1

ASSIGNMENT 3

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Download all python codes from

https://github.com/CRAMYATULASI/
ASSIGNMENT_3/tree/main/ASSIGNMENT3/
CODES

and latex-tikz codes from

https://github.com/CRAMYATULASI/ ASSIGNMENT_3/tree/main/ASSIGNMENT3

1 Question No 2.56

Construct a tangent to a circle of radius 4 units from a point on concentric circle of radius 6 units.

2 Solution

Data from the given question

	Symbols	Circle1	Circle2
Centre	0	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
Radius	r_1,r_2	4	6

Let P be a point on circle with radius 6.

$$\therefore \mathbf{P} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \tag{2.0.1}$$

Let PQ and PR be tangents from point P on circle with radius 6 to the points Q and R on circle with radius 4.

We know a tangent is always perpendicular to the radius.

$$\therefore OQ \perp QP \qquad (2.0.2)$$

Now,

$$(\mathbf{O} - \mathbf{Q})^T (\mathbf{Q} - \mathbf{P}) = 0 \quad (:: OQ \perp QP) \quad (2.0.3)$$

$$\mathbf{Q}^{T}(\mathbf{Q} - \mathbf{P}) = 0 \quad \left(:: \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) \tag{2.0.4}$$

$$\mathbf{Q}^T \mathbf{Q} - \mathbf{Q}^T \mathbf{P} = 0 \tag{2.0.5}$$

$$\|\mathbf{Q}\|^2 = \mathbf{Q}^T \mathbf{P} \tag{2.0.6}$$

$$\|\mathbf{Q}\|^2 = \mathbf{P}^T \mathbf{Q} \quad \left(:: \mathbf{Q}^T \mathbf{P} = \mathbf{P}^T \mathbf{Q}\right)$$
(2.0.7)

$$\mathbf{P}^T \mathbf{Q} = 16 \quad (: \|\mathbf{Q}\|^2 = 16) \quad (2.0.8)$$

$$\begin{pmatrix} 6 & 0 \end{pmatrix} \mathbf{Q} = 16 \quad \left(:: \mathbf{P} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \right) \tag{2.0.9}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{Q} = \frac{8}{3} \tag{2.0.10}$$

$$\mathbf{Q} = \begin{pmatrix} \frac{8}{3} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.11}$$

$$\mathbf{Q} = \mathbf{q} + \lambda \mathbf{m} \tag{2.0.12}$$

$$\mathbf{q} = \begin{pmatrix} \frac{8}{3} \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.13}$$

We know,

$$\|\mathbf{q} + \lambda \mathbf{m}\|^2 = 16 \tag{2.0.14}$$

$$(\mathbf{q} + \lambda \mathbf{m})^T (\mathbf{q} + \lambda \mathbf{m}) = r^2$$
 (2.0.15)

$$\lambda^2 = \frac{r^2 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2}$$
 (2.0.16)

$$\lambda = \pm 2.98 \tag{2.0.17}$$

Substitute λ value in (2.0.11) we get

$$\mathbf{Q} = \begin{pmatrix} \frac{8}{3} \\ 2.98 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} \frac{8}{3} \\ -2.98 \end{pmatrix} \tag{2.0.18}$$

Plot of Tangents PQ and PR:

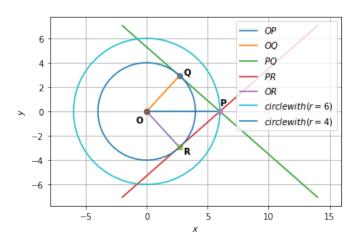


Fig. 2.1: Tangent lines to circle of radius 4 units.