1

ASSIGNMENT 6

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Download all python codes from

https://github.com/CRAMYATULASI/ ASSIGNMENT6/tree/main/ASSIGNMENT6/ CODES

Latex-tikz codes from

https://github.com/CRAMYATULASI/ ASSIGNMENT6/tree/main/ASSIGNMENT6

1 Question No 2.42

Find the point at which the tangent to the curve $y = \sqrt{4x - 3} - 1$ has its slope $\frac{2}{3}$.

2 SOLUTION

Given curve,

$$y = \sqrt{4x - 3} - 1 \tag{2.0.1}$$

$$\implies (y+1)^2 = 4x - 3$$
 (2.0.2)

$$\implies y^2 - 4x + 2y + 4 = 0 \tag{2.0.3}$$

From above equation,

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -2 & 1 \end{pmatrix}, \mathbf{f} = 4 \tag{2.0.4}$$

$$|\mathbf{V}| = 0 \tag{2.0.5}$$

:. Given curve (2.0.1) is parabola. In standard form,

$$\mathbf{P} = \mathbf{I} \implies \mathbf{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.6}$$

Given slope of tangents is $\frac{2}{3}$ then direction vectors and normal vectors are,

$$\mathbf{m} = \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix} \implies \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$
 (2.0.7)

$$\kappa = \frac{\mathbf{p}_1^{\mathrm{T}} \mathbf{u}}{\mathbf{p}_1^{\mathrm{T}} \mathbf{n}} \implies = -1 \tag{2.0.8}$$

.. Point of contact for tangent of parabola is,

$$\begin{pmatrix} \mathbf{u} + \kappa \mathbf{n}^{\mathrm{T}} \\ \mathbf{V} \end{pmatrix} \mathbf{q} = \begin{pmatrix} -\mathbf{f} \\ \kappa \mathbf{n} - \mathbf{u} \end{pmatrix}$$
 (2.0.9)

$$\Longrightarrow \begin{pmatrix} -4 & 4\\ 0 & 0\\ 0 & 1 \end{pmatrix} \mathbf{q} = \begin{pmatrix} -4\\ 0\\ 2 \end{pmatrix} \tag{2.0.10}$$

$$\implies \mathbf{q} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{2.0.11}$$

.. Point of contact for tangent of given curve is

$$\mathbf{q} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{2.0.12}$$

Plot of Tangent to the given curve -

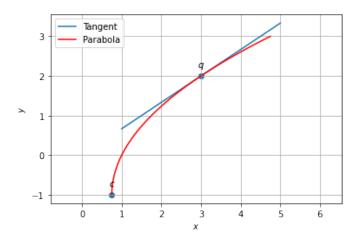


Fig. 2.1: Tangent to Parabola.