Harmonic Balance Method for Turbomachinery Applications

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Abstract

This work presents the Harmonic Balance method for incompressible turbulent periodic flows. The method is implemented and tested in foam-extend, a community–driven fork of the open source software OpenFOAM. One stage centrifugal pump is simulated using the Harmonic Balance method with 1 and 2 harmonics. Results are presented and compared with conventional transient simulation.

1. Introduction

The Harmonic Balance method is a quasi-steady state method developed for simulating non-linear temporally periodic flows. It is based on the assumption that each primitive variable can be accurately presented by a Fourier series in time, using first n harmonics and the mean value. Such assumption allows replacing the time derivative term in transport equations with coupled source terms, thus transforming the transient equations into a coupled set of quasi steady equations. The benefit compared to steady state methods is that Harmonic Balance is able to describe the transient effects of periodic flows, while providing significant speed-up compared to transient simulation.

2. Mathematical Model

Primitive variables are expressed by a Fourier series in time, with n harmonics. Substituting the variables in transport equations with Fourier series, $2n+1$ coupled equations are obtained:

- Harmonic Balance continuity equation:

$$\nabla\bullet\mathbf{u}_{t_j} = 0,$$

- Harmonic Balance momentum equation:

$$\nabla\bullet(\mathbf{u}_{t_j}\mathbf{u}_{t_j}) - \nabla\bullet(\gamma\nabla\mathbf{u}_{t_j}) = -\frac{2\omega}{2n+1}\left(\sum_{i=1}^{2n}\mathsf{P}_{(i-j)}\mathbf{u}_{t_i}\right),$$

where

$$\mathsf{P}_i = \sum_{k=1}^{n} k\sin(k\omega i\Delta t), \qquad \text{for} \quad i = \{1,2n\}.$$

Corresponding to the Fourier series expansion, for n harmonics $2n+1$ equally spaced time steps within a period are obtained. Each of the $2n+1$ equations represents one time instant. Equations without the time derivative term in its original form (continuity equation) remain the same, using variables corresponding to the time instant currently calculated.

3. Numerical Procedure

Second order accurate, polyhedral Finite Volume Method is used. Segregated solution algorithm SIMPLE is adopted. Each of the $2n+1$ time steps is resolved in its own SIMPLE loop.

4. ERCOFTAC Centrifugal Pump

ERCOFTAC Centrifugal Pump is simulated using Harmonic Balance method and compared against conventional transient solver. The geometry is a 2D simplified model of a centrifugal turbomachine, discretised with 93 886 hexahedral cells. The pump consists of rotor (inner) and stator (outer) part. The rotation speed is 2000 rpm with inlet velocity set to 11.4 m/s and k-Epsilon model used for turbulence. Additionally, in Harmonic Balance simulations multiple frequency approach was used to account for different frequencies in the stator and rotor.

Harmonic Balance simulations were run using 1 and 2 harmonics, comparing the efficiency, head and torque, and pressure on the rotor blade surface with transient simulation results. Figure 1 shows the comparison of pressure on the rotor blade surface for the time instant $t = T/3$. Figures 2 and 3 show the comparison at time instants $t = 2T/3$ and $t = T$, respectively. The results for 2 harmonics agree better with the transient solution then in case of 1 harmonic.

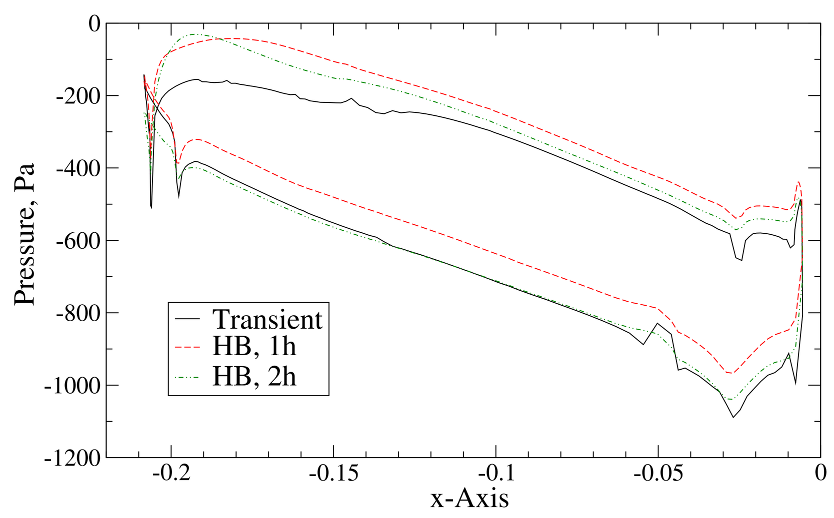

Figure 1: Pressure on rotor blade surface at $t = T/3$.

