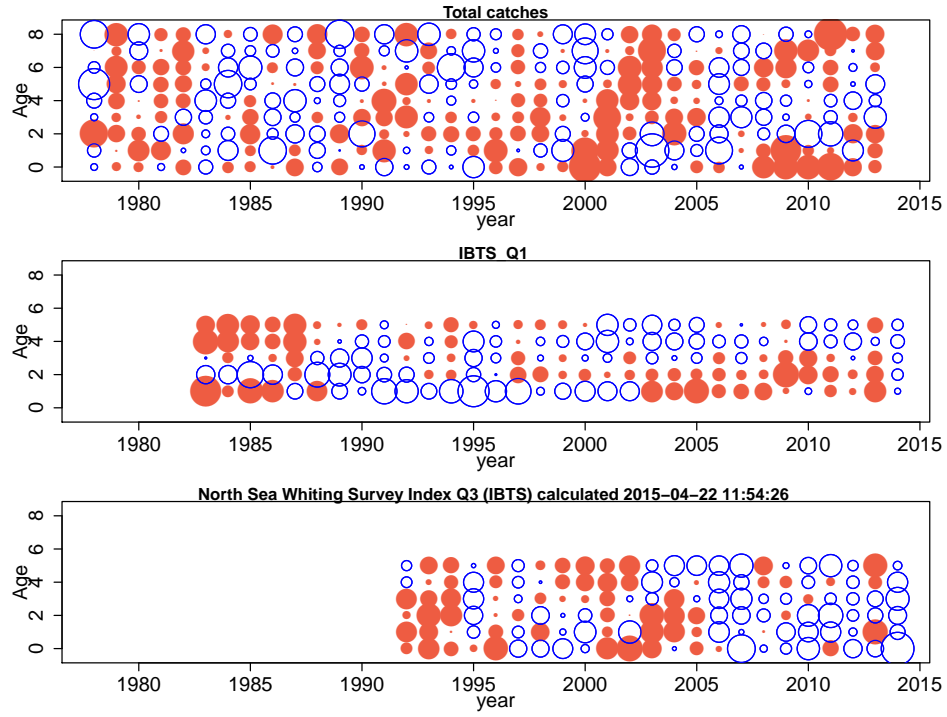


Figure 7: Whiting: Normalized one-step ahead residuals for model 1 (independent observations).

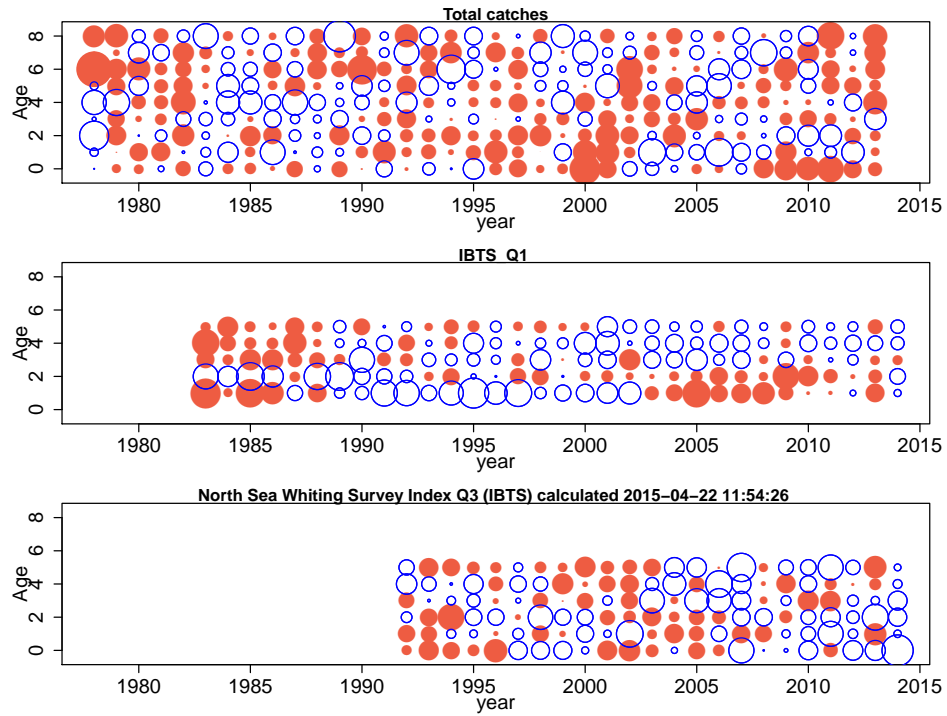


Figure 8: Whiting: Normalized one-step ahead residuals for model 4 (final model).

