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Turbulence in two-phase systems

(Task 1.3 : Turbulence Modelling
at High Phase Fraction)

Project report III-14

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Abstract

This report summarises the work done so far on formulating a new model for multiphase turbulence. It begins with a review of the physics of multiphase turbulence as understood in the literature. A derivation of the turbulence equations using conditional averaging follows. Separate k and ϵ equations for each phase are postulated, and terms representing the interaction between phases modelled. A simple demo calculation of a shear layer is included, together with an analysis of issues related to the modelling. A summary of the work so far, together with suggestions for future work is provided.

1 Introduction

Multi-phase flows are extremely common in both natural and industrial fluid flow processes. They come in a wide variety of types, depending on whether the phases are gas, liquid or solid, aggregated or dispersed, reacting or non-reacting, and much else besides. Turbulence as a phenomenon is equally widespread. Methodologies for modelling turbulence in a single-phase flow are well-developed, with Reynolds Averaged models such as the $k - \epsilon$ model widely used in industry. The effects of the presence of a second phase on the system turbulence are less well understood. In this paper we seek to lay the foundations for extending the standard $k - \epsilon$ models to situations where a second dispersed phase is present. In particular we focus on gas-liquid and liquid-liquid systems. Here the dispersed elements are droplets (liquid) or bubbles (gas) with deformable interfaces, capable of breakup and coalescence. In this respect they share similar behaviour which is not shared with solid particles, and so it is reasonable to look for turbulence models to cover bothe droplets and bubbles – we will refer to these as 'fluid particles' from now on. In addition, the fluid particles are also of similar density to or much less dense than the continuous medium surrounding them, contrary to the case for solid particles in air.

2 Physics of Multiphase Turbulence

2.1 Review of multiphase turbulence : Physics

The interaction between solid particles and turbulence has been extensively studied for low phase fraction and small (μm) particles. Both turbulence enhancement and suppression effects have been noted, sometimes in contradictory ways. Hetsroni [1] suggested that the particle $\mathcal{R}e_p$ was critical here, with low $\mathcal{R}e_p$ leading to suppression and higher $\mathcal{R}e_p$ to enhancement. Gore & Crowe[2] suggested that the behaviour was dependent on the parameter d/L, where d is the diameter of the particle and L the integral length scale. Their suggestion was that turbulence is enhanced for $d/L > 0.1$, and suppressed for $d/L < 0.1$. Elghobashi & Truesdell [3] put forward evidence conflicting with both these viewpoints, presenting a DNS study showing enhancement for both low $\mathcal{R}e_p$ and low d/L. All of this work assumes particle number densities which are low, thus avoiding particle-particle interactions, and high density particles. For such cases asymptotic analytical solutions are possible and compare well with the DNS [4].

Although some of the results of this work will still be appropriate for fluid particle systems, these do display significantly different behaviour to solid-gas systems. It is well known that solid particles in a gas tend to accumulate in low-enstrophy regions of the flow, and the accumulation is most pronounced for particles with response times on the order of the Kolmogorov time. Gaseous bubbles in a liquid flow however accumulate in high-enstrophy regions such as vortex cores. As mentioned, fluid particles are also capable of distortion and even disruption due to the flow conditions, permitting a range of additional coupling interactions between the phases not possible for solid particles.

