

ASSIGNMENT 8

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Download all python codes from

<https://github.com/CRAMYATULASI/ASSIGNMENT8/tree/main/ASSIGNMENT8/CODES>

Latex-tikz codes from

<https://github.com/CRAMYATULASI/ASSIGNMENT8/tree/main/ASSIGNMENT8>

where,

$$\kappa = \pm \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - \mathbf{f}}{\mathbf{n}^T \mathbf{V}^{-1} \mathbf{n}}} \quad (2.0.7)$$

$$\kappa = \sqrt{\frac{-1}{4r}} \quad (2.0.8)$$

From, (2.0.6) point of contact of tangent to given curve

$$\mathbf{q} = \begin{pmatrix} -2\sqrt{\frac{-1}{4r}} + 2 \\ 2\sqrt{\frac{-1}{4r}} + 1 \end{pmatrix} \quad (2.0.9)$$

1 QUESTION No 2.75

Find the slope of the tangent to the curve $y = \frac{x-1}{x+1}$, $x \neq -1$ at $x = 10$.

2 SOLUTION

Given curve,

$$y = \frac{x-1}{x+1} \quad (2.0.1)$$

Above equation can be expressed as,

$$yx - 2y - x + 1 = 0 \quad (2.0.2)$$

From the above we can say,

$$\mathbf{V} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \mathbf{V}^{-1} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{u} = \begin{pmatrix} -\frac{1}{2} & -1 \end{pmatrix}, f = 1 \quad (2.0.3)$$

$$|\mathbf{V}| < 0 \quad (2.0.4)$$

\therefore Given curve (2.0.1) is hyperbola. Let, slope of tangent be r , then direction vector and normal vector of tangent to (2.0.1) are

$$\mathbf{m} = \begin{pmatrix} 1 \\ r \end{pmatrix}, \mathbf{n} = \begin{pmatrix} r \\ -1 \end{pmatrix} \quad (2.0.5)$$

For hyperbola, point of contact for tangent is

$$\mathbf{q} = \mathbf{V}^{-1}(\kappa \mathbf{n} - \mathbf{u}) \quad (2.0.6)$$

From given,

$$x = 10 \quad (2.0.10)$$

$$\Rightarrow \mathbf{q}^T \mathbf{e}_1 = 10 \quad (\because x = \mathbf{q}^T \mathbf{e}_1) \quad (2.0.11)$$

$$\Rightarrow -2\sqrt{\frac{-1}{4r}} + 2 = 10 \quad (\because \text{From (2.0.9)}) \quad (2.0.12)$$

$$\Rightarrow -2\sqrt{\frac{-1}{4r}} = 8 \quad (2.0.13)$$

$$\Rightarrow r = -\frac{1}{64} \quad (2.0.14)$$

\therefore The slope of tangent to the given curve at $x=10$ is $-\frac{1}{64}$ and direction vector and normal vector are

$$\mathbf{m} = \begin{pmatrix} 1 \\ -\frac{1}{64} \end{pmatrix}, \mathbf{n} = \begin{pmatrix} -\frac{1}{64} \\ -1 \end{pmatrix} \quad (2.0.15)$$

And From (2.0.8)

$$\kappa = \sqrt{\frac{-1}{4r}} \quad (2.0.16)$$

$$= \pm 4 \quad (2.0.17)$$

\therefore From (2.0.9) Point of contact for tangents of hyperbola are,

$$\mathbf{q}_1 = \begin{pmatrix} 10 \\ \frac{9}{8} \end{pmatrix}, \mathbf{q}_2 = \begin{pmatrix} -6 \\ \frac{7}{8} \end{pmatrix} \quad (2.0.18)$$

Plot of Tangent to the given curve -

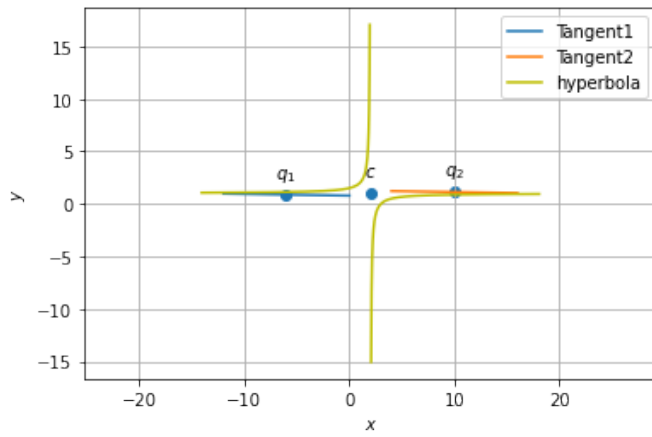


Fig. 2.1: Tangent to HYPERBOLA.