ASSIGNMENT 12

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1 QUESTION No. 2.32

A toy company manufactures two types of dolls, A and B. Market research and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A.Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit?

2 SOLUTION

Let x and y be number of dolls of type A and B respectively such that

$$x \geq 0 \tag{2.0.1}$$
$$y \geq 0 \tag{2.0.2}$$

According to the question,

$$x + y \leq 1200 \tag{2.0.3}$$

$$-x + 2y \leq 0 \tag{2.0.4}$$

$$x - 3y \leq 600 \tag{2.0.5}$$

$\therefore$

$$\max_{\mathbf{x}} Z = \begin{pmatrix} 12 & 16 \end{pmatrix} \mathbf{x} \tag{2.0.6}$$

Using Slack variables,

$$12x + 16y - z = 0 \tag{2.0.7}$$
$$x + y + S_1 = 1200 \tag{2.0.8}$$
$$x - 2y + S_2 = 0 \tag{2.0.9}$$
$$x - 3y + S_3 = 600 \tag{2.0.10}$$

Now,Simplex Table

$$\begin{pmatrix} x & y & S_1 & S_2 & S_3 & b \\ \hline 1 & 2 & 1 & 0 & 0 & 1200 \\ 1 & -2 & 0 & 1 & 0 & 0 \\ 1 & -3 & 0 & 0 & 1 & 600 \end{pmatrix} \tag{2.0.11}$$

resulting,

$$\begin{pmatrix} x & y & S_1 & S_2 & S_3 & b \\ \hline 1 & 0 & \frac{2}{3} & -\frac{1}{3} & 0 & 800 \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} & 0 & 400 \\ 0 & 0 & \frac{1}{3} & \frac{4}{3} & 1 & 1000 \\ \hline 12 & 16 & \frac{40}{3} & \frac{4}{3} & 0 & 0 \end{pmatrix} \tag{2.0.12}$$

This can be expressed in the form of matrix inequality for maximization and minimization respectively as,

$$\max_{\mathbf{x}} \mathbf{c}^T \mathbf{x} \tag{2.0.13}$$

$$s.t. \quad \mathbf{Ax} \leq \mathbf{b}; \mathbf{x} \geq 0 \tag{2.0.14}$$

$$\mathbf{c} = \begin{pmatrix} 12 \\ 16 \end{pmatrix} \tag{2.0.15}$$

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & -2 \\ 1 & -3 \end{pmatrix} \tag{2.0.16}$$

$$\mathbf{b} = \begin{pmatrix} 1200 \\ 0 \\ 600 \end{pmatrix} \tag{2.0.17}$$

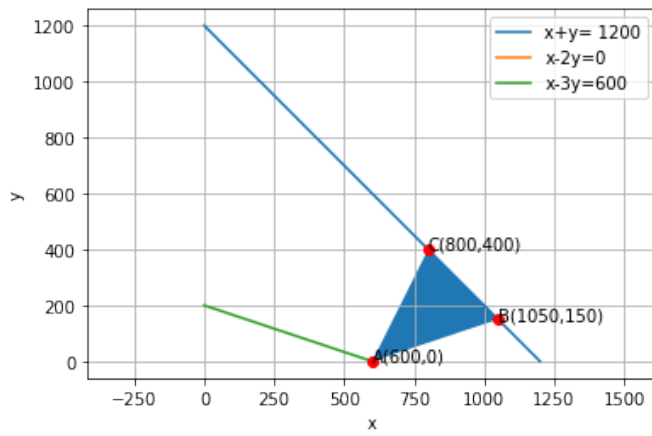Solving for Z by this reduction method,

$$\max Z = 16000 \tag{2.0.18}$$

Fig. 2.1: TOY PROBLEM