The Computational Investigation of the Blood Flow in the Circle of Willis

Steven Daniels and Dr Gavin-Tabor (Supervisor)

Introduction:

The Circle of Willis (CoW) is a ring of arteries located in the base of the brain and is responsible for the distribution of oxygenated blood the cerebral mass. A complex arrangement, the CoW can come in many forms with a small population of the world having a complete CoW. Despite this, the CoW has shown to adapt to these abnormalities making predictions of the blood flow around the geometry even more complicated. Understanding the flow of blood in the CoW is important in understanding the causes and effects of strokes as well as the effects of surgical interventions.

Stroke is the leading cause of death in the developed part of the world, and mortality rates could increase dramatically in the years to come. There are mainly two types of stroke, ischemia caused by obstructions in the blood vessels and subarachnoid haemorrhage caused by the rupture of one or more aneurysms. Aneurysms typically develop in or near the CoW and are relatively common (around 1- 6% of the population during a lifetime). At the same time these can also rupture at an early age, the average age being 52 years **[1]**. An intracranial aneurysm is an expansion of the vessel wall but the reason behind the genesis, growth and rupture of these aneurysms is largely unknown.

Despite the uncertainty of predicting the development of an aneurysm, what is known is that by increasing the *wall- shear-stress* in a vessel increases the chances of the development of an aneurysm. It is also known that the artery walls around the CoW are relatively thin thereby increasing the risk of the development of an aneurysm in that particular area. Furthermore, the arrangements of the cerebral vessels vary greatly. For instance just less than half the population of the world has a 'true' CoW with variations of the arteries such as under-developed vessels or the vessels are missing completely. It has also been noted that the gender, ethnicity and lifestyle have a contributing factor to the development of an aneurysm.

Computational Fluid Dynamics (CFD) is the application of computers to solve the fluid motion equations i.e. the Navier-Stokes equations. The Navier-Stokes equations are a set of nonlinear partial differential equations; collectively these are too complex to be solved in a closed form. However, in some special cases the equations can be simplified and may admit analytical solutions. In this study the aim is demonstrating the feasibility and practicality of CFD calculations on a patient-specific model, with a focus on the modelling of the fluctuating velocity inlet and the resulting wall-shear-stresses.

OpenFOAM:

The chosen computational fluid dynamics code for this investigation is OpenFOAM. OpenFOAM (Open Field Operation and Manipulation) is a C++ toolbox for the development of customised numerical solvers, and pre/post-processing utilities for the solution of continuum mechanics problems, including Computational Fluid Dynamics (CFD). The code is released as free and open source software under the GNU General Public License. For this study OpenFoam 1.7.1 was used. As a result of this, the appeal of using free software rather than paying for commercial licenses would reduce the costs of funding future projects.

<u>Simpleware Ltd:</u>

To convert an MRI scan of a CoW to a model for use of CFD the University of Exeter has one of the world's leading groups in Image Based Meshing (IBM) supported by the company Simpleware Ltd. The software provides facilities for image processing, segmentation (the process of identifying which parts of the scan represent the domain of interest) and subsequent mesh generation. These tools are sophisticated and robust and capable of processing the complex geometries of the CoW. At the same time, exporting meshes for OpenFoam is a new interest to the company and as such this study will be trialling the new OpenFoam meshes from Simpleware Ltd.

<u>Classic Anatomical arrangement of the Circle of Willis:</u>

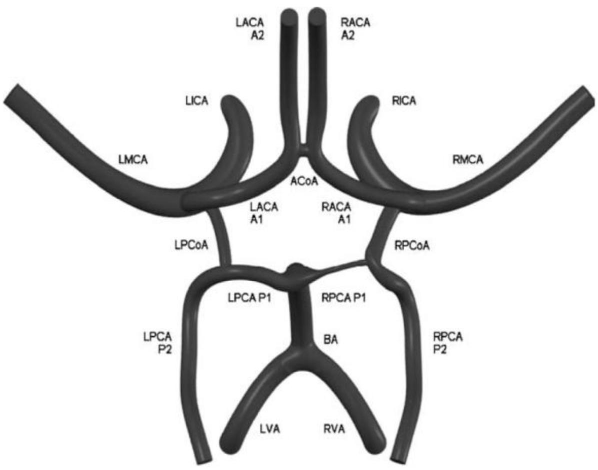


Figure 1: a diagram of a complete Circle of Willis [4]