Experimental or computational work to determine the behaviour of turbulence in liquid/liquid or gas/liquid systems is less common. Souhar[5] used non-invasive electrochemical and optical techniques to investigate near-wall turbulence in bubbly flow at phase fractions up to $\alpha \sim 30\%$. Significant changes in the structure of the continuous phase turbulence were noted, including significant changes (factor of 5) in the longitudinal integral length scale, reduction in the microscale λ, increase in turbulent dissipation (factor of 10) and significant changes in the shape of the energy spectrum. Many of the changes were shown to be functionally dependent on the relative velocity between the phases, with the suggestion being made that the bubbles were dragging some of the larger eddies along as they rose through the liquid. The changes in the shape of the spectrum were in line with those demonstrated by Lance and Bataille [6], demonstrating that the classical $-5/3$ power law behaviour of the spectrum is progressively replaced by a $-8/3$ power law, close to an -3 power law which can be obtained by dimensional analysis. Some early work was done by Serizawa, Kataoka & Michiyoshi [7] on upward-flowing bubbly flows in vertical pipes, measuring mean and rms. velocities in both phases and void fraction. For a constant water velocity the longitudinal turbulence intensity was found to decrease initially with increasing gas velocity. This was attributed to work done moving the bubble and distorting it, and to the decrease in water phase available to support the turbulence. The spectra of the bubble and water velocities were found to be Poisson and Normal, respectively. Further work [8] indicates that the presence of the dispersed phase alters the form of the continuous phase spectrum, boosting the high frequency components, altering the intermediate part of the frequency spectrum and altering the anisotropy of the spectrum, particularly near to the wall. The intermediate and large scale effects were attributed to eddy fragmentation processes involving distortion of the interface. Theoretical work by Serizawa and Kataoka[9] extends this, showing that the distortion of the bubble interface by the turbulence is intrically linked with the turbulence so that energy can easilly be transferred from one to the other. They are in fact able to derive an equation for the conservation of the sum of these effects. As the authors recognise, the overall effect of the interaction will be a reduction in size of the eddy (eddy fragmentation) coupled with a dissipation of energy.

Kashinsky & Randin[10] have investigated bubbly flow for a vertical pipe, with particular reference to near-wall phenomena. Such flows have a different phase fraction distribution to upward (cocurrent) flows, being strongly peaked at the centre. The effect of the dispersed phase varies in the regions close to the wall, with the mean wall shear stress increased over the single phase case. The phase fraction profiles show no bubbles in the region $y^+ < 50$. In the viscous sublayer ($y^+ < 30$) there is no change in the turbulence structure, but surprisingly in the region $20 < y^+ < 50$ the continuous phase turbulence is suppressed relative to single-phase flow. The continuous phase turbulence can show two peaks, with a wall-induced peak at $y^+ \sim 10 - 30$ and a bubble-induced one around $y^+ \sim 100$. Finally, at the centre of the flow the turbulence is much greater than would be expected in since the bulk shear rate there is close to zero, which they attribute to stirring by the bubbles.

Druzhinin and Elghobashi[11] have performed DNS of isotropic decaying turbulence using a two-fluid approach at low phase fraction ($\alpha \leq 10^{-3}$, so as to be able to neglect bubble-bubble interactions) for both one-way and two-way coupling. The DNS was performed for cases where the bubble diameter and response time were much smaller than the respective Kolmogorov scales, and so the bubble accumulation mentioned earlier was not pronounced. They find that the bubble kinetic energy spectrum is reduced with respect to that of the fluid spectrum at small wave numbers due to the bubble inertia, although the difference is small because of the small bubble response time. They also calculate cases with linear concentration gradients in the vertical direction, corresponding to stable and unstable stratification of the bubbles. The modification of the continuous phase energy spectrum is shown to be related to a buoyancy flux term dependent on the concentration gradient, enhancing or reducing the turbulence decay rate according to whether the gradient is stable or unstable.

2.2 Review of multiphase turbulence : Modelling approaches

Crowe, Troutt and Chung authored a review article in Ann.Reviews [12] on numerical models for two-phase flows. This was a general review of approaches to all aspects of turbulent fluid-particle flows, but did contain a section on Eulerian approaches to turbulence via the two-fluid model. A distinction is drawn between dilute and dense dispersions, and the effects of the consequent physics on the modelling discussed. In particular, at low phase fractions $\alpha < 10^{-3}$ particle/particle interactions can be neglected, whilst at very low phase fractions the particles have no effect on the continuous phase at all. At low phase fraction two-fluid models using the Boussinesq approximation are extensively used, but with simple modelling of the dispersed phase [13][14], whilst at higher phase fraction they point out that most modelling is based on kinetic theory (see Reeks, below). There is also some discussion about the modelling of the modulation of the continuous phase turbulence by the presence of the dispersed phase.

