ASSIGNMENT 4

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Download all python codes from

https://github.com/CRAMYATULASI/ASSIGNMENT_4/tree/main/ASSIGNMENT4/CODES

and latex-tikz codes from

https://github.com/CRAMYATULASI/ASSIGNMENT_4/tree/main/ASSIGNMENT4

1 Question No 2.6

Find out whether the lines respresenting the following pairs of linear equations intersect at a point are parallel or coincident.

1)
$$\begin{aligned} \begin{pmatrix} 5 & -4 \end{pmatrix}\mathbf{x} &= -8 \\ \begin{pmatrix} 7 & 6 \end{pmatrix}\mathbf{x} &= 9 \end{aligned} \tag{1.0.1}$$

2)
$$\begin{aligned} \begin{pmatrix} 9 & 3 \end{pmatrix}\mathbf{x} &= -12 \\ \begin{pmatrix} 18 & 6 \end{pmatrix}\mathbf{x} &= -24 \end{aligned} \tag{1.0.2}$$

3)
$$\begin{aligned} \begin{pmatrix} 6 & -3 \end{pmatrix}\mathbf{x} &= -10 \\ \begin{pmatrix} 2 & -1 \end{pmatrix}\mathbf{x} &= -9 \end{aligned} \tag{1.0.3}$$

2 SOLUTION

1)
$$\begin{aligned} \begin{pmatrix} 5 & -4 \end{pmatrix}\mathbf{x} &= -8 \\ \begin{pmatrix} 7 & 6 \end{pmatrix}\mathbf{x} &= 9 \end{aligned} \tag{2.0.1}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix}\mathbf{x} = \begin{pmatrix} -8 \\ 9 \end{pmatrix} \tag{2.0.2}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 5 & -4 & -8 \\ 7 & 6 & 9 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow 7\frac{R_1}{5}} \begin{pmatrix} 7 & \frac{-28}{5} & \frac{-56}{5} \\ 7 & 6 & 9 \end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 7 & \frac{-28}{5} & \frac{-56}{5} \\ 0 & \frac{58}{5} & \frac{101}{5} \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{R_1 \leftarrow 5\frac{R_1}{7}} \begin{pmatrix} 5 & -4 & -8 \\ 0 & \frac{58}{5} & \frac{101}{5} \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_2 \leftarrow 5R_2} \begin{pmatrix} 5 & -4 & -8 \\ 0 & 58 & 101 \end{pmatrix} \tag{2.0.6}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix} 5 & -4 & -8 \\ 7 & 6 & 9 \end{pmatrix} \tag{2.0.7}$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} \tag{2.0.8}$$

is also 2.

$$\begin{aligned} \because Rank\begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} &= Rank\begin{pmatrix} 5 & -4 & -8 \\ 7 & 6 & 9 \end{pmatrix} \\ &= dim\begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} \\ &= 2 \end{aligned} \tag{2.0.9}$$

$\therefore$ Given lines (1.0.1) have unique solution so we can say they intersect.

2)
$$\begin{aligned} \begin{pmatrix} 9 & 3 \end{pmatrix}\mathbf{x} &= -12 \\ \begin{pmatrix} 18 & 6 \end{pmatrix}\mathbf{x} &= -24 \end{aligned} \tag{2.0.10}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 9 & 3 \\ 18 & 6 \end{pmatrix}\mathbf{x} = \begin{pmatrix} -12 \\ -24 \end{pmatrix} \tag{2.0.11}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 9 & 3 & -12 \\ 18 & 6 & -24 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow 7\frac{R_1}{5}} \begin{pmatrix} 7 & \frac{-28}{5} & \frac{-56}{5} \\ 7 & 6 & 9 \end{pmatrix} \quad (2.0.12)$$

$$\xleftrightarrow{R_1 \leftarrow \frac{18R_1}{9}} \begin{pmatrix} 18 & 6 & -24 \\ 18 & 6 & -24 \end{pmatrix} \quad (2.0.13)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 18 & 6 & -24 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.14)$$

$$(2.0.15)$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix} 9 & 3 & -12 \\ 18 & 6 & -24 \end{pmatrix} \quad (2.0.16)$$

results in a matrix with 1 nonzero rows, its rank is 1. Similarly, the rank of the matrix

$$\begin{pmatrix} 9 & 3 \\ 18 & 6 \end{pmatrix} \quad (2.0.17)$$

is also 1.

$$\because Rank \begin{pmatrix} 9 & 3 \\ 18 & 6 \end{pmatrix} = Rank \begin{pmatrix} 9 & 3 & -12 \\ 18 & 6 & -24 \end{pmatrix} = 1$$
$$< dim \begin{pmatrix} 9 & 3 \\ 18 & 6 \end{pmatrix} = 2 \quad (2.0.18)$$

$\therefore$ Given lines (1.0.2) have infinitely many solutions so we can say they coincide.

