ASSIGNMENT 4

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Download all python codes from

https://github.com/CRAMYATULASI/ASSIGNMENT4/tree/main/ASSIGNMENT4/CODES

and latex-tikz codes from

https://github.com/CRAMYATULASI/ASSIGNMENT4/tree/main/ASSIGNMENT4

1 Question No 2.6

Find out whether the lines representing the following pairs of linear equations intersect at a point are parallel or coincident.

1)
$$\begin{pmatrix}5 & -4\end{pmatrix}\mathbf{x} = -8 \\ \begin{pmatrix}7 & 6\end{pmatrix}\mathbf{x} = 9 \tag{1.0.1}$$

2)
$$\begin{pmatrix}9 & 3\end{pmatrix}\mathbf{x} = -12 \\ \begin{pmatrix}18 & 6\end{pmatrix}\mathbf{x} = -24 \tag{1.0.2}$$

3)
$$\begin{pmatrix}6 & -3\end{pmatrix}\mathbf{x} = -10 \\ \begin{pmatrix}2 & -1\end{pmatrix}\mathbf{x} = -9 \tag{1.0.3}$$

2 SOLUTION

1)
$$\begin{pmatrix}5 & -4\end{pmatrix}\mathbf{x} = -8 \\ \begin{pmatrix}7 & 6\end{pmatrix}\mathbf{x} = 9 \tag{2.0.1}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix}5 & -4\\7 & 6\end{pmatrix}\mathbf{x} = \begin{pmatrix}-8\\9\end{pmatrix} \tag{2.0.2}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}5 & -4 & -8\\7 & 6 & 9\end{pmatrix} \xleftrightarrow{R_1\leftarrow\ 7\frac{R_1}{5}} \begin{pmatrix}7 & \frac{-28}{5} & \frac{-56}{5}\\7 & 6 & 9\end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_2\leftarrow R_2-R_1} \begin{pmatrix}7 & \frac{-28}{5} & \frac{-56}{5}\\0 & \frac{58}{5} & \frac{101}{5}\end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{R_1\leftarrow 5\frac{R_1}{7}} \begin{pmatrix}5 & -4 & -8\\0 & \frac{58}{5} & \frac{101}{5}\end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_2\leftarrow 5R_2} \begin{pmatrix}5 & -4 & -8\\0 & 58 & 101\end{pmatrix} \tag{2.0.6}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix}5 & -4 & -8\\7 & 6 & 9\end{pmatrix} \tag{2.0.7}$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix}5 & -4\\7 & 6\end{pmatrix} \tag{2.0.8}$$

is also 2.

$$\begin{aligned}\because Rank\begin{pmatrix}5 & -4\\7 & 6\end{pmatrix} &= Rank\begin{pmatrix}5 & -4 & -8\\7 & 6 & 9\end{pmatrix}\\ &= dim\begin{pmatrix}5 & -4\\7 & 6\end{pmatrix}\\ &= 2\end{aligned} \tag{2.0.9}$$

$\therefore$ Given lines (1.0.1) have unique solution so we can say they intersect.

2)
$$\begin{pmatrix}9 & 3\end{pmatrix}\mathbf{x} = -12 \\ \begin{pmatrix}18 & 6\end{pmatrix}\mathbf{x} = -24 \tag{2.0.10}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix}9 & 3\\18 & 6\end{pmatrix}\mathbf{x} = \begin{pmatrix}-12\\-24\end{pmatrix} \tag{2.0.11}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}9 & 3 & -12\\18 & 6 & -24\end{pmatrix} \xleftrightarrow{R_1\leftarrow 7\frac{R_1}{5}} \begin{pmatrix}7 & \frac{-28}{5} & \frac{-56}{5}\\7 & 6 & 9\end{pmatrix} \tag{2.0.12}$$

$$\xleftrightarrow{R_1\leftarrow \frac{18R_1}{9}} \begin{pmatrix}18 & 6 & -24\\18 & 6 & -24\end{pmatrix} \tag{2.0.13}$$

$$\xleftrightarrow{R_2\leftarrow R_2-R_1} \begin{pmatrix}18 & 6 & -24\\0 & 0 & 0\end{pmatrix} \tag{2.0.14}$$

$$\tag{2.0.15}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix}9 & 3 & -12\\18 & 6 & -24\end{pmatrix} \tag{2.0.16}$$

results in a matrix with 1 nonzero rows, its rank is 1. Similarly, the rank of the matrix

$$\begin{pmatrix}9 & 3\\18 & 6\end{pmatrix} \tag{2.0.17}$$

is also 1.

$$\begin{aligned}\because Rank\begin{pmatrix}9 & 3\\18 & 6\end{pmatrix} = Rank\begin{pmatrix}9 & 3 & -12\\18 & 6 & -24\end{pmatrix} = 1\\ < dim\begin{pmatrix}9 & 3\\18 & 6\end{pmatrix} = 2\end{aligned} \tag{2.0.18}$$

$\therefore$ Given lines (1.0.2) have infinitely many solutions so we can say they coincide.

