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Author(s): Gudmundur Gudmundsson
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Time Series Analysis of Catch-at-age Observations

By GUDMUNDUR GUDMUNDSSON†

Central Bank of Iceland, Reykjavik, Iceland

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SUMMARY

Catches, stocks and fishing mortality rates are connected by given non-linear relationships. Multivariate time series models of fishing mortality rates are proposed and linear approximations to the Kalman filter algorithm are used to estimate the unobserved series of stocks and fishing mortality rates from observed series of catches at each age. The model parameters are estimated by the likelihood function of the catch prediction errors. Time series estimation does not require fishing effort measurements as well as the catch-at-age data, but it is also possible to include auxiliary information. Estimates of stocks and fishing mortality rates with acceptable accuracy can be obtained from exceptionally short series. They are not unduly sensitive to misspecifications and inaccurate estimation of model parameters.

Keywords: Catch at age; Fish stock assessment; Kalman filter; State-dependent models; Structural time series analysis

1. Introduction

Contrary to what readers of statistical journals might conclude, recaptures of tagged fish are not the main source of information about the size of most fish stocks. That is provided by catch-at-age observations. An example of such data is presented in Table 1.

For large and valuable stocks the collection of catch-at-age data is a substantial enterprise. The number of fish collected for age determination is often of the order of 1000 from each age and year. Much larger numbers of length measurements can be obtained at relatively low cost. Detailed reports about the weights of the catches from each vessel are collected throughout the year. There are seasonal and geographical variations in the age distribution and also in the relationship between age and length. Errors in physical measurements, deviations from random sampling and misreporting of catches produce errors in the catch-at-age data. The relative importance of these factors is widely different between stocks.

Stock numbers and fishing mortality rates are connected with the observed values by the following relationships:

$$C(a,t) = \{F(a,t)/Z(a,t)\}[1-\exp\{-Z(a,t)\}]\,N(a,t) + \epsilon(a,t) \qquad (1)$$

and

†*Address for correspondence*: Central Bank of Iceland, 150 Reykjavik, Iceland.

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TABLE 1
Catches of cod in Icelandic waters†

Year	Catches (million fish) for the following ages in years: 4	5	6	7	8	9	10
1977	42.66	32.46	12.16	13.02	2.81	1.77	0.42
1978	16.29	43.93	17.63	8.73	4.12	0.98	0.35
1979	28.43	13.77	34.44	14.13	4.43	1.43	0.35
1980	28.53	32.50	15.12	27.09	7.85	2.23	0.65
1981	13.30	39.19	23.35	12.71	26.45	4.80	1.67
1982	20.81	24.46	28.35	14.01	7.67	11.52	1.91
1983	10.91	24.30	18.94	17.38	8.38	2.05	2.73
1984	31.55	19.42	15.33	8.08	7.34	2.68	0.51
1985	24.55	35.39	18.27	8.71	4.20	2.26	1.06
1986	20.33	26.64	30.84	11.41	4.44	1.77	0.80
1987	62.13	27.19	15.13	15.69	4.16	1.46	0.59
1988	39.32	55.89	18.66	6.40	5.88	1.35	0.45
1989	28.04	50.16	31.52	6.02	1.92	0.88	0.22
1990	12.29	27.13	44.45	17.00	2.57	0.61	0.32

†From Hafrannsóknastofnun (1991).

$$N(a+1,\ t+1) \;=\; \exp\{-Z(a,t)\}N(a,t), \tag{2}$$

where a is the age in years, $a=1, 2, \ldots, A$ (counting from the first age included), t is the time in years, $t=1, 2, \ldots, T$ (counting from the first year included), $C(a,t)$ is the catch in numbers of age a in year t, $N(a,t)$ is the stock of age a at the beginning of year t, $F(a,t)$ is the annual rate of fishing mortality, $Z(a,t)$ is the total mortality rate (equal to $F(a,t)\,+\,M(a,t)$), $M(a,t)$ is the annual rate of natural mortality and $\epsilon(a,t)$ are measurement errors.

The fishing mortality rates obviously depend on fishing effort. But variations in catchability, which is affected by changes in conditions in the sea, can be an important source of randomness in this subject.