The early work[15][16] [17] of Simonin is on modelling of the dispersion of particles by turbulence. The approach is to construct equations for the slip velocity of the particles in an Eulerian framework, and then validate the model using LES data. The continuous phase turbulence is described using the standard $k - \epsilon$ model with additional terms describing the effect of fluid-particle interaction on the continuous phase turbulence. The dispersed phase velocity fluctuations are related to the continuous phase turbulence using time and spatial scales from the continuous phase. The two EDF reports [18][19] contain the analysis of the interaction to derive the appropriate time scales. The 1993 TSF paper [20] is rather more useful ; it derives a Reynolds Stress equation for the particulate phase, and makes one or two suggestions for the functional form of some of the terms. In particular, dispersed phase turbulence is taken as being generated by the shear in the mean dispersed phase velocity.

The papers in the Third International Conference on Multiphase Flow (Lyon, France, 1998) authored by Simonin in collaboration with various authors extend this approach. The most useful is the paper with Février [21], which starts with the transport equations for the two Reynolds stresses (the continuous phase and the dispersed phase : the authors refer to the latter as the kinetic stress) and the fluid particle velocity correlation, and then propose various algebraic and differential models for various of the terms in these tensor equations. The modelling proposed is intended for solid particle mechanics, but the reasoning could be applied to other systems. The paper with Sakiz [22] concentrates on the modelling of collisional integral terms in this approach, whilst the one with Gourdel and Brunier [23] is entirely about the alterations to the modelling necessary for a binary solid phase.

The paper with Hyland and Reeks [24] considers a pdf approach to the modelling, starting by showing that the continuum equations derived from the kinetics (i.e. from the Boltzmann transport equation) and those from a generalised Langevin equation are the same, except for slight differences in the modelling of certain terms. This is hardly very surprising given the links between the Langevin equation and the Boltzmann transport equation. Results are then compared with statistics from LES work. This really just correlates Simonin's work in this area with that of Reeks, who has done extensive work on deriving continuum equations for solid particles in a turbulent gas from the basic kinetics [25][26]. The final paper of interest in the ICMF98 conference is that of Zhou [27] which is a review of work done on turbulent gas/particle flows in China. He mentions the k_p model of Zhou and Huang in which a turbulent kinetic energy equation for the particle turbulence k_p is solved for. Once again, the assumption is made that dispersed phase turbulence is generated by the shear in the dispersed phase velocity, whilst the dissipation is related to the physics of the particle/fluid interaction. Coupled with a $k - \epsilon$ system for the continuous phase this provides a good model for the particle/gas systems he is considering. He then discusses PDF approaches to the problem.

2.3 Physical Interactions.

The cartoon (figure 1) shows the interactions between the two phases present in the system. k_b represents the continuous phase turbulence, k_a the dispersed phase turbulence. The first point to note is the exact meaning of the dispersed phase turbulence. Turbulence in a continuous medium consists of a cascade of eddies at scales from large (the integral scale) to small (the kolmogorov scale), and this is representative of the turbulence in the continuous phase here. It is also conceivable that the fluid in the second phase would posess similar turbulence phenomena, which if the second phase were aggregated it would. However the second phase here is dispersed in fluid particles small enough that any internal motion can largely be ignored. On the other hand, the fluid particles themselves move in space. This motion can be considered to have mean and random components, the random component of which we choose to call turbulence. Under the assumption that internal motions of the fluid inside the particle can be ignored, both types of motion of the interface can be related to the bulk motion of the phase within the interface.

