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An Analytical Method for Determining C_d at High Phase Fraction

(Task 1.2 : Dynamics of Two-Phase Flow at High Volume Fractions)

Project report III-13

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Abstract

This project report outlines a suggested analytical methodology for evaluating the drag coefficient C_d for a fluid particle at high phase fraction. The aim is to provide an analytical route to determine the variation of C_d as a function of phase fraction α due to pairwise interactions of the test particle with other particles in its immediate neighbourhood.

1 Introduction

Task 1.2 of the project is to look at the dynamics of bubbles and droplets (collectively referred to as fluid particles), and particularly to investigate how these change as the phase fraction increases and inter-particle interactions become important (and eventually dominant) in their dynamics. Direct calculation of fluid particle motion is possible for a single particle case using interface tracking techniques [1], but would be too expensive to perform on an ensemble of bubbles of any meaningfull size. Given that the aim of our work is to produce some form of correlation for the dynamics of the fluid particles as a function of the phase fraction, which would require repeated calculations, such an approach is likely to remain impractical for quite some time to come. The main approach is therefore likely to be experimental, which for us means mining the literature for correlations whose use can be evaluated. This will be the subject of a future BRITE-III project report. However it might be possible to construct a model for the behaviour of the fluid particles based on the dynamics of pair-wise interacting particles, which could be evaluated computationally, then integrating the effect of the interaction over each pair of particles in the ensemble. This approach, applied to the problem of evaluating the variation of the drag coefficient C_d as a function of α, is the subject of this report. If correct it would give a phemenological model for the fluid particle behaviour which could be applied to a number of situations and parameters. Such an approach has been suggested before in the literature, most notably by Silverman and Sirignano[2]. Their paper does not go far enough, however, concentrating on analysing the properties of the interaction functions that they derive describing the angular dependence of the particle interactions. Moreover they only consider nearest-neighbour interactions. If the Reynolds number is high enough, the particles will have extended recirculation zones behind them, which might mean that non-nearest-neighbours could also affect the chosen particle.

1.1 Mathematical Preliminaries

Consider an isolated fluid particle : the drag coefficient for this particle we will write as $C_{d,\infty}$. If we now consider the effect of one neighbouring particle i we can write the new drag coefficient as

$$\frac{C_{d,i}}{C_{d,\infty}} = \xi_i = 1 + \zeta_i \tag{1}$$

For a particle in a swarm of similar particles we can introduce a function Ξ

$$\frac{C_d}{C_{d,\infty}} = \Xi \tag{2}$$

relating the drag coefficient to that of the single particle. If we make the fundamental assumption here that the effects of multiple particles are in some sense superposable, i.e.

