ASSIGNMENT 7

C.RAMYA TULASI

Download all python codes from

https://github.com/CRAMYATULASI/ASSIGNMENT7/tree/main/ASSIGNMENT7/CODES

Latex-tikz codes from

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1 Question No 2.79

Find points on the curve $\mathbf{x}^T \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{16} \end{pmatrix} \mathbf{x} = 1$ at which tangents are

1) parallel to x-axis
2) parallel to y-axis.

2 SOLUTION

Given curve,

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{16} \end{pmatrix} \mathbf{x} = 1 \tag{2.0.1}$$

where,

$$\mathbf{V} = \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{16} \end{pmatrix}, \mathbf{V}^{-1} = \begin{pmatrix} 9 & 0 \\ 0 & 16 \end{pmatrix} \mathbf{u} = 0, \mathbf{f} = -1 \tag{2.0.2}$$

$$|\mathbf{V}| > 0 \tag{2.0.3}$$

$\therefore$ Given curve (2.0.1) is ellipse.

For an ellipse, point of contact for tangent is

$$\mathbf{q} = \mathbf{V}^{-1}(\kappa\mathbf{n} - \mathbf{u}) \tag{2.0.4}$$

$$= \mathbf{V}^{-1}\kappa\mathbf{n} (\because \mathbf{u} = 0). \tag{2.0.5}$$

where,

$$\kappa = \pm \sqrt{\frac{\mathbf{u^T V^{-1} u - f}}{\mathbf{n^T V^{-1} n}}} \tag{2.0.6}$$

$$= \pm \sqrt{\frac{\mathbf{-f}}{\mathbf{n^T V^{-1} n}}} (\because \mathbf{u} = 0) \tag{2.0.7}$$

1) Tangents are parallel to x-axis then direction and normal vectors are,

$$\mathbf{m_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{n_1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \kappa_1 = \pm \sqrt{\frac{\mathbf{-f}}{\mathbf{n_1^T V^{-1} n_1}}} \tag{2.0.8}$$

$$= \pm \frac{1}{4} \tag{2.0.9}$$

$\therefore$ By Substituting $\kappa_1, \mathbf{n_1}, \mathbf{V}^{-1}$ in (2.0.5)

$$\mathbf{q} = \mathbf{V}^{-1}\kappa_1\mathbf{n_1} \tag{2.0.10}$$

$$= \begin{pmatrix} 0 \\ \pm 4 \end{pmatrix} \tag{2.0.11}$$

$\therefore$ Point of contact for tangents of ellipse are,

$$\mathbf{q_1} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \mathbf{q_2} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \tag{2.0.12}$$

2) Tangents are parallel to y-axis then direction and normal vectors are,

$$\mathbf{m_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mathbf{n_2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \kappa_2 = \pm \sqrt{\frac{\mathbf{-f}}{\mathbf{n_2^T V^{-1} n_2}}} \tag{2.0.13}$$

$$= \pm \frac{1}{3} \tag{2.0.14}$$

$\therefore$ By Substituting $\kappa_2, \mathbf{n_2}, \mathbf{V}^{-1}$ in (2.0.5)

$$\mathbf{q} = \mathbf{V}^{-1}\kappa_2\mathbf{n_2} \tag{2.0.15}$$

$$= \begin{pmatrix} 0 \\ \pm 3 \end{pmatrix} \tag{2.0.16}$$

$\therefore$ Point of contact for tangents of ellipse are,

$$\mathbf{q_3} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{q_4} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \tag{2.0.17}$$

Plot of Tangents to the given curve -

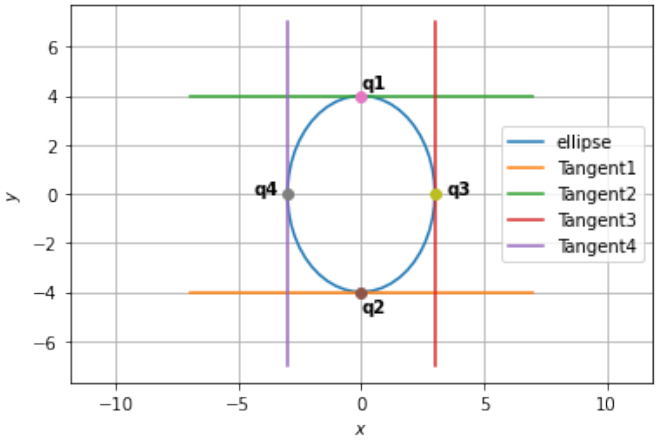

Fig. 2.1: Tangents to ELLIPSE.