It seems reasonable to assume that turbulence in the continuous phase is generated in the same way as turbulence in a single-phase medium, i.e. by shear in the continuous phase, whilst being dissipated by the normal processes. However in addition the continuous phase turbulence is being modified by the presence of the dispersed phase, and at the same time driving this turbulence. The relative size of the fluid particles relative to

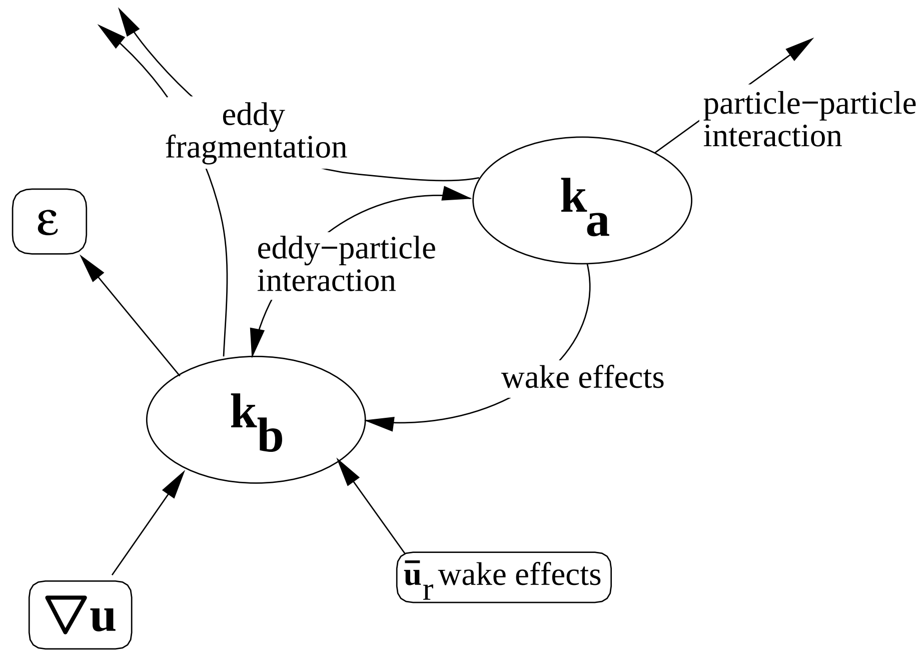

Figure 1: Cartoon demonstrating the interactions between dispersed and continuous phases in a two-phase system.

the eddies is important, as it is for solid particles (c.f. [2]).

Firstly individual fluid particles will be advected by the eddies, leading to a direct link between the phases, and generating the dispersed phase turbulence. Fluid particles moving (for instance due to buoyancy) into a higher turbulence region of the flow will be accellerated by the new eddies, representing an attempted equilibration in the energy between the two phases. Complete equilibration may not be possible if the fluid particle response time τ_s (the Stokes timescale for the fluid particle) is large compared with the duration of the interaction, τ_i, probably best represented as $\tau_i^{-1} = \frac{v_r}{l_e} + \tau_e(l_e)^{-1}$, where l_e is the eddy scale, v_r the relative velocity and τ_e the lifetime of the eddy. If complete equilibration is possible with individual eddies, interaction with subsequent eddies will be uncorrelated over distances greater than the integral length scale Λ (or times greater than the integral timescale T_Λ), and so the coupling between the phases will be incomplete. The interaction will thus result in energy from large-scale eddies in the flow being transmitted via the motion of the fluid particles to small-scale eddies in the flow (vortex shedding due to the stochastic relative velocity) or dissipated directly as surface friction. Similarly, if the fluid particles have a higher turbulent energy than the fluid phase around them, the energy flow will be in the other direction. This illustrates the need for our turbulence model to contain history effects.

Relative motion between the phases may generate continuous phase turbulence via vortex shedding, if the fluid particle Reynolds number is high enough. Although this will be a unified effect in reality, for the purposes of modelling we must split it into a random component due to the fluid particle turbulence and a mean component due to the mean relative velocity, which could for instance be driven by buoyancy. The other phase coupling effect is that of eddy fragmentation. If the eddy is of a similar size to the fluid particle then it will distort the particle, which takes energy from the eddy, leaving it smaller. For this reason, Michiyoshi and Serizawa [28] call this effect 'eddy fragmentation'. Important to this process will be the Capillary and Bond numbers of the fluid particle, being the ratios of viscous stress and hydrostatic pressure variation, respectively, to the interfacial tension stress. The visocsity ratio will also be of importance.