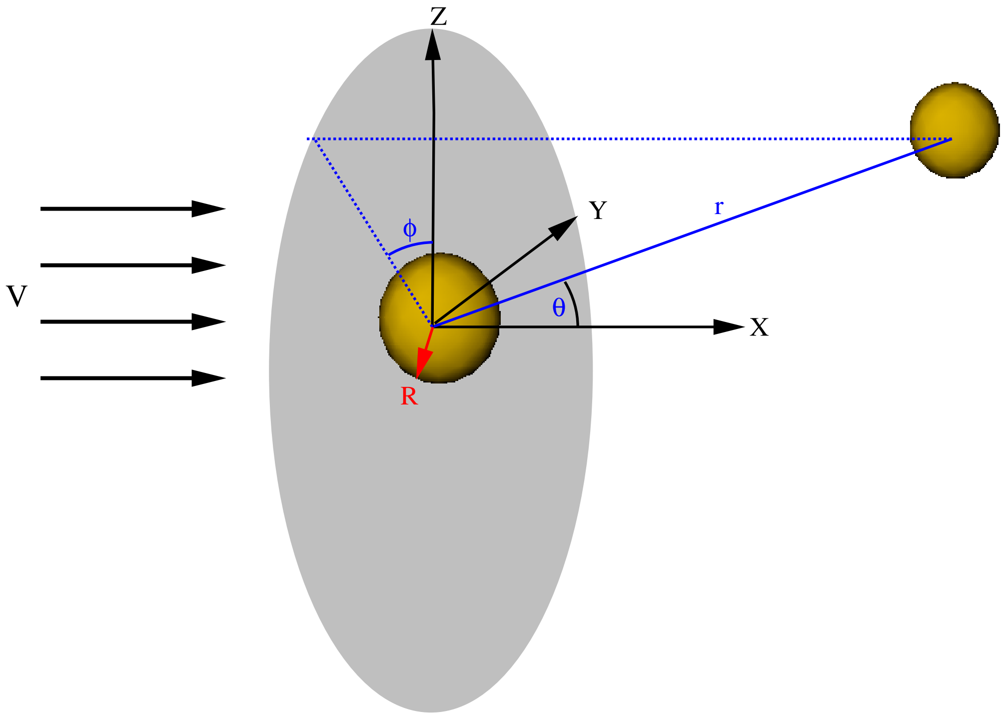

Figure 1: Cartoon showing the interaction between two fluid particles. This is primarily provided to introduce the coordinate system

we will not consider multiparticle interactions, we can write

$$\begin{aligned} \Xi &= \prod_i \xi_i \\ &= 1 + \sum_i \zeta_i + (\text{h.o.t.}) \\ &= 1 + \int_V \zeta(r,\theta,\phi)\, n(r,\theta,\phi)\, dV = 1 + I \end{aligned} \tag{3}$$

The geometric variables r, θ, ϕ are defined in figure1. $n(r,\theta,\phi)$ is the number density of neighbouring fluid particles within a volume δV. In order to evaluate this integral we need to make assumptions about the angular (θ) variation of ζ.

It will be useful to make a list of the assumptions at this point. The global assumptions of the model are as follows :

A1. The angular (θ) variation has a prescribed functional form

A2. Bubble-bubble correlations will be neglected

A3. Flow superposition is assumed (C_d multiplicative)

A4. Multibody interactions will be neglected

In addition, for the purposes of this preliminary report, we will make the following additional assumptions

B1. Use spheres throughout – no deformation

B2. Non-slip conditions on sphere (appropriate for contaminated system)

B3. Correlations are being considered for $\mathcal{R}e < 200$

These assumptions correspond to the experimental data presented in the next section. Assumption A2 implies that we can write the number density as

$$n(r, \theta, \phi)\ dV = n_m r^2\ d\cos\theta\ dr\ d\phi \tag{4}$$

i.e. assuming that the number density is constant at the mean density.

2 Evaluation of ξ

In order to evaluate the integral in eqn.(3) we need a functional form for ξ. Both experimental and computational results are available in the literature, as detailed in the following table, for flow around pairs of smooth spheres at low Reynolds numbers [3][4][5]. This data can be supplimented with in-house computations as necessary.

Authors		$\mathcal{R}e$	Comments
Kim, Elghobashi & Sirignano, JFM **246** pp.465 - 488 (1993) [3]	KES	50–150	Computation w. spheres side-by-side
Legendre & Magnaudet ICMF'98 (1998)[4]	L&M	0.1–500	Computation w. spheres side-by-side
Zhu, Liang & Fan. IJMF **20** pp.117 - 129 (1994)[5]	ZLF	2–130	Experimental, spheres in line

2.1 Angular variation of ξ

The first thing to do is to look at the angular variation of ξ. The computational and experimental data highlighted above is for two specific angles : $\theta = 0°$, i.e. in line, and $\theta = 90°$, i.e. side-by-side. In-house calculations were performed for various other angles at a Reynolds number of 50 and a separation $r = 4D$ (D = diameter of particle). Streamlines for the various different cases are shown in figure 2. We note how the stable recirculation zone behind the individual sphere is modified by the presence of the second sphere behind. The variation of the drag coefficient for this case, represented by ξ_i, is shown in figure 3. This was not the best case to pick, as the spheres are sufficiently far apart that they have little effect on each other unless one is directly in the wake of the other : hence over much of the range of θ, $\xi \sim 1$ (although the data from Legendre and Magnaudet is slightly higher, and further refinement of the calculations may be necessary to eliminate this discrepancy). This does suggest that as a first approximation there are three states to be considered, 1.when the second particle is leading the main one, 2. when they are side-by-side, and 3. when the second particle is trailing the main one. This third case can probably be ignored for the moment, leaving us with 2 contributions to the integral, $I = I_1 + I_2$. The wake is taken to be $2D$ in width, as shown in figure 4. The angular dependence is therefore represented as a series of step functions with splits at $\cos\theta = \pm\sqrt{1 - \frac{4R^2}{r^2}}$.