A complete CoW is composed of pairs of arteries (left and right) which join to form a ring which circulates oxygenated blood throughout the tissues in the brain. The Internal Carotid and Vertebral arteries (ICA's, VA's) supply blood into the circle and are termed afferent arteries (where a prefix 'L' and 'R' in front of the artery abbreviation indicates the left and right hand of the CoW respectively). The ICA's and VA's are located in the front and back of the neck respectively. The Anterior, Middle and Posterior Cerebral arteries (ACA, MCA, PCA) transport blood away from the circle and are termed efferent arteries. The efferent arteries are connected by the Anterior and Posterior Communicating arteries (ACoA, PCoA's) collectively named the anastomosing arteries. These allow the blood to be re-routed to maintain oxygen supply to the whole cerebral mass should the oxygen supply be reduced. Described by Cieslicki nad Ciesla **[1, 2]**:

"The Circle of Willis is an extremely complicated flow system, with the physical nodes causing substantial disturbances along the outgoing branches, and the tortuousity of the channels causing forces which give rise to secondary flows perpendicular to the main stream. The pressure-flow relations are nonlinear in all sections."

The complexity of the velocity and pressure distributions around the CoW are not helped by the fact that the a 'true' CoW is the simplest case which as stated earlier only appeals to a small majority of the world. In the absence of a vessel the CoW compensates for its absence which may or may-not decrease the risk of the development of an aneurysm. In a study carried out by Alastruey et al . **[3]**, the CoW with the absent ICA (either left or right) has the highest chance of developing cerebral stroke.

Modelling the CoW in a CFD simulation has been attempt many times beforehand. A common aim in these studies is the attempt in creating a ‘patient specific’ model while considering the many abnormalities the CoW can sustain. Most commonly **[4, 5, 6]** an estimation of the inlet/outlets from the CoW were calculated using a CFD simulation followed by a comparison to data collected from the patient using Transcranial Doppler scanning (TCD). In this study this method was attempted using a true CoW created from an MRI scan with an initial study using a equivalent fluctuating velocity inlet to that of the blood flow from the heart, this was then followed by substituting the patient data into the model. It should be noted that the patient data is not that from the subject from this study but from a patient from another study **[3]** who also has a ‘true’ CoW.

Methodology

Blood properties:

For this study the fluid in question contained some of the properties of blood but for simplicity the fluid is considered to be an unsteady, incompressible, Newtonian fluid. In reality however blood is considered to be a non-Newtonian fluid due to its shear thinning and viscoelasticity. It is assumed that the shear thinning has a negligible effect on the wall-shear-stress. Although this study doesn’t include dynamic vessel walls or non-Newtonian fluids the value of the kinematic viscosity (υ) was set to $2.83 \times 10^{-6}\mathrm{m}^2\ \mathrm{s}^{-1}$ with a corresponding density of 1060 Kg m^{-3} in accordance to the study by Alastruey et al.**[3]**. Due to the complexity of the model (even with simplifications/assumptions of the fluid) the PISO algorithm was used with the turbulence model of k-ε, a widely used turbulence model when modelling blood flow despite the low Reynolds numbers. The SIMPLE algorithm was tested but was found to be inadequate when trying to obtain convergence for the solution of the Navier-Stokes equations.

Meshes:

Part of the Simpleware package ScanIP is an image processing program which can work with 2D and 3D data sets. Images can be imported in different file formats (most importantly DICOM (Digital Imaging and Communications in Medicine)). Unsurprisingly it is extremely difficult to create a structured mesh from an image, especially when the image is as complex as the one used in this study.

