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I2 Report

Investigating and validating current wake models for use in creating a Mathematical Model to predict power output and optimise the layout of Offshore Wind Farm Arrays

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4th year MEng Group Project

I certify that all material in this thesis that is not my own work has been identified and that no material has been included for which a degree has previously been conferred on me.

Signed..

College of Engineering, Mathematics, and Physical Sciences
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I2 Report

ECMM102

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Abstract

This project formed part of a group project that aims to both identify discrepancies with current techniques that analyse the power output of wind turbines and to develop new techniques. As part of this, this individual project has analysed Engineering Wake Models that are currently used in industry and created a mathematical model that can be used in future analyses.

This project has discovered that aside from differences in prediction of the wake velocity deficit, the models that have been employed mainly differ in their predictions of the downstream wake expansion. This has a large effect on the total power output of an array as downstream wake interactions are responsible for the majority of power losses.

It has also been identified that changes in wind direction massively affect the power output of wind farms, ±10 degrees results in the largest power output however any angle between ± 30 degrees results in an increase in power compared to the 0 degree condition.

Finally this project has resulted in the creation of a functioning mathematical model utilising the three wake models typically employed in industry. The model can predict the power output for a wind farm with any layout up to 20 turbines and can also be used for any wind direction. It is fully editable and only requires a few starting inputs and the wind farm layout.

It is recommended that further works includes the acquisition of LiDAR or other experimental data to further validate the model. It is also recommended that the optimisation code that is given is refined so that a module to optimise the layout of a wind farm can be added to the mathematical models.

Keywords: Engineering Wake Models, Wind Turbines, Wake Deficit, Wake interaction, Wake Expansion, Mathematical Model

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1. Introduction and background

As alluded to in the I1 report submitted previously, public attention is being increasingly drawn to issues of sustainability, the impact of our use of fossil fuels and the more environmentally friendly alternatives. One of the major sources of renewable energy is from harnessing the power available from the wind, however this technology comes with inaccuracies in the prediction of the power output of wind turbines and therefore wind farms.

Before a wind turbine array is built various calculations and optimisation studies are performed in order to ascertain the likely power outputs, and therefore profits, from a particular array. It has been found however that these calculations are consistently over predicting the amount of energy produced by the arrays.

This individual project will investigate a range of different algebraic Engineering Wake Models (EWMs) typically employed in industry, these will then be incorporated into a mathematical model to predict the power output of a wind farm. Furthermore these models will be compared to the CFD simulations carried out by other members of this group. A basic optimisation study will be implemented and compared to work carried out in parallel by a further group member which focuses on using more complicated optimisation techniques.

Finally conclusions will be drawn in the effectiveness of using the models employed in industry and the CFD analysis that has also been carried out.

1.1. Project evolution

This project has undergone several changes since the submission of the I1 report, and while the aims of the project (outlined below) have remained broadly the same the objectives that need to be met, and the process through which they are achieved have been altered.

Initially the aim of the group project was to provide analysis of the Rodsand II wind farm in Denmark, however the group was unable to get hold of the relevant data that would have enabled an accurate characterisation of the wind farm layout and the turbines situated in it. Instead the group have decided to carry out a case study on a non-descript wind farm, with a simple straight row and column design. This layout will enable more practical comparisons to be drawn from the range of studies carried out. Some elements of this project will also carry out layout optimisation studies using the same number of turbines as in the simple studies.

The lack of data from the Rodsand II wind farm has also resulted in a lack of real world validation for this project. Instead this project has used other similar studies that have been carried out to validate the models and findings within this report.

In the I1 report it was proposed that this project would carry out CFD simulations of wind farms as well as mathematical model calculations. However as the project progressed it was discovered that the setting up of the models was more complicated than previously thought, and so the running of CFD calculations as well was not feasible. Instead the models created in this report will compared to the simulations carried out by other members of the group.

1.2. Aims

- Identify discrepancies between typically employed wake models and observed effects
- Create useable and editable mathematical model for use in future research
- Link the evaluation of currently used techniques with new approaches and provide validation of these techniques

1.3. Objectives

In order to meet the project aims the following list of objectives has been identified:

- Identify and evaluate most commonly used wake models
- Identify wake summing methods
- Set up robust and customisable mathematical models
- Create basic optimisation algorithm
- Compare gathered data to other group members, including performance of mathematical models to CFD and optimisation techniques

1.4. Report Structure

This report is to serve as a stand-alone report to detail the work that has been carried out individually on this project. It is an extension of the I1 report previously submitted, going in to more detail and recording the advances in work carried out. This report also compliments the G2 report that will be submitted with it and focuses on the specifics of this individual project which will be referenced in the G2.

The report contains an extensive literature review, which has been built from the review submitted in I1. The underlying theory of this project will be examined before the results are presented. Any conclusions from these results will then be drawn along with comparisons to other work and the direction that future work should take. Finally this report will end with a description of the project management tools that have been used and an analysis of how this project contributed to the group as a whole.

2. Literature review

As is to be expected there is a huge range of literature available on wake models and wind farm layout optimisation. This review will be limited to explore previously carried out work on the relevant wake models, the methods used to combine the wake deficits of wind turbines, the specific papers used to validate the models and finally an investigation into wake meandering and the current state of the art of wake deficit prediction.

2.1. Wake Models

If an object is placed in a fluid flow it leaves a wake behind it, this principle is applicable to a wind turbine and its effect on the air flow around it. The wake behind a wind turbine is characterised by a stream-wise velocity deficit and this wake deficit results in a reduction in power for the downstream turbines [2]. As stated earlier *Herbert-Acero et al* [1] claim that wake effects can result in power losses of up to 41% (the extent of the loss will vary with the parameters of each scenario including turbine spacing, wind direction and site complexity. They also report that the effects can be seen in the atmospheric boundary layer which can have an influence on neighbouring turbine arrays as well as other turbines within the same array.

With such a potentially large effect on the performance of an array it is important to predict the wake effects as accurately as possible. In an effort to predict these wake effects sub models are often used because the mathematical model that is used to predict the performance of the farm must be as accurate as possible [1]. These models are used to "*translate the implications of adverse effects into performance measures"*[1], or in other words are used to consider factors of importance and varying complexity, such as wake effects.

As *Herbert-Acero et al* [1] state the investigation into wind turbine wake modelling has been conducted for over four decades and has resulted in two different approaches. The first approach is based on what are known as EWMs, which are algebraic expressions for the wake conversion and evolution that takes place behind the turbines.

The second approach is based on Reynolds-Averaged Navier stokes (RANS) equations or Large Eddy Simulations (LES). For an analysis of these techniques refer to the reports of the other members of the Large Scale Simulation team within this group project.

It is generally agreed that the second approach, known as Computational Fluid Dynamics (CFD), is the more accurate of the two approaches when describing the evolution and effects of the wakes [1] however they are more computationally expensive. As a result of this EWMs are generally used in industry when predicting and optimising the power outputs of wind farms.

It has been suggested that EWMs model the far wake velocity deficit accurately but tend to over predict wake effects [1]. This is an area that this project aims to investigate by comparing various different EWMs with CFD simulations.

2.1.1. Jensen-Katic

The Jensen Katic model, as proposed by *Katic et al* [3] in 1986 and was specifically designed to optimise the turbine locations for a given site [3]. Despite being one of the simplest EWMs it claims good agreement with many of the more complicated models [3] and as a result of this is one of the most widely used [1, 4]. The model assumes a linearly expanding wake with the velocity deficit being dependant only on the distance behind the rotor [5,6].

The simplicity of this model stems from both the equation which predicts the velocity deficit and from the prediction of the wake expansion. The equation can be seen in the theory section below, while the wake is considered to be axisymmetric and has a top-hat profile [1,4,5].

This model does have limitations [1], as it relies on momentum conservation in a fully developed wake it is valid only in the far wake region. The definition of far wake however is often contested [1] starting between 2 and 4 rotor diameters downstream [1,4,6].

However, as *Duckworth* [4] and *Tong et al* [6] state wind turbines in arrays are rarely if even situated in the near wake region, therefore validity in the far wake region is all that is required for use in predicting velocity deficits in wind farms.

Jeon et al [7] also note that this model is inaccurate in the prediction of the wake form when compared to other models and experimental data due to the simplicity of the expression through which the prediction is made.

2.1.2. Frandsen

The Frandsen model, as proposed by *Frandsen et al* [8] in 2006 was designed to be able to be used on wind farms of all sizes. It is primarily designed for use in wind farms with straight rows and columns [8]. The model shows good agreement with the Jensen Katic model and the Schlichting model for a single turbine [8].

The Schlichting model refers to the 'classic' wake theory that was developed by Schlichting in the 1940's and is considered to be a suitable approximation for the wake deficit observed behind a single turbine.

The Frandsen model differs from the Jensen Katic model in its approximation of the wake diameter as it evolves downstream. Like the Jensen Katic it considers the wake to be axisymmetric and have a constant transversal profile (i.e. top-hat profile) [1,5]. The equations for the velocity deficit and the wake expansion can be seen later in this report.

This model is an empirical model and is used mainly in determining the fatigue loading across a turbine [4], however it has been included in this study as according to *Duckworth et al* it is also represents the current state of the art of velocity deficit prediction.

2.1.3. Larsen

The Larsen model was first proposed in 1988 [9], in this paper Larsen proposed a first and second order model that could be used to predict the velocity deficit in the wake of wind turbines. Both of these models required experimental data to be used. It also requires the Prandtl mixing length to be used to define a non-dimensional mixing length [9].

The Prandtl mixing length is a length scale that is related to the turbulence of a flow and can be used to define the flow when combined with a turbulent velocity scale [10]. The Larsen model was updated in 2009 [1] and the dependency on the Prandtl mixing length was removed.

Instead the wake radius at 9.5 rotor diameters downstream of the turbine is required. This can be calculated in one of two ways, the first is to use another model to approximate the wake radius at this location. A better way however was set out by Larsen in the EWTS II report, in which the distance can be approximated using some empirical expressions [5,11].

Like the Frandsen model earlier this model is based on the Schlichting ‘classic’ wake theory [4] and, as stated earlier uses empirical relations to predict the turbulent intensity and turbulent length scale. As with the other models it assumes an axisymmetric wake and it also neglects ground effects (which again is common to all the models investigated here) [4].

2.2. Wake interaction

Due to space and practicality constraints wind turbines are often placed relatively close to each other. This inevitably leads to the wakes of the individual merging and affecting downstream turbines. This section of the literature review will investigate the two main problems that arise when modelling the merging of wakes and the resulting velocity deficit.

The first of these issues is how to calculate the total velocity deficit within a merged wake. It has been observed that this deficit depends mainly on the closet wind turbine [1]. The interaction of multiple wakes is not yet fully understood and as *Herbert-Acero et al* [1] state is still the subject of many studies however it is agreed that the merging process is based on the concept of constructive superposition.

The current best practice, and most widely used method, is to sum the kinetic energy deficits of all the wind turbines which contribute to the wake, in other words the sum of the square of the velocity deficits [1]. This method was proposed after the original method of adding the velocity deficits and turbulence kinetic energy was found to be at fault as it led to the overestimation of velocity deficits and can lead to negative velocities where many wakes merge [12]. There are many other wake merging methods but, as stated previously, the sum of the squares of the velocity deficits, is the most widely used in industry [1,5,12,13]

The second issue of wake merging is that the downstream turbine might not be completely in the wake of an upstream turbine. This can be solved by calculating the area of the turbine that is in the wake and averaging the wake velocity with the free stream velocity that is present on the rest of the turbine [14]. This method can also be employed where a turbine experiences the effects of more than one upstream wake.

2.3. *Wake model validation and comparison*

2.3.1. Model Validation

To ensure that the mathematical models were set up correctly two different sources were used to validate them. The first [8] was published in 2006 and is the paper in which the Frandsen model was first proposed. As part of the justification and validation *Frandsen et al* [8] compared their model to the Jensen Katic model as well as some other individual wake models.

