Final Examination 2

1. Let $A=\begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 2 & -1 & 1 \end{pmatrix}$, find the inverse matrix of *A*.

2. The linear system is $\begin{cases} x_1 + x_3 = 1 \\ 4x_1 + x_2 + 2x_3 = 3 \\ 6x_1 + x_2 + 4x_3 = 4 + \lambda \end{cases}$.

(1).Write the coefficient matrix *A* and the augmented matrix $\overline{A}$ of the system.

(2).Compute the dimension of the null space of *A* and the rank of *A*.

(3).Determine the value of λ such that the system is consistent.

(4).When the system is consistent, solve it.

3. Let $A=\begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$, $\lambda_1 = \lambda_2 = 2, \lambda_3 = 6$ are eigenvalues of *A*.

Diagonalize the matrix of *A*, if possible.

4. Let quadratic form $f(x_1, x_2, x_3) = 5x_1^2 + 5x_2^2 + 2x_3^2 - 8x_1x_2 - 4x_1x_3 + 4x_2x_3$.

(1).Write the matrix of the quadratic form $f(x_1, x_2, x_3)$.

(2).Suppose $\xi_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \xi_2 = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$,verify that ξ_1 and ξ_2 are eigenvectoes of *A*.

(3).Make a change of variable *X=PY*, that transforms the quadratic form $f(x_1, x_2, x_3)$ into a quadratic form with no cross-product term. Give *P* and the new quadratic form.

(4).Determine whether the quadratic form $f(x_1, x_2, x_3)$ is positive definite.

5. Let *A* be a $n \times n$ matrix, the number 0 is not the eigenvalue of *A*. Show that *A* is invertible.