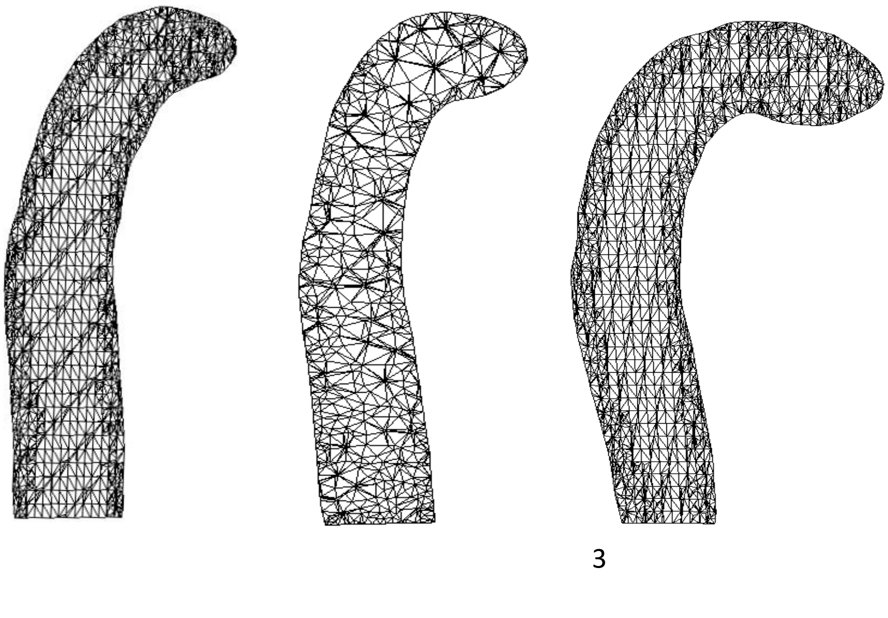


Figure 2: a slice of the RMCA showing the mesh used in this investigation - (far left) a simple mesh, (Middle) a ‘Free’ mesh and (far right) ‘Grid’ mesh with focus on creating a boundary layer around the Circle of Willis

The simple mesh was the first mesh to be created using ScanIP with the aim of this mesh was to create a fairly coarse mesh without compromising the accuracy of the result. The Free mesh was a more refined version of the basic mesh. The Grid mesh is based on the Free mesh but with a focus on a fine mesh along the walls of the artery (creating a boundary layer mesh). The values for table 1 were obtained using the command '*checkMesh*'.

Mesh	No. of cells - hexahedra	No. of cells - tetrahedral	No. of cells - polyhedral	Minimum element size (m)	Maximum Skewness	Maximum Aspect Ratio
Simple	12007	386138	20186	1.034121e-09	1.882454	12.82259
Free	0	109569	0	3.643976e-09	1.843254	16.7208
Grid	12003	385202	20169	1.034121e-09	1.882454	12.82259

Table 1: characteristic values for the quality of the meshes created using ScanIP

For this investigation the analysis was performed on the basic mesh (far left, figure 2). It was found that the remaining two meshes were two coarse to obtain a reliable result. However despite this, the analysis is still being attempted on these two but the results of these will not be included in this report.

<u>The Circle of Willis model:</u>

Rather than the simplified case expressed in the literature review (figure 1) the model created for this investigation include the many branches from the outlets. These have been labelled according to the boundary conditions shown in figure 3.