Annual recruitment is highly variable. The relationship between stock size and recruitment is usually weak and will not be considered here at all.

In all methods of assessment discussed in this paper the rate of natural mortality, $M(a,t)$, is supposed to be known. Variations in $M(a,t)$ with time are rarely considered and usually the same constant value is assumed for most ages.

The most widely applied method for estimating the stocks and fishing mortality rates from these data is called virtual population analysis (VPA), introduced by Gulland (1965). For any given set of positive fishing mortality rates for the last catch from each cohort, $F(A,t)$ and $F(a,T)$, a solution can be calculated which fits the observed catches at each age exactly. One such solution is selected in accordance with auxiliary information, usually an effort index or catch per unit effort data. A survey of VPA algorithms has been presented by Pope and Shepherd (1985).

The other main approach to catch-at-age analysis has been to fit the separable model

$$\log F(a,t) \;=\; U(a) \;+\; V(t) \tag{3}$$

to the data by weighted least squares or related procedures. One stock value from

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each cohort is estimated as a parameter and the remaining values are calculated by equation (2). The variations in $U(a)$ are often constrained by assuming that they follow some simple function or are constant above a given age below A. If this model is fitted without constraints on $V(t)$ the estimated values of $F(a, t)$ in the last year are so inaccurate that they are of no practical use. In serious applications constraints based on effort data are therefore introduced. (See for example Deriso *et al.* (1985).)

The argument for the separable model is plausible; $U(a)$ is determined by the technology employed in fishing and $V(t)$ by the effort. But technology is not constant and many important stocks are exploited by more than one fleet, using fishing gear with different selectivity and operating on fishing grounds with different age compositions. Changes in the relative proportion caught by each fleet upset the separability. The problem can sometimes be avoided by working with catch-at-age data disaggregated by fleets, but this entails problems with the appropriate weighting of fleet data of different accuracy.

An estimation of the separable model assigns all randomness to the measurement errors, which is incorrect unless variations in catchability are negligible (or follow the separability assumption). VPA, in contrast, could capture any kind of variability in fishing mortality rates but is vulnerable to measurement errors as it follows the observed catches at age exactly.

For practical purposes the estimates of stocks and fishing mortality rates in the last year are most important. With both methods sketched above they are much less accurate than the estimates obtained for earlier years. The true values are unknown, but comparisons of estimates from the whole data set with estimates ending in year $T-k$, $k=1, 2, \ldots$, provide useful information about the quality of the estimates in the last year of observations. Errors of the order of 0.5 on a natural logarithm scale seem to be common (Working Group on Methods of Fish Stock Assessments, 1991).

Both VPA and least squares estimation of the separable model rely heavily on effort indices. But it is difficult to measure the effect of technological development and changes in regulations. Errors in effort measurements have been a major source of errors in fish stock assessment.

In the present paper fishing mortality rates are regarded as unobserved time series, modelled as a multivariate random walk process. In this way irregular variations in catchability are modelled separately from measurement errors. We apply structural time series analysis (Harvey, 1989), with state-dependent parameters (Priestley, 1980), to estimate the unobserved series of stocks and fishing mortality rates from the observed catches according to the given non-linear relationships (1) and (2). With the criteria applied in the time series analysis useful estimates can be obtained without auxiliary information. The accuracy appears to be comparable with the best VPA algorithms or least squares methods (Working Group on Methods of Fish Stock Assessments, 1988, 1989, 1991). The time series analysis can also be extended to include the auxiliary data required by the other methods.

2. Time Series Models

Selection of models is constrained by the fact that the series from which they must be estimated are very short. Experimentation with alternative formulations is also

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hampered by the non-linear structure which implies that most changes require extensive programming. Our present programs (Gudmundsson, 1991) can estimate the following model.