Finally there are also bubble-bubble interactions to take account of. These are considerably less important than in the case of solid particle/gas flows, since the momentum of the fluid particles is similar or less than that of the continuous phase, as against considerably larger for solid particles in air. However, particularly at high phase fraction, there will be particle/particle interactions taking place, in the form of inelastic collisions which will therefore dissipate k_a.

3 Formulation of $k - \epsilon$ equations

3.1 Fluctuation

To formulate the $k-\epsilon$ equations we need to derive an equation for some form of fluctuating velocity $\boldsymbol{u'}$. This causes some conceptual problems. We use the conditioned averaging techniques first developed by Dopazo[29], which is based on ensemble averaging. An indicator functional χ_k is introduced which projects out those realisations of the ensemble which have phase k at the point of interest. Considering the velocity $\boldsymbol{u}$ as the variable of interest here, we can define the mean velocity in phase k as

$$\overline{\boldsymbol{u}}_k = \frac{\overline{\chi_k \boldsymbol{u}}}{\overline{\chi_k}} \tag{1}$$

(Further details of the conditioned averaging and surface averaging techniques are given in the Appendix). We can define an overall mean velocity as

$$\overline{\boldsymbol{u}} = \alpha_k \overline{\boldsymbol{u}}_k + \alpha_{\neg k} \overline{\boldsymbol{u}}_{\neg k}, \qquad \alpha_{\neg k} = 1 - \alpha_k \tag{2}$$

(Here, $\neg k$ is everything execpt k. For a two-phase system, this will be just the other phase). Using these definitions we can define three differing versions of the fluctuating velocity :

$$\boldsymbol{u}' = \boldsymbol{u} - \overline{\boldsymbol{u}} \tag{3}$$

$$\boldsymbol{u}'_k = \boldsymbol{u} - \overline{\boldsymbol{u}}_k \tag{4}$$

$$\boldsymbol{u}^{k'} = \boldsymbol{u}_k - \overline{\boldsymbol{u}}_k, \qquad \boldsymbol{u}_k = \{\boldsymbol{u}_i : i \in k\} \tag{5}$$

Of these definitions, eqn.(3) takes no account of phase. Eqn.(4) is a phase-specific velocity fluctuation which is easy to evaluate, but incorporates a contribution due to the difference in mean velocities between the phases. Eqn.(5) uses the velocity only in those ensembles which have phase k at the point of interest, and so is the most phase-specific, but this phase projection makes it difficult to evaluate mathematically. However it is reasonable to assert that

$$\overline{(\boldsymbol{u}' \otimes \boldsymbol{u}')}_k \equiv \overline{\boldsymbol{u}'_k \otimes \boldsymbol{u}'_k} \equiv \overline{\boldsymbol{u}^{k'} \otimes \boldsymbol{u}^{k'}} \tag{6}$$

We will need to consider the variable density case here since even if the fluids are individually incompressible, the mixture density will be a function of the phase fraction and thus spatially (and temporally) varying. Thus we use Favré averaging, where

$$\widetilde{\phi} = \frac{\overline{\rho\phi}}{\overline{\rho}} \tag{7}$$

and we define a fluctuating velocity as

$$\boldsymbol{u}'' = \boldsymbol{u} - \widetilde{\boldsymbol{u}} \tag{8}$$

This concept can be extended to cover phase-weighted variables in the obvious manner, by conditioning $\rho\phi$ and ρ separately before averaging, giving the result $\overline{\chi_k \rho\phi} = \alpha_k \overline{(\rho\phi)}_k = \alpha_k \overline{\rho}_k \widetilde{\phi}_k$, i.e.

$$\widetilde{\phi}_k = \frac{\overline{\chi_k \rho\phi}}{\overline{\chi_k \rho}} \tag{9}$$