This then provides an excellent opportunity to validate the mathematical models created in this project and to ensure that the single wake models have been implemented correctly. This paper is from a from a peer review journal and as such can be taken as a very reliable source against which to validate. Furthermore the model that was proposed (Frandsen model) has since become one of the industry standards in wake modelling, which lends further credence to the findings and therefore in its ability to verify the mathematical models implemented.

The second paper [6] that has been used for validation is also from a reputable source having been published in a peer reviewed journal as well. *Tong et al* [6] provide comparisons of 4 different wake models including the Frandsen, Larsen and Jensen Katic model. As well as providing comparisons of the wake speed behind a turbine, as with the previous paper, *Tong et al* [6] compare the wake expansions of each model.

All of this data will be useful in ensuring that the wake models have been implemented correctly and in validating the mathematical model.

2.3.2. Angles

The angle study that will be carried out will also be compared to a range of different sources, *Barthelmie et al* [15] have used a range of different pieces of software to analyse the effect of varying wind direction on the Horn's Rev wind farm. *Porté-Agel et al* [16] have also studied the effect of wind direction on the Horn's Rev wind farm using Large Eddy Simulations (LES).

Both of these papers provide good opportunity to compare the wake models employed in this study against previous studies using a variety of techniques. Both of these papers have been published in peer reviewed journals and so can be regarded as valuable sources.

Unfortunately due to the nature of the reports and the details given therein it has not been possible to completely replicate the experiments that have been carried out. Nevertheless it is still possible to compare results as both studies plot the normalised power of the wind farm which allows comparisons to be drawn irrespective of the set up.

The angle study will be compared with the results obtained by *March* [17] who used LES to study the effect of wind direction for the same case as this study. The results obtained have also been compared to other literature which increases the confidence in any conclusions made.

2.4. State of the art

The current commonly used approach in wind farm design codes is to combine single wake calculations with a method for wake superposition to determine the merged wake deficit [13]. As mentioned earlier the fluid mechanics behind the merging of wakes is a subject that is still not yet fully understood [1].

Despite this multi-wake models are beginning to be formulated but as of yet have not received proper validation to be adopted by industry. While this would have been an interesting topic to investigate it would have required an entire group project on its own and would have also necessitated the acquisition of LIDAR or other experimental data. The same can be said for investigation the wake summing which falls into this sub category.

Machefaux et al [13] have also investigated 4 different wake superposition techniques against validated CFD simulations. They discovered that the Dynamic Wake Meandering model and G.C. Larsen superposition method (two of the newest proposed wake model techniques) show a good degree of accuracy. It was also noted that quadratic technique that will be employed in this project showed acceptable performance albeit with higher uncertainties.

They concluded that a further model, based on a combination of currently used techniques would be developed to improve prediction accuracy. Sideways wake merging is also an area that will receive more attention. This paper was published in 2015 in the Journal of Physics and so represents the current state of the art of wake modelling.

3. Theory and Set up

3.1. Spreadsheet

As has been explained previously, this project has two main aims, the second of which is to create a useable and editable mathematical model that will describe the evolution of the wake deficit behind a field of wind turbines. The first aim is to identify the cause of discrepancies between industry predictions of these wake deficits and the observed results.

In order to meet both of these aims an Excel spreadsheet was created to serve as the mathematical model required for the second aim, it also allowed the first aim to be evaluated as the model will use some of the equations and theory that is currently used throughout the industry. This background theory has been briefly touched on in the literature review but will be investigated more extensively in the following sections.

The creation of this spreadsheet was of vital importance to the project as such several sub-objectives were created to ensure that it functioned as it should:

- The model should use the most prevalent industry techniques to give an accurate representation of the results obtained (these techniques will also be the most validated techniques)
- The model should be fully editable and therefore able to model any wind farm up to the number of turbines provided for

The first objective has been met by using different wake models and wake merging techniques these models will be compared later on in this report which may help to identify discrepancies. As expressed in the literature review wake merging is a difficult subject and so the sum of squares method has been used as it is the most common and most validated technique.

The second objective set out above has been achieved by making the model as simple for the user as possible. Each turbine (of which the model can include up to 20) has its own tab within the spreadsheet. Once the turbines have been assigned positions each tab calculates the influence of the wakes from upstream turbines on the turbine to which the tab has been assigned. The tab then calculates the wake deficit at a distance downstream that has been specified by the user. Furthermore the power output for the turbine is also calculated, the total power output for the farm is displayed on the input tab which also allows the user to specify the location of the turbines, the angle of the wind on the farm and other base values that are needed to characterise the model.

As well as showing the velocity deficit for each turbine (and subsequently the power output for the farm) the spreadsheet has a tab that shows the location of each turbine and the wake expansion so that it is easy to view which wakes interact with the downstream turbines.

A basic optimisation tool has been programmed into the spreadsheet using Visual Basic for Applications (VBA) which is Excel's programming tool. A piece of code was written that places the turbines in the farm to in random locations within a set domain. The total power of the wind farm layout is calculated as normal before another random location is chosen.

If the second location results in a larger power output than the first then the new location is remembered along with its power output, if not the original is kept. This is a very basic form of optimisation that relies on the program being able to quickly run a large number of iterations. As this model is a driven by Excel it is far quicker than many other type of simulations such as CFD. The results of the optimisation studies will be investigated later in this report.

3.2. Wake Models

This section of the report will describe the theory behind the different wake models, including how they calculate the velocity deficit and the downstream wake expansion. *Herbert-Acero et al* [2] explain that all wake models have a basic mathematical structure as shown below:

$$(1) U_w(x,r) = U_0\left(1 - U_{Def}(x,r)\right) \quad (2) U_{Def}(x,r) = 1 - \frac{U_w(x,r)}{U_0}$$

$U_w(x,r)$, which is sometimes simple referred to as U, is the horizontal wind speed in the wake at some distance x (m) downstream of the turbine and some radial distance r (m) from the centre line. U_0 is the free stream velocity and $U_{Def}(x,r)$ is the velocity deficit which, as can be see above, can be used to express the wake velocity as a function of the free stream velocity.

The wake velocity is typically lowest just behind the turbine and recovers to the free stream velocity downstream of the turbine. As the distance downstream increases the width of the wake also expands, in general wake models differ by how they approximate the wake velocity recovery and the expansion rate of the wake. Most studies rewrite equation (1) in the form shown in equation (2), this is because the velocity deficit is normally of particular interest to the study, it also allows studies with different initial conditions to be compared as a ratio of velocities is used rather than the absolute values that will vary between studies.

3.2.1. Jensen Katic

$$(3)\ U_{Def} = 1 - \frac{U_w\,(x)}{U_0} = \left(1 - \sqrt{1 - C_t}\right)\left(1 + \frac{2kx}{D}\right)^{-2}$$

As with the previous equations U_w (x) is the velocity in the wake and U_0 is the velocity in the free stream. Equation 3 also uses Ct, k, x and D, which are the thrust coefficient, wake decay constant, downstream distance and rotor diameter respectively to calculate the velocity deficit.

The wake decay constant, k, is used to approximate rate at which the wake recovers, it is used in this model instead of attempting to model the turbulent intensity of the scenario. It has been suggested in literature [3] that this value should be 0.075 for onshore farms and 0.04 for offshore farms. It is expected that onshore farms require a higher value as the surrounding terrain for onshore farms is generally less flat which results in higher turbulence compared to offshore farms.

In equation 3 seen above x represents the distance downstream of the turbine, it can be seen then that as the downstream distance increases the wake velocity also increases, this shows how the wake recovers downstream of the turbines which matches basic assumptions and observations.

D, represents the diameter of the wind turbines rotor, this varies between different makes of wind turbines and should be ascertained before the calculation is carried out.

$$(4)\quad D_w = D(1 + 2ks) \quad (5)\ \ s = \frac{x}{D}$$

The Jensen Katic model assumes that the wake behind a turbine expands linearly as shown in equation 4 above. D_w is the wake diameter at a distance (x) downstream of the turbine. As before k is the wake decay constant (set to 0.04), s, as can be seen in equation 5, is the ratio between the distance downstream and the diameter of the turbine rotor.

As can be seen the wake expands with increasing distance downstream, this is normally referred to as a “top hat profile”. This is because the profile seen is similar to that of a top hat, being circular and wider at one end than the other.

3.2.2. Frandsen

$$(6)\quad \frac{U_w(x)}{U_0} = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 2\frac{A}{A_{w\,(X)}}\,C_t}$$

The $\pm$ seen in equation 6 is either + or – depending on the value of Ct, this is logical as there can only be one speed at a given location in the wake and so having both values that the $\pm$ implies would not make sense.

Frandsen et al [8] state that for $0 \leq C_t \geq 0.75$ the $\pm$ in equation 6 becomes + (for values higher than 0.75 – is used). For this project values of less than 0.75 were used for Ct. Combining this with equation 2 gives:

$$(7)\qquad U_{Def} = 1 - \left(\frac{1}{2} + \frac{1}{2}\sqrt{1 - 2\frac{A}{A_{w(x)}}Ct}\right)$$

This is the equation that was used in the spreadsheet, combing equation 6 with equation 2, puts this model in the same form as the others, having a common form aids the construction of the mathematical model while simultaneously making it easier to compare the models to each other. In equation 7 shown above A is the swept area of the wind turbine rotor and $A_{w(x)}$ is swept area of the wake some distance x downstream of the turbine. This can be calculated as shown in equation 8 below:

$$(8)\quad A_{w(x=0)} = \beta A \qquad\qquad (9)\qquad \beta = \frac{1 + \sqrt{1 - Ct}}{2\sqrt{1 - Ct}}$$

At any distance other than x = 0 $A_{w(x)}$ can be calculated if the wake diameter $D_{w(x)}$ is known:

$$(10)\qquad D_{W(x)} = \left(\beta^{\frac{k}{2}} + \alpha_s\right)^{\frac{1}{k}} D \qquad\qquad (11)\qquad \alpha_s = \beta^{\frac{k}{2}}\left(\left(1 + 2\alpha_{noj} s\right)^k - 1\right) s^{-1}$$

The value for k varies depending on which approximations and variance of the model that is being used. *Frandsen et al* [8] use values of k of 3 in their validation and 2 for normal use of the model. It has been found that other studies [6] have also used a value of 2 for k and so this study has followed the example of *Frandsen et al* [8] and has used to a value of 3 to validate the model and has used 2 to perform comparisons and predictions.

The model also uses α_{noj}, as seen above, which is the decay constant used in the Jensen Katic model (described earlier as k). The same values will be used in both models.

3.2.3. Larsen

$$(12) \qquad U_{w\,(x,r)} = \frac{U_\infty}{9}\left(CtA(x+x_0)^{-2}\right)^{\frac{1}{3}}\left[r^{\frac{3}{2}}\left(3C_1CtA(x+x_0)^{-\frac{1}{2}}\right)-\left(\frac{35}{2\pi}\right)^{\frac{3}{10}}\left(3{C_1}^2\right)^{-\frac{1}{5}}\right]^2$$

As with the previous models the Larsen model can be combined with equation 2 and rearranged to give the velocity deficit downstream of the turbine as shown below.

$$(13) \qquad U_{Def} = 1-\left(\frac{1}{9}\left(CtA(x+x_0)^{-2}\right)^{\frac{1}{3}}\left[r^{\frac{3}{2}}\left(3C_1CtA(x+x_0)^{-\frac{1}{2}}\right)-\left(\frac{35}{2\pi}\right)^{\frac{3}{10}}\left(3{C_1}^2\right)^{-\frac{1}{5}}\right]^2\right)$$

The wake radius for this model is calculated as shown:

$$(14) \qquad R_{w(x)} = \left(\frac{35}{2\pi}\right)^{\frac{1}{5}}\left(3{C_1}^2\right)^{\frac{1}{5}}\left(CtA(x+x_0)\right)^{\frac{1}{3}}$$

Both equation 13 and 14 use the term C_1 and x_0 which have to be evaluated separately, it was originally suggested by Larsen that the value of C_1, which is a coefficient that can vary for each scenario, can be calculated using the Prandtl mixing length.

The Prandtl mixing length hypothesis was first suggested as a basic turbulence model, the mixing length is the distance it takes at which eddies will have dispersed within the surrounding fluid. This theory was described by Prandtl himself as own a rough approximation despite its extensive use on a range of topics.