The measurement errors $\epsilon(a, t)$ are serially uncorrelated, $N\{0;\ \sigma^2 \mathbf{Q}(t)\}$. The covariance matrix must be largely specified *a priori* and we use

$$\operatorname{var}\{\epsilon(a, t)\} = \{\sigma B(a)\, k_{\mathrm{a}}(a)\, k_{\mathrm{c}}(t-a)\}^2.$$

The values of $k_{\mathrm{a}}(a)$ and $k_{\mathrm{c}}(t-a)$ are obtained by a least squares fit of

$$\log C(a, t) = \log k_{\mathrm{a}}(a) + \log k_{\mathrm{c}}(t-a)$$

so that the standard deviation of the measurement error is proportional to factors corresponding to age and to cohort. The values of $B(a)$ and the correlation between measurement errors at different ages are predetermined. This method is closely related to working with $\log C(a, t)$. However, we had more trouble with outliers in the logarithmic versions when we used the least squares methodology (Gudmundsson, 1986).

Our model of $F(a, t)$ adds random disturbances to the separable model (3) and allows the selectivity to change with time:

$$\log F(a, t) = U(a, t) + V(t) + \delta_1(a, t), \tag{4}$$

$$U(a, t) = U(a, t-1) + \delta_2(a, t), \qquad \text{for } a \leqslant a_{\mathrm{m}} < A, \tag{5}$$

$$U(a, t) = U(a_{\mathrm{m}}, t) \qquad \text{for } a > a_{\mathrm{m}},$$

with the constraint that

$$\sum_{a=1}^{a_{\mathrm{m}}} U(a, t) = 0; \tag{6}$$

$$V(t) = Y(t) + \delta_3(t), \tag{7}$$

$$Y(t) = Y(t-1) + \Theta_1 + \delta_4(t). \tag{8}$$

The residuals δ_i are mutually independent, normally distributed with mean 0 and serially uncorrelated. The covariance matrices of δ_1 and δ_2 are $\sigma^2 h_1^2 \mathbf{H}_1$ and $\sigma^2 h_2^2 \mathbf{H}_2$ where $\mathbf{H}_1$ and $\mathbf{H}_2$ are matrices. The variances of δ_3 and δ_4 are $\sigma^2 h_3^2$ and $\sigma^2 h_4^2$.

Variations in $\log F(a, t)$ produced by δ_1 and δ_3 do not affect subsequent values, but δ_2 and δ_4 produce permanent variations. Considering actual causes of variations in fishing mortality rates, the weather and irregular variations in conditions in the sea are mainly transitory, but technological development and changes in fleet size induce more permanent variations.

By estimation with this model the restrictions of separability and proportionality between effort and fishing mortality rates in earlier methods are replaced by assuming that the first differences of $\log F(a, t)$ follow a multivariate moving average (MA(1)) model.

The matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ are predetermined as well as $\mathbf{Q}(t)$. The simplest specification of $\mathbf{Q}(t)$ is that the measurement errors are uncorrelated and all $B(a) = 1$. $\mathbf{H}_1$ and $\mathbf{H}_2$ are most simply defined as unit matrices.

Arbitrary assumptions about covariances of residuals are inevitable in any statistical estimation of $F(a, t)$ and $N(a, t)$ from these data, as indeed in most applied

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time series analyses involving more than two or three series. It is easier to introduce such prior knowledge as we may have about the fishing and data collection when random variations in fishing mortality rates are defined separately from the measurement errors. We can often do better than to let all non-diagonal elements of the residual correlations be 0 by inserting some positive correlations in $\mathbf{H}_1$ and negative correlations in $\mathbf{Q}(t)$. But we know little about the magnitudes and fortunately the results are not sensitive to this.

3. Estimation

Parameters in a structural time series model are usually estimated by maximizing the likelihood function of prediction errors one step ahead. $N(a,t)$, $\log F(a,t)$, $U(a,t)$, $V(t)$ and $Y(t)$ are state variables. They are predicted by equations (2) and (4)–(8) and inserted in equation (1) to predict the catches. The state variables are then updated in accordance with the difference between observed and predicted catches before proceeding to the next year. For linear models the Kalman filter provides the updating which leads to the lowest mean-square prediction errors. Details of all these calculations for linear models are presented by Harvey (1989).