3)

$$\begin{aligned}\begin{pmatrix}6 & -3\end{pmatrix}\mathbf{x} = -10\\ \begin{pmatrix}2 & -1\end{pmatrix}\mathbf{x} = -9\end{aligned} \tag{2.0.19}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix}6 & -3\\2 & -1\end{pmatrix}\mathbf{x} = \begin{pmatrix}-10\\-9\end{pmatrix} \tag{2.0.20}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}6 & -3 & -10\\2 & -1 & -9\end{pmatrix} \xleftrightarrow{R_1\leftarrow \frac{2R_1}{6}} \begin{pmatrix}2 & -1 & \frac{-10}{3}\\2 & -1 & -9\end{pmatrix} \tag{2.0.21}$$

$$\xleftrightarrow{R_2\leftarrow R_2-R_1} \begin{pmatrix}2 & -1 & \frac{-10}{3}\\0 & 0 & \frac{-17}{3}\end{pmatrix} \tag{2.0.22}$$

$$\xleftrightarrow{R_1\leftarrow 3R_1} \begin{pmatrix}6 & -3 & -10\\0 & 0 & \frac{-17}{3}\end{pmatrix} \tag{2.0.23}$$

$$\tag{2.0.24}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix}6 & -3 & -10\\2 & -1 & -9\end{pmatrix} \tag{2.0.25}$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix}6 & -3\\2 & -1\end{pmatrix} \tag{2.0.26}$$

is 1.

$$\because Rank\begin{pmatrix}6 & -3\\2 & -1\end{pmatrix} \neq Rank\begin{pmatrix}6 & -3 & -10\\2 & -1 & -9\end{pmatrix} \tag{2.0.27}$$

$\therefore$ Given lines (1.0.3) have no solution so we can say they are parallel.

PLOT OF GIVEN LINES -

Plot of (1.0.1) -

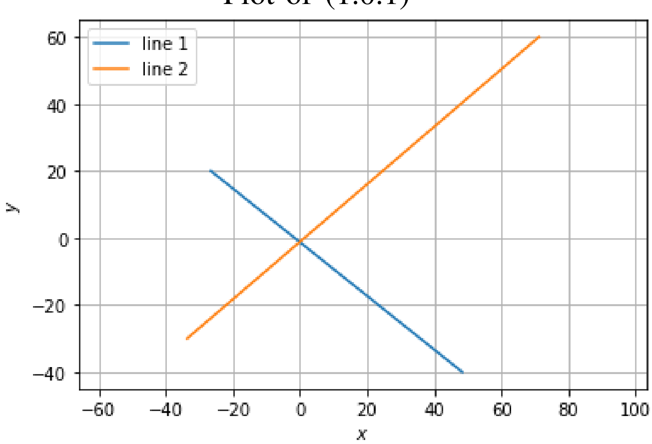

Fig. 2.1: INTERSECTING LINES.

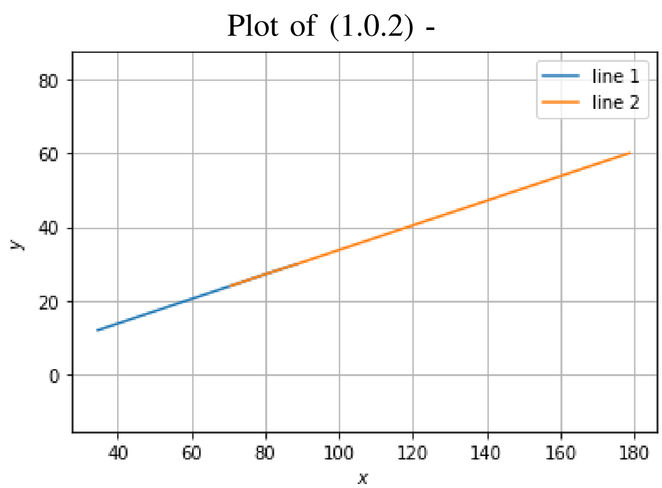

Fig. 2.2: SAME LINES

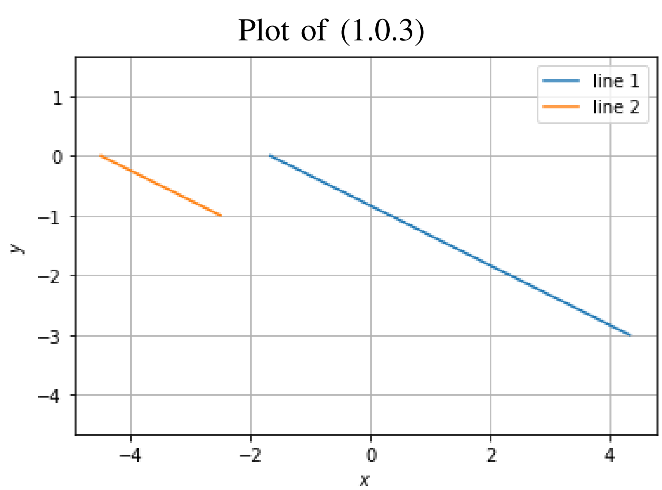

Fig. 2.3: Parallel lines