Modeling Watercolor by Simulating Diffusion, Pigment, and Paper Fibers

David Small

Visible Language Workshop, MIT Media Lab
Massachusetts Institute of Technology
20 Ames St, Cambridge, MA 02139

Abstract

This paper explores a parallel approach to the problem of predicting the actions of pigment and water when applied to paper fibers. This work was done on the Connection Machine II, whose parallel architecture allows one to cast the problem as that of a complex cellular automata. One defines simple rules for the behavior of each cell based on the state of that cell and its immediate neighbors. By repeating the computation for each cell in the paper over many time steps, elaborate and realistic behaviors can be achieved. The simulation takes into account diffusion, surface tension, gravity, humidity, paper absorbency and the molecular weight of each pigment. At each time step a processor associated with each fiber in the paper computes water and pigment gradients, surface tension and gravitational forces, and decides if there should be any movement of material. Pigment and water can be applied and removed (blotting) with masks created from type or scanned images. Use of a parallel processor simplifies the creation and testing of software, and variables can be stored and manipulated at high-precision. The resulting simulation runs at approximately one-tenth real time.

1. Introduction

Digital paint systems have changed the way that we think about the creation of images. Artists can control, manipulate and modify photographic quality images with an ever-increasing variety of tools. In moving from paper to the computer screen, the artist gains an incredible amount of flexibility. However, this is often in exchange for the kind of subtlety and presence that are found in fine art tools and papers. One way to improve the realism of digital imagery is to simulate real-world processes. This paper explores the application of a simulation technique to the domain of paper, pigment and water. By simulating the movement of water and pigments on a paper substrate, one can create images that have many of the same qualities found in traditional watercolor.

In contrast to traditional paintbox tools[2,5,6], the computational power resides in the paper, or substrate, rather than in the tools or graphic objects. This approach is based on cellular automata[7] and the field of transport phenomena[1] Simply put, one can simulate complex-seeming behavior by describing it as the aggregate actions of many tiny, definable cells. Each cell must accurately compute its actions based solely on information it knows about itself and its immediate neighbors. The Connection Machine II's parallel architecture[4] is ideally suited to computing such models. By taking advantage of this specialized hardware it is possible to simulate the way that pigment and water interact with each other and the paper substrate in close to real time. This means that the artist can create images by interacting with the simulation in real time and by controlling factors which he or she already knows about from real-world experience. An advantage of a simulation is that the artist can edit the image by modifying high-level properties, such as gravity or humidity, without having to edit every pixel.

2. Method

The simulation can be broken down into three parts. First, pigment and water are applied to the paper in a variety of ways. The paper characteristics are specified, as well as environmental variables such as humidity and gravity. Second, the movement of pigment and water in response to various forces at discrete time steps is computed. Finally, given the state of the simulation at some discrete time, the image can be rendered in a variety of ways.

The connection machine allows us to have a processor for each pixel in the image. The basic unit of the simulation becomes the pixel, or paper cell, which can be thought of as a group of paper fibers and the spaces between the fibers. The water and pigments are assumed to be evenly distributed across the area of a paper cell, which has constant absorbency, and an initial color. The cell can communicate only with its immediate neighbors. Figure 1 illustrates the topology of the paper cells.

Real paper is, of course, made up of many long, tangled and intertwined fibers held together by glue. Fibers intersect with many other fibers and create small cavities where water and pigment can accumulate, as in Figure 2. The length of the fibers and the amount and type of glue used in making the paper determine its absorbency and diffusion characteristics. This simulation assumes that on the scale of a pixel the aggregate actions of the fibers and empty spaces can be expressed by relatively simple formulas.

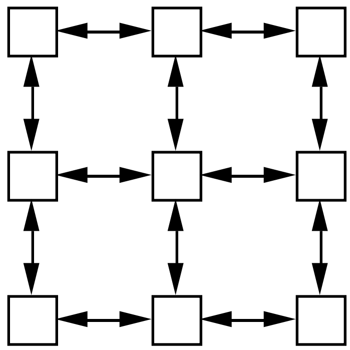

Figure 1

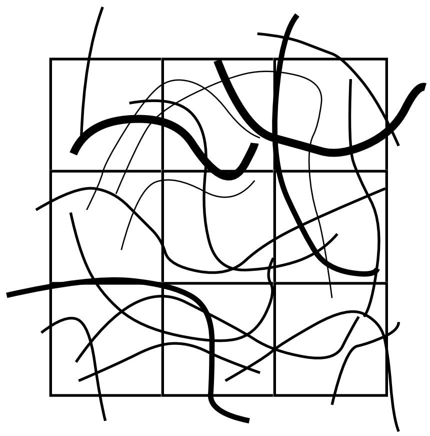

Figure 2

2.1. Setup

Each processor (or cell) has its own memory where the information needed to describe its state is stored. The Connection Machine manipulates these fields as if they were single values. Because the memory allocated to each processor is limited, it is important to be conservative with the use of fields. In this implementation, each cell knows its:

