ASSIGNMENT 5

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Latex-tikz codes from

https://github.com/CRAMYATULASI/ASSIGNMENT5/tree/main/ASSIGNMENT5

1 Question No 2.56

Find the equation of the plane which contains the line of intersection of the planes

$$\begin{pmatrix}1 & 2 & 3\end{pmatrix}\mathbf{x} = 4 \tag{1.0.1}$$

$$\begin{pmatrix}2 & 1 & -1\end{pmatrix}\mathbf{x} = -5 \tag{1.0.2}$$

and which is perpendicular to the plane

$$\begin{pmatrix}5 & 3 & -6\end{pmatrix}\mathbf{x} = -8 \tag{1.0.3}$$

2 SOLUTION

Equation (1.0.1),(1.0.2) and (1.0.3) can be written as,

$$\mathbf{n_1^T x} = c_1 \tag{2.0.1}$$

$$\mathbf{n_2^T x} = c_2 \tag{2.0.2}$$

$$\mathbf{n_3^T x} = c_3 \tag{2.0.3}$$

Where,

$$\mathbf{n_1} = \begin{pmatrix}1\\2\\3\end{pmatrix}, \mathbf{n_2} = \begin{pmatrix}2\\1\\-1\end{pmatrix}, \mathbf{n_3} = \begin{pmatrix}5\\3\\-6\end{pmatrix}$$

$$c_1 = 4, c_2 = -5, c_3 = -8 \tag{2.0.4}$$

Required equation of plane containing (2.0.1) and (2.0.2) is,

$$\mathbf{n_1^T x} + \mathbf{n_2^T x}\lambda = c_1 + c_2\lambda \tag{2.0.5}$$

$$\implies (\mathbf{n_1^T} + \mathbf{n_2^T}\lambda)\mathbf{x} = c_1 + c_2\lambda \tag{2.0.6}$$

But (2.0.6) is perpendicular to (2.0.3).So,

$$\cos 90^\circ = \frac{a^T b}{\|a\|\,\|b\|} \implies a^T b = 0 \tag{2.0.7}$$

$$\implies (\mathbf{n_3^T})^T(\mathbf{n_1^T} + \mathbf{n_2^T}\lambda) = 0 \tag{2.0.8}$$

$$\implies \lambda = \frac{-\mathbf{n_3 n_1^T}}{\mathbf{n_3 n_2^T}} \tag{2.0.9}$$

$$\implies \lambda = \frac{7}{19} \tag{2.0.10}$$

$\therefore$ By substituting λ,n_1,n_2,c_1,c_2 values in (2.0.6) we get required plane equation as,

$$\begin{pmatrix}33 & 45 & 50\end{pmatrix}\mathbf{x} = 41 \tag{2.0.11}$$