3)

$$\begin{pmatrix} 6 & -3 \end{pmatrix} \mathbf{x} = -10$$
$$\begin{pmatrix} 2 & -1 \end{pmatrix} \mathbf{x} = -9 \quad (2.0.19)$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 6 & -3 \\ 2 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -10 \\ -9 \end{pmatrix} \quad (2.0.20)$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 6 & -3 & -10 \\ 2 & -1 & -9 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \ frac2R_16} \begin{pmatrix} 2 & -1 & \frac{-10}{3} \\ 2 & -1 & -9 \end{pmatrix} \quad (2.0.21)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 2 & -1 & \frac{-10}{3} \\ 0 & 0 & \frac{-17}{3} \end{pmatrix} \quad (2.0.22)$$

$$\xleftrightarrow{R_1 \leftarrow 3R_1} \begin{pmatrix} 6 & -3 & -10 \\ 0 & 0 & \frac{-17}{3} \end{pmatrix} \quad (2.0.23)$$

$$(2.0.24)$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix} 6 & -3 & -10 \\ 2 & -1 & -9 \end{pmatrix} \quad (2.0.25)$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 6 & -3 \\ 2 & -1 \end{pmatrix} \quad (2.0.26)$$

is 1.

$$\because Rank \begin{pmatrix} 6 & -3 \\ 2 & -1 \end{pmatrix} \neq Rank \begin{pmatrix} 6 & -3 & -10 \\ 2 & -1 & -9 \end{pmatrix} \quad (2.0.27)$$

$\therefore$ Given lines (1.0.3) have no solution so we can say they are parallel.

PLOT OF GIVEN LINES -

Plot of (1.0.1) -

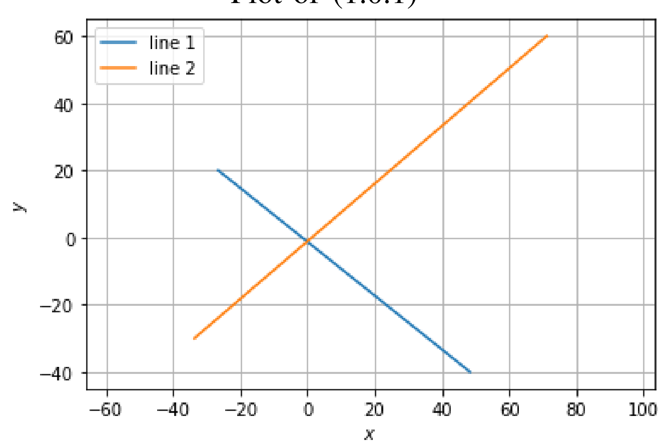

Fig. 2.1: INTERSECTING LINES.

Plot of (1.0.2) -

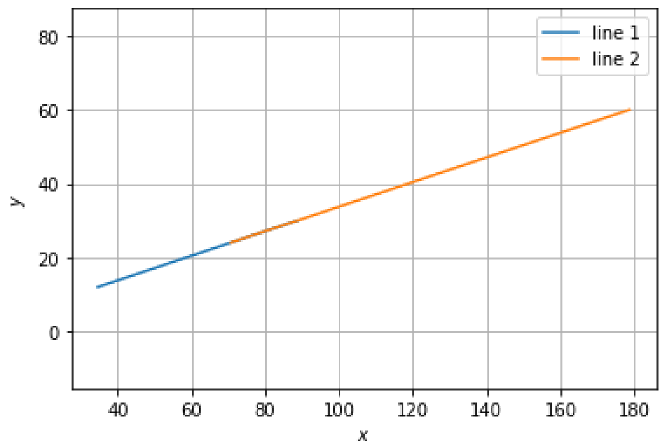

Fig. 2.2: SAME LINES

Plot of (1.0.3)

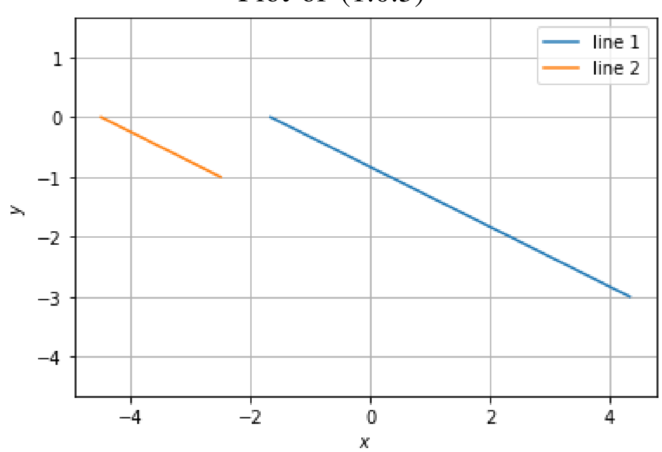

Fig. 2.3: Parallel lines