

ASSIGNMENT 3

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Download all python codes from

https://github.com/CRAMYATULASI/ASSIGNMENT_3/tree/main/ASSIGNMENT3/CODES

and latex-tikz codes from

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1 QUESTION No 2.56

Construct a tangent to a circle of radius 4 units from a point on concentric circle of radius 6 units.

2 SOLUTION

Data from the given question

	Symbols	Circle1	Circle2
Centre	\mathbf{O}	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
Radius	r_1, r_2	4	6

Let P be a point on circle with radius 6.

$$\therefore \mathbf{P} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (2.0.1)$$

Let PQ and PR be tangents from point P on circle with radius 6 to the points Q and R on circle with radius 4.

We know a tangent is always perpendicular to the radius.

$$\therefore OQ \perp QP \quad (2.0.2)$$

Now,

$$(\mathbf{O} - \mathbf{Q})^T (\mathbf{Q} - \mathbf{P}) = 0 \quad (\because OQ \perp QP) \quad (2.0.3)$$

$$\mathbf{Q}^T (\mathbf{Q} - \mathbf{P}) = 0 \quad \left(\because \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) \quad (2.0.4)$$

$$\mathbf{Q}^T \mathbf{Q} - \mathbf{Q}^T \mathbf{P} = 0 \quad (2.0.5)$$

$$\|\mathbf{Q}\|^2 = \mathbf{Q}^T \mathbf{P} \quad (2.0.6)$$

$$\|\mathbf{Q}\|^2 = \mathbf{P}^T \mathbf{Q} \quad (\because \mathbf{Q}^T \mathbf{P} = \mathbf{P}^T \mathbf{Q}) \quad (2.0.7)$$

$$\mathbf{P}^T \mathbf{Q} = 16 \quad (\because \|\mathbf{Q}\|^2 = 16) \quad (2.0.8)$$

$$\begin{pmatrix} 6 & 0 \end{pmatrix} \mathbf{Q} = 16 \quad \left(\because \mathbf{P} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \right) \quad (2.0.9)$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{Q} = 2.7 \quad (2.0.10)$$

$$\mathbf{Q} = \begin{pmatrix} 2.7 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.11)$$

$$\mathbf{Q} = \mathbf{q} + \lambda \mathbf{m} \quad (2.0.12)$$

$$\mathbf{q} = \begin{pmatrix} 2.7 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.13)$$

We know,

$$\|\mathbf{q} + \lambda \mathbf{m}\|^2 = 9 \quad (2.0.14)$$

$$(\mathbf{q} + \lambda \mathbf{m})^T (\mathbf{q} + \lambda \mathbf{m}) = r^2 \quad (2.0.15)$$

$$\lambda^2 = \frac{r^2 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} \quad (2.0.16)$$

$$\lambda^2 = \frac{4^2 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} \quad (\because r = 4) \quad (2.0.17)$$

$$\lambda^2 = \frac{16 - 7.29}{1} \quad (2.0.18)$$

$$\lambda = \pm 2.95 \quad (2.0.19)$$

Substitute λ value in (2.0.11) we get

$$\mathbf{Q} = \begin{pmatrix} 2.7 \\ 2.95 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 2.7 \\ -2.95 \end{pmatrix} \quad (2.0.20)$$

Plot of Tangents PQ and PR :

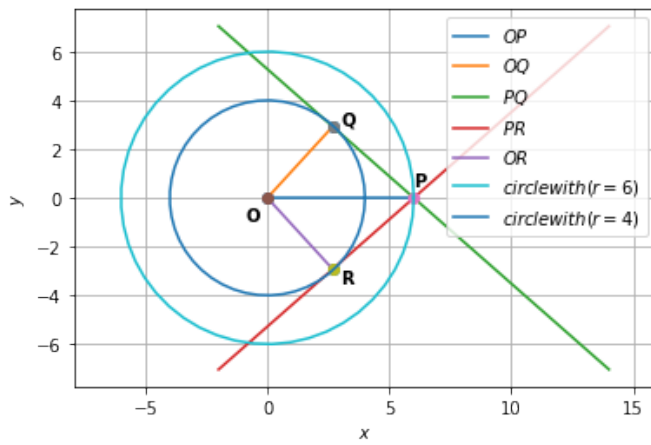


Fig. 2.1: Tangents to a circle of radius 4 units from a point on concentric circle of radius 6 units.