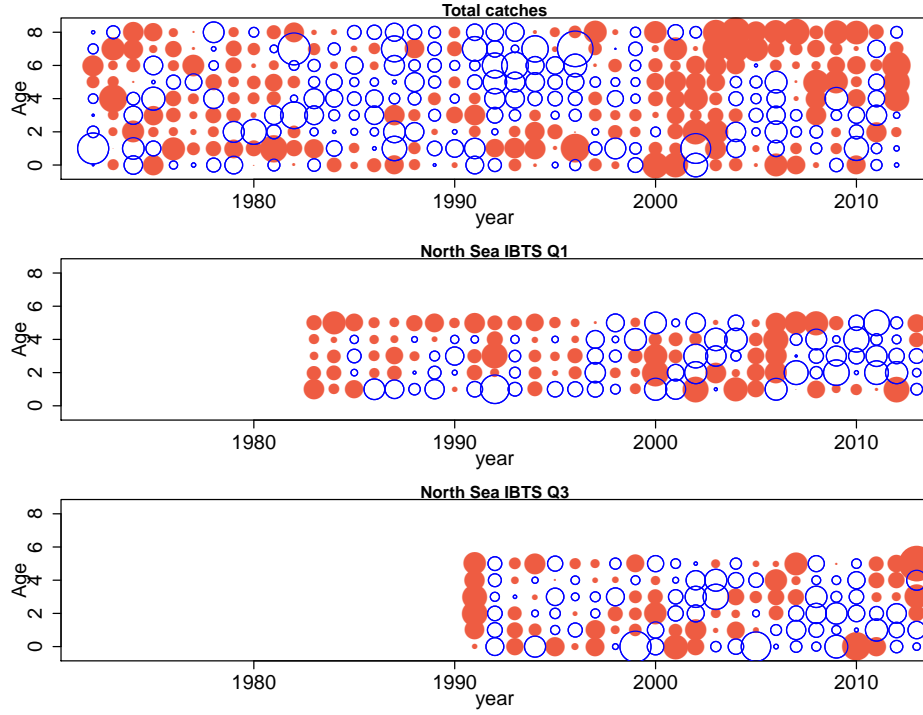


Figure 9: Haddock: Normalized one-step ahead residuals for model 1 (independent observations).

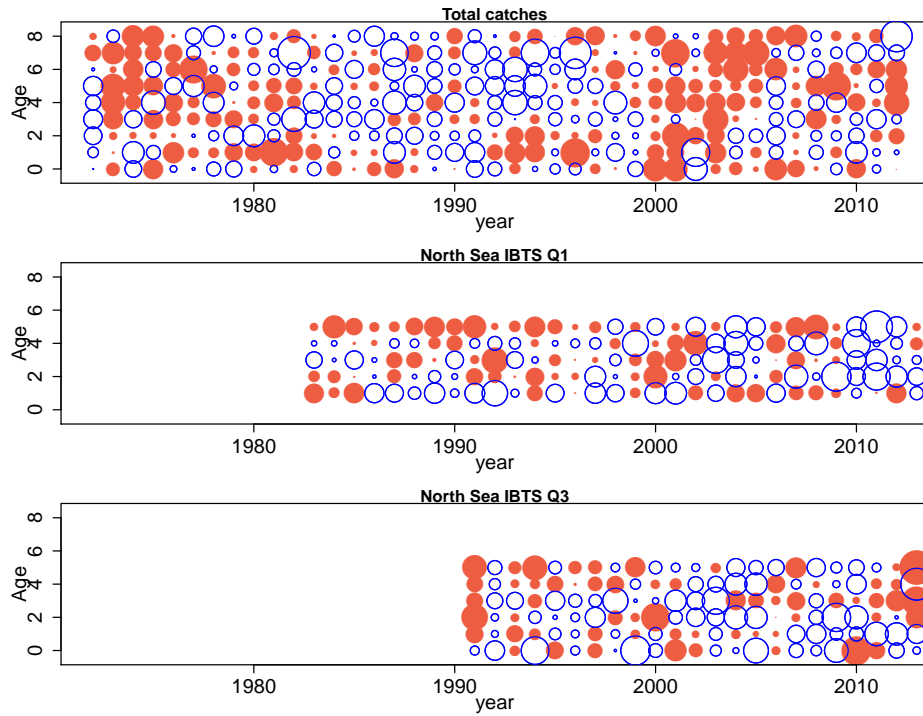


Figure 10: Haddock: Normalized one-step ahead residuals for model 4 (final model).

