ASSIGNMENT 8

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Download all python codes from

https://github.com/CRAMYATULASI/ASSIGNMENT8/tree/main/ASSIGNMENT8/CODES

Latex-tikz codes from

https://github.com/CRAMYATULASI/ASSIGNMENT8/tree/main/ASSIGNMENT8

1 Question No 2.75

Find the slope of the tangent to the curve $y = \frac{x-1}{x+1}$,$x \neq 2$ at x= 10.

2 SOLUTION

Given curve,

$$y = \frac{x-1}{x+1} \tag{2.0.1}$$

Above equation can be expressed as,

$$yx - 2y - x + 1 = 0 \tag{2.0.2}$$

From the above we can say,

$$\mathbf{V} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \mathbf{V}^{-1} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \mathbf{u} = \begin{pmatrix} -\frac{1}{2} & -1 \end{pmatrix}, f = 1 \tag{2.0.3}$$

$$|\mathbf{V}| < 0 \tag{2.0.4}$$

$\therefore$ Given curve (2.0.1) is hyperbola. Let,slope of tangent be r ,then direction vector and normal vector of tangent to (2.0.1) are

$$\mathbf{m} = \begin{pmatrix} 1 \\ r \end{pmatrix}, \mathbf{n} = \begin{pmatrix} r \\ -1 \end{pmatrix} \tag{2.0.5}$$

For hyperbola, point of contact for tangent is

$$\mathbf{q} = \mathbf{V}^{-1}(\kappa\mathbf{n} - \mathbf{u}) \tag{2.0.6}$$

where,

$$\kappa = \pm\sqrt{\frac{\mathbf{u^T V^{-1} u - f}}{\mathbf{n^T V^{-1} n}}} \tag{2.0.7}$$

$$\kappa = \sqrt{\frac{-1}{4r}} \tag{2.0.8}$$

From, (2.0.6) point of contact of tangent to given curve

$$\mathbf{q} = \begin{pmatrix} -2\sqrt{\frac{-1}{4r}} + 2 \\ 2\sqrt{\frac{-r}{4}} + 1 \end{pmatrix} \tag{2.0.9}$$

From given,

$$x = 10 \tag{2.0.10}$$

$$\implies \mathbf{q^T e_1} = 10 \qquad (\because x = \mathbf{q^T e_1}) \tag{2.0.11}$$

$$\implies -2\sqrt{\frac{-1}{4r}} + 2 = 10 \qquad (\because From \quad (2.0.9)) \tag{2.0.12}$$

$$\implies -2\sqrt{\frac{-1}{4r}} = 8 \tag{2.0.13}$$

$$\implies r = -\frac{1}{64} \tag{2.0.14}$$

$\therefore$ The slope of tangent to the given curve at $\mathbf{x}$=10 is $-\frac{1}{64}$ and direction vector and normal vector are

$$\mathbf{m} = \begin{pmatrix} 1 \\ -\frac{1}{64} \end{pmatrix}, \mathbf{n} = \begin{pmatrix} -\frac{1}{64} \\ -1 \end{pmatrix} \tag{2.0.15}$$

And From (2.0.9)

$$\kappa = \sqrt{\frac{-1}{4r}} \tag{2.0.16}$$

$$= \pm 4 \tag{2.0.17}$$

$\therefore$From (**??**) Point of contact for tangents of hyperbola are,

$$\mathbf{q_1} = \begin{pmatrix} 10 \\ \frac{9}{8} \end{pmatrix}, \mathbf{q_2} = \begin{pmatrix} -6 \\ \frac{7}{8} \end{pmatrix} \tag{2.0.18}$$

Plot of Tangent to the given curve -

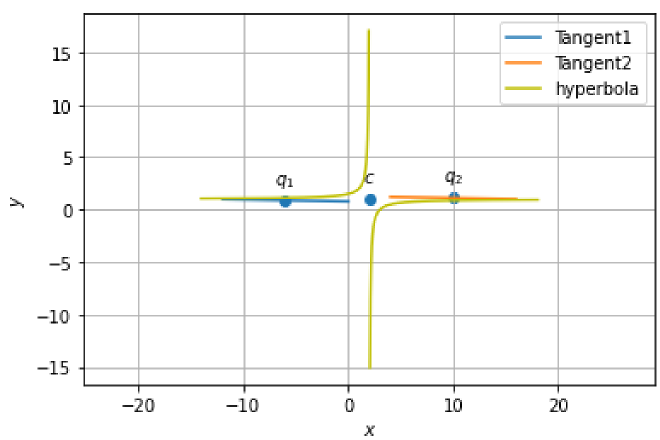

Fig. 2.1: Tangent to HYPERBOLA.