However for this project a different method was chosen to evaluate C_1 as used by *Larsen et al* [11]. C_1 can be found using:

$$(15) \qquad C_1 = \left(\frac{105}{2\pi}\right)^{-\frac{1}{2}}\left(d_1\frac{D}{2}\right)^{\frac{5}{2}}\left(CtAx_0\right)^{-\frac{5}{6}} \qquad\qquad (16) \qquad d_1 = \left(\frac{\left(1+\frac{1}{\sqrt{1-Ct}}\right)}{2}\right)^{\frac{1}{2}}$$

Where d1 is shown in equation 16 above and x_0 can be seen in equation 17 below:

$$(17) \qquad x_0 = \frac{9.6D}{\left(\frac{2R_{w(9.6D)}}{d_1D}\right)^3-1}$$

$R_{w(9.6D)}$ refers to the wake radius at a distance of 9.6 rotor diameters downstream, the literature varies between using 9.5D and 9.6D, this is down to the issue of the near and far wake regions and the inability to accurately define them.

Larsen et al [11] also suggest that the wake radius at the 9.6D downstream should be recorded from observations, however this is not possible for this project as there is no wind farm that it is based off (as has been mentioned previously there was an issue with the LIDAR data which the group was meant to receive).

As a result of this the radius at the point required has been approximated using the wake width equations from the other models.

3.3. Wake adding

3.3.1. Intersection Area

One of the most important aspects of this mathematical model is creating a robust way to calculate whether a downstream turbine is affected by the wake of an upstream turbine. There are three states that a downstream turbine can be in, the first is when the turbine is completely in the wake of the upstream turbine, depending on the wake expansion this can either be directly behind (as is normal in a regular turbine layout) or can vary by some distant in the y direction as long as the turbine inlet is still within the wake.

For this first case it is known that the inlet velocity for the downstream turbine is simply the recovered wake velocity at the downstream turbines location, and so can be simply evaluated using the models explained previously.

The second case occurs when the downstream turbine is not affected by the upstream turbine, this case is simple as the inlet velocity of the turbine is the same as the free stream velocity.

The third case occurs when the turbine is partially in the wake of the upstream turbine, in this case the inlet velocities of the recovered wake from the upstream turbine and the free stream velocity are averaged over the area of the turbine that they respectively cover.

As mentioned earlier it is important to create a set of boundary conditions and equations that can identify which of the three states each turbine is in with respect to all of the other turbines in the array and from there calculate the inlet velocity at each turbine.

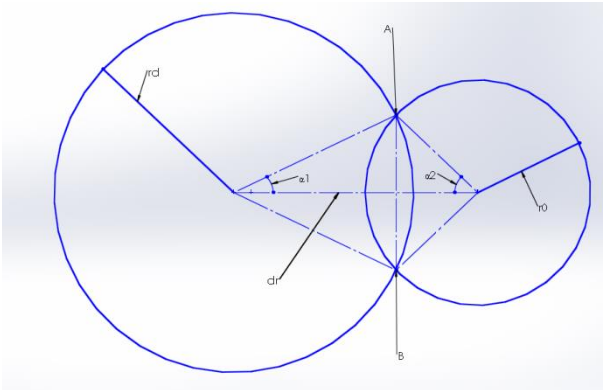

Figure 1: Sketch showing the interaction between the wake of an upstream turbine (Right hand circle) and the turbine (left hand circle)

Figure 1 above illustrates the third scenario, where the wake from an upstream turbine can partially interact with a downstream turbine. As is to be expected the wake has a larger area than the turbine, since it has had a chance to expand downstream of the turbine that created it.

The area of intersection can be seen as the sum of the two segments, the area of a circle made by a line and an arc [18], either side of the chord which runs between points A and B. As can be seen in Figure 1, rd is the radius of the wake at the distance, x, of the downstream turbine, r0 is the radius of the turbine blades and dr is the distance between the centre of the wake and the centre of the downstream turbine.

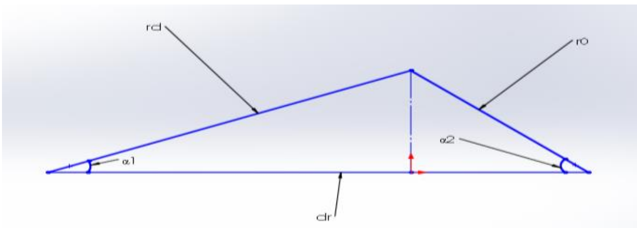

Figure 2: Simplified illustration of important distances from Figure 1

The angles α_1 and α_2 can be calculated simply using rd, r0 and dr as shown in Figure 2 above.

Applying the Cosine rule to the left hand triangle in Figure 2 results in the equation below:

$$(18) \quad {r_0}^2 = {r_d}^2 + dr^2 - 2r_d dr cos(\alpha_1)$$

Which can be rearranged to solve for α_1:

$$(19) \quad \alpha_1 = \cos^{-1}\left(\frac{{r_d}^2 + dr^2 - {r_0}^2}{2r_d dr}\right)$$

A sector of a circle is defined as “the area between two radii and the connecting arc of a circle” [19] and the area of the sector is given by:

$$(20) \qquad A = \frac{\theta}{2} r^2$$

Where Θ is the angle between the two radii which in this study is equal to 2α. The area of the triangle, which along with the intersection segment makes up the sector, can now be calculating using:

$$(21) \qquad A = \frac{1}{2} bh$$

Where b is equal to the length of the chord AB and h is a fraction of the distance between the centre of the wake area and turbine area, dr.

$$(22) \qquad \sin\alpha_1 = \frac{b/2}{r_d} \qquad b = 2r_d \sin\alpha_1$$

$$(23) \qquad \cos\alpha_1 = \frac{h}{r_d} \qquad h = r_d \cos\alpha_1$$

These can be subbed back into the previous equation to calculate the area of the triangle:

$$(24) \qquad A = \frac{1}{2} bh = \frac{1}{2}\big((2r_d \sin\alpha_1)(r_d \cos\alpha_1)\big) = r_d{}^2 \sin\alpha_1 \cos\alpha_1$$

Combining this equation with the equation that calculates the area of the sector it is possible to calculate the area of the right hand segment of the overlap of the two areas:

$$(25) \qquad Area_{Segment} = Area_{Sector} - Area_{Triangle}$$

$$(26) \qquad Area_{wake\ overlap} = (\alpha_1 r_d{}^2) - (r_d{}^2 \sin\alpha_1 \cos\alpha_1) = r_d{}^2(\alpha_1 - \sin\alpha_1 \cos\alpha_1)$$

This shows how the segment that overlaps from the wake area can be calculated and the same procedure can be used to calculate the area of the second segment, from the wind turbine area.

This uses the right hand triangle from Figure 2, applying the Cosine rule gives the angle as:

$$(27) \qquad \alpha_2 = \cos^{-1}\left(\frac{dr^2 + r_0{}^2 - r_d{}^2}{2drr_0}\right)$$

The method to calculate the area of the segment which intersects is also the same and with the radii of the circles being the only difference, such that:

$$(28) \qquad Area_{Turbine\ overlap} = (\alpha_1 r_0{}^2) - (r_0{}^2 \sin\alpha_1 \cos\alpha_1\ ) \ = r_0{}^2(\alpha_1 - \sin\alpha_1 \cos\alpha_1\ )$$

The total area of intersection can be seen as the sum of these two areas and so:

$$(29) \qquad Area_{Intersection} = r_d{}^2(\alpha_1 - \sin\alpha_1 \cos\alpha_1\ ) + r_0{}^2(\alpha_1 - \sin\alpha_1 \cos\alpha_1\ )$$

This is a relatively simple equation that only requires three inputs, the wake radius r_d, the turbine radius r_0 and the distance between the centres of the wake and the turbine dr. As will be shown later this equation is a fundamental piece of the mathematical model as it allows the boundary conditions for the three interaction states to be determined.

3.3.2. Summing Method

The wake summing method that is used is, in theory, a relatively simple method to implement. The governing equation can be seen below:

$$(30) \qquad U_{Def}{}^2 = U_{Def\ 1}{}^2 + U_{Def\ 2}{}^2$$

Where U_{Def} is the total deficit and $U_{Def\ 1}$ and $U_{def\ 2}$ are the deficits of the contributing wakes. This equation can be built upon so that it includes the deficits for however many wakes are interacting with the downstream turbine. The total velocity deficit of the wake will enable the inlet velocity of the downstream turbine to be calculated using a combination of the wake models and the wake overlap as detailed above.

The “free stream velocity” for each turbine will also vary depending on the positioning and wake expansion of upstream turbines. The EWMs that have been employed in this project are all designed for the single wake case which means that their free stream velocity detailed in the equations will be equal to the inlet velocity. However for the case of a wind farm it would be wrong to set the free stream velocity of a downstream turbine as the free stream velocity of the whole farm if it is influenced by the wake of an upstream turbine.

Instead the free stream velocity, which is the velocity the wake of the turbine will recover to, will be influenced by the recovery of any wakes that it interacts with. As such the wake deficit equation above can be used to model the recovery of the wake of a turbine that may be subject to interactions from many other wakes.

3.4. *Wind Direction*

This model was initially set up to be used with the wind approaching the turbines head on, that is to say that the wind was assumed to hit the array in the direction that the farm was orientated, an angle of 0 Degrees.

However it is known that in real world applications wind direction is rarely constant, despite its tendency to have a prevailing direction. It is fair to assume that the wind direction will vary during the life of a wind array and this will have a huge effect on the performance of the wind array as a whole.

Other members of this group project are investigating the change in wind direction using CFD techniques, the wind direction is also playing an important part in the genetic algorithm project that is also being carried out. As a result of this it is vital that the mathematical model that has been designed and created for this project incorporates the ability to analyse the effects of varying wind directions so that it can be used to validate the other projects against industry standards. This is in keeping with one of the main aims of this project to provide a link between currently used industry techniques and the methods that are being evaluated for this group project.

In order to achieve wind directions of various angles it was decided to rotate the points of the turbine locations so that the x axis always remained in the downstream direction. This was necessary to retain the simplicity of the model as the EWMs employed all require the downstream distance, x, to calculate the velocity deficit and wake expansions.

The spreadsheet was designed so that the desired angle of the wind direction could be input in either degrees or radians along with the original turbine locations. These locations were then rotated around the point (0,0) using the equations shown below. The Origin was chosen as the rotation point as it simplified the equations used as well as ensuring that the rotated farm looked the same for any turbine layout.

$$x_{Rotated} = x\cos(\theta) - y\sin(\theta) \tag{31}$$

$$y_{Rotated} = x\sin(\theta) + y\cos(\theta) \tag{32}$$

3.5. Optimisation

```
Sub Optimisation()

Dim h As Long

For h = 1 To 200

Dim i As Integer

For i = 17 To 36
    Cells(i, 4).Value = Rnd * 5000

Next i

If Range("L3").Value > Range("P3").Value Then _
    Range("P3").Value = Range("L3").Value

If Range("L3").Value = Range("P3").Value Then _
    Range("P17:P36").Value = Range("D17:D36").Value

For j = 17 To 36
    Cells(j, 5).Value = Rnd * 2000

Next j

If Range("L3").Value > Range("P3").Value Then _
    Range("P3").Value = Range("L3").Value

If Range("L3").Value = Range("P3").Value Then _
    Range("Q17:Q36").Value = Range("E17:E36").Value

Range("P4").Value = h

Next h

End Sub
```

Figure 4: Optimisation code that was written in VBA

Optimisation is an important aspect of wind farm array design as the turbine layout can drastically affect the power output of the farm. As a result of this it has been decided to include basic optimisation functionality within the mathematical model.

The code required for the task was written in Excel VBA (Visual Basic Applications) which is Excel's coding application. The code which was written runs an optimisation study that uses random numbers to assign different turbine locations, the power from the array is then evaluated and compared to the highest power output that has been achieved so far. If the new layout results in a greater yield then the new configuration is stored, if not it is discarded, the turbines are then placed in a new random configuration and the process is continued.

