

Lurie Networks with Robust Convergent Dynamics

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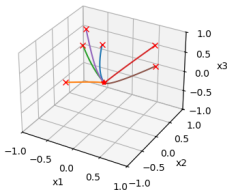
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Motivation I

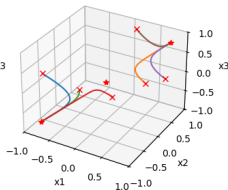
Many dynamical systems exhibit some form of convergence. These include convergence to unique or non-unique equilibrium points and limit cycles. The examples below (from left to right) illustrate:

- ▶ how three agents converge to the same opinion
- ▶ the evolution of three neurons in a Hopfield network
- ▶ autocatalytic chemical reactions (e.g., of a Rössler system)

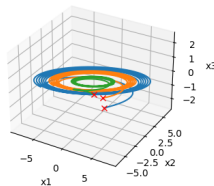
1-Contracting Opinion Dynamics



2-Contracting Hopfield Network



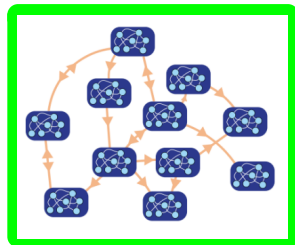
3-Contracting Simple Attractor



Motivation II

Of course, the brain is also a dynamical system formed of interacting neural circuits implementing a myriad of functions. Some of which leverage convergent (attractor) dynamics for:

- ▶ learning
- ▶ memory storage
- ▶ de-noising representations
- ▶ processing information over extended periods



Research Questions

1. Does encoding convergence as an inductive bias lead to more robust models of dynamical systems?
2. Inspired by the brain, can convergence be harnessed to develop a general and robust ML model?

Outline

1. Propose the *Lurie network* as a unifying recurrent model
2. Derive constraints on its weights to ensure convergence
3. Parametrise the weights so these constraints are unconditionally satisfied
4. Empirical results

Lurie Network

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