Equations (1) and (2) are not linear, but we use linear approximations to estimate the covariance matrices required by the Kalman filter:

$$\begin{aligned}\Delta N(a+1,\ t+1) \;=\; & \exp\{-Z(a,t)\}\Delta N(a,t) \\ & - F(a,t)N(a,t)\exp\{-Z(a,t)\}\Delta\{\log F(a,t)\},\end{aligned}$$

where $\Delta N(a+1,\ t+1)$ is the prediction error and $\Delta N(a,t)$ and $\Delta\{\log F(a,t)\}$ are the errors of the corresponding updated estimates. The coefficients are functions of the values of F and N in year t, i.e. they are state dependent. They are calculated by the updated estimates. The catch prediction errors are treated similarly, but there the coefficients are calculated from predicted state variables before updating.

The proportionality factor σ^2 in the variance of all the residuals is concentrated out of the likelihood function, but the parameters from the time series models, h_1, h_2, h_3, h_4 and Θ_1, are estimated.

The values of $N(1,t)$ cannot be predicted from previous stocks by equation (2). The simplest prediction of $N(1,t)$ is a constant value, i.e.

$$N(1,t) \;=\; N_0 \;+\; \delta_5(t) \tag{9}$$

where $\delta_5(t)$ are independent and identically distributed $N(0;\sigma^2 h_5^2)$.

The Kalman filter needs initial values of the state variables. In estimation of linear models with constant coefficients the calculations can be started with arbitrary values, provided that the associated variances are assumed sufficiently high. But our coefficients depend on the values of the state variables. We must therefore start with sensible values to ensure that the procedure has approximately the properties of a Kalman filter in linear models. For this we represent $F(a,1)$ by a function of a few parameters which are estimated from the likelihood function ($\log F(1,1)$ is estimated separately, $\log F(2,1)$–$\log F(a_{\mathrm{m}},1)$ by a low order polynomial and $\log F(a,1) = \log F(a_{\mathrm{m}},1)$ for $a > a_{\mathrm{m}}$). The initial values of U and V are calculated in accordance with $F(a,1)$ and the stocks at the beginning of the first

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year are determined so that they correspond exactly to the catches with these fishing mortality rates.

Let us now pretend that the values of the state variables were obtained from estimates of previous values with given rather high variances. We proceed to calculate the covariance matrix of the 'updated' state variables in the first year although no updating would in fact take place because the $N(a, 1)$ were chosen to fit the observed catches exactly. The procedure is repeated once more, but with the covariance matrix of the updated state variables in the first year as an estimate of the covariances of the non-existent estimates in year 0. The likelihood function of the catch prediction errors is calculated for the prediction errors from the second to the last year.

It is sometimes useful to add regressors to the right-hand side of the models of the state variables. Thus, if a change in selectivity is effected in a given year by regulation, a suitable function of age can be added to $U(a, t)$ (Gudmundsson, 1987a). Recruitment indices are often available and can be included as regressors in equation (9). For the Icelandic cod in Table 1 we estimate additions to the stocks by immigration from Greenland of 8-year-old cod in 1981 and 6-year-old cod in 1990. (The age and year must be known *a priori*.)

The various residuals defined in our models are unobservable, but calculation of standardized catch prediction errors is a valuable aid in the specification. Estimation with the simple specifications of $\mathbf{Q}$ and $\mathbf{H}_1$ usually produces abnormally large standardized residuals for the oldest and youngest fish. Smaller samples provide an obvious explanation of high relative errors in the older ages and this is adjusted for by higher values of $B(a)$ in these ages. But the numbers caught of the youngest fish are often large. Variations in size and catchability are an important cause of irregular catches of the smallest fish which implies that the first diagonal element in $\mathbf{H}_1$ should be increased.

In statistical analysis of catch-at-age data it is necessary to have ways of reducing the influence of outliers. With the Kalman filter this is effected by increasing the variance of the corresponding catch prediction error.

The calculations sketched above provide an approximation to the mean-squared error estimates of the state variables at time t, given the observations up to and including that time. Only in the last year are updated estimates of stocks and fishing mortality rates based on all information in the data. Final estimates in earlier years are obtained by a recursive procedure, called 'smoothing', starting from the estimates in the last year (Harvey, 1989).

