

ASSIGNMENT 11

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Download all python codes from

<https://github.com/CRAMYATULASI/ASSIGNMENT11/tree/main/ASSIGNMENT11/CODES>

and latex-tikz codes from

<https://github.com/CRAMYATULASI/ASSIGNMENT11/tree/main/ASSIGNMENT11>

1 QUESTION No. 2.47

Solve $5x+4y \leq 20, x \geq 1, y \geq 2$

2 SOLUTION

The given system of inequality can be written in matrix form as

$$\begin{pmatrix} -5 & -4 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -20 \\ 1 \\ 2 \end{pmatrix} \quad (2.0.1)$$

Let the surplus vector be

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq 0 \quad (2.0.2)$$

1)

$$\begin{pmatrix} -5 & -4 \\ 1 & 0 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -20 \\ 1 \end{pmatrix} \quad (2.0.3)$$

$$\Rightarrow \begin{pmatrix} -5 & -4 \\ 1 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -20 \\ 1 \end{pmatrix} + \mathbf{u} \quad (2.0.4)$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -5 & -4 \\ 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -20 \\ 1 \end{pmatrix} + \begin{pmatrix} -5 & -4 \\ 1 & 0 \end{pmatrix}^{-1} \mathbf{u} \quad (2.0.5)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} 1 \\ 15/4 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -1/5 & -4/5 \end{pmatrix} \mathbf{u} \quad (2.0.6)$$

2)

$$\begin{pmatrix} -3 & -2 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -12 \\ 2 \end{pmatrix} \quad (2.0.7)$$

$$\Rightarrow \begin{pmatrix} -3 & -2 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -12 \\ 2 \end{pmatrix} + \mathbf{u} \quad (2.0.8)$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -5 & -4 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -20 \\ 2 \end{pmatrix} + \begin{pmatrix} -5 & -4 \\ 0 & 1 \end{pmatrix}^{-1} \mathbf{u} \quad (2.0.9)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} 12/5 \\ 2 \end{pmatrix} + \begin{pmatrix} -1/5 & -4/5 \\ 0 & 1 \end{pmatrix} \mathbf{u} \quad (2.0.10)$$

Now, solution region which is common to regions of eq. (2.0.6) and eq. (2.0.10), is given by

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1/20 & -21/20 \end{pmatrix} \mathbf{u} \quad (2.0.11)$$

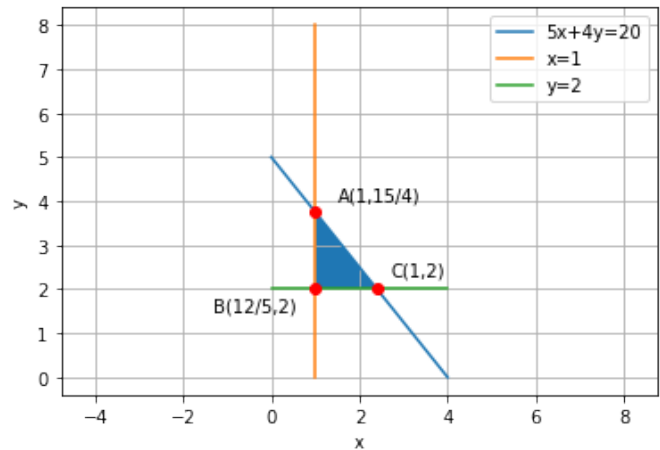


Fig. 2.1: Solution Region

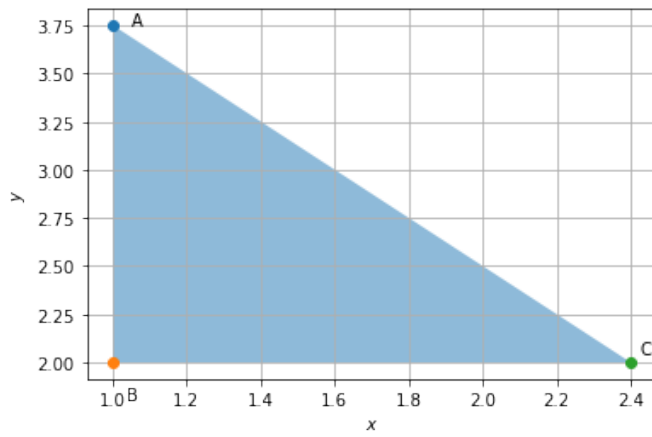


Fig. 2.2: Magnified Solution Region