QUANTUM PHASE ESTIMATION WITH QUANTUM EXPONENTIAL LEAST SQUARES MINIMIZATION

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QUANTINUUM

PHASE ESTIMATION

- Algorithm to obtain the eigen phase of a given unitary

$$U|\phi_j\rangle = e^{i\phi_j}|\phi_j\rangle$$

- When Applied to a given eigen state $|\phi_j\rangle$

PHASE ESTIMATION

- For Hamiltonians

$$e^{iHt}|\phi_j\rangle = e^{iE_jt}|\phi_j\rangle$$

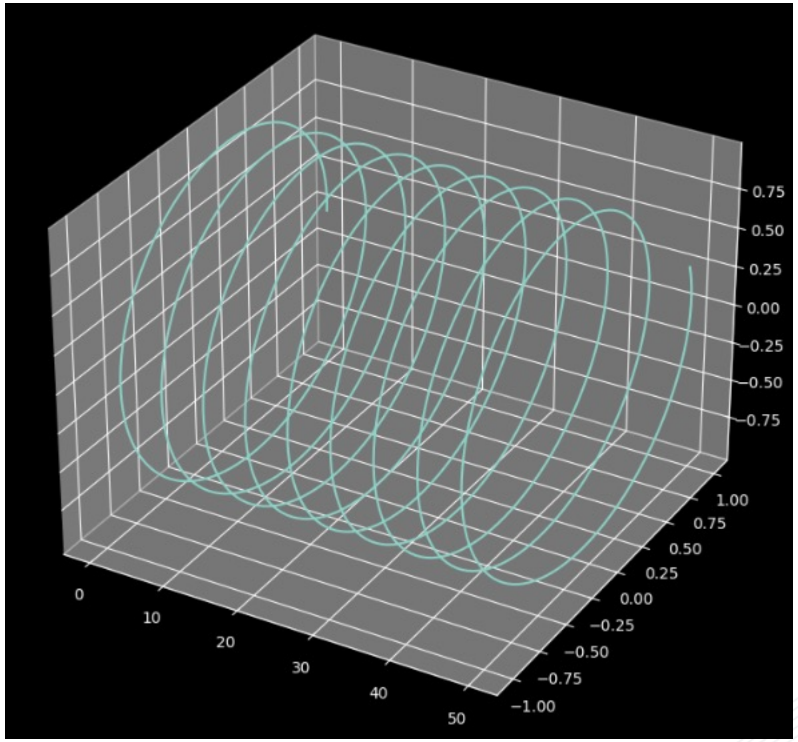
- When Applied to a given eigen state $|\phi_j\rangle$

PHASE ESTIMATION

- Transition matrix elements

$$\langle \phi_j | e^{-iHt} | \phi_j \rangle = e^{-iE_j t} \langle \phi_j | \phi_j \rangle$$