The Kalman filter and smoothing algorithm provide covariance matrices of the estimated state variables in each year, but these are calculated with state-dependent coefficients and conditional on the estimated parameters. I have no analytical examination to offer of the quality of these values, but they are quite useful in practice. The estimates of stocks and fishing mortality rates are unreliable, especially in the last year, and the Kalman filter shows this. It is not of crucial importance whether the actual standard deviations are a few per cent higher or lower than the calculations suggest.

In the least squares methods the variations associated with δ_1 and δ_2 in our models are assumed to be 0. In VPA methods measurement errors are, implicitly or explicitly, assumed to be negligible. I have analysed about 15 sets of catch-at-age data from various parts of the world with the present method. The assumption of

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no permanent variations in selectivity is often, but not always, consistent with the data. Estimates of h_1 are rather inaccurate, but the data neither support the assumption that it is usually practically 0 nor extremely large. The uncertainty about h_1 has little effect on the estimated values of stocks and fishing mortality rates or the standard deviations in the last year. But the estimation of the accuracy of the smoothed values is sensitive to the value of h_1.

4. Example

An analysis of the data in Table 1 was carried out, assuming a constant rate of natural mortality of 0.2 year^{-1}. After adjusting the covariance matrices of the residuals for the youngest and oldest fish and reducing the effect of the large catch of 10-year-old cod in 1981 the following time series parameters were obtained:

$$h_1,\ 2.05\ (0.51);\quad h_2,\ 0.01\ (0.47);\quad h_3,\ 0.03\ (1.01);\quad h_4,\ 3.32\ (1.06);$$
$$\Theta_1,\ 0.042\ (0.039);\quad \sigma = 0.047.$$

Standard deviations, calculated by the Hessian matrix, are presented in parentheses after the corresponding parameter although they are hardly very precise indicators of the accuracy.

A low and insignificant value of h_2 indicates that permanent variations in selectivity were negligible.

The value of Θ_1 is insignificant but indicates a large increase in fishing mortality rates from 1977 to 1990. When Θ_1 is left out the log-likelihood decreases by 0.50 and the parameters are as follows:

$$h_1,\ 1.69\ (1.02);\quad h_2,\ 0.00\ (0.30);\quad h_3,\ 0.02\ (0.98);$$
$$h_4,\ 2.93\ (1.46);\quad \sigma = 0.055.$$

The series are too short for calculation of serial correlations at each age. The estimated first-order correlation for all ages is 0.02 and the cross-correlation

TABLE 2
Stock of cod†

Year	*Stock of cod (million fish) for the following ages in years:* 4	5	6	7	8	9	10
1977	292.92 (9.35)	120.06 (3.99)	43.36 (2.11)	25.47 (0.96)	5.65 (0.35)	3.27 (0.23)	0.79 (0.13)
1978	118.73 (3.16)	201.38 (7.08)	69.36 (2.36)	22.39 (1.03)	9.65 (0.51)	2.07 (0.19)	1.12 (0.14)
1979	175.33 (5.81)	82.16 (2.30)	125.04 (4.69)	39.59 (1.50)	11.03 (0.64)	3.92 (0.36)	0.88 (0.12)
1980	193.28 (5.66)	118.27 (3.42)	52.64 (1.51)	71.74 (2.77)	20.03 (0.85)	5.06 (0.40)	1.81 (0.21)
1981	114.28 (2.96)	131.74 (3.54)	69.20 (2.01)	28.94 (0.81)	52.17 (1.54)	9.01 (0.52)	2.20 (0.23)
1982	112.77 (3.06)	80.09 (2.17)	72.37 (2.19)	34.20 (1.23)	12.45 (0.50)	18.70 (1.19)	3.20 (0.25)
1983	108.59 (2.61)	73.27 (2.07)	42.64 (1.34)	33.50 (1.31)	14.64 (0.78)	3.33 (0.35)	4.94 (0.83)
1984	185.87 (4.62)	77.75 (1.92)	38.62 (1.31)	18.50 (0.76)	12.65 (0.77)	4.92 (0.48)	1.05 (0.20)
1985	103.87 (2.45)	121.29 (2.72)	45.41 (1.19)	18.96 (0.74)	8.12 (0.42)	4.34 (0.42)	1.79 (0.23)
1986	110.55 (3.05)	63.08 (1.64)	66.94 (1.69)	20.78 (0.63)	7.7 (0.39)	2.87 (0.23)	1.46 (0.21)
1987	265.78 (11.30)	72.05 (2.19)	28.31 (1.06)	26.49 (0.96)	6.74 (0.36)	2.26 (0.23)	0.82 (0.12)
1988	204.91 (13.39)	148.68 (6.20)	34.80 (1.29)	11.01 (0.56)	8.46 (0.46)	1.88 (0.18)	0.60 (0.09)
1989	124.79 (12.83)	132.20 (10.30)	70.74 (4.22)	12.04 (0.75)	3.24 (0.30)	1.75 (0.26)	0.40 (0.08)
1990	98.96 (31.19)	76.58 (9.81)	95.60 (7.66)	28.81 (2.53)	4.07 (0.37)	0.87 (0.12)	0.47 (0.08)

