Project 1

Deadline: Sunday, 26 March 2023

In this project you will make use of strings, lists and functions in Python.

Any natural number $n \in \mathbb{N}$ has a unique representation $(n)_b$ with respect to a given base $b \in \mathbb{N}_{\geq 2}$. For example, $(42)_2 = 101010$, $(42)_8 = 52$ and $(42)_{10} = 42$. For simplicity, in this project you may always assume that $2 \leq b \leq 10$.
In addition, we call a string s admissible, if it does not contain any two neighbouring identical and nonempty substrings. For example, all of AA, ABAB and ABCBCA are not admissible, while ABCACB is. Observe that the longest admissible strings made up of only the latters A and B are ABA and BAB.
Finally, for a given $2 \leq b \leq 10$ we say a number $n \in \mathbb{N}$ is b-admissible if the representation $(n)_b$ interpreted as a string is an admissible string. Hence the largest 2-admissible number is 5, since $(5)_2 = 101$. For $b > 2$ there are infinitely many b-admissible numbers.

Write Python functions that (a) can compute $(n)_b$ for given $n \in \mathbb{N}$ and $2 \leq b \leq 10$, (b) can determine if a given number n is b-admissible, (c) can count all the b-admissible numbers within a range, (d) can count all the b-admissible numbers whose b-representation has a given number of digits, (e) can find the largest number within a range that is b-admissible for all $b \in L$, where L is a given list.

In particular:

- Write a function `number2base_rep(n, b)` that takes two integers and returns $(n)_b$ as a string.
- Write a function `admissible(n, b)` that takes two integers and returns if n is b-admissible.
- Write a function `count_admissible(b, start, end)` that takes three integers and returns the number of b-admissible numbers n with `start` $\leq n <$ `end`.
- Write a function `count_admissible_width(b, width)` that takes two integers and returns the number of b-admissible numbers n whose b-representation has exactly `width` digits.
- Write a function `largest_multi_admissible(L, start, end)` that takes a list `L` and two integers and returns the largest integer n with `start` $\leq n <$ `end` that is b-admissible for all $b \in$ `L`. The function must return `None` if no such number exists.

Your functions may use all available standard types and methods in Python 3, but must work without importing any packages or modules.

Together with your code you should submit a 1- or 2-page report with a very brief description of your implementations. For an increasing sequence of integers `k`, report on the results obtained from, and the CPU time necessary for, your function calls

- `count_admissible(5, 10**k, 10**(k+1))`
- `count_admissible_width(3, k)`
- `largest_multi_admissible([3, 5, 7, 10], 1, 10**k)`

In each case, formulate a hypothesis on how the CPU time depends on k as k becomes large. Finally, using the world wide web, can you identify some of the sequences of numbers your code has produced?

Submission via Moodle

Prepare a Python source file called `project1.py` that contains your function `complete_shuffle`, but nothing else. The docstring of the file `project1.py` must contain your full name. In addition, prepare a report in PDF format, called `project1.pdf`, as outlined above.