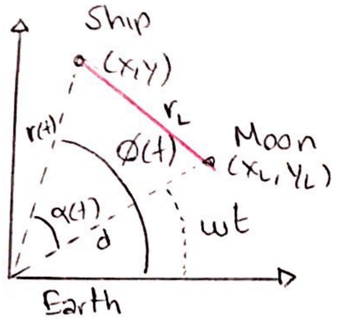
c)



$$\alpha(t) = \phi - \omega t$$

$\alpha(t)$ $\downarrow$

Usando ley de cosenos

$$r_L^2(r,\phi,t) = r^2(t) + d^2 - 2r(t)d\cos(\phi - \omega t)$$

$$r_L(r,\phi,t) = \sqrt{r^2(t) + d^2 - 2r(t)d\cos(\phi - \omega t)}$$

d) Dado que

$$H = p_r\dot{r} + p_\phi\dot{\phi} - L \xrightarrow{\text{donde}} L = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}m\dot{\phi}^2 + \frac{GmM_T}{r} + \frac{Gmm_L}{r_L(r,\phi,t)}$$

Sabemos que

$$m\dot{r} = p_r \rightarrow \frac{p_r}{m} = \dot{r} \quad ; \quad rm\dot{\phi} = p_\phi \rightarrow \dot{\phi} = \frac{p_\phi}{mr}$$

Por lo tanto

$$L = \frac{\cancel{m}}{2}\frac{p_r^2}{m^2} + \frac{\cancel{m}}{2r^2}\frac{p_\phi^2}{m^2} + \frac{Gmm_T}{r} + \frac{Gmm_L}{r_L(r,\phi,t)}$$

$$L = \frac{1}{2m}p_r^2 + \frac{p_\phi^2}{2mr^2} + \frac{Gmm_T}{r} + \frac{Gmm_L}{r_L(r,\phi,t)}$$

$$H = \frac{p_r^2}{m} + \frac{p_\phi^2}{mr^2} - \frac{p_r^2}{2m} - \frac{p_\phi^2}{2mr^2} - \frac{Gmm_T}{r} - \frac{Gmm_L}{r_L(r,\phi,t)}$$

$$H = \frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} - \frac{Gmm_T}{r} - \frac{Gmm_L}{r_L(r,\phi,t)}$$

e)

$$\dot{r} = \frac{\partial H}{\partial p_r} = \frac{\cancel{2}p_r}{\cancel{2}m} = \frac{p_r}{m} \quad ; \quad \dot{\phi} = \frac{\partial H}{\partial p_\phi} = \frac{\cancel{2}p_\phi}{\cancel{2}mr^2} = \frac{p_\phi}{mr^2}$$

$$\dot{p}_r = -\frac{\partial H}{\partial r} = -\left[-\frac{2p_\phi^2}{2mr^3} + \frac{Gmm_T}{r^2} + \frac{1}{\cancel{2}}\frac{Gmm_L(\cancel{2}r(t) - \cancel{2}d\cos(\phi-\omega t))}{\sqrt[3]{r^2(t)+d^2-2r(t)d\cos(\phi-\omega t)}}\right]$$

$$\dot{p}_r = \frac{p_\phi^2}{mr^3} - \frac{Gmm_T}{r^2} - \frac{Gmm_L(r(t)-d\cos(\phi-\omega t))}{r_L^3(r,\phi,t)}$$

$$\dot{p}_\phi = -\frac{\partial H}{\partial \phi} = -\left[\frac{1}{\cancel{2}}\frac{Gmm_L(\cancel{2}rd\sin(\phi-\omega t))}{\sqrt[3]{r^2+d^2-2rd\cos(\phi-\omega t)}}\right] = -\frac{Gmm_L rd\sin(\phi-\omega t)}{r_L^3(r,\phi,t)}$$

f)

$$\dot{\tilde{r}} = \frac{\dot{r}}{d} = \frac{p_r}{md} = \tilde{p}_r$$

$$\dot{\phi} = \frac{p_\phi}{mr^2} = \frac{\tilde{p}_\phi \cancel{m} \cancel{d^2}}{\cancel{m}\tilde{r}^2 \cancel{d^2}} = \frac{\tilde{p}_\phi}{\tilde{r}^2}$$

$$\dot{\tilde{p}}_r = \frac{\dot{p}_r}{md} = \frac{1}{md}\left[\frac{p_\phi^2}{mr^3} - \frac{Gmm_T}{r^2} - \frac{Gmm_L}{r_L^3(r,\phi,t)}[r - d\cos(\phi - \omega t)]\right]$$

$$\dot{\tilde{p}}_r = \frac{1}{md}\left[\frac{\tilde{p}_\phi^2 m^2 d^4}{\cancel{m}\tilde{r}^3 \cancel{d^3}} - \frac{Gmm_T}{\tilde{r}^2 d^2} - \frac{Gm_T m m_L [\tilde{r}d - d\cos(\phi - \omega t)]}{m_T d^3 \tilde{r}'^3}\right]$$

$$\dot{\tilde{p}}_r = \frac{\tilde{p}_\phi^2}{\tilde{r}^3} - \Delta\left\{\frac{1}{\tilde{r}^2} + \frac{\mu}{\tilde{r}'^3}[\tilde{r} - \cos(\phi - \omega t)]\right\}$$

$$\dot{\tilde{p}}_\phi = \frac{\dot{p}_\phi}{md^2} = \frac{-1}{md^2}\,\frac{Gmm_L m_T r d \sin(\phi - \omega t)}{r_L^3 m_T} = -\frac{\Delta\mu\tilde{r}\sin(\phi - \omega t)}{\tilde{r}'^3}$$