The code written can be seen in Figure 4 above. As can be seen h refers to the number of iterations that will be carried out, this example shows 200 different locations however tests were carried out up to 3.6 million random locations. The first For loop controls the x direction locations for all 20 turbines, the region was bounded at 5000m and the If statements control whether the new location is retained or not.

The second For loop and If statement that can be seen control the y direction location for the 20 turbines, this time it was bounded to 2000m, once the code has run through once in its entirety it is counted as one iteration, the code will run until the pre-set number of iterations has been achieved or if it is stopped prematurely.

3.6. Power

The power output of each individual turbine can be calculated as described by *Kulunk* [20]. The wind velocity through the turbine (U_R) can be defined as the average between the inlet velocity (U) and the velocity in the near wake region (U_w). For this study a distance of 100m or 1 rotor diameter was used to define the near wake region.

$$(33) \qquad U_R = \frac{U + U_w}{2}$$

Furthermore the axial induction factor (a) is defined as

$$(34) \qquad a = \frac{U - U_R}{U}$$

This induction factor can be used to define the Coefficient of power (Cp)

$$(35) \qquad Cp = 4a(1-a)^2$$

This can then be used to calculate the power output of the turbine:

$$(36) \qquad P = \frac{1}{2}\rho A U^3$$

Where ρ is the density of air, taken as 1.204 kg/m^3, and A is the swept area of the turbine. This series of equations was input into the model so that it would adapt with each scenario, i.e. turbine location or wind angle. Furthermore the power outputs for the turbines were then summed to give the total power output for the entire wind farm array.

4. Validation and results

This section details the results obtained from the mathematical models compared to a range of sources including work carried out by other group members and previously carried out work.

4.1. Spreadsheet

Three different mathematical models have been created, which all use the same underlying principles and only vary with respect to the wake models that have been employed. The models require a set of inputs such as turbine diameter, location, wind speed and direction as well as a series of constants and coefficients that have been detailed in the theory section. From this the power output for the wind farm can be obtained immediately. The model will also show the positions of the turbines that have been specified and the wakes of the turbines. This allows the layout of the turbines to be optimised by reviewing the wake effects on each turbine.

4.2. Validation of Models

As stated in the literature review earlier, the first validation carried out was by comparing the results of the mathematical model against the work previously carried out by *Frandsen et al*[8]. They initially proposed two versions of their model, with the differences being the values used for k as can be seen in equations 10 and 11 (see section 3.2.2). The model that was decided as being the best fit and subsequently adopted has been referred to as "Frandsen" in Figure 5 and "U_(1/2)" in Figure 6 which used a k value of 2. The dropped model used a k value of 3 and has been referred to as "Frandsen (Schlichting)" in Figure 5 and "U_(1/3)" in Figure 6.

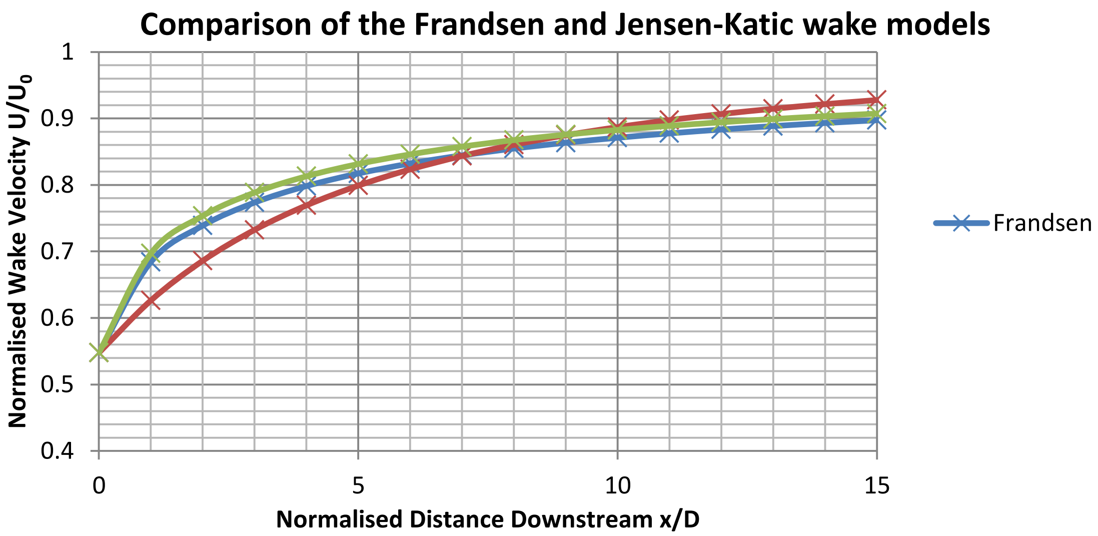

Figure 5: Results of the mathematical model for validation against *Frandsen et al* [8]

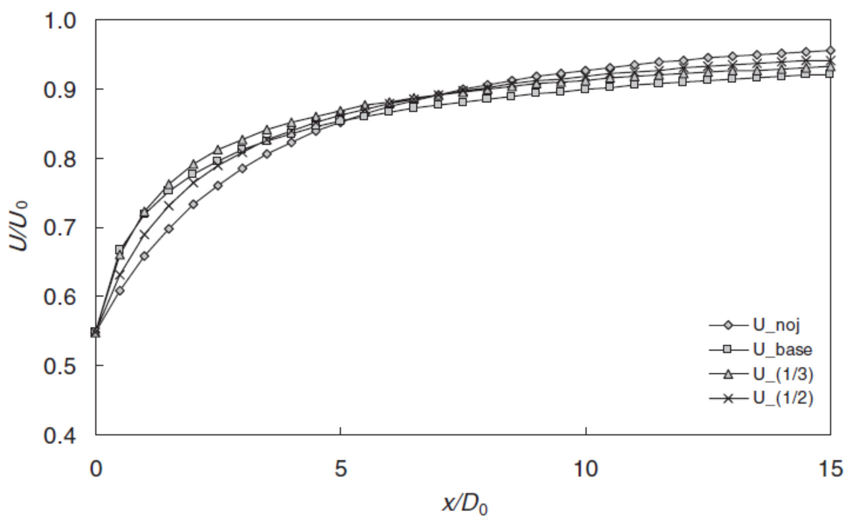

Figure 6: Results from *Frandsen et al* [8] to validate the mathematical model

The models were set up with a wake decay constant (k) of 0.04 as recommended by *Katic et al* [3] for off shore wind turbines. The thrust coefficient was set at 0.7 as has been justified by other members of this group project. The diameter of the wind turbines was also set as 100m as is typical of offshore turbines while the free stream velocity was specified as 13m/s.

Equation 11 shows how to calculate the term α for the Frandsen model which is the wake decay constant, it uses the decay constant from the Jensen-Katic model (0.04) as well being calibrated when the far wake region has started. The difference between the near wake region and far region is still a subject of debate, as well as the transition period between the two. However *Frandsen et al* [8] calibrated their model initially at a distance of 7 rotor diameters downstream of the turbine. The version of the Frandsen model employed in this report has also been calibrated against the Jensen-Katic model at a distance of 7 rotor diameters downstream.

Figures 5 and 6 above show very similar shaped graphs for all three of the plots that have been tested. As is expected the models converge around the distance $\frac{x}{D} = 7$ before separating off again. Equally it can be seen that the final values obtained are very similar obtaining between 90% and 95% of the free stream inlet velocity.

The drawbacks of this method of validation are that it relies on comparing two graphs and so accuracy of data can be an issue, for example Figure 6 does not contain any grid relies which further reduces the confidence that can be had in the accuracy of this validation.

However the nature of the equations that are being tested means that the comparison between the graphs is still useful. As the graphs have similar shapes, including cross over points and final values it can be reasoned that the wake models have been correctly input otherwise graphs of different shapes would be obtained. As such reasonable confidence can be placed in the validation of the wake models through this source.

Tong et al[6] compared the Jensen-Katic, Larsen and Frandsen models, using different conditions to the previous paper. These conditions have been replicated and graphed so that the models employed in this investigation can be more thoroughly validated. They also used a turbine with a diameter of 100m and a Jensen-Katic wake decay coefficient of 0.04, however they used a thrust coefficient of 0.82 which has been replicated in the mathematical model. It was also reasoned that the near wake region was present up to a distant of 2 diameters downstream. Since the wake models are only effective in the far and transition regions the graphs start at this distance.

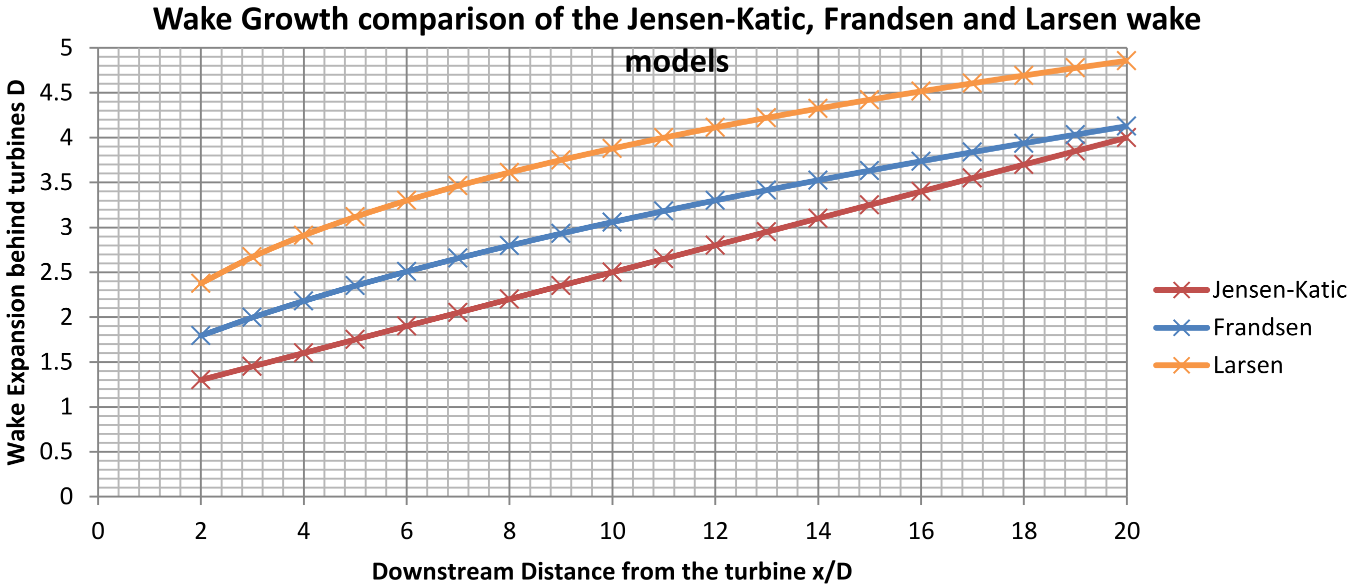

Figure 7: Results of the mathematical model for validation against *Tong et al* [6]