- location
- initial color
- absorbency
- water content
- pigment content (cyan, magenta, and yellow)

The location of a cell is stored in two integer fields (x and y) of sufficient length for the size of the paper (for example if the simulation is 2048 x 1024 pixels in size, it will have location fields 11 and 10 bits long respectively). The fiber color is the color of the paper fiber without any pigment added to it. This is used when one wants to begin with a non-white or unevenly colored surface. Every cell has its own rate of absorbency, which describes a fiber's propensity soak up water, and the pigment it carries, from its environment. If the absorbency were exactly the same for every cell, the pigment would diffuse in a very smooth manner. The amount and type of variation in this field has a strong effect on the perceived texture of the paper. Finally, we have the water and pigment content of the cell. Currently, the simulation uses only three base pigments: cyan, yellow and magenta, although more are certainly possible. A black smudge will eventually separate out into its component pigments as water is applied and time progresses. The artist may want to deal with a pure black pigment which does not separate. The use of such “spot” pigments is allowed and linearly increases the time needed to calculate a time step, or to render the paper. The absorbency, water and pigment fields are all represented as 32-bit floats, both to avoid the accumulation of errors and to take advantage of hardware floating point accelerators.

In addition to the specific information stored with each cell, there is also certain global information. This includes humidity, gravity, the surface tension of the pigment-carrying medium (i.e. water), and the weight of the pigments used. This last term is simply the propensity of a pigment to diffuse relative to all the other pigments, and can be thought of as the molecular weight of the pigment molecule. The user sets up the simulation by giving each of these values an initial state. The processor fields are initialized. Pigment and water can be applied to the paper in a variety of ways as in most paint systems, remembering that most tools can be used to both apply and remove materials from the paper.

2.2. Simulation

Once an initial state has been described one can begin to simulate how various forces such as diffusion, gravity, surface tension, and humidity will interact with the paper, pigment and pigment medium. One limitation of this system is that it only handles two of the three dimensions. Real paint will also move into and out of the paper. Although there is a continuum of states, the simulation deals with only the two extreme cases. The first state is when the fluid is on the surface of the paper. Imagine that you have taken an eye-dropper and applied a drop to a piece of paper. Before it is absorbed, it will act like a drop of water on a pane of glass. Surface tension will cause the drop to ball up and gravity may pull it down to form a drip. Depending on the type of paper and the fluid used, the drop will be absorbed into the paper itself. Now it will act in a different way. Surface tension forces will no longer have much of an effect. The paper fibers will allow the fluid to diffuse much faster in all directions, and gravity will have less of an effect. The simulation assumes that the paint can accurately be described as some combination of these states. The simulation can then be broken down into three steps: first the movement of water and pigments are calculated for the surface; then they are calculated for the infused material; and finally any movement of material between these two states is computed.

2.2.1 Surface effects

First we will deal with the paint on the surface of the paper which has not yet soaked in. We can combine all of the forces acting on the water to create a displacement force **D**, which is used to determine the proportion and direction of any water displacement. These forces are gravity, surface tension, and a spreading force, which is simply the component of gravity perpendicular to the surface which prevents surface tension from pulling all of the water together. The following equation defines the displacement force, which is divided into horizontal and vertical components, where $\mathbf{D_x}$ is the horizontal component of the displacement force and **g**, **s**, and **sp** are the coefficients that define the relative strengths of gravity, surface tension, and spreading and $\mathbf{water_x}$ is the surface component of the water at some location x. This equation must be solved at each location for both the x axis and the y axis.

$$\mathbf{D_x} = g\ \mathbf{water}_x + s\left(\sum_{n=1}^{n=10}\frac{1}{n}\,\mathbf{water}_{(x+n)} - \sum_{n=1}^{n=10}\frac{1}{n}\,\mathbf{water}_{(x-n)}\right) + sp\ \left(\mathbf{water}_{(x-1)} - \mathbf{water}_{(x+1)}\right) \qquad [1]$$

Although not yet implemented, one may also add an inertial term so that fluid in motion will tend to stay in motion. Similarly, a frictional term would reduce the displacement in proportion to the dampness of the paper in the direction of displacement (the infused water).