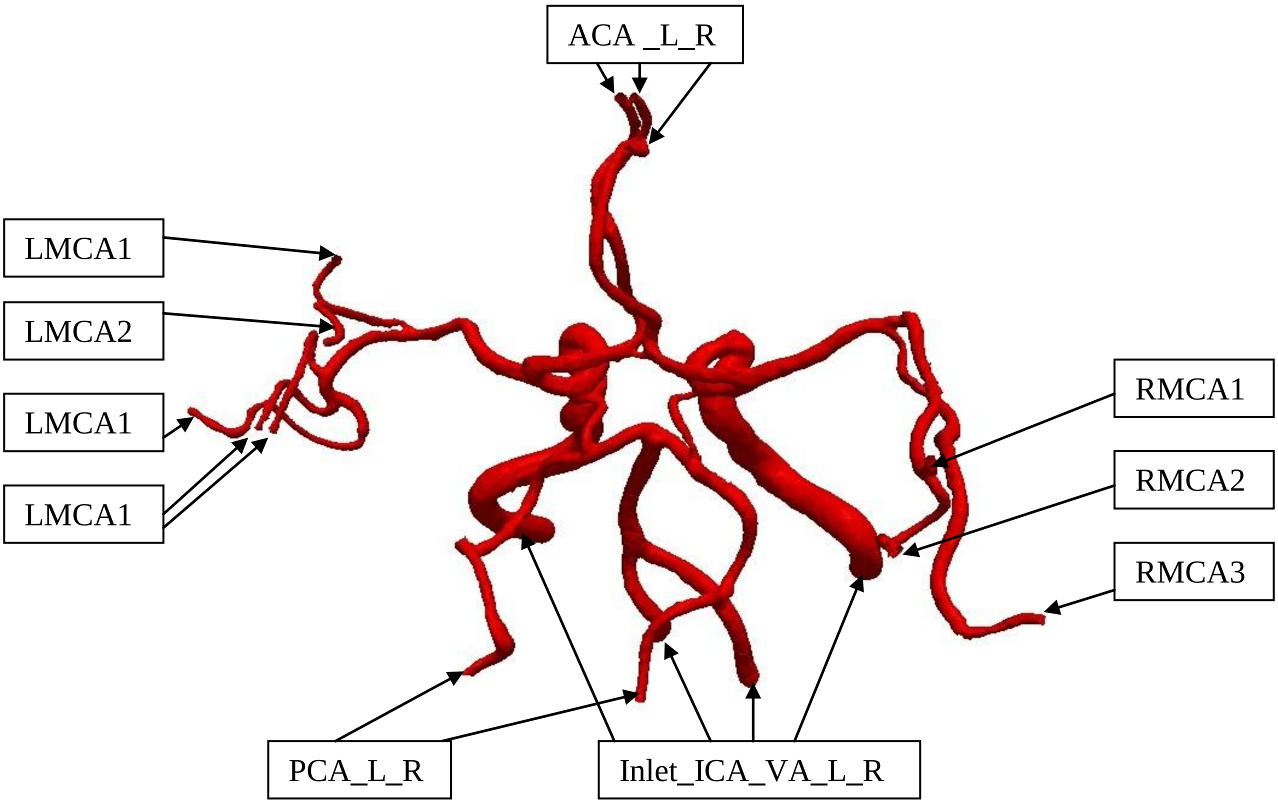


Figure 3: labelled model of the Circle of Willis using in this study

Fluctuating velocity inlet using 'groovyBC':

One of the many assets of OpenFoam are the free 'add ons' (compiling codes) which are available on a dedicated website *www.openfoamwiki.net* **(note:** this website is not the official OpenFoam website). A relatively new code that has become available is the coding which modifies the inlet/outlet patches on OpenFoam giving the user the option of producing a more specific setting such as parabolic profiles. In similar studies the fluctuating velocity values due to the pulsating heart have required a sinusoidal inlet. One particular study **[7]** suggested that the characteristic inlet fluctuation for a series of arteries around the body obeys the equation

$$U_{inlet} = \sqrt{\frac{214\sin\omega t + 1069}{0.48\pi\rho}} \qquad \textit{Eq 1}$$

Equation 1 was modified for the use in this study with the reference velocity (mean value) to be 0.25m/s ($Re_{equivalent} \approx 88$). This is the average value used in the Alastruey study **[2]**. The angular frequency ω is that of the average beating heart i.e. 1 cycle per second or 2π radians. The density of the fluid was set to 1060 Kg m^{-3}. Combined with a parabolic profile the new modified inlet condition was applied to the model and run with one cycle for 10seconds with a time step Δt of 1×10^{-5} seconds. The reason for this small time step was to satisfy the CFL number, which will be addressed in the discussion section of this report. Figure 4 shows the resulting contour diagram of the velocity magnitude at time 2×10^{-5}seconds ($CFL_{mean} \approx 0.041$).

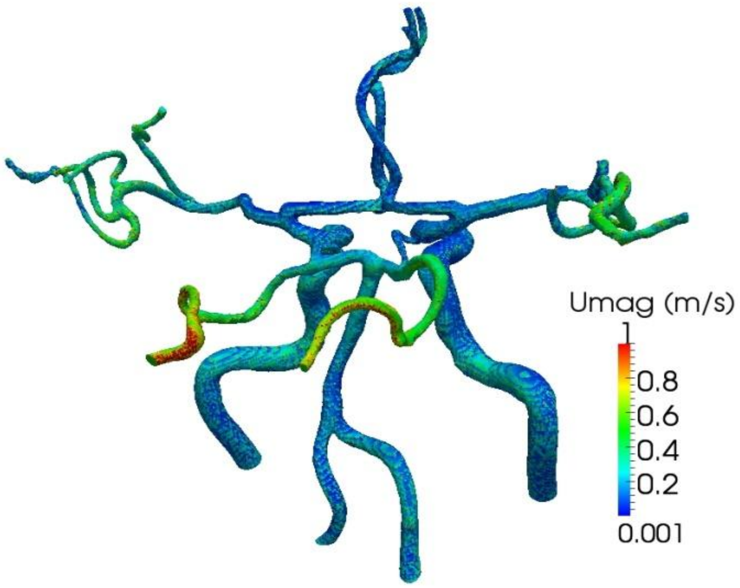


Figure 4: a velocity contour diagram of the Circle of Willis (basic mesh) with the groovyBC patch as the inlet condition

In comparison to the Alastruey study, although the velocity contour (figure 4) looked promising the wall shear stresses were found to be too large in comparison, reaching a magnitude of 1500Pa. Possible reasons for this include the quality of the mesh, as well as the magnitude of the fluctuations. Compared to the patient data (next section) the velocity fluctuations of the inlet of the CoW aren't periodic nor do the fluctuations reach more than 0.4m/s. The Womersley number (α) is a dimensionless expression which describes the magnitude of the vicious shear effects along the wall of an arterial/pipe flow under an oscillating inlet. The range was empirically derived by Womersley and found that when the number is greater than 10 the inlet profile is distorted and becomes a 'flat plane' which has little effect on the velocity gradient. Less than 10 the inlet profile has time to develop creating a larger velocity gradient.