2.2 Superposition of solutions

One of the prime assumptions made in this report is A3 : that in some sense the flow solutions are superposable. What is assumed is that the correction to the value of C_d due to two neighbouring particles is a composite of that due to each separately. An effort can be made to check this by calculating the flow around multiple spheres. This is most easilly done by calculating the flow around a single sphere on a computational domain with symmetry boundary conditions, which is equivalent to calculating the flow around an infinite 2d lattice of spheres. The geometry of such a case is shown in figure 5. The data from Legendre and Magnaudet (L&M) is shown in figure 6 as a function of $\mathcal{R}e$. L&M's data is for two spheres side-by-side, and shows the complex nature of the functional relationship between C_d and $\mathcal{R}e$. Two calculations have been performed on lattices of spheres, and are shown as triangles on the graph rather than diamonds. They certainly show an increase in drag over the corresponding pair-wise values, consistent with a superposition principle being used.

2.3 Evaluation of Integrals I_1, I_2

For in-line spheres, ZLF fit a function

$$\begin{aligned}
\xi &= 1 - A_1(\mathcal{R}e)\mathrm{e}^{-\frac{B_1(\mathcal{R}e)r}{2R}} \\
A_1(\mathcal{R}e) &= \mathrm{e}^{-0.483+3.45\times10^{-3}\mathcal{R}e-1.07\times10^{-5}\mathcal{R}e^2} \\
B_1(\mathcal{R}e) &= 0.115 + 8.75\times10^{-4}\mathcal{R}e - 5.61\times10^{-7}\mathcal{R}e^2 \\
\mathcal{R}e &= \frac{2RV}{\nu}
\end{aligned} \qquad (5)$$

This is quite an appropriate functional form, since it gives ζ as a negative exponential, i.e. the effect of a neighbouring sphere dies off as it gets further away. Subsituting this in the integral I_1 we have

$$\begin{aligned} I_1 &= -\int_0^{2\pi}\int_{2R}^{\infty}\int_{-\sqrt{1-\frac{4R^2}{r^2}}}^{-1} \zeta(r,\theta,\phi)\, n_m r^2 \, d\cos\theta \, dr \, d\phi \\ &= -2\pi \int_{2R}^{\infty} A_1 \mathrm{e}^{-\frac{B_1 r}{2R}} \left[1-\sqrt{1-\frac{4R^2}{r^2}}\right] r^2 dr \\ &= \frac{16\pi R^3 A_1 n_m}{B_1^3}\left[B_1^2 K_2(B_1) - (2+2B_1+B_1^2)\mathrm{e}^{-B_1}\right] \end{aligned} \tag{6}$$

Data from both KES and L&M is shown as a function of r in figure 7. It seems to have much the same functional form, and a negative exponential might well be an appropriate function. However there is a certain ammount of spread on the values (particularly at $r/R = 10$) which will require a certain ammount of care with the fitting.

If we assume the same functional form for the fit for this data, we postulate a function

$$\xi = 1 + A_2(\mathcal{R}e)\mathrm{e}^{-\frac{B_2(\mathcal{R}e)r}{2R}} \tag{7}$$

Substituting this into I_2 gives

$$I_2 = \frac{32\pi n_m A_2 R^3}{B_2} K_2(B_2) \tag{8}$$

Finally, since $\alpha = \frac{4}{3}\pi R^3 n_m$, we can write

$$\begin{aligned} \Xi &= 1 + \alpha \left[\frac{24K_2(B_2)A_2}{B_2}\right. \\ &\quad \left. + \frac{12A_1}{B_1^3}\left\{B_1^2 K_2(B_1) - (2+2B_1+B_1^2)\mathrm{e}^{-B_1}\right\}\right] \end{aligned} \tag{9}$$

3 Conclusions

This report suggests a methodology for calculating dynamical parameters of particle flow for cases where the particle density is quite high. The methodology rests primarily on the assumption that the effect of n particles interacting with the test particle is equivalent to the effect of 1 particle interacting with the test particle, repeated n times. As is shown here, this can be used to produce a correction function for the drag coefficient C_d as a function of phase fraction α, of the form $1 + K\alpha$. This seems plausible at low phase fraction, at least. At higher phase fraction, multiple-particle interactions would have to be taken into account : this could be done by extending the expansion of eqn.(3) to the next term. However this would require the calculation of 3-body effects. The work presented here is not complete, being more of an outline of the suggested method. Further work would be required in order to tighten up some of the calculations, and in addition the resulting functional form should be compared with data for the experimental variation of C_d with α. This would show whether the basic concept is valid. If it is, then this would represent a useful technique for evaluating not only the variation in drag due to phase fraction, but also variations in other parameters of multiphase flow.