The displacement force is then used to determine the amount of water that will move from each cell, Δwater. Note that a positive displacement indicates that the materiel has moved to the right (or higher numbered cell). Therefore the amount of water in each cell is diminished by absolute value of the displacement and then increased by any positive displacement from the cell to the left and any negative displacement from the cell to the right.

$$\Delta\mathbf{water}_x = \mathbf{D}_x\ \mathbf{water}_x,\ \Delta\mathbf{water}_y = \mathbf{D}_y\ \mathbf{water}_y \qquad [2]$$

$$\text{if } \left(\Delta\mathbf{water}_{(x-1)} > 0\right) \mathbf{from\_left} = \Delta\mathbf{water}_{(x-1)}, \text{ else } \mathbf{from\_left} = 0 \qquad [3]$$

$$\text{if } \left(\Delta\mathbf{water}_{(x+1)} < 0\right) \mathbf{from\_right} = \Delta\mathbf{water}_{(x+1)}, \text{ else } \mathbf{from\_right} = 0 \qquad [4]$$

$$\mathbf{water}_X = \mathbf{water}_X - \left|\Delta\mathbf{water}_X\right| + \mathbf{from\_left} + \mathbf{from\_right} \qquad [5]$$

Equations [3], [4], and [5] show the displacement in the horizontal direction only. The displacement force $\mathbf{D_x}$ is used to calculate the movement of each pigment in the same manner. Note that the displacement force is determined by the water content, regardless of the pigments. The pigments are in solution and will flow in equal proportion with the water.

2.2.2 Substrate effects

Now we can compute displacement of the water and pigment which have become infused in the paper. First, gradient fields are calculated for the water and each pigment. The gradient, denoted by the symbol ∇, is simply the difference between a cell and its neighbor. Again, the horizontal and vertical components can be calculated separately.

$$\nabla \mathbf{water} = \mathbf{water}_{x} - \mathbf{water}_{(x-1)}\ , \quad \nabla \mathbf{cyan} = \mathbf{cyan}_{x} - \mathbf{cyan}_{((x-1)}\ , \ \text{etc.} \qquad [6]$$

The displacement field, Δ, is then computed for each component by the following formulas,

$$\Delta \mathbf{water} = g\ \mathbf{a}\ \nabla \mathbf{water} \qquad [7]$$

$$\Delta \mathbf{cyan} = g\ w_{cyan}\ \mathbf{a}\ \nabla \mathbf{cyan}\ , \quad \text{etc} \qquad [8]$$

Where g is the gravity constant and **a** is the field which describes the absorbency of each cell. The amount and type of variation in this field contributes to the texture of the image. Note that the displacement formula for the pigments differs from the water. The ability of pigment to move along a gradient is proportional to the dampness of the paper, as well as the weight of the pigment. By controlling the pigment weights, one can select which pigment components will diffuse faster or slower. Once the displacement fields are generated, the water and pigments may be adjusted as in the surface materials (Equations 3-5).

2.2.3 Paper Absorption

So far we have been looking at the surface and infused components of the water and pigments separately. In this step we determine how much of the surface material will be absorbed by the paper. This is affected by the absorbency of the cell **a** and the fluid capacity of the cell **c** and a constant **k** which is used to set the overall speed of the absorption. For each cell the amount of fluid absorbed is described by the following formulas:

$$\mathbf{A} = k\ \mathbf{a}\ \mathbf{water}_{surface},\ \text{if}\ \mathbf{A} > (c - \mathbf{water}_{infused})\ \text{then}\ \mathbf{A} = c - \mathbf{water}_{infused} \qquad [9]$$

$$\mathbf{water}_{infused} = \mathbf{water}_{infused} + \mathbf{A}\ , \ \mathbf{water}_{surface} = \mathbf{water}_{surface} - \mathbf{A} \qquad [10]$$

Pigment will be absorbed in the same proportions to the water as exist in the surface paint. In addition to being constantly absorbed by the paper, water is being constantly absorbed by the air. The amount of water is simply reduced at a constant rate proportional to the humidity.

2.3. Rendering

Once we know how much pigment and water are at each paper cell, we can compute an image of the simulation. The equation for the color of each cell is as follows:

$$\mathbf{pixel}_{r} = \mathbf{initial\_color}_{r} - (\mathbf{cyan}_{surface} + \mathbf{cyan}_{infused}) \qquad [11]$$

$$\mathbf{pixel}_{g} = \mathbf{initial\_color}_{g} - (\mathbf{magenta}_{surface} + \mathbf{magenta}_{infused}) \qquad [12]$$

$$\mathbf{pixel}_{b} = \mathbf{initial\_color}_{b} - (\mathbf{yellow}_{surface} + \mathbf{yellow}_{infused}) \qquad [13]$$

Note that the pigment is subtractive. This means that the application of pigment will always cause the image to darken. For this reason, watercolor is generally done on light papers. One advantage of simulated watercolor is that one can invert the color rule and create images with luminous pigments on dark papers.