The Womersley equation is as follows *Eq 2*

$$\alpha = R\sqrt{\frac{\omega}{\upsilon}}$$

The length scale R in this case is the diameter of the artery, ω is the angular frequency and υ is the kinematic viscosity. Applying this formula to the groovyBC study shows that the resulting Womersley number is 2.22. As this value is much smaller than 10, the velocity profile at the inlet lets the parabolic profile develop along the artery creating a larger velocity gradient, thus increasing the value of the shear-wall-stress.

Fluctuating velocity inlet using TimeVaryingUniformFixedValue:

Taking the perspective of a more patient-specific model an application that OpenFoam has is the TimeVaryingUniformFixedValue. This patch lets the user apply a constant value at an inlet/outlet patch at a defined time. The patient data collected from the Alastruey study (figure 5) was used for the inlet and outlet values at defined time spacing's of 0.05seconds (20 samples). As there are many outlets used in this study (RMCA1, RMCA2 etc.) the equivalent velocity values for each outlet were calculated by determining the equivalent volumetric flow rate from the patient in the Alastruey study, calculating the cross-sectional area of the CoW used in this study and then deriving the velocity accordingly.

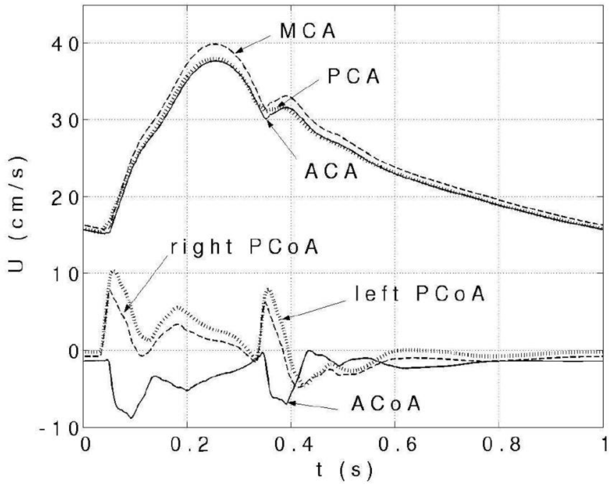
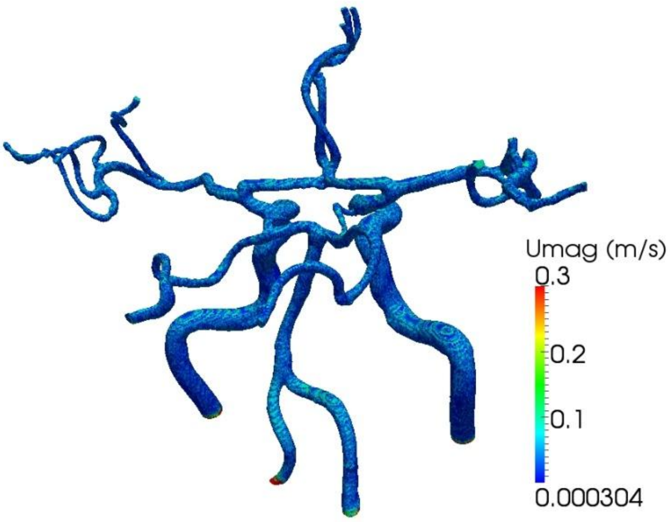


Figure 5: the patient data of a true Circle of Willis (collected using Transcranial Doppler scanning) [3] used for the inlet/outlet data in this study

The time step Δt was set to 1×10^{-5} seconds for 10 seconds. Figure 6 shows the resulting contour diagram of the velocity magnitude at time 2×10^{-5}seconds ($\mathrm{CFL_{mean}} \approx 0.053$).

Figure 6: A velocity contour diagram of the Circle of Willis (basic mesh) with the TimeVaryingUniformFixedValue patch as the inlet condition



The effect of specifying all the inlets and outlets of the model has had an unusual effect on the flow of the fluid around the geometry as various back-flows along the RMCA begin to develop as shown in figure 7. At the same time, the solution to the model becomes more and more unstable as the CFL number increases (discussed in more detail in the 'Discussion' section of this report).