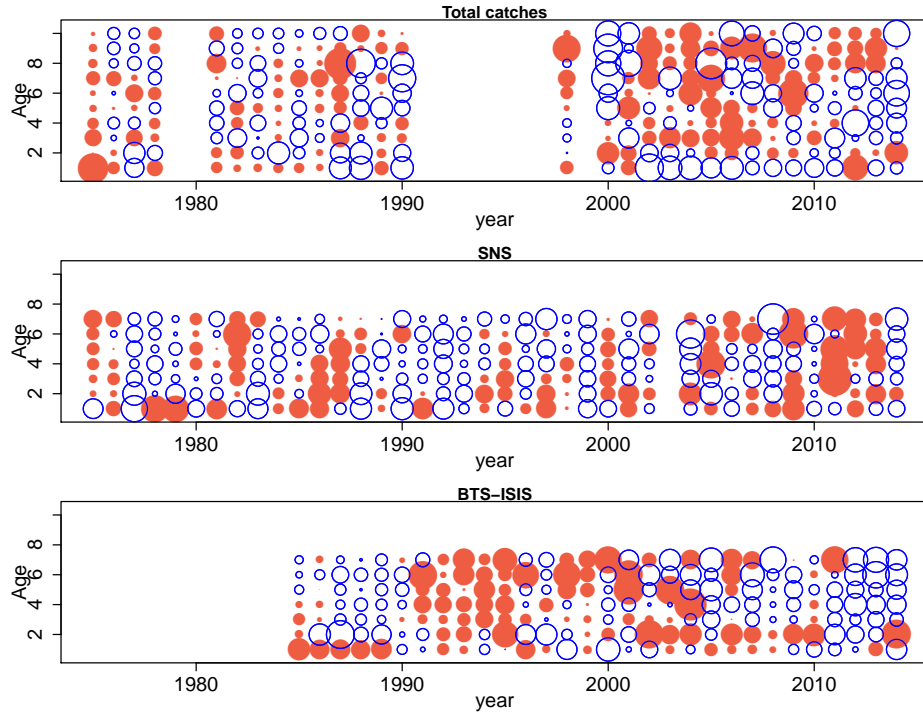


Figure 11: Turbot: Normalized one-step ahead residuals for model 1 (independent observations).

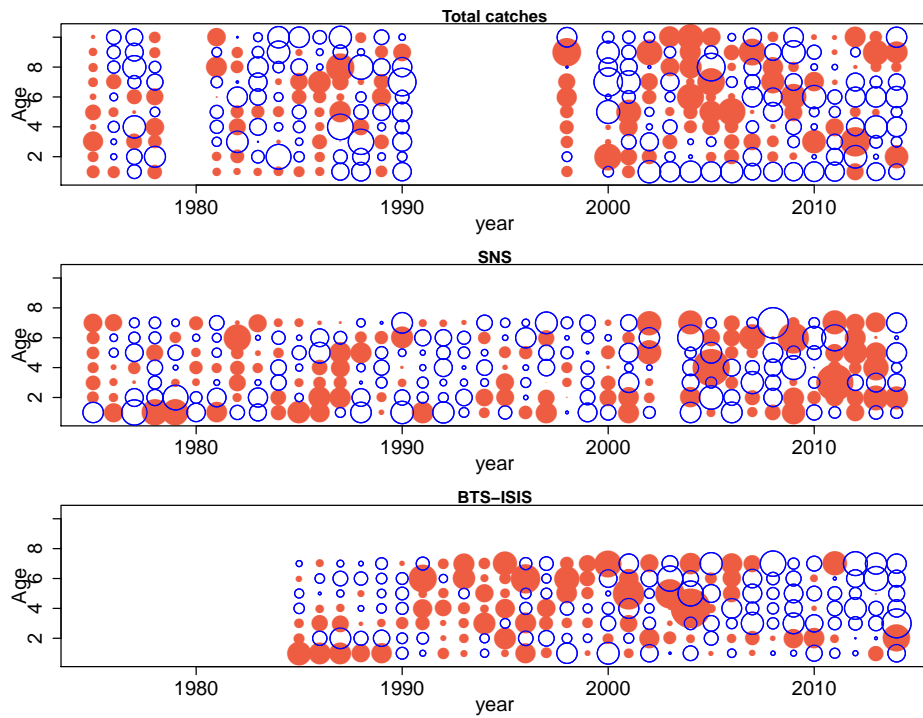


Figure 12: Turbot: Normalized one-step ahead residuals for model 4 (final model).

