Mid-term Examination

一． Mark each statement True or False.

1. ( ) If a system of linear equations $AX=b$ has no free variables, then it has an unique solution.
2. ( ) If AB=O then A=O or B=O.
3. ( ) If AB=AC then B=C.
4. ( ) If A and B are $n \times n$ matrix, then $A^2 - B^2 = (A-B)(A+B)$.
5. ( ) If two rows of a $n \times n$ matrix A are the same, then det A=0.
6. ( ) If A is invertible, then det A^{-1}=(det A)$^{-1}$

二． Fill vacancies.

1. If A= $\begin{pmatrix} -4 & -5 \\ 5 & 6 \end{pmatrix}$, then A^{-1}=______________________

2.If A= $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 5 & 0 & 0 \\ 3 & 6 & 8 & 0 \\ 4 & 7 & 9 & 10 \end{pmatrix}$ then det A=____________________.

3. Let α_1= (1, 2, 3), α_2 = (4, 5, 6), α_3=(2, 1, 0), then α_1, α_2 and α_3 are linearly__________.

4. A= $\begin{pmatrix} 1 & 2 \\ -2 & 5 \\ -5 & 6 \end{pmatrix}$, b= $\begin{pmatrix} 7 \\ 4 \\ -3 \end{pmatrix}$, the system of linear equations $AX=b$ is____________.

三．. Compute the determinants of the matrices.

1. $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ 2. $B = \begin{pmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{pmatrix}$

四． Find the inverses of the matrices.

1. $A = \begin{pmatrix} 1 & -2 & 0 & 0 \\ -2 & 5 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ 2. $B = \begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & 1 \end{pmatrix}$

五．Solve the system. $\begin{cases} x_1 + 2x_2 = 4 \\ -x_1 + 3x_2 + 3x_3 = -2 \\ x_2 + x_3 = 0 \end{cases}$

六．Determine the value of a such that the matrix is the augmented matrix of a consistent linear system. $\begin{pmatrix} -2 & 1 & 1 & -2 \\ 1 & -2 & 1 & a \\ 1 & 1 & -2 & a \end{pmatrix}$.

七．Show that the columns of $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 5 & 0 & 0 \\ 3 & 6 & 8 & 0 \\ 4 & 7 & 9 & 6 \end{pmatrix}$ are linearly independent.

八. Suppose A^k=O for some k>1. verify that $(I - A)^{-1} = I + A + A^2 + \cdots + A^{k-1}$

I is identity matrix. A and I are $n \times n$ matrices.