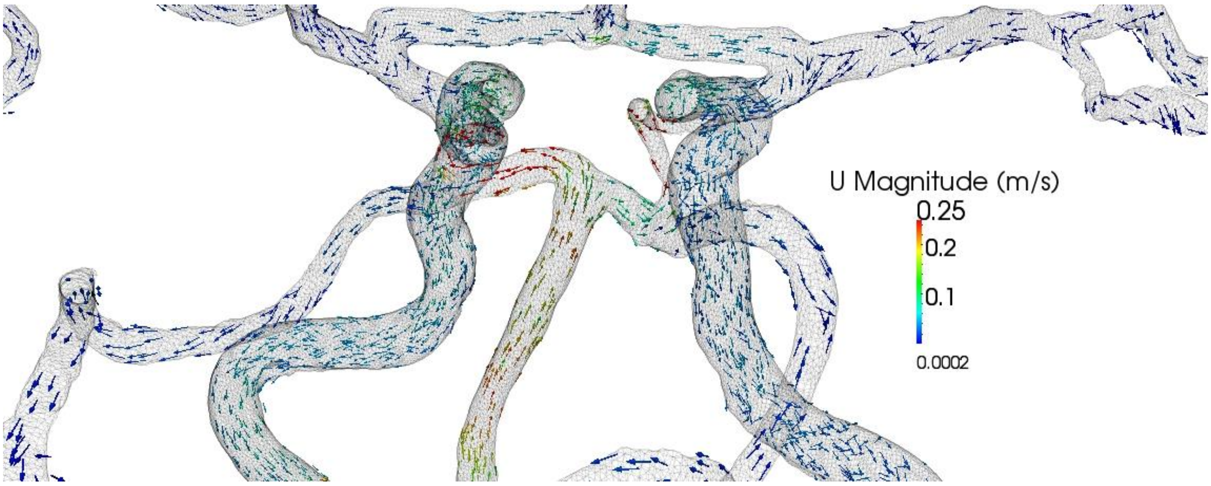


Figure 7: the vector plot of the Circle of Willis (basic mesh) with the TimeVaryingUniformFixedValue patch as the inlet condition at time $\mathbf{2 \times 10^{-5}}$seconds

In the case of this user defined study, the maximum Reynolds number (Re_{max}) fluctuates around the range 88-140. As this range is below the turbulent range for pipe flow ($Re_{laminar} < 2300$) the wall shear stresses can simple be obtained using the equation

$$\tau_{wall} = \mu \frac{dU}{dy} \qquad \textit{Eq 3}$$

The equation 3 shows the linear relationship between the wall shear stresses with the dynamic viscosity ($\mu = \upsilon * \rho$) and the velocity gradient perpendicular to the wall (not necessarily in the y direction). OpenFoam already has an application to calculate the wall shear stresses with the command '*wallShearStress*'. Figure 8 shows the resulting shear wall stress contour.

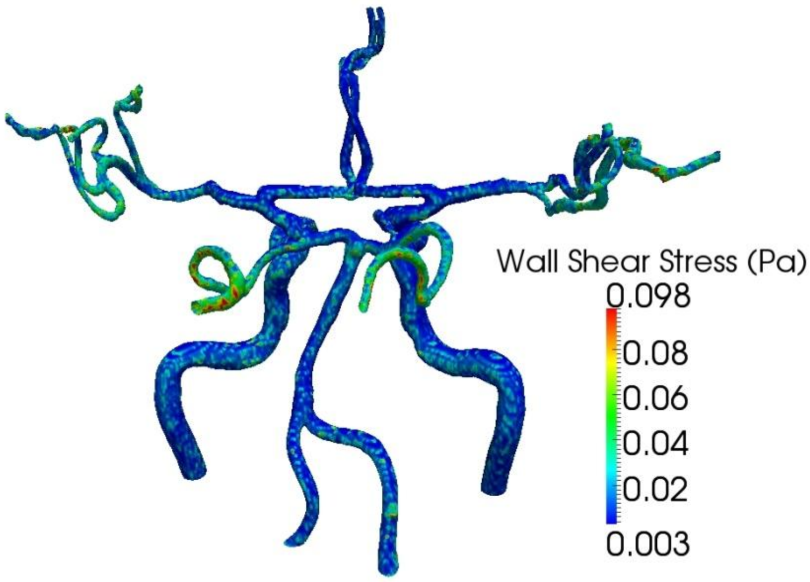


Figure 8: A wall-shear-stress contour diagram of the Circle of Willis (basic mesh) with the TimeVaryingUniform FixedValue patch as the inlet condition at time $\mathbf{2 \times 10^{-5}}$seconds

Figure 8 shows that the main areas of 'high risk' of developing a cerebral stroke are in the PCA arteries (it is well known that increasing the wall shear stress increases the risk of a cerebral stroke). Similarly to this investigation the previous study using the groovyBC had also shown that the PCA's are also have the highest wall shear stresses. These areas also have a high velocity magnitude. Overall it can be deduced from the contour diagrams (figures 6 and 8) that the arteries with the smaller cross-sectional area have a higher velocity and wall shear stress.