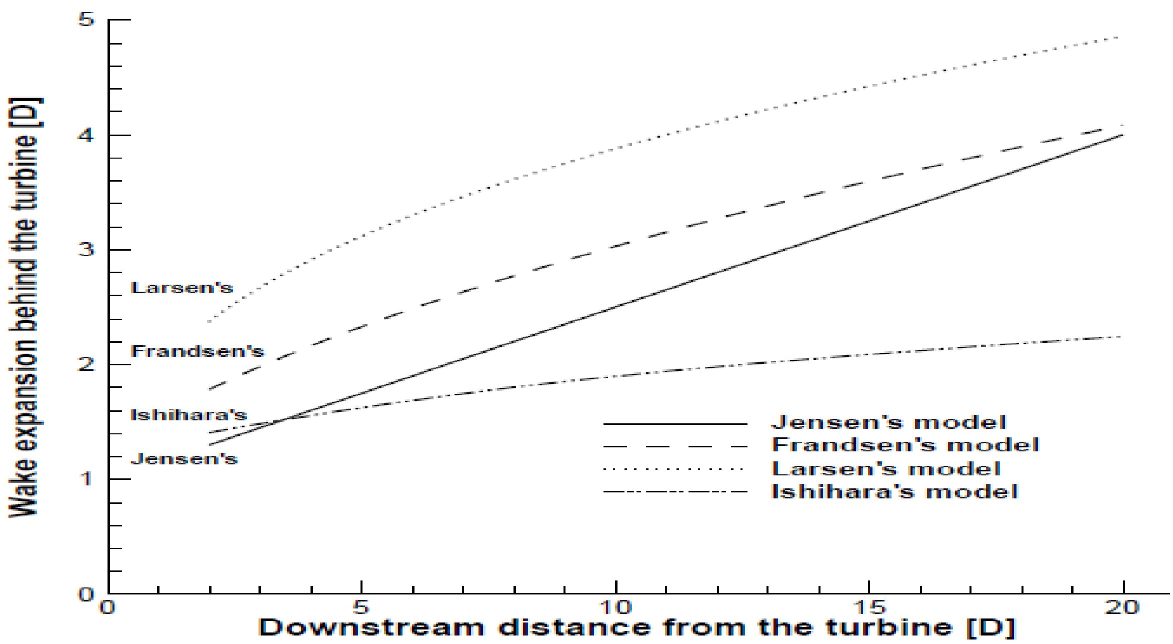

Figure 8: Results from *Tong et al* [6] to validate the mathematical model

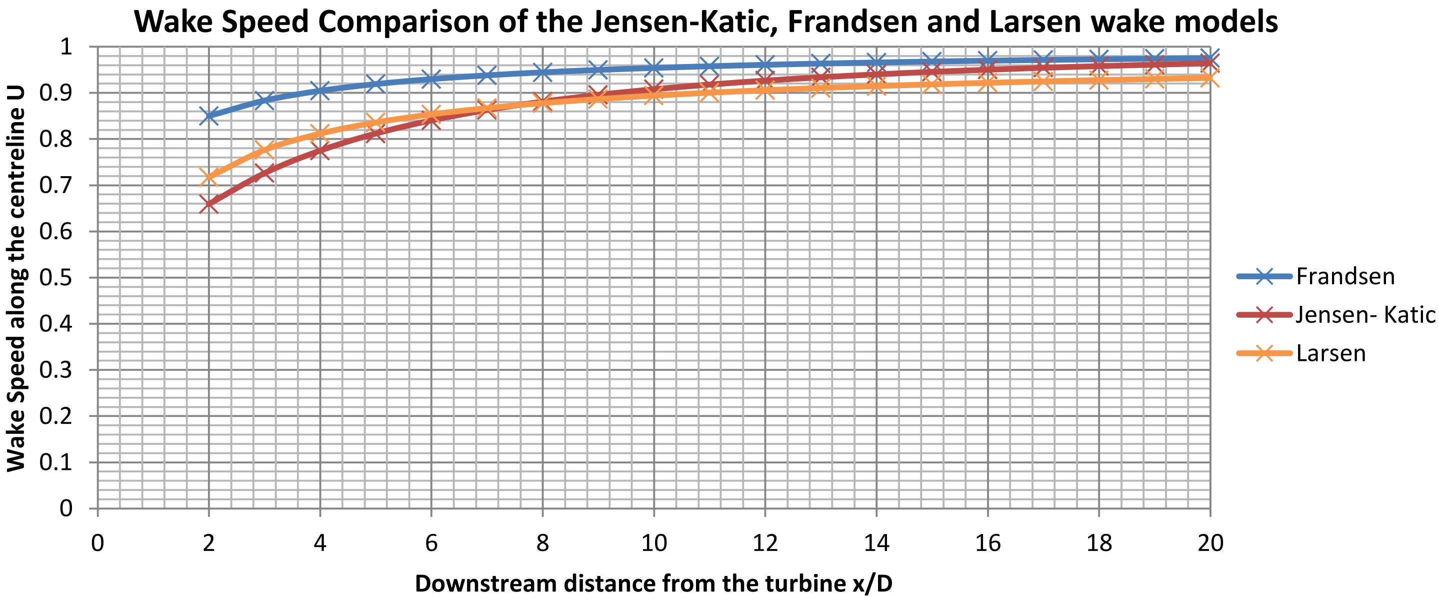

Figure 9: Results of the mathematical model for validation against *Tong et al* [6]

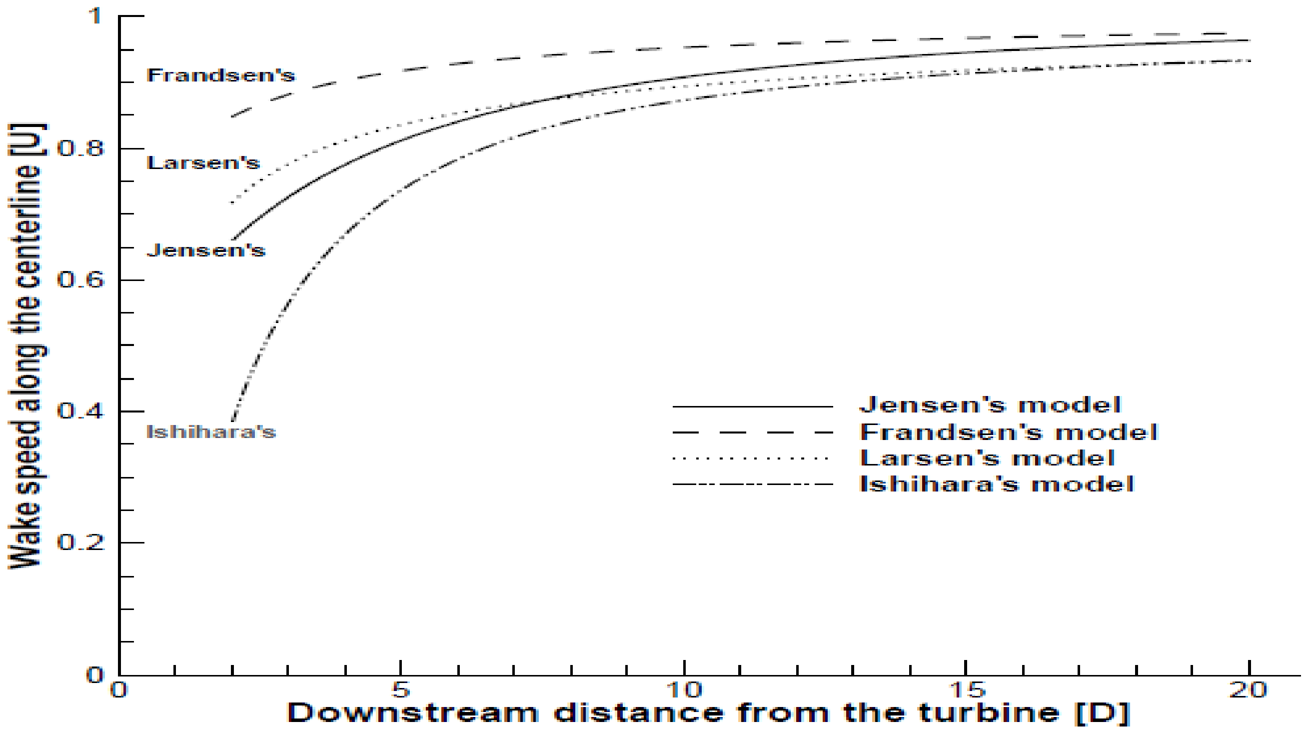

Figure 10: Results from *Tong et al* [6] to validate the mathematical model

As with the previous validation study it can be seen that the results obtained from the mathematical model closely mirror those obtained by *Tong et al*[6]. Figures 7 and 8 show how the wakes expand, the Jensen-Katic model has linear wake expansion and this can be clearly seen on the graphs. It can also be seen that while the wakes Frandsen and Jensen-Katic models appear to converge with increasing distance the Larsen model remains significantly higher.

Figures 9 and 10 also show the same shapes and finishing points and that as downstream distance increases the wake speeds predicted by the models converge. The similarity in results that has been demonstrated between the mathematical model and previous studies, as demonstrated in Figures 5-10, enables a strong degree of confidence to be placed in the mathematical model and the results that it predicts.

4.3. Discrepancies

One of the original key aims of this project was to attempt to identify why in industry the potential power output for a wind farm is consistently over predicted and Figures 5-10 show one of the possible reasons. While it can be seen that the wake speed predicted by the models converges at around 12 rotor diameter downstream (Figure 9) the turbines placed in an array are rarely spaced at this distance. Instead they are typically placed around 8 rotor diameters apart and as can be seen the wake speed at this point varies significantly.

The set-up of the models can also have a large impact on the results obtained, for example Figure 5 shows that the Jensen-Katic and the Frandsen model show almost identical results at 8D downstream. However the set up used by *Tong et al* [6] results in the Frandsen model predicting a much higher wake velocity (Figure 9).

The last source of discrepancies that will have a large effect on the power output of a wind farm array is the wake expansion predicted by each model. Figures 11 and 12 below shows how the wake expansions vary for a single turbine. Figure 11 shows how the wake expands for a single turbine up to 8 rotor diameters downstream, this is the distance at which the turbines in an average wind farm array are spaced. While the Jensen-Katic and Frandsen models have near identical results the expansion of the Larsen model can be seen to be nearly twice the others.

This will have a huge impact on whether downstream turbines are affected by the wakes of upstream turbines. Figure 12 shows that at a distance 5000m, which is a typical distance for a wind farm array, the Jensen-Katic wake expansion has expanded more rapidly than the Frandsen model such that it is between the Larsen and the Frandsen mode.

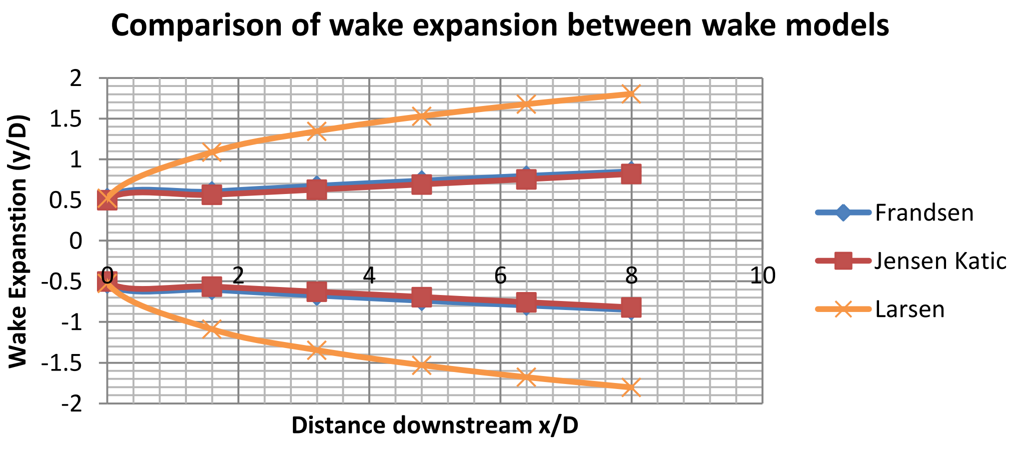

Figure 11: Comparison of wake expansion between EWMs from the mathematical model

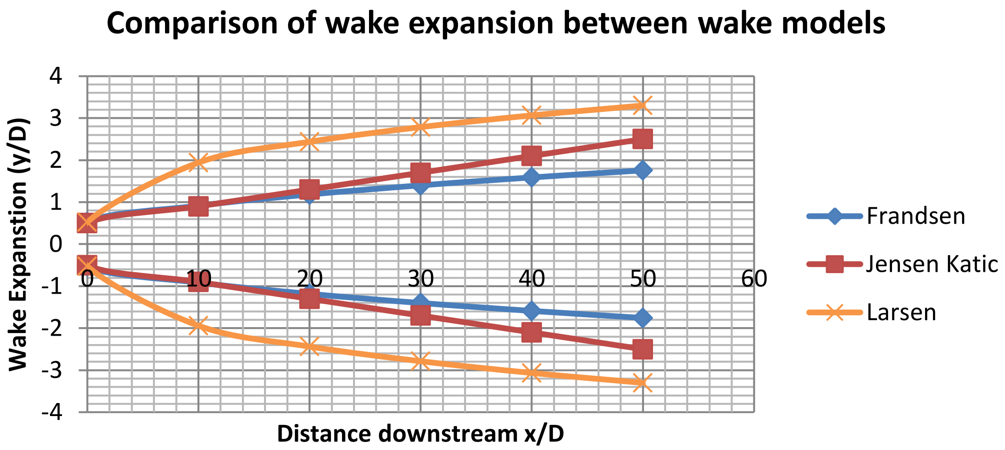

Figure 12: Comparison of wake expansion between EWMs from the mathematical model

4.4. Wind Angles

In order to investigate the effect that the wind direction has on the power output of the wind farm two different cases have been studied, the first looks how the average power across the wind farm as a whole varies. The second analyses average power across each row of turbines which results in a more depth study of the wind farm as it is possible to identify which rows are more several affected by the wakes of upstream turbines.