References

[1] Ubbink, O. and Issa, R. I.: "A method for capturing sharp fluid interfaces on arbitrary meshes", *J.Comp.Phys*, 153(1):26 – 50, July 1999.

[2] Silverman, I. and Sirignano, W. A.: "Multi-droplet interaction effects in dense sprays.", *Int.J.Multiphase Flow.*, 20(1):99 – 116, 1994.

[3] Kim, I., Elghobashi, S., and Sirignano, W. A.: "Three-dimensional flow over two spheres placed side by side.", *J.Fluid Mech.*, 246:465 – 488, 1993.

[4] Legendre, D. and Magnaudet, J.: "Interaction between two spherical bubbles rising side by side", In *Proceedings of Third International Conference on Multiphase Flow, Lyon, France.* ICMF98, 1998.

[5] Zhu, C., Liang, S.-C., and Fan, L.-S.: "Particle wake effects on the drag force of an interactive particle", *Int.J.Multiphase Flow.*, 20(1):117 – 129, 1994.

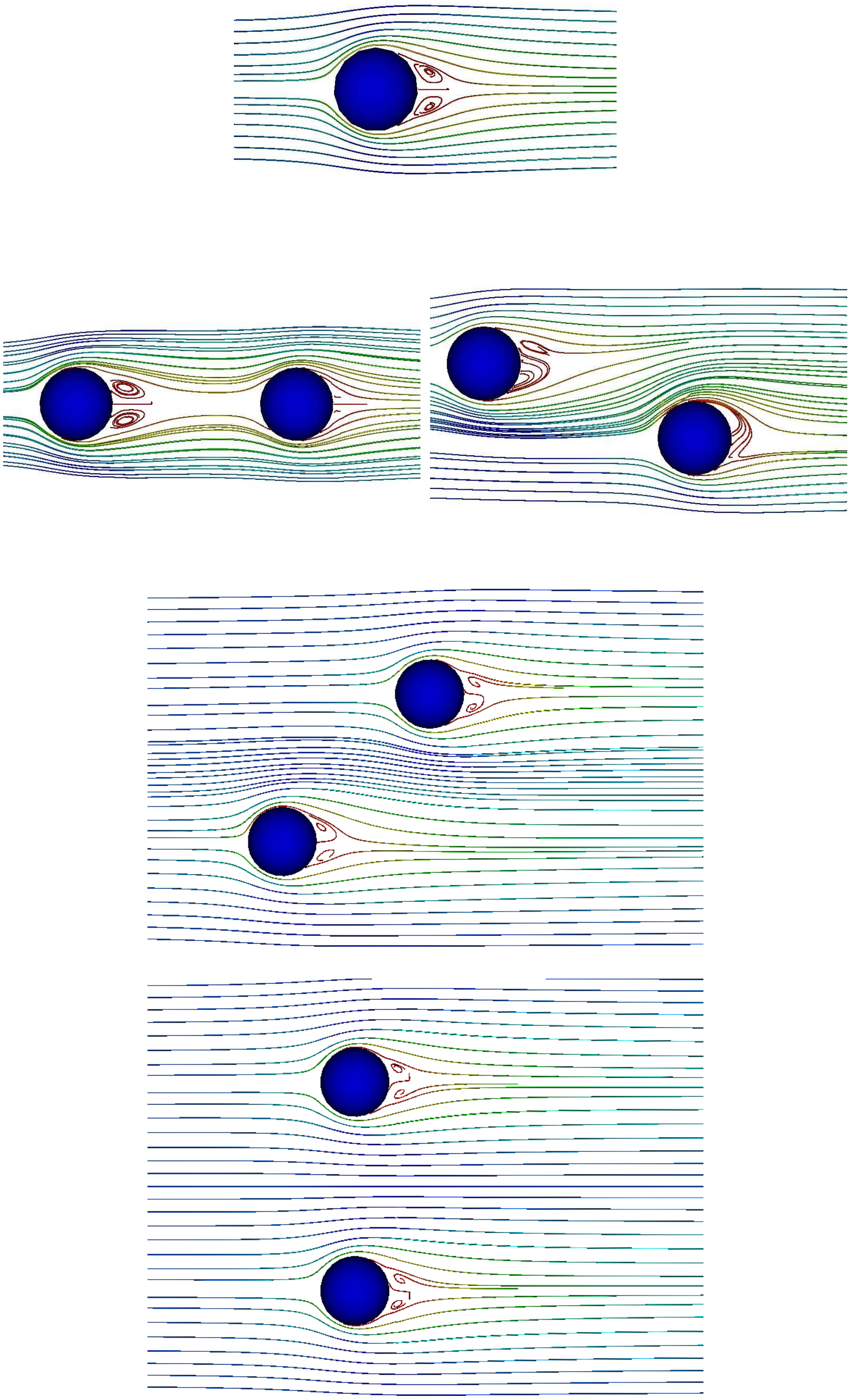

Figure 2: Streamlines around spheres at various angles, $\mathcal{R}e = 50$. The top figure shows the streamlines for an isolated sphere : the others are for pairs of spheres at a separation $r = 4D$.

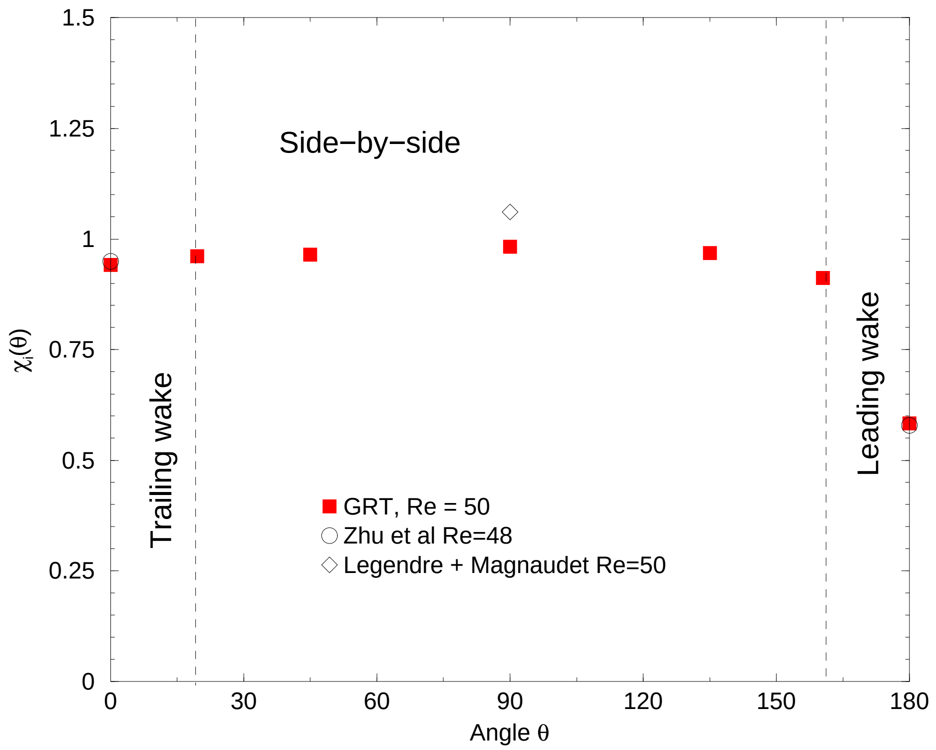

Figure 3: Angular variation of ξ for a pair of spheres at $\mathcal{R}e = 50$ and $r = 4d$. Data from in-house calculations is shown as red squares, with the other symbols referring to data from the literature

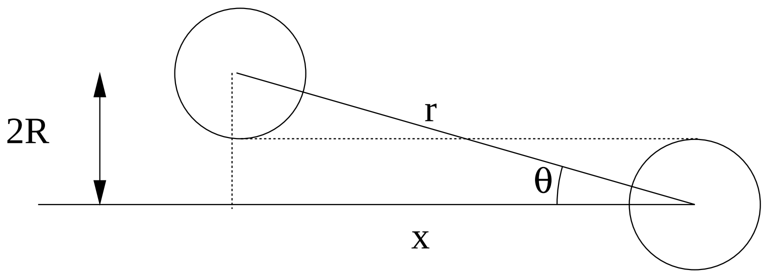

Figure 4: Geometry of the case of two spheres one just in the wake of the other. This defines the coordinate system for the integrals I_1, I_2

