

# 1 List of symbols

(1 indicates a dimensionless quantity and – indicates not applicable).

Symbol	Variable name	Units	Description
$\arctan2(y, x)$	–	–	The four quadrant inverse tangent which returns values in the interval $[-\pi, \pi]$ , inclusive.
$c$	c	$ms^{-1}$	Sound speed in water.
$D(v)$	filter_v[n] ['D']	1	Integer filter decimation factor.
$DFT_{N_{DFT}}(x)$	–	–	The discrete Fourier transform of length $N_{DFT}$ of $x$ .
$f$	f_m	Hz	Frequency.
$f_c$	f_c	Hz	Frequency at centre of linear upswing chirp pulse. Equivalent to $f_{start} + \frac{ f_{stop} - f_{start} }{2}$ .
$f_n$	f_n	Hz	Nominal operating frequency of a transducer.
$f_s$	f_s	Hz	Analogue to digital sampling frequency.
$f_{s,dec}$	f_s_dec	Hz	Decimated sampling frequency.
$f_{start}$	f_1	Hz	Start frequency of linear upswing chirp pulse.
$f_{stop}$	f_2	Hz	Stop frequency of linear upswing chirp pulse.
$g(\theta, \phi, f)$	g_theta_phi_m	1	Transducer gain as a function of echo arrival angle in the beam and frequency.
$g_0(f)$	g_0_m	1	Transducer gain along the main acoustic axis, i.e. $g(\theta, \phi, f)$ where $\theta = \phi = 0$ .
$h_{bp}(i, v)$	filter_v[n] ['h_fl_i']	1	Complex valued receiving filter coefficients.
$i$	i	1	Generic integer index.
$\Im(x)$	–	–	The imaginary part of $x$ .
$j$	j	–	The square root of $-1$ .
$m$	m	1	Sample index in frequency domain.

$N_u$	$N_u$	1	Total number of transducer sectors/transceiver channels that are used to receive and process the acoustic signal.
$N_v$	$N_v$	1	Total number of filter stages.
$N_w$	$N_w$	1	Number of samples used in the sliding Hanning window.
$n$	$n$	1	Sample index in time domain.
$P_{rx,e,t}(f)$	–	W	FT of the received electric power in a matched load for the signal from a single target at frequency $f$ .
$P_{rx,e,t}(m)$	$P_{rx_e_t_m}$	W	DFT of the received electric power in a matched load for the signal from a single target.
$P_{rx,e,v}(f)$	–	W	FT of the received electric power in a matched load for the signal from a volume at frequency $f$ .
$P_{rx,e,v}(m)$	$P_{rx_e_v_m}$	$Wm^2$	DFT of the received electric power in a matched load for the signal from a volume.
$p_{rx,e}(n)$	$p_{rx_e_n}$	W	Received electric power in a matched load.
$p_{tx,e}$	$p_{tx_e}$	W	Transmitted electric power.
$p_{tx,auto}(n)$	$p_{tx\_auto}$	1	Square of the absolute value of the matched filter autocorrelation function.
$\Re(x)$	–	–	The real part of $x$ .
$r$	$svf\_range$	m	Distance from transducer.
$r_0$	–	m	Reference distance.
$r_c$	–	m	Distance from the transducer to the centre of the range volume covered by $t_w$ .
$r_c(n)$	$r\_c\_n$	m	$range_c$ at sample number $n$ .
$r(n)$	$r\_n$	m	Distance from transducer.
$S_p(n)$	$Sp\_n$	dB re 1 m <sup>2</sup>	Point scattering strength at sample index $n$ .
$S_v$	–	dB re 1 m <sup>-1</sup>	Volume backscattering strength.
$S_v(f)$	–	dB re 1 m <sup>-1</sup>	Volume backscattering strength at frequency $f$ .
$S_v(n)$	$Sv\_n$	dB re 1 m <sup>-1</sup>	Volume backscattering strength at sample index $n$ .

$S_V(m)$	<b>Sv_m</b>	dB re 1 m <sup>-1</sup>	Volume backscattering strength at frequency index $m$ .
TS	–	dB re 1 m <sup>2</sup>	Target strength.
TS( $f$ )	–	dB re 1 m <sup>2</sup>	Target strength at frequency $f$ .
TS( $m$ )	<b>TS_m</b>	dB re 1 m <sup>2</sup>	Target strength at frequency index $m$ .
$t$		s	Time.
$t_w$	<b>t_w</b>	s	Duration of sliding window for calculating volume backscattering strength.
$u$	<b>u</b>	1	Receiver channel number and transducer sector number.
$u(i)$	–	1	The Heaviside step function.
$V$	–	m <sup>3</sup>	Volume occupied by scattering targets.
$v$	–	1	Filter stage.
$w$	–	1	The Hann window function.
$w(i)$	<b>w_i</b>	1	The Hann window function for index $i$ , defined by $w(i) = 0.5(1 + \cos(2\pi i/N_w))$ , $-N_w/2 \leq i \leq N_w/2$ .
$\tilde{w}(i)$	<b>w_tilde_i</b>	1	Normalised Hann window.
$x^*$	–	–	The complex conjugate of $x$ .
$\ x\ _2$	–	–	The $l^2$ -norm of $x$ , also known as the Euclidean norm.
$Y_{\text{mf,auto}}(m)$	<b>Y_mf_auto_m</b>	1	Discrete Fourier transform (DFT) of the autocorrelation function for the matched filter.
$Y_{\text{mf,auto,red}}(m)$	<b>Y_mf_auto_m_red</b>	1	DFT of the reduced autocorrelation function for the matched filter.
$Y_{\text{pc,t}}(m)$	<b>Y_pc_t_m</b>	V	DFT of the pulse compressed signal from a single target.
$\tilde{Y}_{\text{pc,t}}(m)$	<b>Y_tilde_pc_t_m</b>	Vm	DFT of the pulse compressed signal from a single target normalized by the DFT of the reduced autocorrelation function for the matched filter.
$Y_{\text{pc,v}}(m)$	<b>Y_pc_v_m</b>	Vm	DFT of the pulse compressed signal from a volume. Compensated for spreading loss.