†Standard deviations are given in parentheses.

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between residuals at (a, t) and $(a-1,\ t-1)$ is 0.22.

The smoothed values of stocks and fishing mortality rates estimated by the Kalman filter with these parameters are presented in Tables 2 and 3. Immigration of 8-year-old cod in 1981 was estimated at 17.3 million fish and 31.3 million fish 6 years old in 1990 with standard deviations 4.2×10^6 and 11.9×10^6 respectively. The standard deviations in Table 2 do not include the uncertainty in these parameters. The average annual increase in $\log F(a, t)$ from 1977 to 1990 is 0.022 but was 0.026 with Θ_1 included. The values of $\log F(a, t)$ are on average 0.056 higher in the last year when Θ_1 is included, and in the earlier years the difference between the two estimates is smaller. The random walk model alone thus provides almost the same account of the trend in fishing mortality rates in this period.

The estimation was repeated with the same data, but ending in 1984, 1985. . . . The last years' results are presented in Table 4. No standard deviations are presented, but they are virtually the same as for the last year in Table 3. The parameter values change little ($h_1 = 2.60$, $h_2 = 0.04$, $h_3 = 0.02$, $h_4 = 3.64$ and $\sigma = 0.048$ for

TABLE 3
Annual fishing mortality rates, smoothed estimates†

Year	*Mortality rates for the following ages in years:* 4	5	6	7	8	9	10
1977	0.174 (0.07)	0.349 (0.06)	0.461 (0.08)	0.740 (0.04)	0.805 (0.07)	0.860 (0.08)	0.817 (0.11)
1978	0.166 (0.07)	0.276 (0.06)	0.361 (0.06)	0.508 (0.06)	0.678 (0.05)	0.653 (0.09)	0.602 (0.11)
1979	0.194 (0.12)	0.236 (0.05)	0.353 (0.06)	0.476 (0.05)	0.570 (0.07)	0.573 (0.08)	0.573 (0.11)
1980	0.181 (0.10)	0.336 (0.05)	0.390 (0.06)	0.511 (0.06)	0.572 (0.06)	0.608 (0.10)	0.604 (0.11)
1981	0.155 (0.07)	0.391 (0.05)	0.487 (0.05)	0.640 (0.04)	0.820 (0.05)	0.830 (0.08)	0.834 (0.11)
1982	0.221 (0.07)	0.413 (0.05)	0.535 (0.04)	0.618 (0.04)	1.016 (0.05)	1.019 (0.07)	0.951 (0.10)
1983	0.134 (0.07)	0.413 (0.04)	0.590 (0.04)	0.723 (0.04)	0.863 (0.05)	0.877 (0.07)	0.836 (0.10)
1984	0.225 (0.08)	0.335 (0.05)	0.508 (0.04)	0.623 (0.04)	0.863 (0.05)	0.812 (0.08)	0.792 (0.10)
1985	0.282 (0.05)	0.394 (0.04)	0.582 (0.04)	0.700 (0.05)	0.838 (0.06)	0.889 (0.09)	0.908 (0.11)
1986	0.226 (0.06)	0.550 (0.03)	0.712 (0.03)	0.904 (0.04)	1.017 (0.06)	1.049 (0.09)	1.066 (0.11)
1987	0.379 (0.10)	0.528 (0.04)	0.736 (0.03)	0.941 (0.04)	1.074 (0.05)	1.123 (0.09)	1.173 (0.11)
1988	0.226 (0.13)	0.542 (0.06)	0.855 (0.05)	1.017 (0.05)	1.356 (0.07)	1.337 (0.09)	1.299 (0.11)
1989	0.287 (0.12)	0.507 (0.12)	0.658 (0.09)	0.837 (0.08)	1.044 (0.11)	1.042 (0.12)	1.065 (0.13)
1990	0.221 (0.26)	0.485 (0.19)	0.681 (0.16)	0.887 (0.19)	1.088 (0.20)	1.091 (0.20)	1.089 (0.19)