Conclusion:

Presented in this report is a demonstration of the practicality of CFD when developing a model of a 'true' Circle of Willis. With the use of the OpenFoam this investigation was able to plot the contour diagrams of the velocity magnitude and resulting wall shear stresses around the model.

This investigation focused on modelling the velocity fluctuations at the inlet and outlets of a 'true' Circle of Willis, and resulting wall shear stresses using patient data at specified time steps and an equivalent sinusoidal model. From these two models the similar result of predicting a high risk (by high risk the wall shear stress are higher than the other areas around the model) in the arteries where the cross-sectional areas are relatively small (roughly 1-2mm).

Despite the promising results presented in this report the model itself is still in a primitive stage as the model does not include the effects of having a non-Newtonian fluid on the wall shear stress or the effects of the dynamic (elastic) walls of the vessel. This report also hasn't so far reported the effects of the pressure distribution around the Circle of Willis as this is not the focus of this report. However this was not completely forgotten, the blood pressure in this investigation used the Hagen- Poiseuille law to calculate the inlet and outlet boundary conditions and is depicted with the equation

$$\Delta P = \frac{8\mu LU}{r^2} \qquad \text{Eq 4}$$

Overall this report and accompanying data could be used as a basis for future work, allowing the research to focus on the larger scope of the project, limiting the time spent on setting up and segmenting the data, and focussing on more patient specific models with an 'incomplete' Circle of Willis. The next section discusses a more accurate representation of the inlet and outlet conditions to produce a more accurate patient specific model.

Discussion:

A main challenge in this investigation was maintaining stability in the solution of the Navier-Stokes equations. This is mainly due to the explicit/semi-implicit methods used to achieve a solution, which were the SIMPLE and PISO algorithms. The condition that must be fulfilled for convergence of an explicit algorithm (marching or finite-element) is the **Courant–Friedrichs–Lewy condition** (CFL condition). This takes the form of a dimensionless number which for convergence needs to be less than 1 (≤ 1) The CFL equation is as follows:

$$CFL = \frac{u\,\Delta t}{\Delta x} \qquad \text{Eq 5}$$

From the *checkMesh* application in OpenFoam the minimum value (producing the largest CFL number) was found to be $\Delta x = 1.034 \times 10^{-9}$m. The largest velocity (producing the largest CFL number) can be derived from the fluctuating velocity at the inlet of the CoW. In the case of the timeVaryingUniformFixedValue this value was found to be 0.4m/s, in the case of the groovyBC this value was found to be 0.5m/s. Overall to maintain a reliable result the value of Δt needs to be 2.6×10^{-9}seconds. In figures 4, 6, 7 and 8 the time step was set to 1×10^{-5}seconds as the velocity values at the inlet were smaller at that time step.

In the case of the timeVaryingUniformFixedValue, the result of this is to apply a uniform distribution in the area of the defined boundary condition. However in order to create a non-uniform distribution (such as a parabolic inlet) a possible solution is to apply the timeVaryingUniformMappedFixedValue application. This application uses a velocity value for each point on the patch, opening the possibility of producing a non-uniform inlet/outlet.

<u>Notes for future research:</u>

In this study, the patient specific inlet and outlets were specified using the timeVaryingUniformFixedValue to specify the inlet and outlets. To run this, firstly the boundary condition representing the inlet/outlet needs to be specified as ‘patch’. The boundary condition needs to be specified at the 0 directory for that specific patch in the following form:

```
type            timeVaryingUniformFixedValue;
fileName        “name of file containing data either .txt or .dat”;
outOfBounds     clamp;
```

The condition ‘Clamp’ in this case stops the calculation after running out of data in the file under ‘fileName’ however other conditions include ‘repeat’ which starts back at the beginning after running out of inlet/outlet data. In the run directory (containing constant, system and 0) a new file needs to be created containing the data with the time and the corresponding velocity values for example

```
(
(0 (0 0 0.15)
(0.1 (0 0 0.25))
)
```

The above example shows that when the time value is equal to 0.1seconds, the velocity value in the z direction is equal to 0.25m/s and between 0 and 0.1secs the velocity value is a constant 0.15m/s. Past this point of 0.1seconds the velocity is 0.25m/s only.

