

ASSIGNMENT 6

C.RAMYA TULASI

Download all python codes from

<https://github.com/CRAMYATULASI/ASSIGNMENT6/tree/main/ASSIGNMENT6/CODES>

Latex-tikz codes from

<https://github.com/CRAMYATULASI/ASSIGNMENT6/tree/main/ASSIGNMENT6>

1 QUESTION No 2.42

Find the point at which the tangent to the curve $y = \sqrt{4x-3} - 1$ has its slope $\frac{2}{3}$.

2 SOLUTION

Given curve,

$$y = \sqrt{4x-3} - 1 \quad (2.0.1)$$

$$\Rightarrow (y+1)^2 = 4x-3 \quad (2.0.2)$$

$$\Rightarrow y^2 - 4x + 2y + 4 = 0 \quad (2.0.3)$$

From above equation,

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = (-2 \ 1), \mathbf{f} = 4 \quad (2.0.4)$$

$$|\mathbf{V}| = 0 \quad (2.0.5)$$

\therefore Given curve (2.0.1) is parabola.

In standard form,

$$\mathbf{P} = \mathbf{I} \Rightarrow \mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.6)$$

Given slope of tangents is $\frac{2}{3}$ then direction vectors and normal vectors are,

$$\mathbf{m} = \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix} \Rightarrow \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad (2.0.7)$$

$$\kappa = \frac{\mathbf{p}_1^T \mathbf{u}}{\mathbf{p}_1^T \mathbf{n}} \Rightarrow -1 \quad (2.0.8)$$

\therefore Point of contact for tangent of parabola is,

$$\begin{pmatrix} \mathbf{u} + \kappa \mathbf{n}^T \\ \mathbf{V} \end{pmatrix} \mathbf{q} = \begin{pmatrix} -\mathbf{f} \\ \kappa \mathbf{n} - \mathbf{u} \end{pmatrix} \quad (2.0.9)$$

$$\Rightarrow \begin{pmatrix} -4 & 4 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} = \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix} \quad (2.0.10)$$

$$\Rightarrow \mathbf{q} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad (2.0.11)$$

\therefore Point of contact for tangent of given curve is

$$\mathbf{q} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad (2.0.12)$$

Plot of Tangent to the given curve -

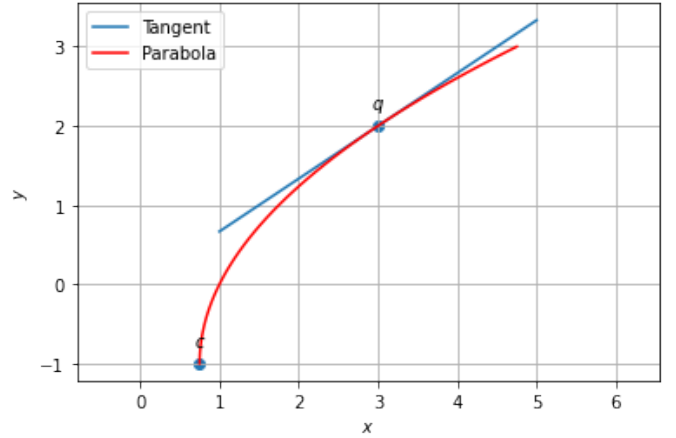


Fig. 2.1: Tangent to Parabola.