†Standard deviations of $\log F(a, t)$ are given in parentheses.

TABLE 4
Last years' estimates of fishing mortality rates

Year	*Estimates for the following ages in years:* 4	5	6	7	8	9	10
1984	0.235	0.355	0.454	0.584	0.799	0.828	0.797
1985	0.214	0.414	0.589	0.685	0.895	0.897	0.941
1986	0.183	0.382	0.566	0.724	0.884	0.914	0.891
1987	0.476	0.530	0.662	0.910	1.121	1.168	1.183
1988	0.327	0.563	0.762	0.933	1.201	1.227	1.227
1989	0.317	0.673	0.908	1.112	1.421	1.407	1.421
1990	0.221	0.485	0.681	0.887	1.088	1.091	1.089

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1977–84). The results in Table 4 are similar to the Kalman filter results for the whole data set before smoothing, regardless of whether they are calculated with the parameters from 1977–84 or 1977–90.

The biggest discrepancies between the last years' estimates and the smoothed values appear in 1989. They are large from a practical point of view, but the Kalman filter provides adequate warning that errors of this magnitude can occur.

5. Auxiliary Information

With the time series method we can include effort or catch per unit effort data without assuming proportionality between effort measurements and fishing mortality rates. Let

$$\mathrm{CP}(a, t) = \Phi(a)\,\Omega(t)\,N(a, t)\exp\{-\tau Z(a, t)\} + \epsilon_{\mathrm{cp}}(a, t),$$

where $\mathrm{CP}(a, t)$ is the catch per unit effort at time τ in the year, $[0 \leqslant \tau \leqslant 1]$, e.g. data from a research vessel survey. The age-specific selectivity $\Phi(a)$ is estimated by a low order polynomial as the initial values of $F(a, 1)$. Joint variations in catchability are represented by the state variable $\Omega(t)$ and we include both permanent and transitory variations:

$$\Omega(t) = \beta(t) + \delta_6(t);$$
$$\beta(t) = \beta(t-1) + \delta_7(t),$$

where the residuals are defined in the same way as in equations (7) and (8). It is not possible to distinguish measurement errors from random variations in catchability in the catch per unit effort data and $\epsilon_{\mathrm{cp}}(a, t)$ represents both.

The main difference between this formulation and the premises of VPA or least squares methods is that we can allow permanent changes in the ratio between effort and fishing mortality rates. If only transitory variations in the ratio are present the variance of δ_7 is 0. Examples of the application of these models are presented by Gudmundsson (1987b) and Working Group on Methods of Fish Stock Assessments (1991).

6. Discussion

The most important practical advantage of the present method is that by using information in the data about their time series properties it can afford to ignore or give appropriate weight to auxiliary data, which are commonly less reliable than the catch-at-age data. The series are exceptionally short so that there is little scope for examination of the validity of estimated models. However, some restrictions which are imposed in other methods can be tested here against more general specifications.

Catch-at-age series are often available for more than 14 years. In our example stretching the analysis further back would require elaborate modelling to account for changes in selectivity caused by changes in fleet composition and regulations. Similar circumstances are common in other fisheries. The best auxiliary data are often from research vessel surveys, but consistent series of those are usually short.

Although it is customary to assume known constant rates of natural mortality there is considerable uncertainty about this. A wrong assumption about natural

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mortality produces bias in estimates of fishing mortality in the opposite direction and of similar magnitude. The estimation of natural mortality rates and the consequences of misspecifying it are an important subject, but outside the scope of the present paper.

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