ASSIGNMENT 7

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Download all python codes from

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https://github.com/CRAMYATULASI/
ASSIGNMENT7/tree/main/ASSIGNMENT7/
CODES
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Latex-tikz codes from

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https://github.com/CRAMYATULASI/
ASSIGNMENT7/tree/main/ASSIGNMENT7
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1 Question No 2.79

Find points on the curve $\mathbf{x}^T\begin{pmatrix}\frac{1}{9} & 0\\ 0 & \frac{1}{16}\end{pmatrix}\mathbf{x} = 1$ at which tangents are

1) parallel to x-axis
2) parallel to y-axis.

2 SOLUTION

Given curve,

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{9} & 0\\ 0 & \frac{1}{16}\end{pmatrix}\mathbf{x} = 1 \tag{2.0.1}$$

where,

$$\mathbf{V} = \begin{pmatrix}\frac{1}{9} & 0\\ 0 & \frac{1}{16}\end{pmatrix}, \mathbf{V}^{-1} = \begin{pmatrix}9 & 0\\ 0 & 16\end{pmatrix}\mathbf{u} = 0, \mathbf{f} = -1 \tag{2.0.2}$$

$$|\mathbf{V}| > 0 \tag{2.0.3}$$

$\therefore$ Given curve (2.0.1) is ellipse.

For an ellipse, point of contact for tangent is

$$\mathbf{q} = \mathbf{V}^{-1}(\kappa\mathbf{n} - \mathbf{u}) \tag{2.0.4}$$
$$= \mathbf{V}^{-1}\kappa\mathbf{n} \qquad (\because \mathbf{u} = 0). \tag{2.0.5}$$

where,

$$\kappa = \pm\sqrt{\frac{\mathbf{u^TV^{-1}u - f}}{\mathbf{n^TV^{-1}n}}} \tag{2.0.6}$$
$$= \pm\sqrt{\frac{\mathbf{-f}}{\mathbf{n^TV^{-1}n}}} \qquad (\because \mathbf{u} = 0) \tag{2.0.7}$$

1) Tangents are parallel to x-axis then direction and normal vectors are,

$$\mathbf{m_1} = \begin{pmatrix}1\\0\end{pmatrix}, \mathbf{n_1} = \begin{pmatrix}0\\1\end{pmatrix}, \kappa_1 = \pm\sqrt{\frac{\mathbf{-f}}{\mathbf{n_1^TV^{-1}n_1}}} \tag{2.0.8}$$
$$= \pm\frac{1}{4} \tag{2.0.9}$$

$\therefore$ By Substituting $\kappa_1, \mathbf{n_1}, \mathbf{V}^{-1}$ in (2.0.5)

$$\mathbf{q} = \mathbf{V}^{-1}\kappa_1\mathbf{n_1} \tag{2.0.10}$$
$$= \begin{pmatrix}0\\ \pm4\end{pmatrix} \tag{2.0.11}$$

$\therefore$ Point of contact for tangents of ellipse are,

$$\mathbf{q_1} = \begin{pmatrix}0\\4\end{pmatrix}, \mathbf{q_2} = \begin{pmatrix}0\\-4\end{pmatrix} \tag{2.0.12}$$

2) Tangents are parallel to y-axis then direction and normal vectors are,

$$\mathbf{m_2} = \begin{pmatrix}0\\1\end{pmatrix}, \mathbf{n_2} = \begin{pmatrix}1\\0\end{pmatrix}, \kappa_2 = \pm\sqrt{\frac{\mathbf{-f}}{\mathbf{n_2^TV^{-1}n_2}}} \tag{2.0.13}$$
$$= \pm\frac{1}{3} \tag{2.0.14}$$

$\therefore$ By Substituting $\kappa_2, \mathbf{n_2}, \mathbf{V}^{-1}$ in (2.0.5)

$$\mathbf{q} = \mathbf{V}^{-1}\kappa_2\mathbf{n_2} \tag{2.0.15}$$
$$= \begin{pmatrix}0\\ \pm3\end{pmatrix} \tag{2.0.16}$$

$\therefore$ Point of contact for tangents of ellipse are,

$$\mathbf{q_3} = \begin{pmatrix}3\\0\end{pmatrix}, \mathbf{q_4} = \begin{pmatrix}-3\\0\end{pmatrix} \tag{2.0.17}$$

Plot of Tangents to the given curve -

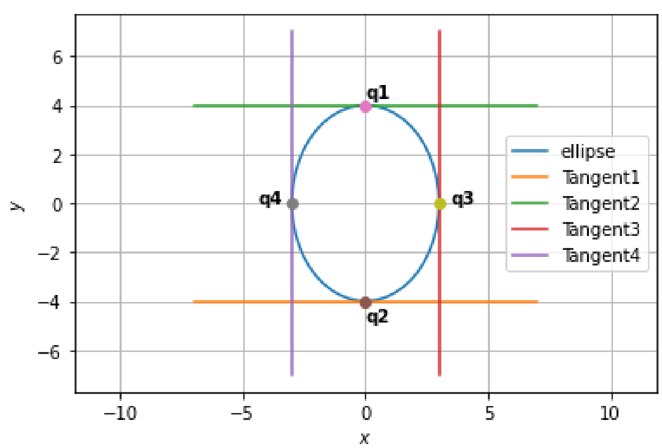

Fig. 2.1: Tangents to ELLIPSE.