Clearly the concepts of Favré and conditional averaging commute. The earlier comments about the definitions of phase-specific fluctuations carry over. Thus to evaluate the Reynolds stress, we expand

$$\begin{aligned}
\overline{\chi_k \rho \boldsymbol{u} \otimes \boldsymbol{u}} &= \overline{\chi_k \rho (\widetilde{\boldsymbol{u}}_k + \boldsymbol{u}''_k) \otimes (\widetilde{\boldsymbol{u}}_k + \boldsymbol{u}''_k)} \\
&= \overline{\chi_k \rho \widetilde{\boldsymbol{u}}_k \otimes \widetilde{\boldsymbol{u}}_k} + \overline{\chi_k \rho \boldsymbol{u}''_k \otimes \boldsymbol{u}''_k} \\
&= \alpha \overline{\rho}_k \widetilde{\boldsymbol{u}}_k \otimes \widetilde{\boldsymbol{u}}_k + \alpha_k \overline{(\rho \boldsymbol{u}'' \otimes \boldsymbol{u}'')}_k
\end{aligned} \tag{10}$$

3.2 The Navier-Stokes Equations.

The Navier-Stokes Equations for a variable-density fluid are

$$\frac{\partial \rho}{\partial t} + \nabla.\rho \boldsymbol{u} = 0 \tag{11}$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla.(\rho \boldsymbol{u} \otimes \boldsymbol{u}) = \nabla.\boldsymbol{\tau} \tag{12}$$

where

$$\boldsymbol{\tau} = -p\boldsymbol{I} + \mu \boldsymbol{S}, \qquad \boldsymbol{S} = \mathrm{dev}\, \boldsymbol{u} \tag{13}$$

Conditionally averaging the momentum equation gives

$$\begin{aligned}\frac{\partial}{\partial t}\alpha_k\overline{\rho}_k\widetilde{\boldsymbol{u}}_k+\nabla.(\alpha_k\overline{\rho}_k\widetilde{\boldsymbol{u}}_k\otimes\widetilde{\boldsymbol{u}}_k)+\nabla.(\alpha\overline{\rho}_k\widetilde{\boldsymbol{u}''\otimes\boldsymbol{u}''}) &= \nabla.\alpha_k\overline{\tau}_k\\ +\,\widetilde{\rho\boldsymbol{u}\otimes(\boldsymbol{u}-\boldsymbol{v}_i).\boldsymbol{n}_k}\Sigma &- \widetilde{\boldsymbol{\tau_k}.\boldsymbol{n}_k}\Sigma\end{aligned}\tag{14}$$

If we assume that there is no interphase mass transfer, the interface moves with the phase velocity at the interface, ie $\widetilde{\boldsymbol{u}}=\boldsymbol{v}_i$, and so the second term on the rhs. is zero.

3.3 Reynolds Stress equation

If we take the outer product of $\boldsymbol{u}$ with eqn.(12) and add on its transpose, then noting that

$$\begin{aligned}\nabla.\rho\boldsymbol{u}\otimes\boldsymbol{u}\otimes\boldsymbol{u} &= \mathrm{sym}(\boldsymbol{u}\otimes\nabla.\rho\boldsymbol{u}\otimes\boldsymbol{u})+\mathrm{sym}(\rho\boldsymbol{u}\otimes\boldsymbol{u}.\nabla\boldsymbol{u})\\ \frac{\partial(\rho\boldsymbol{u}\otimes\boldsymbol{u})}{\partial t} &= \mathrm{sym}\left(\rho\boldsymbol{u}\otimes\frac{\partial\boldsymbol{u}}{\partial t}\right)+\mathrm{sym}\left(\boldsymbol{u}\otimes\frac{\partial\rho\boldsymbol{u}}{\partial t}\right)\end{aligned}\tag{15}$$

we obtain

$$\frac{\partial\rho\boldsymbol{u}\otimes\boldsymbol{u}}{\partial t}+\nabla.\rho\boldsymbol{u}\otimes\boldsymbol{u}\otimes\boldsymbol{u} = 2\,\mathrm{sym}\nabla.(\boldsymbol{u}\otimes\boldsymbol{\tau})-2\,\mathrm{sym}(\boldsymbol{\tau}.\nabla\boldsymbol{u})\tag{16}$$