In the case of the timeVaryingMappedFixedValue the same idea of the timeVaryingUniformFixedValue is applied however there a few differences between the two. In the 0 directory the form that the timeVaryingMappedFixedValue should be

```
Type          timeVaryingMappedFixedValue;
 setAverage    off;
```

In the case of the timeVaryingMappedFixedValue application the fluctuating velocity depends on the points at that particular inlet/outlet. To find these points, this investigation found that by sampling each of the surfaces or patches individually and specifying that the output data should be ‘raw’ (xyz data) and using the format ‘foamFile’ the data is exported as a directory containing ‘faces’, ‘faceCells’ etc. but most importantly a ‘points’ file. An example of the sampleDict file is as follows

```
interpolationScheme cell;
setFormat raw;
surfaces
(
RMCA3
   {
      type          patch;
      patchName      RMCA3;
   }
);
fields (U);
sets ();
surfaceFormat foamFile;
```

With the points collected for each of the patches the timeVaryingMappedFixedValue requires a new directory to be created in the ‘constant’ dictionary where the vector/scalar and time data is read. This dictionary should be entitled ‘boundaryData’. Within this dictionary the dictionary containing the patch name should be created. In this dictionary more dictionaries should be created (entitled the time value e.g. 0.05). Within the time dictionary the points file should be added with another file specifying the vector/scalar at each of the points (e.g. with 45 points the file should contain 45 entries for the values at each of the points). Overall the layout should look like figure 9.

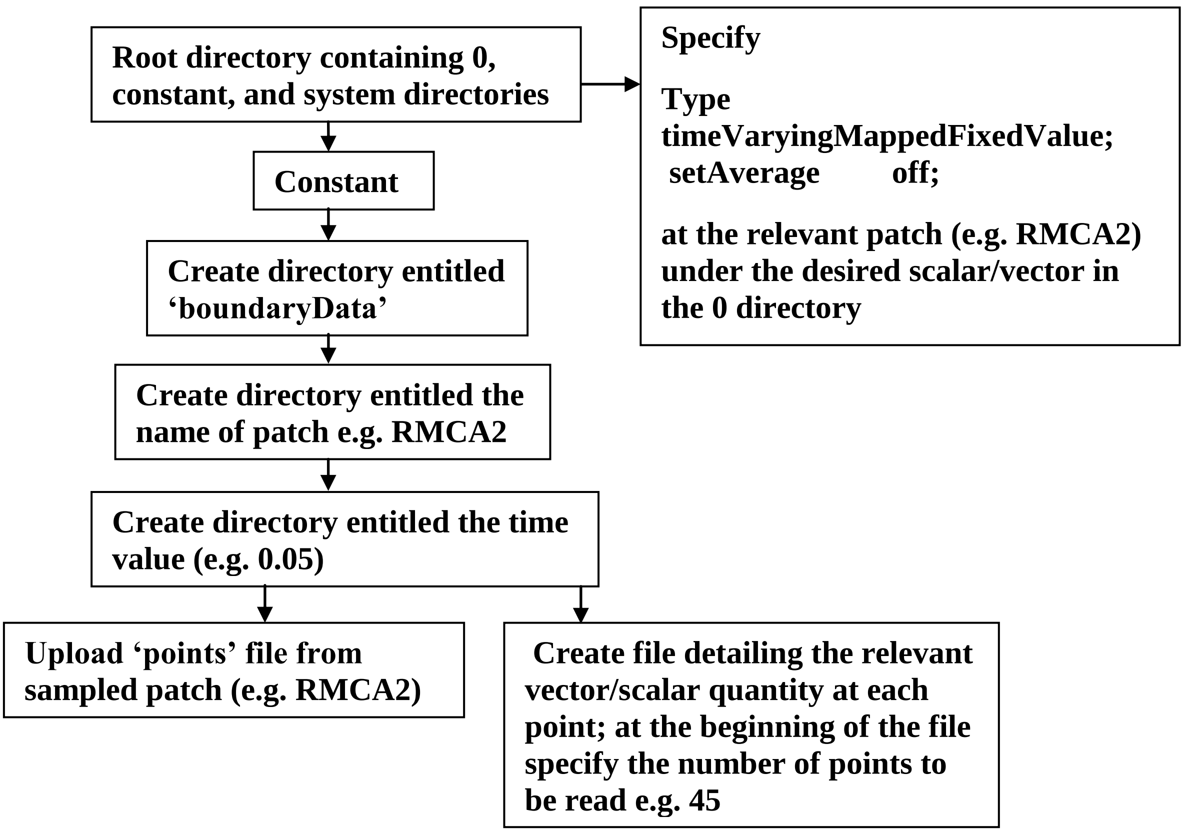


Figure 9: the layout to execute the timeVaryingMappedFixedValue application in OpenFoam

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