ASSIGNMENT 6

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Download all python codes from

https://github.com/CRAMYATULASI/ASSIGNMENT6/tree/main/ASSIGNMENT6/CODES

Latex-tikz codes from

https://github.com/CRAMYATULASI/ASSIGNMENT6/tree/main/ASSIGNMENT6

1 Question No 2.42

Find the point at which the tangent to the curve $y = \sqrt{4x-3} - 1$ has its slope $\frac{2}{3}$.

2 SOLUTION

Given curve,

$$y = \sqrt{4x-3} - 1 \tag{2.0.1}$$

$$\implies (y+1)^2 = 4x - 3 \tag{2.0.2}$$

$$\implies y^2 - 4x + 2y + 4 = 0 \tag{2.0.3}$$

From above equation,

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, f = 4 \tag{2.0.4}$$

$$|\mathbf{V}| = 0 \tag{2.0.5}$$

$\therefore$ Given curve (2.0.1) is parabola.

In standard form,

$$\mathbf{P} = \mathbf{I} \implies p_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.6}$$

Given slope of tangents is $\frac{2}{3}$ then direction vectors and normal vectors are,

$$m = \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix} \implies \begin{pmatrix} 3 \\ 2 \end{pmatrix}, n = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \tag{2.0.7}$$

$$\kappa = \frac{p_1^T \mathbf{u}}{p_1^T n} \implies = -1 \tag{2.0.8}$$

$\therefore$ Point of contact for tangent of parabola is,

$$\begin{pmatrix} \mathbf{u} + \kappa n^T \\ \mathbf{V} \end{pmatrix} \mathbf{q} = \begin{pmatrix} -f \\ \kappa n - \mathbf{u} \end{pmatrix} \tag{2.0.9}$$

$$\implies \begin{pmatrix} -4 & -4 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} = \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix} \tag{2.0.10}$$

$$\implies \mathbf{q} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{2.0.11}$$

$\therefore$ Point of contact for tangent of given curve is

$$\mathbf{q} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{2.0.12}$$

Plot of Tangent to the given curve -

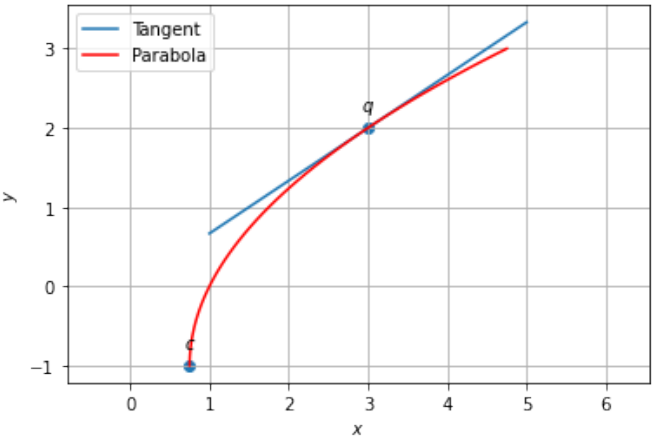

Fig. 2.1: Tangent to Parabola.