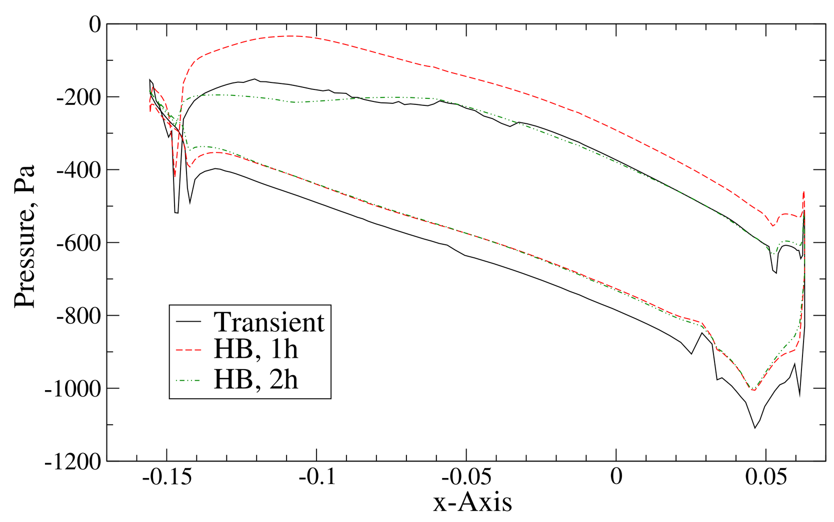

Figure 2: Pressure on rotor blade surface at $t = 2T/3$.

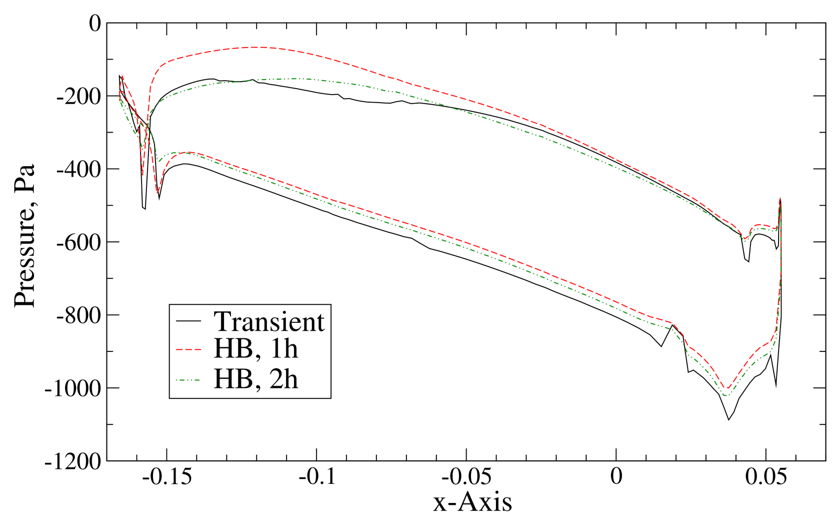

Figure 3: Pressure on rotor blade surface at $t = T$.

Table 1 presents a comparison of pump characteristics obtained using the Harmonic Balance and a transient solver. Results are presented for efficiency, head and torque at three time instants. In the compared features and time instants the error is lower than 5%, showing the capability and accuracy of the Harmonic Balance method.

Table 1: Pump characteristics comparison

		Transient solver	HB, 1h	error, %	HB, 2h	error, %
$t = \frac{T}{3}$	Efficiency	89.72	93.55	4.26	90.07	0.39
	Head	81.48	83.37	2.32	83.04	1.92
	Torque	0.0297	0.0305	2.65	0.0303	2.03
$t = \frac{2T}{3}$	Efficiency	89.92	92.07	2.38	93.85	4.36
	Head	81.48	83.45	2.41	83.13	2.02
	Torque	0.0296	0.0304	2.64	0.0303	2.28
$t = T$	Efficiency	89.83	89.63	0.22	91.68	2.07
	Head	81.49	83.09	1.96	82.94	1.77
	Torque	0.0297	0.0304	2.65	0.0303	2.28

ERCOFTAC centrifugal pump geometry and velocity field is presented in Figure 4. Figures a) and b) present the Harmonic Balance solutions for 1 and 2 harmonics, respectively. Figure c) presents the transient solution velocity field.

