Final Examination

一. Let $A=\begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 2 & -1 & 1 \end{pmatrix}$

1. Compute the determinant of *A*.
2. Find the rank of *A*.
3. Determine if *A* is invertible. If *A* is invertible, find A^{-1}.

二. If $A=\begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix}$ and $AX=A-X$. Find *X*.

三. The linear system is $\begin{cases} x_1+x_2+4x_3=4 \\ -x_1+4x_2+x_3=16 \\ x_1-x_2+2x_3=\lambda \end{cases}$.

1. Write the augmented matrix $\overline{A}$ of the system.
2. Determine the value of λ such that the system is consistent.
3. Find the dimension of the null space of the coefficient matrix *A*.
4. When the system is consistent, solve it.

四. Let $A=\begin{pmatrix} 1 & -2 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

1. Find the eigenvalues and eigenvectors of *A*.
2. Orthogonally diagonalize *A*.

五. Let quadratic form $f(x_1,x_2,x_3)=x_1^2+x_2^2+x_3^2-4x_1x_2$

1. Write the matrix of the quadratic form $f(x_1,x_2,x_3)$.

2. Make a change of variable *X*=*PY*, that transforms the quadratic form $f(x_1,x_2,x_3)$ into a quadratic form with no cross-product term. Give *P* and the new quadratic form.

3. Determine whether the quadratic form $f(x_1,x_2,x_3)$ is positive definite.

六. If *A* is 3×3 and $A-I, A-2I, A-3I$ are all not invertible (sometimes called singular). Show that *A* can be diagonalizable.

七. If you do the subject below, you can get 5 extra points.

If $A^2 = A$. Show that matrix $A+I$ is invertible and find $(A+I)^{-1}$.