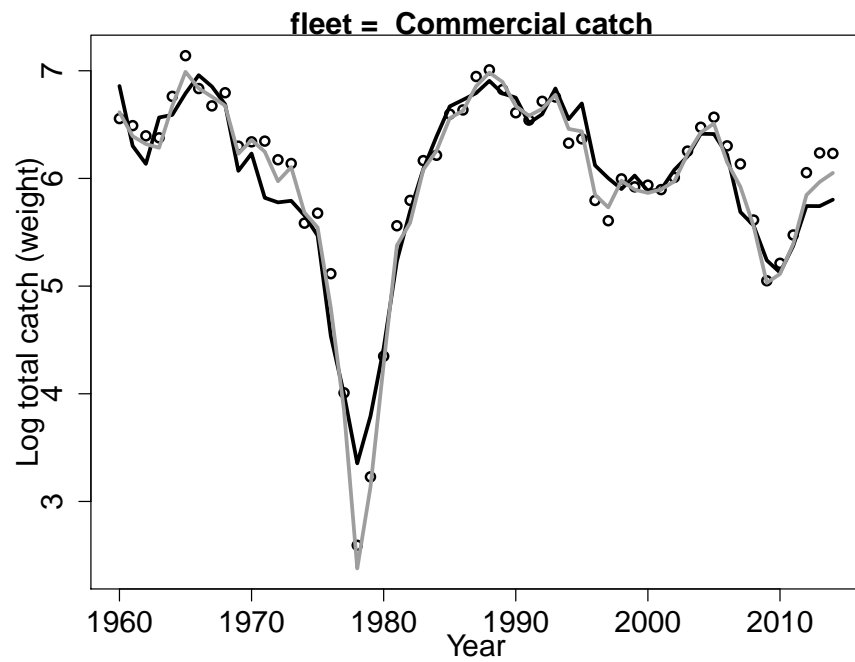


Figure 13: Herring: Observed and fitted total catch in weight. Circles are observed values, black lines are predictions from Model 4 and grey lines from Model 1.

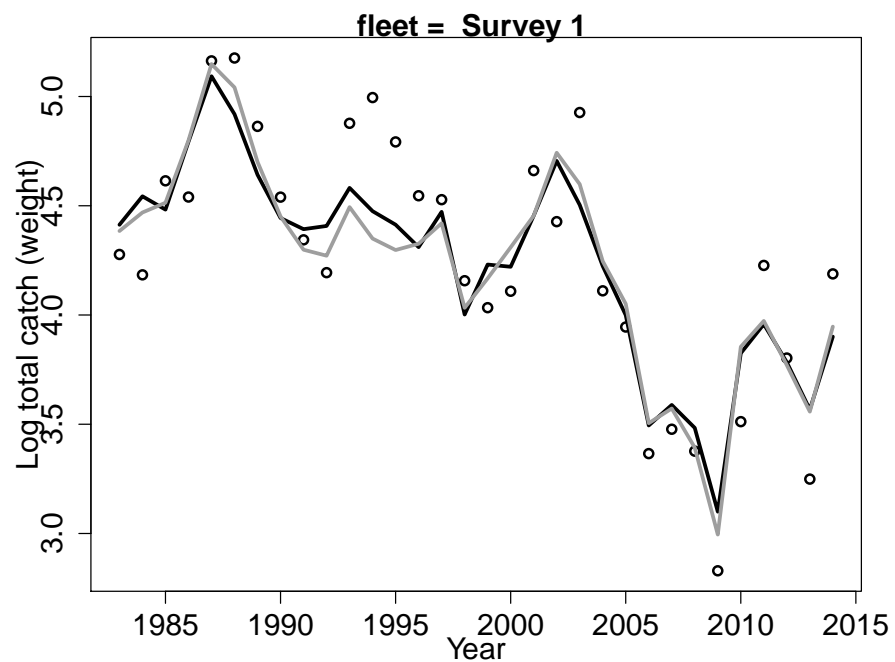


Figure 14: Herring: Observed and fitted total catch in weight. Circles are observed values, black lines are predictions from Model 4 and grey lines from Model 1.