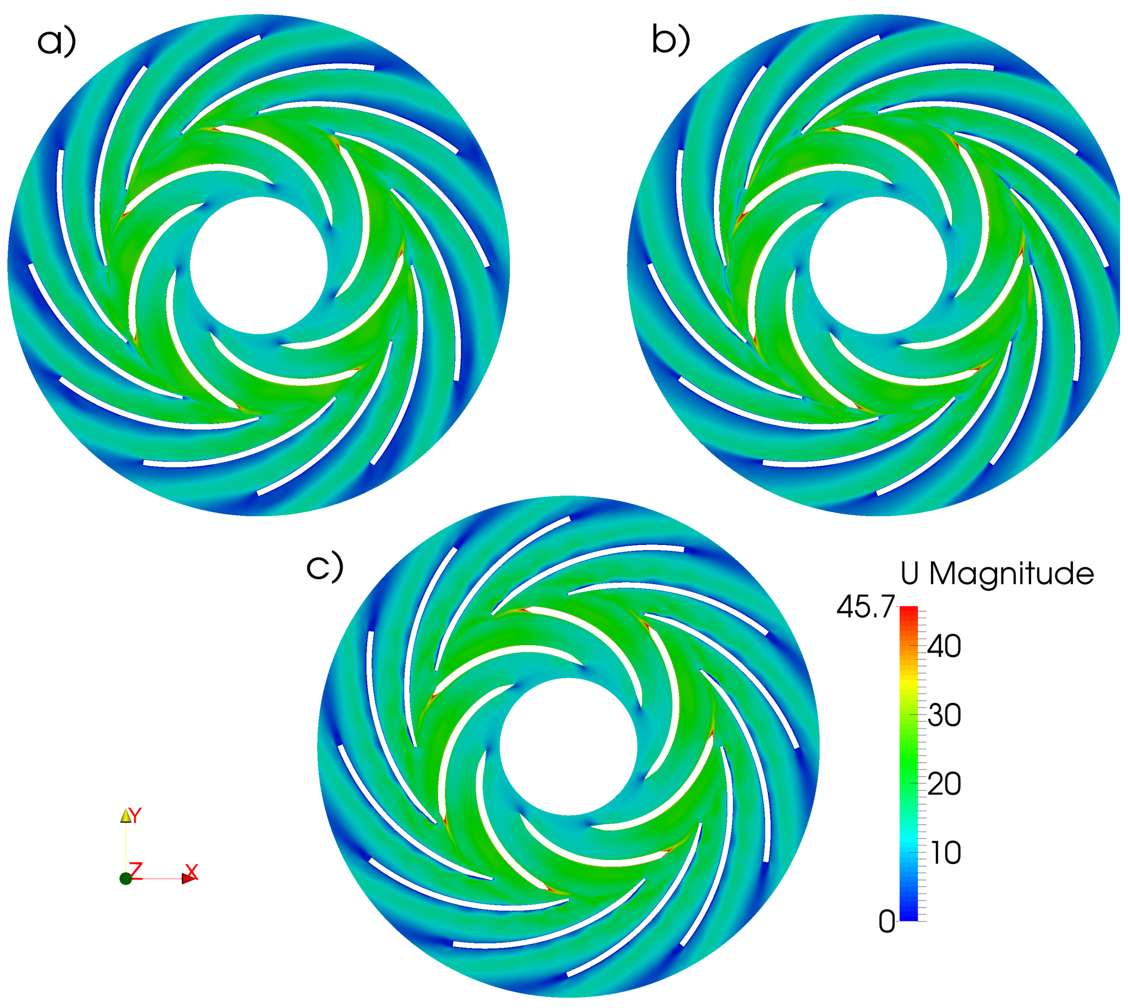

Figure 4: a) Harmonic Balance with 1 harmonic; b) Harmonic Balance with 2 harmonics; c) transient solver flow field at $t = T$.

5. Performance and hardware

The simulations were run in parallel using four cores on an Intel Core i5-3570K, 3.4 GHz computer. The significant CPU time reduction from transient to Harmonic Balance simulation can be noticed: one period of transient simulation took ~5 hours of CPU time, while Harmonic Balance simulation with 1 harmonic took ~52 minutes and nearly 3000 iterations. The 2 harmonics Harmonic Balance simulation took ~78 minutes of CPU time, converging in approximately 2400 iterations. In transient runs, a number of periods have to be run before reaching fully periodic steady state. Thus, CPU time of 1 period should be multiplied.

6. Conclusion

The Harmonic Balance method is presented for unsteady periodic non–linear flows in turbomachinery applications. The comparison of pressure contours around the rotor blade shows that the Harmonic Balance method is capable of capturing the transient flow field accurately even in multi-frequential environment. Additional comparison of pump characteristics with highest error being 4.36% shows that the Harmonic Balance method can be used as a part of a design process with accurate flow predictions and significant CPU time savings.