- When Applied to a given eigen state $|\phi_j\rangle$

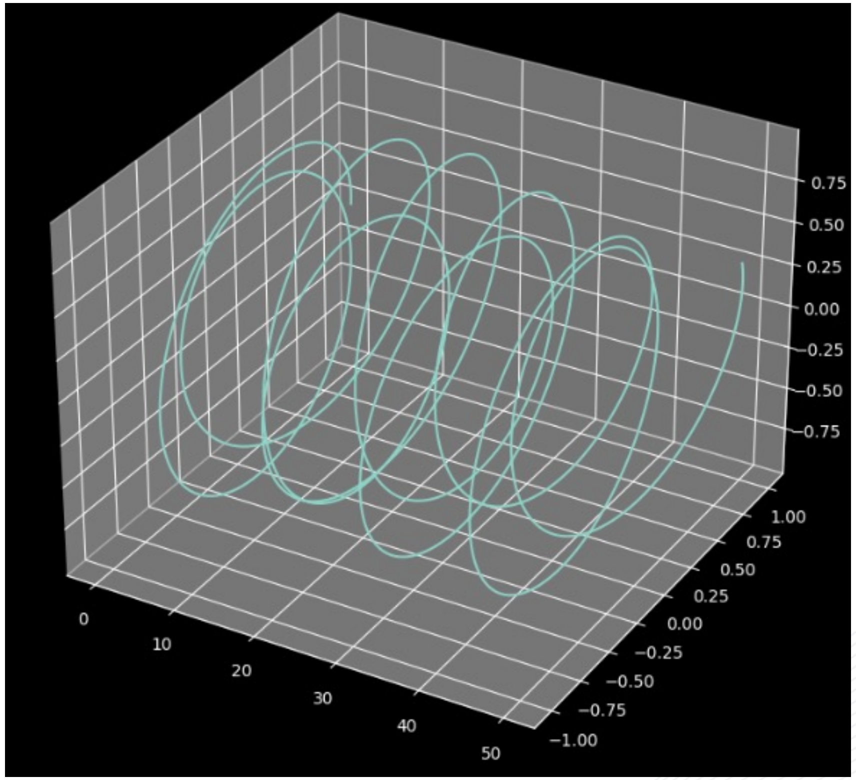


PHASE ESTIMATION

- Transition matrix elements

$$|\psi\rangle = \sum_j c_j |\phi_i\rangle$$

$$\langle\psi|e^{-i{\color{red}H}t}|\psi\rangle = \sum_j |c_j|^2 e^{-i{\color{red}E_j}t}\langle\phi_j|\phi_j\rangle$$



- When Applied to a linear combination eigen states $|\psi\rangle$

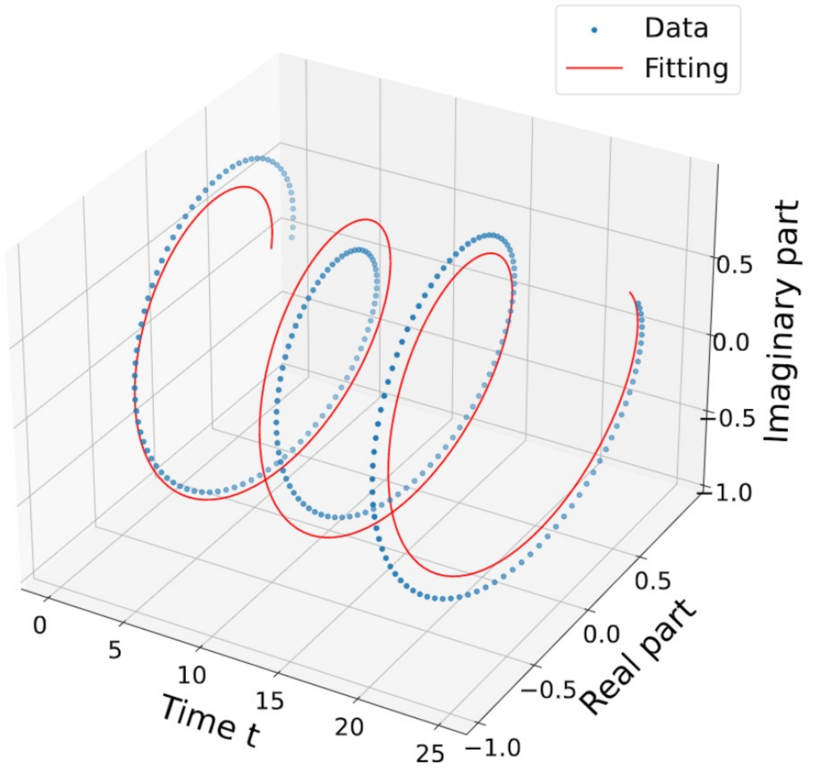
QCELS

- Transition matrix elements

$$L(r,\theta) = \frac{1}{N}\sum_{n=0}^{N-1} |Z_n - re^{-i\theta n\tau}|^2$$

$$Z_n = \langle\psi|e^{-iHn\tau}|\psi\rangle$$

- For n time slices τ find θ in: $re^{-i\theta n\tau}$



QCELS

- Transition matrix elements can be measured with the Hadamard test

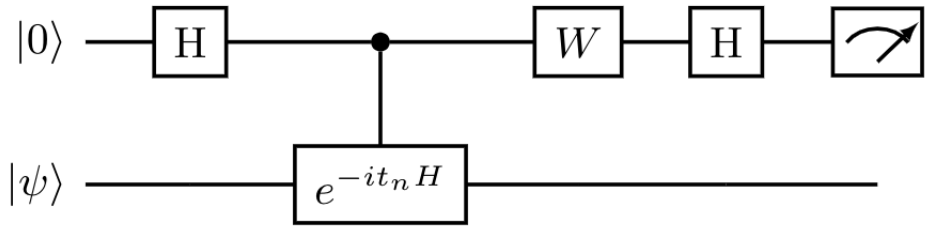


Figure 1. Quantum circuit used for collecting the input data. H is the Hadamard gate, $t_n = n\tau$. Choosing $W = I$ or $W = S^\dagger$ (S is the phase gate) allows us to estimate the real or the imaginary part of $\langle\psi|\exp(-it_n H)|\psi\rangle$.