where $\mathrm{sym}(\boldsymbol{T})=\frac{1}{2}(\boldsymbol{T}+\boldsymbol{T}^T)$ for a second rank tensor $\boldsymbol{T}$. Conditionally averaging this equation gives

$$\begin{aligned}\frac{\partial}{\partial t}\alpha\overline{(\rho\boldsymbol{u}\otimes\boldsymbol{u})}_k \;+\; \nabla.\alpha\overline{(\rho\boldsymbol{u}\otimes\boldsymbol{u}\otimes\boldsymbol{u})}_k &= 2\,\mathrm{sym}\left(\nabla.\alpha\overline{(\boldsymbol{u}\otimes\boldsymbol{\tau})}_k\right)\\ &-2\,\mathrm{sym}\left(\alpha\overline{\boldsymbol{\tau}.\nabla\boldsymbol{u}_k}\right)-2\,\mathrm{sym}\left(\widetilde{\boldsymbol{u}\otimes\boldsymbol{\tau}.\boldsymbol{n}_k}\right)\Sigma\end{aligned}\tag{17}$$

Performing a similar operation on eqn.(14) we find

$$\begin{aligned}\frac{\partial}{\partial t}\alpha\overline{\rho}_k\widetilde{\boldsymbol{u}}_k\otimes\widetilde{\boldsymbol{u}}_k \;+\; &\nabla.\alpha\overline{\rho}_k\widetilde{\boldsymbol{u}}_k\otimes\widetilde{\boldsymbol{u}}_k\otimes\widetilde{\boldsymbol{u}}_k+2\,\mathrm{sym}\left(\nabla.\alpha\overline{\rho}_k\widetilde{\boldsymbol{u}}_k\otimes\widetilde{\boldsymbol{u}''\otimes\boldsymbol{u}''}\right)\\ = \;&2\,\mathrm{sym}(\alpha\overline{\rho}_k\widetilde{\boldsymbol{u}''\otimes\boldsymbol{u}''}.\nabla\widetilde{\boldsymbol{u}}_k)+2\,\mathrm{sym}(\nabla.\alpha\widetilde{\boldsymbol{u}}_k\otimes\overline{\boldsymbol{\tau}}_k)-2\,\mathrm{sym}(\alpha\overline{\boldsymbol{\tau}}_k.\nabla\widetilde{\boldsymbol{u}}_k)\\ &-2\,\mathrm{sym}(\widetilde{\boldsymbol{u}}_k\otimes\widetilde{\overline{\boldsymbol{\tau}}_k.\boldsymbol{n}_k})\Sigma\end{aligned}\tag{18}$$

Subtracting eqn.(18) from eqn.(17),

$$\begin{aligned}\frac{\partial}{\partial t}\alpha\overline{\rho}_k(\widetilde{\boldsymbol{u}''\otimes\boldsymbol{u}''})_k \;+\; &\nabla.\alpha\overline{\rho}_k(\widetilde{\boldsymbol{u}''\otimes\boldsymbol{u}''})_k\otimes\widetilde{\boldsymbol{u}}_k+\nabla.\alpha\overline{\rho}_k(\boldsymbol{u}''\otimes\widetilde{\boldsymbol{u}''}\otimes\boldsymbol{u}'')_k\\ = \;&2\,\mathrm{sym}\left\{\left(\alpha\overline{\rho}_k(\widetilde{\boldsymbol{u}''\otimes\boldsymbol{u}''})_k.\nabla\widetilde{\boldsymbol{u}}_k\right)\right.\\ &+\nabla.\alpha\left[\overline{(\boldsymbol{u}\otimes\boldsymbol{\tau})}_k-\widetilde{\boldsymbol{u}}_k\otimes\overline{\boldsymbol{\tau}}_k\right]-\alpha\left[\overline{(\boldsymbol{\tau}.\nabla\boldsymbol{u})}_k-\overline{\boldsymbol{\tau}}_k.\nabla\widetilde{\boldsymbol{u}}_k\right]\\ &\left.-\left[\widetilde{\boldsymbol{u}\otimes\boldsymbol{\tau}.\boldsymbol{n}_k}-\widetilde{\boldsymbol{u}}_k\otimes\widetilde{\overline{\boldsymbol{\tau}}.\boldsymbol{n}}\right]\Sigma\right\}\end{aligned}\tag{19}$$