For these studies the mathematical model was set up with a thrust coefficient of 0.7 and a Jensen-Katic wake decay coefficient of 0.04. As before the wind turbines have a diameter of 100m and the free stream velocity was given as 13m/s.

Figures 14-16 show the results of varying the wind direction for the three wake models used in this investigation, 0 degrees is taken as the wind approaching the farm from head on from the left. The array was arranged as shown in Figure 13 below, with 800m downstream between the rows and 500m between each turbine in a row.

This layout is taken as an approximation from the layout of the Rodsand II wind farm. The project was going to be based around this wind farm however the necessary data was not made available and so approximations were made. The farm is then rotated, as detailed earlier, to simulate the wind approaching from different directions with negative values rotating the farm clockwise and positive values rotating the farm anti-clockwise.

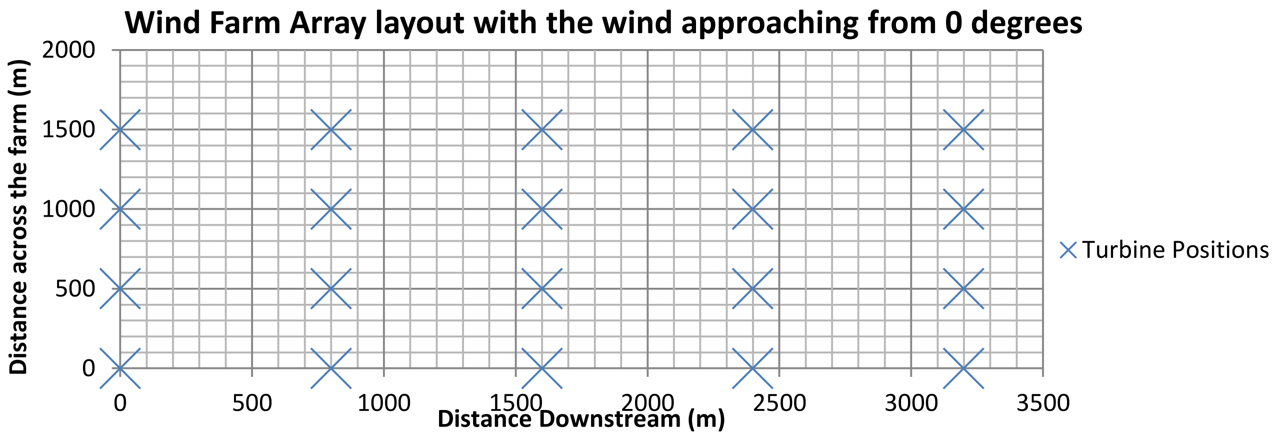

Figure 13: Illustration of wind turbine layout for this study

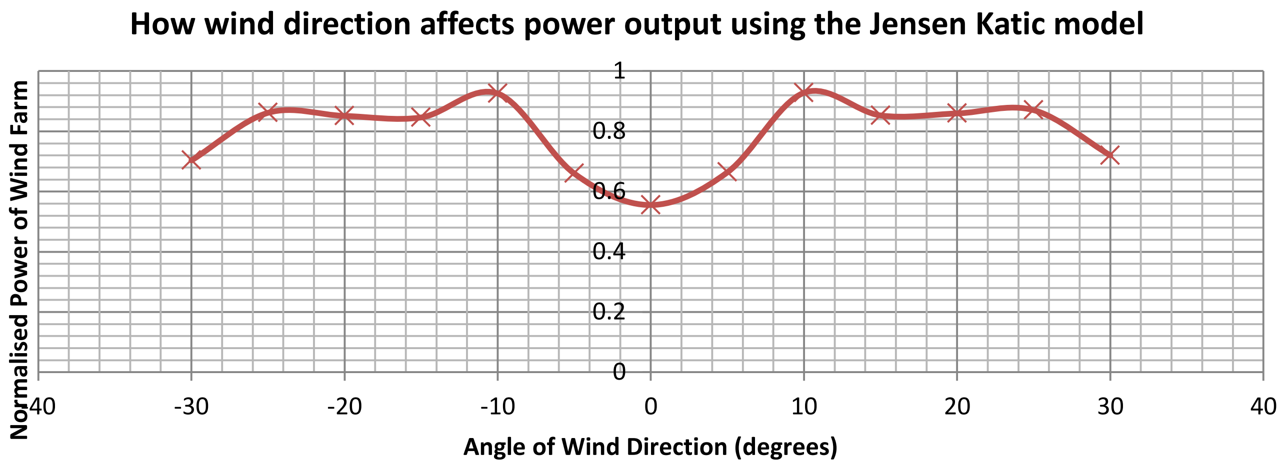

Figure 14: Change in power of the wind farm with change in wind direction using the Jensen-Katic model

Figure 14 above shows the effect of different wind directions on the power output for the entire farm. The output has been normalised against maximum wind turbine power observed in the first row of turbines (which experienced no wake interactions).

As can be seen in Figure 14 the 0 degrees case results in the lowest total power output for the wind farm, with the peak occurring at 10 degrees and -10 degrees. Subsequently changes in angles also result in an increase in power from the 0 degrees condition albeit with a lower overall power compared to ±10 degrees.

The reason for the change in power output can be attributed to the wake effects of the turbines, by changing the angle that the wind approaches the farm the turbines do not have as many subsequent turbines downstream that will be affected by the wake. This results in fewer turbines being affected by velocity deficits and so a larger power output.

Figure 15 below shows the same study using the Frandsen model, while the model shows similar results to the Jensen-Katic model above there are notable differences. The minimum power output can still be found at 0 degrees, yet it is significantly higher than the value found in the Jensen-Katic model (67% of the maximum turbine power rather than 56%).

The power output also remains more constant than shown in Figure 14, this can be attributed to how the different models calculate the wake expansion. As explained earlier, Figure 12 shows how over the length of the wind farm array the Jensen-Katic model predicts a larger wake expansion. This in turn will affect how many of the downstream turbines experience wake interaction and therefore the power output.

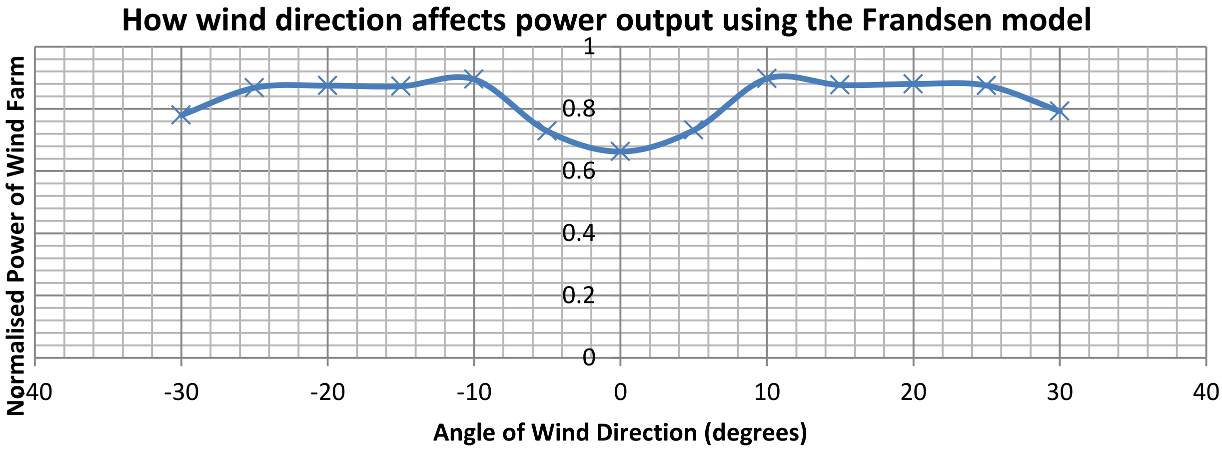

Figure 15: Change in power of the wind farm with change in wind direction using the Frandsen model

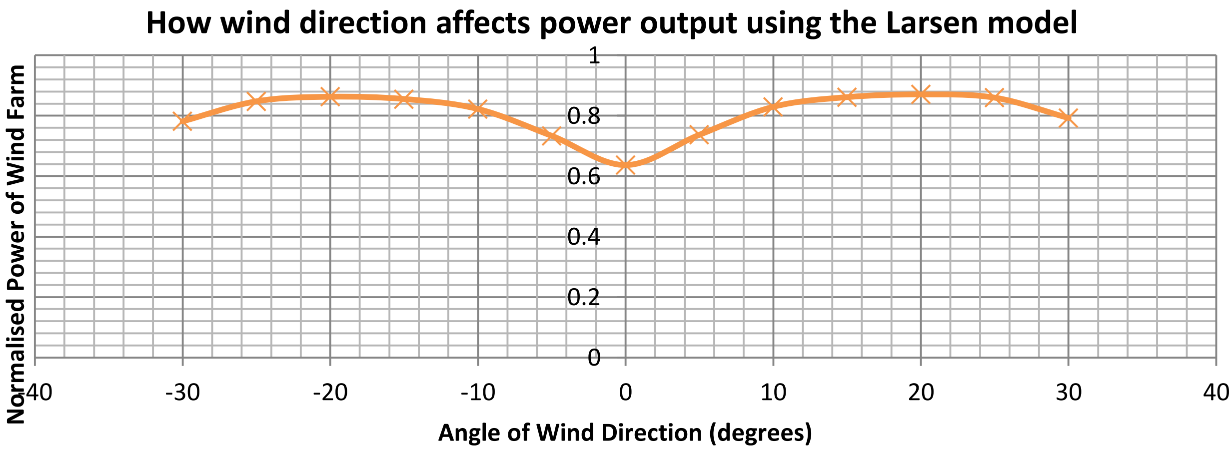

Figure 16: Change in power of the wind farm with change in wind direction using the Larsen model

Figure 16 above shows the results of the study for the Larsen model, it has similarities to the previous Figures, with the 0 degrees case again having the lowest output power (64%) and with a significant increase being seen with the change in angles. However unlike the previous figures the Figure 16 shows that the Larsen model does not predict a significant dip in power after 10 degrees. The power output for ±10 degrees is also significantly lower than the previous two models.