DERIVATIVES

$$L(r,\theta) = \frac{1}{N}\sum_{n=0}^{N-1} |Z_n - re^{-i\theta n\tau}|^2$$

- For a fixed value of $\boldsymbol{\theta}$ the minimisation of $\boldsymbol{L}$ wrt to $\boldsymbol{r}$ $\boldsymbol{GIVES}$:

$$r(\theta) = \frac{1}{N}\sum_{n=0}^{N-1} e^{i\theta n\tau} Z_n,$$

- Sub back in:

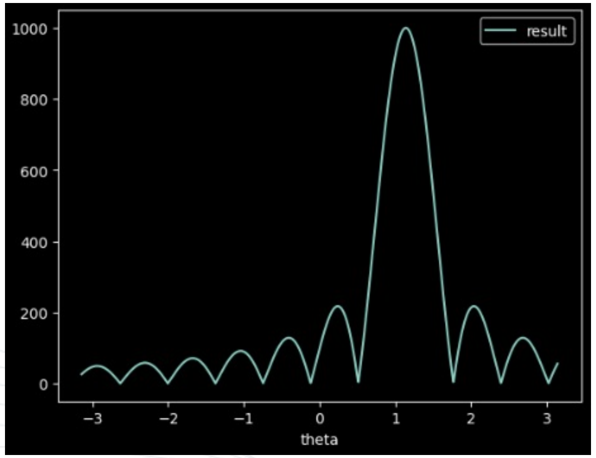
$$\min_{r\in\mathbb{C}} L(r,\theta) = \frac{1}{N}\sum_{n=0}^{N-1} |Z_n|^2 - \frac{1}{N}\left|\sum_{n=0}^{N-1} Z_n e^{i\theta n\tau}\right|^2$$

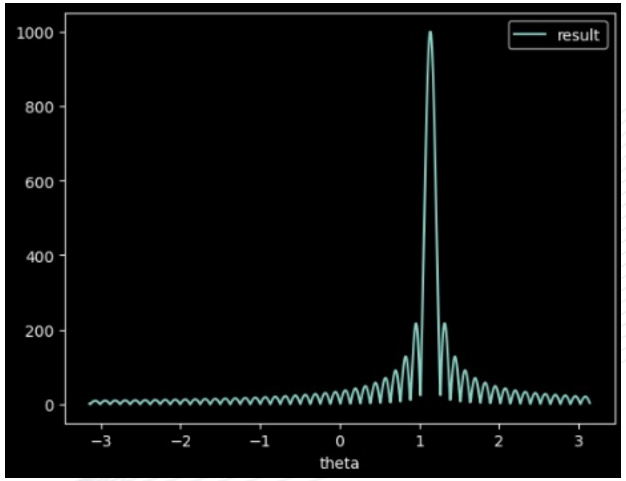
DERIVATIVES

$$\min L(\theta) = \frac{1}{N}\sum_{n=0}^{N-1} |Z_n|^2 - \frac{1}{N^2}\left|\sum_{n=0}^{N-1} Z_n e^{i\theta n\tau}\right|^2$$

- Minimization of L equivalent to maximizing

$$f(\theta) = \left|\sum_{n=0}^{N-1} Z_n e^{i\theta n\tau}\right|^2$$





Larger n

COMPLEXITY ARGUMENTS

- The algorithm satisfies the Heinsberg scaling limit $O(\varepsilon^{-1})$
- Only single ancilla phase estimation method to do so
- When the overlap with the exact GS is > 0.71 repeat until success can be used
- When the overlap with the exact GS is < 0.71 Filtering and signal processing methods can be used (Kentaro, Sam)
- Everything formally proved in the paper arXiv:2211.11973

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