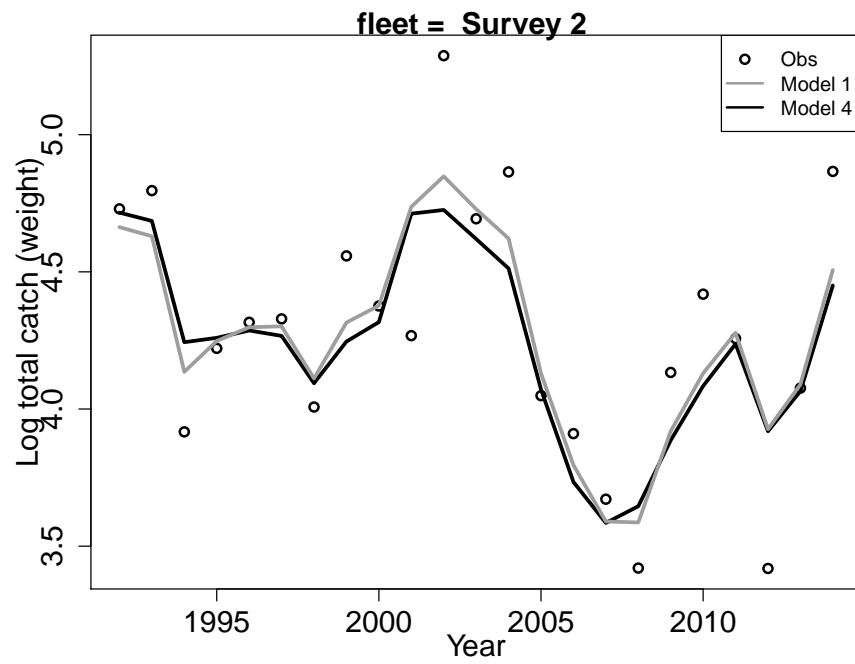


Figure 15: Herring: Observed and fitted total catch in weight. Circles are observed values, black lines are predictions from Model 4 and grey lines from Model 1.

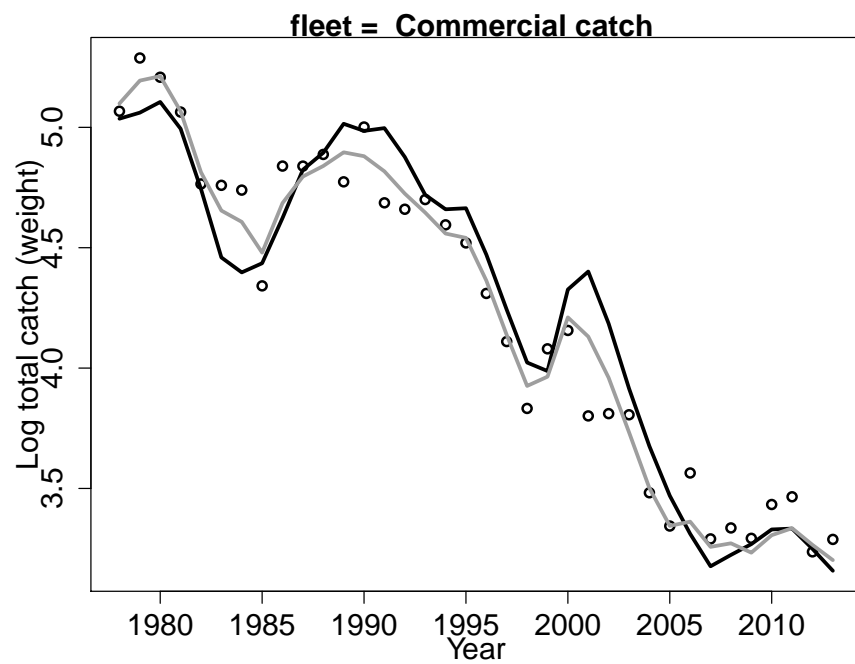


Figure 16: Whiting: Observed and fitted total catch in weight. Circles are observed values, black lines are predictions from Model 4 and grey lines from Model 1.



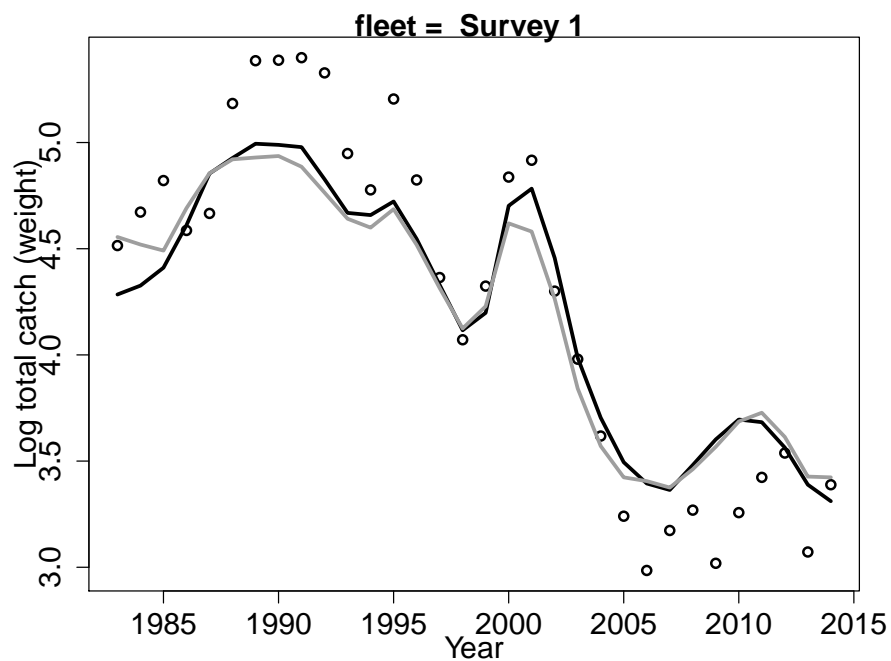


Figure 17: Whiting: Observed and fitted total catch in weight. Circles are observed values, black lines are predictions from Model 4 and grey lines from Model 1.