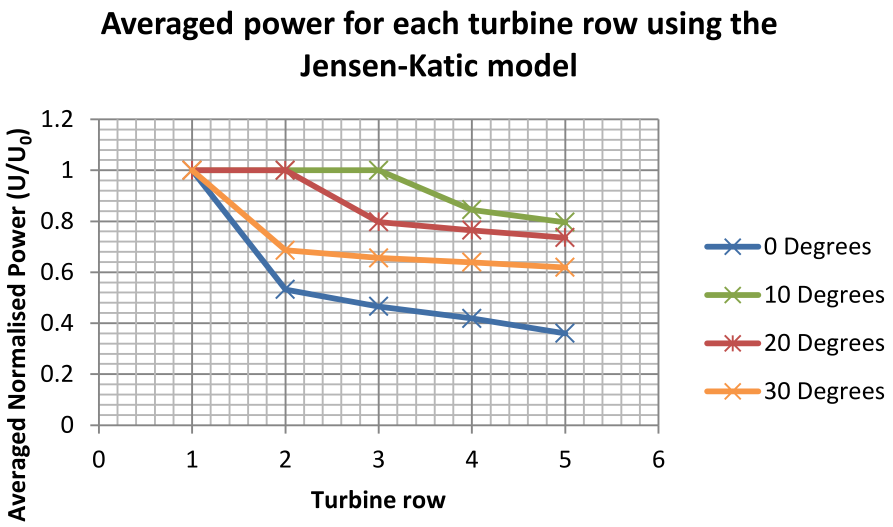

Figure 17: Average power for each row using the Jensen-Katic model

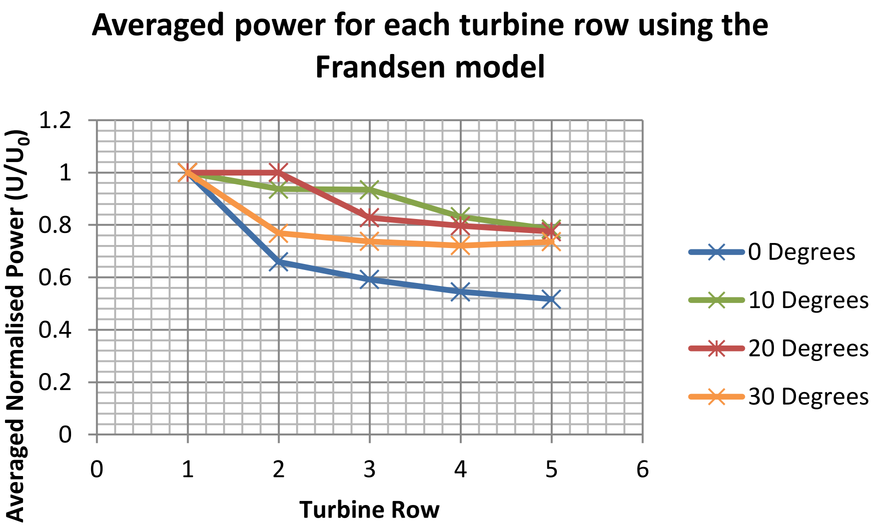

Figure 18: Average power for each row using the Frandsen model

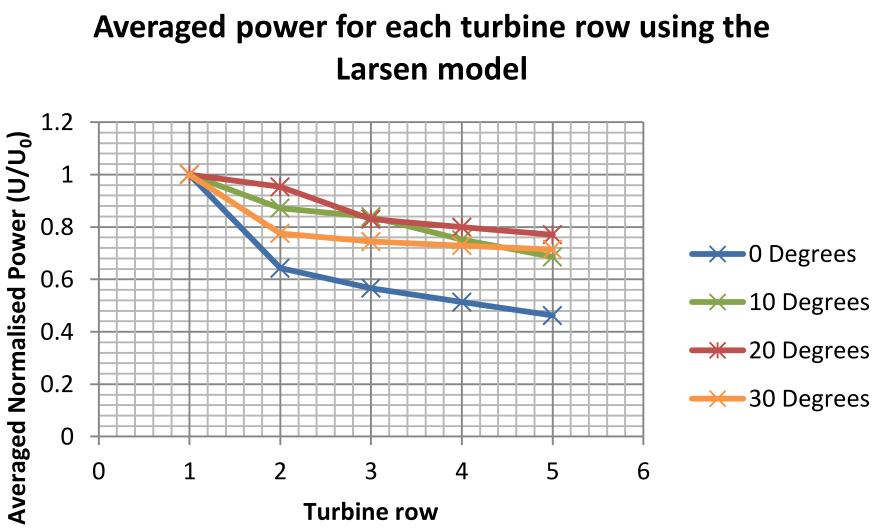

Figure 19: Average power for each row using the Larsen model

Again this can be explained through a more in depth analysis of the wake expansions of the three models. The spacing between the wind turbines is such that for 10 degrees condition the Jensen-Katic model predicts that the wakes of the upstream turbines will pass through the gaps in the first three rows so that the first turbines to experience wake deficits are in the fourth row as shown in Figure 17.

The Frandsen model (Figure 18) predicts that the second and third rows will experience wake interaction, but that it is small compared to the effects on further rows.

Finally the Larsen model (Figure 19) predicts the wake expansions such that only the first row experience the free stream velocity, that is to say beyond the first row all of the rows of turbines experience wake interaction of some variety.

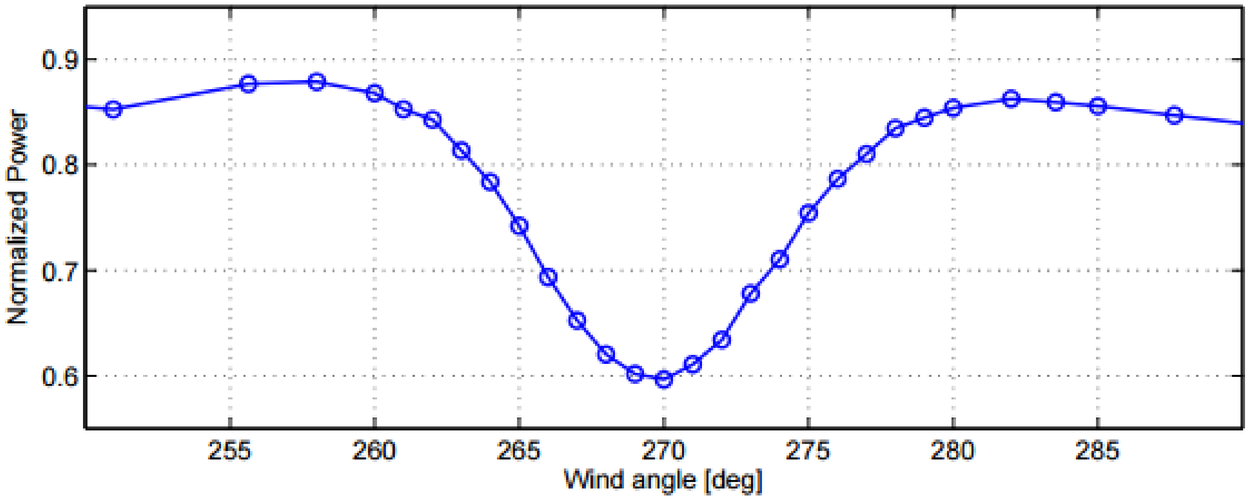

Figure 20: Results from *Porté-Agel et al* [16] to be compared against the study using the mathematical model

Porté-Agel et al [16] used LES to simulate the normalised power of an array as shown in Figure 20 above. It can be seen that this shows a very close resemblance to the power prediction of the Larsen Model. It too shows a peak power prediction around ± 10 degrees and doesn't show the dip at subsequent angles that is apparent in the Jensen-Katic and Frandsen models.

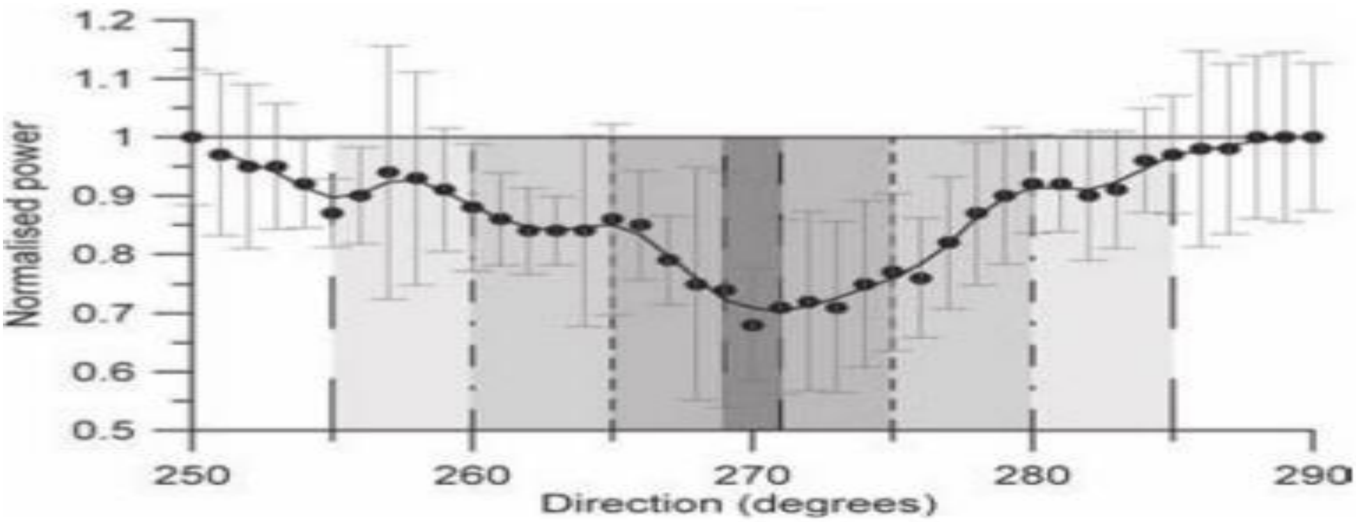

Figure 21: Results from *Barthelmie et al* [15] to be compared against the study using the mathematical model

Barthelmie et al [15] used a variety of different software packages that incorporate different forms of EWMs which account for the error bars shown in Figure 20 above. The general shape seen in Figure 21 is closer to that seen in the Jensen-Katic and Frandsen models (Figures 14 and 15 respectively).

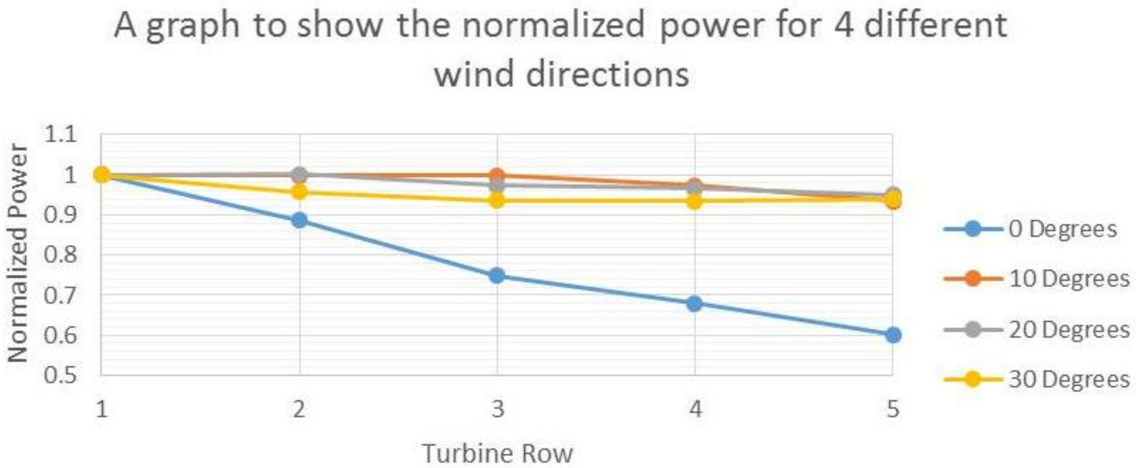

Figure 22: Results from *March* [16] to be compared against the study using the mathematical model

March [17] worked as part of the group that carried out this project, using CFD techniques to analyse the same wind farm as has been used in this report. Figure 22 shows the results from running the k-ε turbulence models at different wind directions.

Comparing to Figures 17-19 it can be seen that the 0 degrees case is very similar to all three EWMs. The results for the different angles however show results that are far closer to the maximum possible power. 10 Degrees is still shown as resulting in the highest power output, however the reason for the discrepancies between the EWMs and Figure 22 is likely to lie in how the wake expansion is modelled. A lack of wake expansion will lead to a lack of wake interaction in downstream turbines which is an explanation for the results in Figure 22.

4.5. Optimisation results

The optimisation study was carried out using the method described previously which involved using a random generator to place the turbines. However the results obtained from the simulations cannot be said to be optimum locations due to a number of different issues.

The code that was written (Figure 4) did not restrict the locations of the turbines other than to ensure they remained within the defined field array of 5000m x 2000m. Subsequently the locations chosen could vary by as little as 10cm, for the size of wind farm that was being studied this was both impractical and massively increased the computational expense.