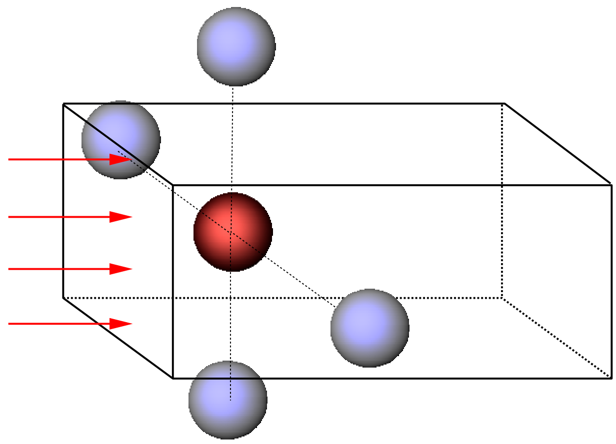

Figure 5: Geometry for calculating flow around a lattice of spheres.

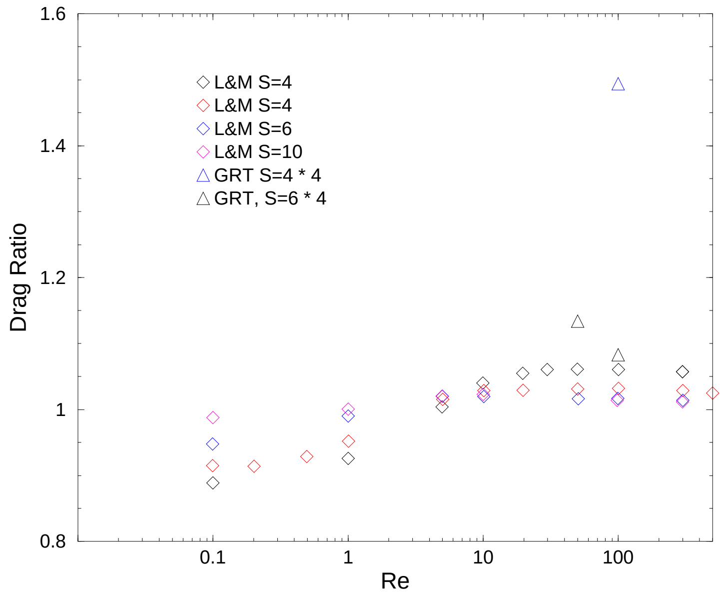

Figure 6: Variation of function ξ as a function of $\mathcal{R}e$.

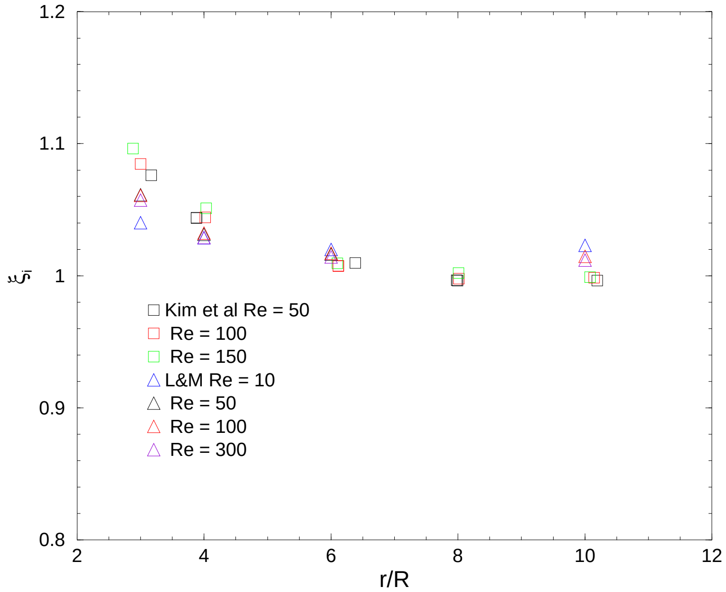

Figure 7: Variation of ξ as a function of distance for spheres in line. All available data has been plotted.