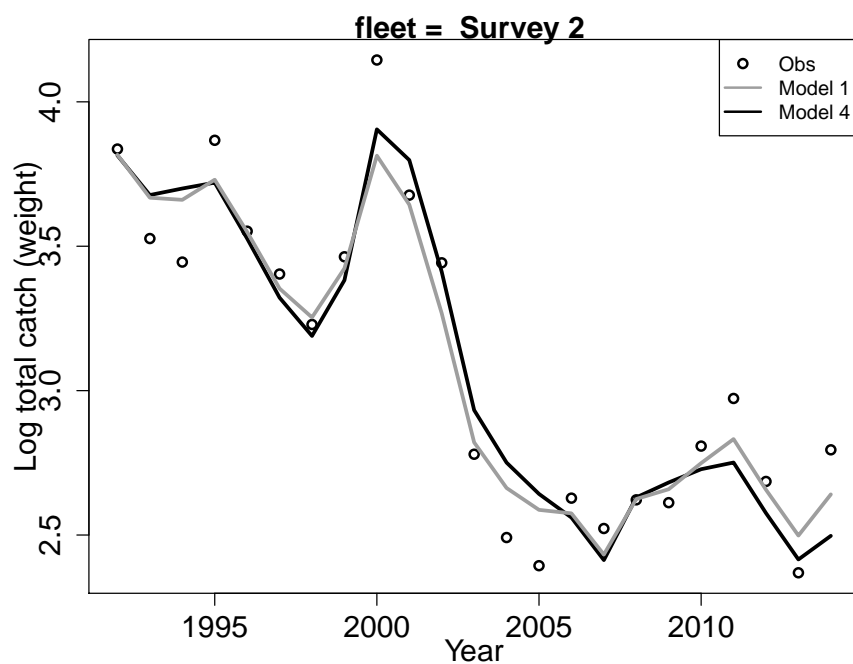


Figure 18: Whiting: Observed and fitted total catch in weight. Circles are observed values, black lines are predictions from Model 4 and grey lines from Model 1.

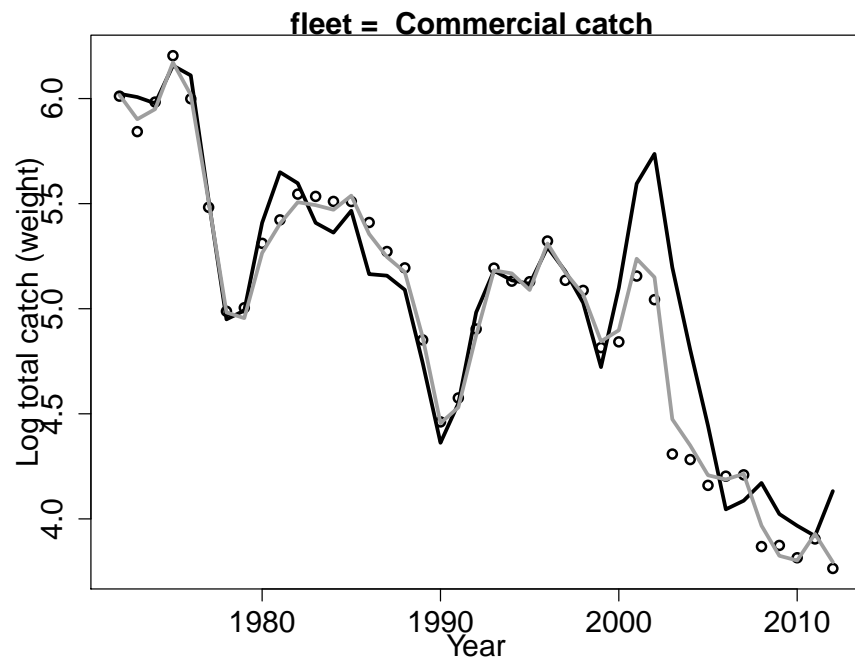


Figure 19: Haddock: Observed and fitted total catch in weight. Circles are observed values, black lines are predictions from Model 4 and grey lines from Model 1.