$\tilde{Y}_{pc,v}(m)$	<code>Y_tilde_pc_v_m</code>	V	DFT of the pulse compressed signal from a single volume normalized by the DFT of the reduced autocorrelation function for the matched filter. Compensated for spreading loss.
$y_{mf}(n)$	<code>y_mf_n</code>	1	Matched filter. Signal used for pulse compression.
$y_{mf,auto}(n)$	<code>y_mf_auto_n</code>	1	Autocorrelation function for the matched filter.
$y_{mf,auto,red}(n)$	<code>y_mf_auto_red_n</code>	1	Reduced autocorrelation function for the matched filter.
$y_{pc}(n)$	<code>y_pc_n</code>	V	Pulse compressed signal averaged over all transducer sectors.
$y_{pc}(n, u)$	<code>y_pc_nu</code>	V	Pulse compressed signal from channel $u$ .
$y_{pc,aft}(n)$	<code>y_pc_aft_n</code>	V	Pulse compressed signal from the aft transducer half.
$y_{pc,fore}(n)$	<code>y_pc_fore_n</code>	V	Pulse compressed signal from the forward transducer half.
$y_{pc,port}(n)$	<code>y_pc_port_n</code>	V	Pulse compressed signal from the port transducer half.
$y_{pc,s}(n)$	<code>y_pc_s_n</code>	Vm	Pulse compressed signal compensated for spherical spreading.
$y_{pc,star}(n)$	<code>y_pc_star_n</code>	V	Pulse compressed signal from the starboard transducer half.
$y_{pc,t}(n)$	<code>y_pc_t_n</code>	V	Pulse compressed signal from a single target.
$y_{rx}(n, u)$	<code>y_rx_nu</code>	V	Received digitised, bandpass filtered, decimated complex signal after the final filter stage, $y_{rx}(n, u) = y_{rx}(n, u, N_V)$ .
$y_{rx}(n, u, v)$	–	V	Received digitised, bandpass filtered, decimated complex signal.
$y_{rx,a}(t)$	–	Pa	Analogue acoustic signal received by the transducer.
$y_{rx,e}(t, u)$		V	Analogue electric signal received by each transceiver channel $u$ .
$y_{rx,org}(n, u)$	<code>y_rx_org</code>	V	Received digitised signal before the bandpass filtering and decimation stages, $y_{rx,org}(n, u) = y_{rx}(n, u, 0)$ .

$y_{\text{tx}}(n)$	y_tx_n	V	Ideal transmitted signal generated from transmit signal properties.
$\tilde{y}_{\text{tx}}(n)$	y_tilde_tx_n	1	Ideal normalized transmitted signal generated from transmit signal properties.
$\tilde{y}_{\text{tx}}(n, v)$	y_tilde_tx_nv	1	Ideal normalized transmitted signal generated from transmit signal properties after application of filter stage $v$ .
$y_{\text{tx},\text{a}}(t)$	–	Pa	Analogue acoustic transmit signal.
$y_{\text{tx},\text{e}}(t)$	–	V	Analogue electric transmit signal.
$y_{\theta}(n)$	y_theta_n	rad	Electrical angle along the minor axis of the transducer (alongship when ship-mounted).
$y_{\phi}(n)$	y_phi_n	rad	Electrical angle along the major axis of the transducer (athwartship when ship-mounted).
$z_{\text{rx},\text{e}}$	z_rx_e	$\Omega$	Receiver electric impedance.
$z_{\text{td},\text{e}}$	z_td_e	$\Omega$	Transducer sector electric impedance.
$\alpha(f)$	alpha_m	$\text{dB m}^{-1}$	Absorption coefficient at frequency $f$ .
$\gamma_{\theta}$	gamma_theta	1	Conversion factor between phase difference in signals from the fore and aft transducer halves and the physical arrival angle of an echo.
$\gamma_{\phi}$	gamma_phi	1	Conversion factor between phase difference in signals from the port and starboard transducer halves and the physical arrival angle of an echo.
$\theta$	theta_n	rad	Angle coordinates along the minor axis of the transducer (alongship when ship-mounted).
$\lambda$		m	Acoustic wavelength.
$\lambda_m$	lambda_m	m	Acoustic wavelength at frequency index $m$
$\sigma_{\text{bs}}$		$\text{m}^2$	Backscattering cross-section.
$\tau$	tau	s	Nominal transmit pulse duration.
$\tau_{\text{eff}}$	tau_eff	s	Effective transmit pulse duration.

$\phi$	phi	rad	Angle coordinates along the major axis of the transducer (athwartship when ship-mounted).
$\psi(f)$	psi_m	sr	Two-way equivalent beam angle at frequency $f$ .