# ASSIGNMENT 8

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Download all python codes from

https://github.com/CRAMYATULASI/ASSIGNMENT8/tree/main/ASSIGNMENT8/CODES

Latex-tikz codes from

https://github.com/CRAMYATULASI/ASSIGNMENT8/tree/main/ASSIGNMENT8

## 1 Question No 2.75

Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}$,$x \neq 2$ at $x$= 10.

## 2 SOLUTION

Given curve,

$$y = \frac{x-1}{x-2} \tag{2.0.1}$$

Above equation can be expressed as,

$$yx - 2y - x + 1 = 0 \tag{2.0.2}$$

From the above we can say,

$$\mathbf{V} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \mathbf{V}^{-1} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \mathbf{u} = \begin{pmatrix} -\frac{1}{2} \\ -1 \end{pmatrix}, f = 1 \tag{2.0.3}$$

$$|\mathbf{V}| < 0 \tag{2.0.4}$$

$\therefore$ Given curve (2.0.1) is hyperbola. Let,slope of tangent be $r$ ,then direction vector and normal vector of tangent to (2.0.1) are

$$\mathbf{m} = \begin{pmatrix} 1 \\ r \end{pmatrix}, \mathbf{n} = \begin{pmatrix} r \\ -1 \end{pmatrix} \tag{2.0.5}$$

$$\kappa = \pm \sqrt{\frac{\mathbf{u^T V^{-1} u - f}}{\mathbf{n^T V^{-1} n}}} \tag{2.0.6}$$

$$\kappa = \sqrt{\frac{-1}{4r}} \tag{2.0.7}$$

For hyperbola, point of contact for tangent is

$$\mathbf{q} = \mathbf{V}^{-1}(\kappa\mathbf{n} - \mathbf{u}) \tag{2.0.8}$$

$$\implies \mathbf{Vq} + \mathbf{u} = \kappa\mathbf{n} \tag{2.0.9}$$

$$\implies \begin{pmatrix} \frac{1}{16} \\ 4 \end{pmatrix} = \kappa\mathbf{n} \quad (\because \mathit{From} \quad (2.0.3)) \tag{2.0.10}$$

$$\implies \begin{pmatrix} \frac{1}{16} \\ 4 \end{pmatrix} = \sqrt{\frac{-1}{4r}} \begin{pmatrix} r \\ -1 \end{pmatrix} \tag{2.0.11}$$

$$\implies \begin{pmatrix} \frac{1}{16} \\ 4 \end{pmatrix} = \begin{pmatrix} r\sqrt{\frac{-1}{4r}} \\ -\sqrt{\frac{-1}{4r}} \end{pmatrix} \tag{2.0.12}$$

$$\implies -\sqrt{\frac{-1}{4r}} = 4 \tag{2.0.13}$$

$$\implies r = -\frac{1}{64} \tag{2.0.14}$$

$\therefore$ The slope of tangent to the given curve at **x**=10 is $r = -\frac{1}{64}$.

Plot of Tangent to the given curve -

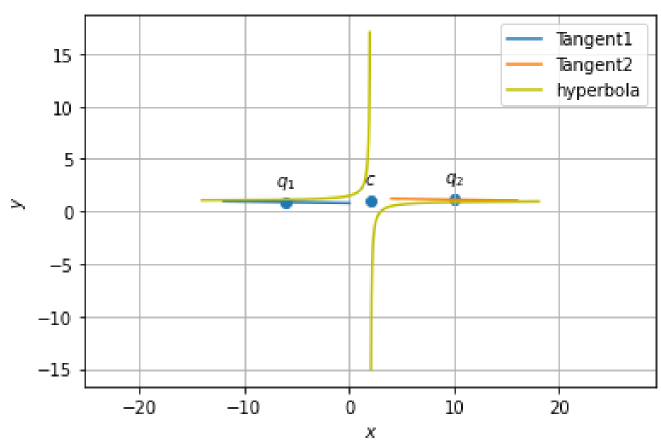


Fig. 2.1: Tangent to HYPERBOLA.