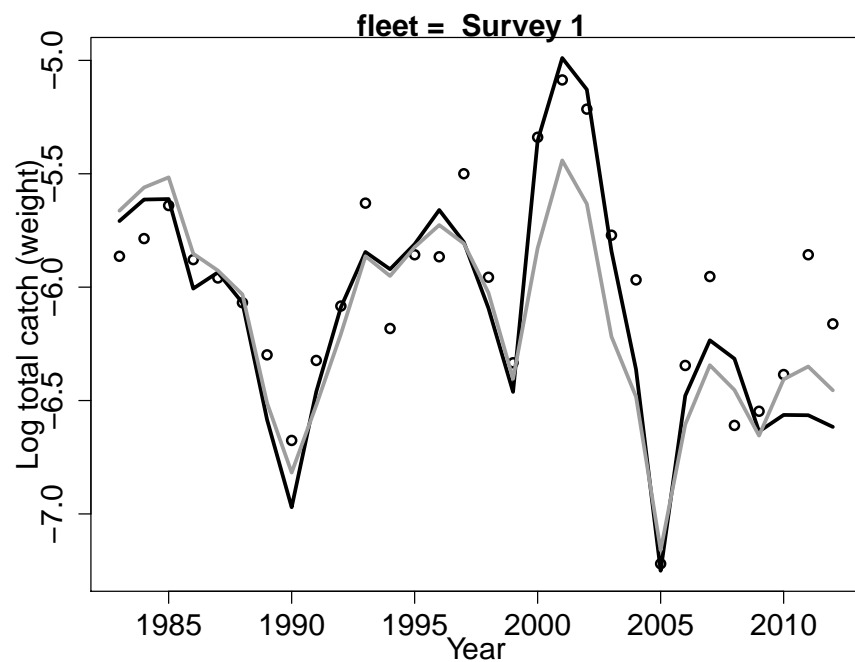


Figure 20: Haddock: Observed and fitted total catch in weight. Circles are observed values, black lines are predictions from Model 4 and grey lines from Model 1.

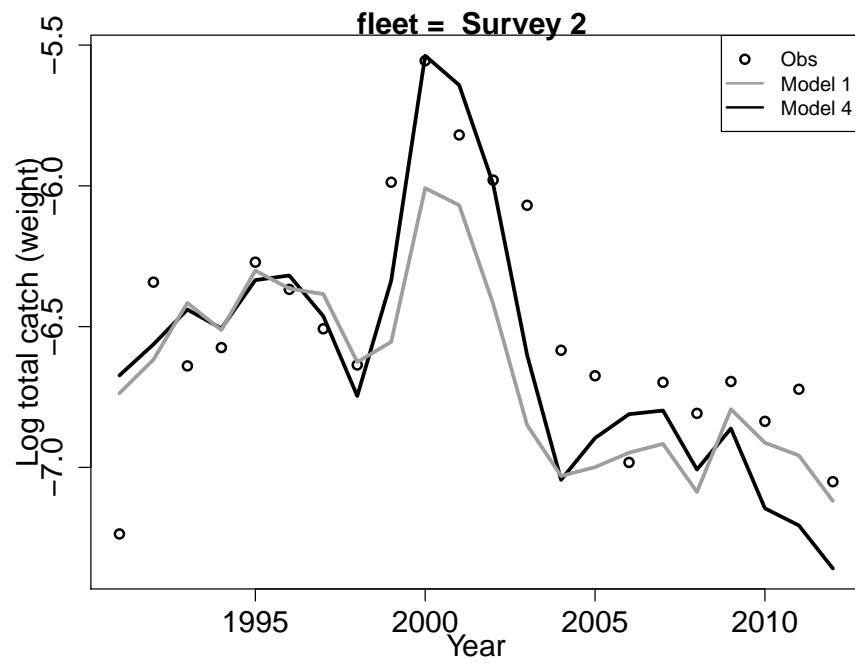


Figure 21: Haddock: Observed and fitted total catch in weight. Circles are observed values, black lines are predictions from Model 4 and grey lines from Model 1.

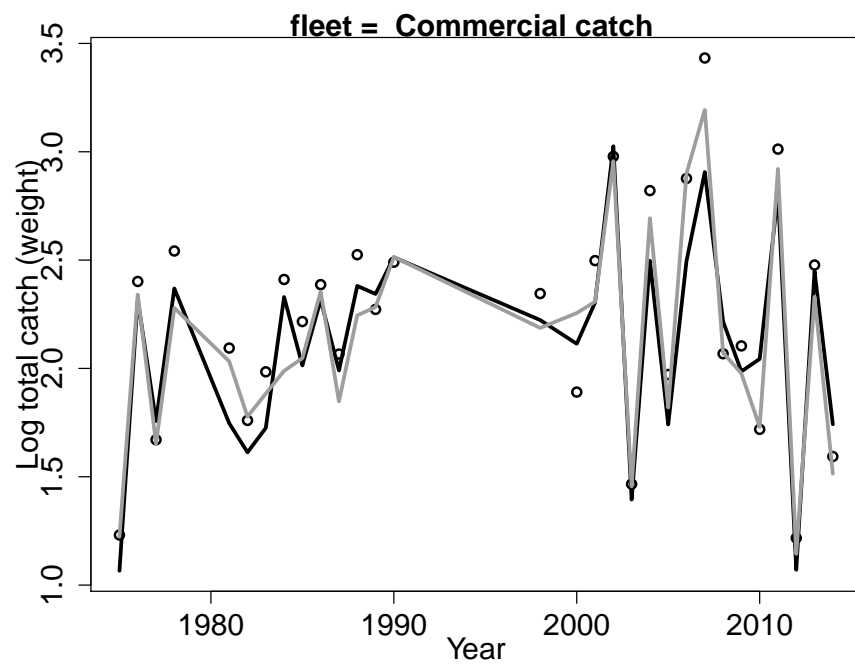


Figure 22: Turbot: Observed and fitted total catch in weight. Circles are observed values, black lines are predictions from Model 4 and grey lines from Model 1.