It took approximately 24 hours to run 1,000,000 different combinations, studies were also run with 3.6 million iterations however despite the increase in run time the power difference between the two studies was 0.03MW across the entire farm. From these issues it can be concluded that the method used to attempt to optimise the layout of the wind farm was not suitable and led to the lack of meaningful results.

5. Conclusions

This section aims to detail the main outcomes of this report, for convenience it has been divided into subsections, while these subsections are by no means mutual exclusive splitting them in this fashion aids the clarity in which the conclusions can be presented.

5.1. Spreadsheet conclusions

Three mathematical models have been successfully implemented, one for the Jensen-Katic, Frandsen and Larsen models. The models are fully adjustably with respect to free stream velocity, turbine location and size and wind direction for a wind farm with 20 or fewer wind turbines. Furthermore the models have been validated against a range of different studies such that a reasonable degree of confidence can be placed in the predictions of these models.

5.2. Wake Model Discrepancies

Two main sources of discrepancies have been identified between the wake models investigated in this study. The first lies within the setup of the individual models, for example it was shown when the Frandsen model was used in two separate studies (*Tong et al* [6] and *Frandsen et al* [8]) the results varied hugely.

The second source of discrepancies lies with the predicted wake expansion of the models, while this has no effect on the single turbine case the implications for the prediction of power output for an entire wind farm array is huge. As discussed variance in the wake expansion massively effects the wake interaction of downstream turbines and their subsequent power output.

The angle study carried out clearly shows this, while all three models show that a change in angle increases the power output, with ± 10 degrees being agreed as one of the optimal angles, the variation in power output is still disputed.

5.3. Layout Optimisation

The layout optimisation code that was written for the mathematical models designed in this project was not suitable to carry out the task it was designed for. The problems came from the use of the random number generator which was used to place the turbines. The generator used meant that the turbine location could vary by as little as 10cm, this meant that the number of possible locations for 20 turbines was too large to reach any sensible form of optimised layout.

5.4. Between other group members work

The ability to identify discrepancies between different techniques of predicting power output for wind farms has been hampered by the lack of real world data which was not forthcoming from the project's partners. As a result of this it has not been possible to conclude which methods are more accurate and so this analysis has not been carried out.

Analysis between the wake models employed in this study and the CFD simulations (*March* [17]) have shown that the Larsen Model most closely resembles the CFD predictions.

This project also aimed to compare the results of its optimisation study to the results of the genetic algorithm optimisation study that was carried out by another group member but for the reasons stated above this was not possible.

5.5. Further Work

There are two areas of work that this project has identified to further improve on the findings of this report. The first is to acquire numerical data, both for the results of wake model analysis and from real world wind farms. This data will make it possible to further validate the mathematical models that have been created and identify further discrepancies between the power output for EWMs the observed power output for wind farm arrays.

The second area of work focuses on improving the mathematical models further and increasing their power and functionality. More investigation in optimisation techniques is required to be able to write a code that can perform simple optimisation tasks. This could be achieved by limiting the areas in which turbines can be placed so as to reduce the number of possible combinations and therefore computational expense of the process.

Once the optimisation process has been improved and the model further validated against numerical data the possibility of releasing the mathematical models as open source wind farm analysis packages can also be investigated.

6. Project management, consideration of sustainability and health and safety

Project management is a vital component of all projects, with good project management aims and objectives can be achieved on time and with minimum disruption. Good project management is even more important within a group project, where delays and disruptions can have large impacts upon other members. To this end it is important to utilise a range of techniques to ensure the proper management of both the individual and group projects.

6.1. Project Management techniques

A range of techniques have been employed within this project, Gantt charts and risk assessment tables have been used on an individual and group basis. The individual techniques table can be seen below while the group techniques are visible in the G2 report.

Further management techniques were also employed that helped improve the communication within the group. Clear communication is a vital part of group projects and so it was necessary to devote specific management techniques to ensure that this could be maintained throughout the duration of the project.

Group meetings were held every week, with Agendas circulated beforehand and Minutes taking during and again circulated after each meeting. This gave the group an important platform to be able to discuss progress and issues that were arising in individual projects. Advanced warning of potential issues which arose from frequent communication enabled any risks to be quickly mitigated.

A further technique that was employed to aid transparency between individual projects was the use of shared area on Google Drive. This allowed each group member to upload all of their work to a shared area where the other group members could also view the progress made. This also allowed for the efficient sharing of data and research.

An individual log book was also kept, this included personal notes from meetings that complemented the minutes taken. The log book also contains research notes and findings and provides an important reference point for project progress.

6.1.1. Risk Assessment Table

A Risk Assessment Table ensures that all project risks have been considered and contains information on how to mitigate the possible risks. There are two main types of risks that can be associated with a project, the first are the risks to individuals and the second refers to risks to the aims of the project. The individual risks will be dealt with in the Health and Safety section below and so the Risk Assessment table seen here only details the risks to the project aims.

Table 1: Risk Assessment Table for the Individual project

Risk Item	Effect	Cause	Chance (0/10)	Severity (0/10)	Importance (0/100)	Action
Lack of LiDAR data	Unable to validate findings against real world data.	Project partners unable to deliver data	(8/10)	(6/10)	(48/100)	Alternative methods to validate the findings of the project, such as other similar studies.
Optimisation studies exceeds allocated run time	Unable to compare to other group members. Spreadsheet loses some functionality	Code is too complicated or computers under powered.	(7/10)	(4/10)	(28/100)	Code is made as simple as possible, studies are started well in advanced of deadline to allow maximum possible run time.
Loss of data	Project overrunning or failing completely	Corruption or loss of files	(5/10)	(10/10)	(50/100)	Back up all data to external hard drive and Google Drive.

As can be seen the most important risk is the loss of project data, however as described above this can be easily mitigated through good project management. The lack of LiDAR data is the next important but has mitigated by finding other methods that can validate the project. The final risk is the optimisation study overrunning this can also be mitigated through good project management using the Gantt chart as detailed below.

Gantt Chart

ACTIVITY	PLAN START	PLAN DURATION	ACTUAL START	ACTUAL DURATION	PERCENT COMPLETE
Term 1 Individual Objectives					
Initial Project Background research	1	2	1	2	100%
Formulation of Individual project scope	1	3	1	2	100%
Literature Review	1	4	1	4	100%
Write section for G1 Report	1	2	1	2	100%
Create Solidworks models	4	2	1	1	100%
Research Wind Farm (Rodsand)	3	2	6	1	100%
Write I1 report	1	9	9	2	100%
Term 2 Individual Objectives					
Algorithm					
Research Algorithm types and construction	15	2	15	3	100%
Design and implement algorithm	16	4	18	2	100%
Wake Models					
Research model types and implementation	15	2	15	2	100%
Design and implement chosen models	16	4	16	6	100%
Compare models to CFD	16	5	20	3	100%
Angle Study					
Create method to simulate different wind angle	15	3	20	3	100%
Implement into mathematical model	16	3	21	2	100%
Evaluate results	21	2	21	2	100%
Comparisons					
Compare Optimised models to each other	23	2	23	2	100%
Evaluate power losses in models and against other memebers	24	2	24	2	100%
Assessments					
Write section for G2 Report	22	8	21	8	100%
Write I2 report	22	8	20	9	100%
Peer Assessments	29	1	28	1	100%
Poster Day	30	4	0	0	0%

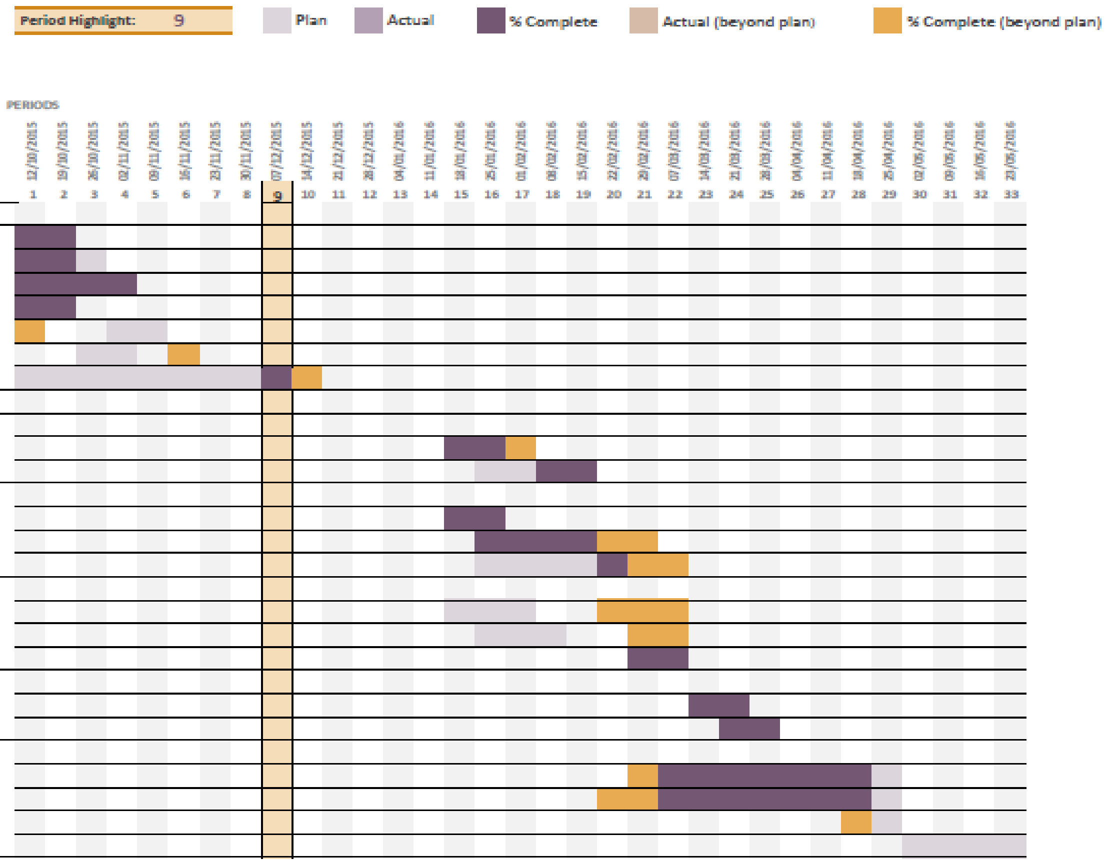

Figure 23: Individual Gantt Chart

6.1.1. Gantt Chart

A Gantt chart is a multi-functional management tool that can be vital to the success of a project. The chart below comprises of the individual tasks that have been completed and the times of when they were completed. During the project the chart was regular updated to ensure that it reflected the current state of the project. This enabled anyone in the group to check on the progress that had been made and the expected delivery date of various deliverables.

6.2. Health & Safety and Sustainability

As has been explained this project has been entirely computational and has such there have been very few if any health & safety or sustainability risks. Few materials were used and the only sustainability issues that could be considered refer to the use of the electricity and other day to day material usages. As such it has been decided to only mention this section in passing as it is not strictly relevant.

The same can be said for the health & safety aspects of this project, the only risks that can be associated to it are the same that can be considered in the day to day workings of an office. As such there are no specific risks to identify as long as good office working practice is adhered to. This also explains why no health & safety issues have been mentioned or mitigated in the methodology sections of this report.

7. Contribution to group functioning

This section aims to detail how the author and the work laid out in this report have contributed to the functioning of the group as a whole. The author was present and contributed heavily at the weekly group meetings during which they acted as Chair and secretary several times.

The work for this individual project was heavily linked to the work carried out by the other members. It was the only project which spanned to the two sub-groups that the group project was split into. This is detailed more fully in the G2 report however in brief, the project was divided into two groups. The first evaluated the current wind farm analysis techniques and the second worked on developing new methods or software.

This report involved elements that could contributed to both of these group aims, furthermore this project investigated the techniques that are most prevalent in industry and so provided a good bench mark against which to validate other members work. The creation of a versatile mathematical model meant that other members could compare their results to the industry standards irrespective of their array layout or input conditions.

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