One can also use the surface water gradient to calculate a highlight and shadow for any drips, or send the fields to a more complex rendering system where they might be used as texture or bump maps[3]. Even if highlights are used, the presence of water can usually only be inferred from the response of the pigments. It is often useful when designing an image to be able to observe the water in isolation. By separating the rendering from the simulation, the artist can view the same data in a variety of ways without modifying the simulation itself.

3. Results and Analysis

The system is implemented on a Connection Machine II with 16k processors and 64k bits of memory for each processor. A simulation of a piece of paper at 1024x1024 resolution runs between two and ten times slower than one would expect for real paper. Speed can be increased by reducing the size of the simulation or by adding more processors (The Connection Machine II supports up to 64k processors). All arithmetic is done at 32-bit float precision, using Wytek 3031 floating point accelerators. The resultant images are displayed on a high resolution, 24-bit display subsystem.

Figures 4a-4c illustrate the the effect of water, which has been added to the top of the paper, as gravity and diffusion pull it down through the pigment and paper. Figures 5a-5c were created by using a less absorbent paper. By preventing the paper from absorbing the paint, long drips were encouraged. The diffusion of paint and formation of drips looks realistic over a wide range of conditions.

4. Future Work

While this work has shown some positive initial results, it will not be very useful unless the power of the model is made available to the artist in a straightforward manner. One reason for simulating paper is to be able to expand the abilities of the artist. In addition to being able to mimic papers that the artist is familiar with, it should be possible to define new kinds of paper which have extremely specific properties unachievable in nature. For example, one could define a paper which diffuses only chrominance in the x direction and only luminance in the y direction, or a paper where the absorbency of the fibers varies in a distinct pattern. One should be able to specify a wide variety of pigments. Additional pigments would diffuse in exactly the same manner as those discussed here, but would contribute to more than one channel of the displayed image.

One of the first issues that need to be addressed is the use of brushes in applying and modifying the paint in the simulation. Currently, we are working towards using a pressure sensitive tablet as a painting interface. Ideally, one would have access not only to the pressure applied, but also the angle and attitude of the brush. A model such as that described by Strassman[6], where the brush is simulated as a collection of many individual bristles, could be modified to work within the system. It would have to define not only how pigment is added to the paper, but also how water would move into and out of the brush. Brushes could be used to add paint, pick up excess paint, and to manually spread paint around on the surface of the paper.

5. Summary

By assuming that the actions of many tiny paper fibers can be modeled by simple rules applied to small paper cells, a realistic simulation of paint and paper can be achieved. The use of a parallel processor makes it possible to calculate the behavior of many (~10^6) cells in close to real time. Each cell is required to know only about itself and its immediate environment. The simulation takes into account diffusion, surface tension, gravity, humidity, paper absorbency and the molecular weight of each pigment. By separating the behavior of surface and infused paint, one can achieve a variety of effects while having the advantages of a two-dimensional model. At each time step every cell computes the forces acting on the pigments and water and decides if there should be any movement of material. The image is then rendered and sent to a display system. The advantages of such a system lie in being able to directly control the paper, the forces acting on the paint, and the behavior of the paper fibers.

6.Acknowledgements

The research was performed in the MIT Media Lab’s Visible Language Workshop, under the direction of Professor Muriel Cooper, Ronald MacNeil, and Henry Lieberman. Mike McKenna, Ali Alivazadeh, Robin Kullberg, Marie Sobalvaro and Anne Russell assisted in the preparation of this manuscript. Jaqueline Casey, a visiting design scholar, provided valuable expertise in the development and testing of the system. This work was supported in part by Bitstream, Hewlett Packard, NYNEX, PAWS, Inc. and Wacom.

7.References

[1] F.A.L. Dullien, Porous Media: Fluid Transport and Pore Structure, Academic Press, New York, 1979.

[2] Paul Haeberli, “Paint By Numbers: Abstract Image Representations”, *Computer Graphics*, Volume 24, Number 4, August 1990.

[3] Pat Hanrahan and Paul Haeberli, "Direct WYSIWYG Painting and Texturing on 3D Shapes", *Computer Graphics*, Volume 24, Number 4, August 1990.

[4] W. Daniel Hillis, The Connection Machine, MIT Press, Cambridge, Massachusetts, 1985.

[5] John-Peter Lewis, "Texture Synthesis for Digital Painting", *Computer Graphics*, Volume 18, Number 3, July 1984.

[6] Steve Strassmann, "Hairy Brushes", *Computer Graphics*, Volume 20, Number 4, August 1986.

[7] Stephen Wolfram, ed. Theory and Applications of Cellular Automata, World Scientific, Singapore, 1986.

Figure 3. Fives, design used for the fifth anniversary of the Media Laboratory. The image was the result of a collaboration betweeen the author and Jaqueline S. Casey, Visiting Design Scholar at the Visible Language Workshop.

Firgure 4, a-c

Figure 5, a-c