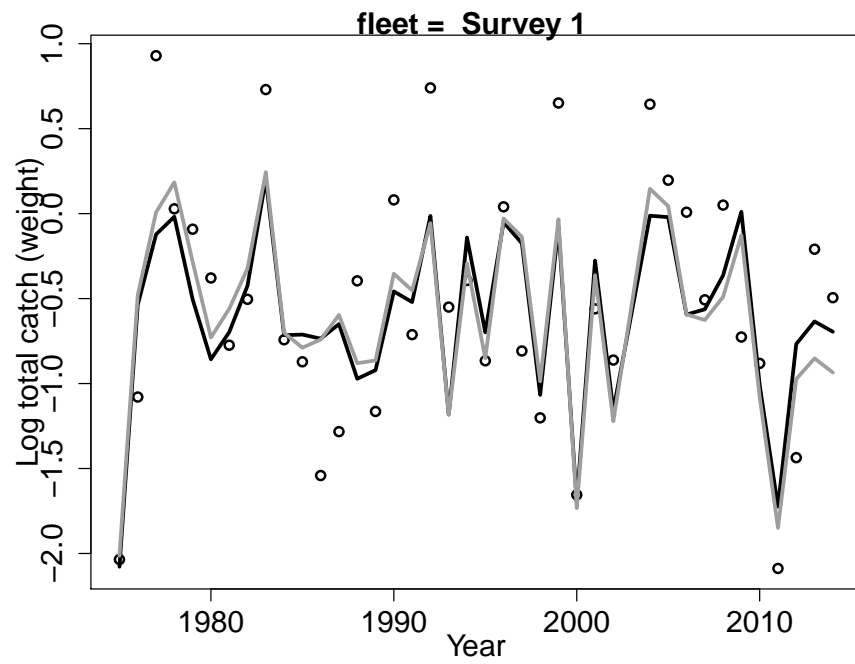


Figure 23: Turbot: Observed and fitted total catch in weight. Circles are observed values, black lines are predictions from Model 4 and grey lines from Model 1.

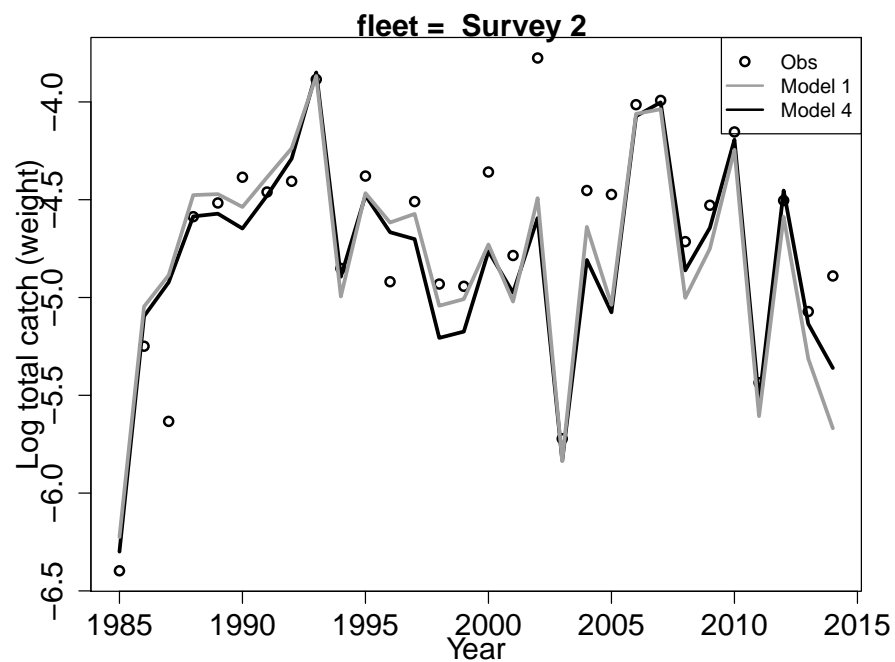


Figure 24: Turbot: Observed and fitted total catch in weight. Circles are observed values, black lines are predictions from Model 4 and grey lines from Model 1.