The triple correlation term comes from splitting the term $\overline{(\rho\boldsymbol{u}\otimes\boldsymbol{u}\otimes\boldsymbol{u})}_k$ using the fact that $\boldsymbol{u}=\widetilde{\boldsymbol{u}}+\boldsymbol{u}''$. Other terms on the rhs. can be treated in a similar manner, showing that

$$\begin{aligned}\nabla.\alpha\left[\overline{(\boldsymbol{u}\otimes\boldsymbol{\tau})}_k-\widetilde{\boldsymbol{u}}_k\otimes\overline{\boldsymbol{\tau}}_k\right] &= \nabla.(\alpha_k\overline{\boldsymbol{u}''}\otimes\overline{\boldsymbol{\tau}})+\nabla.(\alpha\overline{\boldsymbol{u}''\otimes\boldsymbol{\tau}'}) &(20)\\ \overline{(\boldsymbol{\tau}.\nabla\boldsymbol{u})}_k-\boldsymbol{\tau}_k.\nabla\widetilde{\boldsymbol{u}}_k &= \overline{\boldsymbol{u}''}.\nabla\overline{\boldsymbol{\tau}}+\overline{\boldsymbol{u}''.\nabla\boldsymbol{\tau}'} &(21)\end{aligned}$$

Substituting back into eqn.(19), and expanding $\boldsymbol{\tau}$, gives

$$\begin{aligned}\frac{\partial}{\partial t}\alpha_k\overline{\rho}_k\boldsymbol{R}+\nabla.\alpha_k\overline{\rho}_k\boldsymbol{R}\otimes\widetilde{\boldsymbol{u}}_k &= -\nabla.\alpha_k\boldsymbol{D_k}+2\alpha\left(\mathrm{sym}\boldsymbol{P}_k+\mathrm{sym}\boldsymbol{\Phi}_k+\mathrm{sym}\boldsymbol{W}_k\right)\\ &\quad -2\alpha\overline{\rho}_k\mathrm{sym}\boldsymbol{\epsilon}_k+\widehat{\varsigma}_k\Sigma &(22)\end{aligned}$$

Here

$$\begin{aligned}\nabla.\alpha\boldsymbol{D}_k &= \nabla.\alpha\overline{\rho}_k\widetilde{(\boldsymbol{u}''\otimes\boldsymbol{u}''\otimes\boldsymbol{u}'')}_k-2\alpha\mathrm{sym}(\nabla.\overline{(\boldsymbol{u}''\otimes\boldsymbol{\tau}')}_k)\\ \boldsymbol{P}_k &= \overline{\rho}_k\widetilde{(\boldsymbol{u}''\otimes\boldsymbol{u}'')}_k.\nabla\widetilde{\boldsymbol{u}}_k\\ \boldsymbol{\Phi}_k &= -\overline{\boldsymbol{u}''}_k\nabla\overline{p}_k, \qquad \boldsymbol{W}_k=-\overline{\nabla p'\boldsymbol{u}''}_k\\ \boldsymbol{\epsilon}_k &= -\overline{\mu\boldsymbol{S}''.\nabla\boldsymbol{u}''}_k\end{aligned}$$

and

$$\widehat{\varsigma}_k=-2\ \mathrm{sym}\left(\widehat{\overline{\boldsymbol{u}\otimes\boldsymbol{\tau}.\boldsymbol{n}_k}}\right)+2\ \mathrm{sym}(\widetilde{\boldsymbol{u}}_k\otimes\widehat{\overline{\boldsymbol{\tau}_k.\boldsymbol{n}_k}}) \tag{23}$$

are the surface terms. If we set $\alpha=1$ and disregard the surface terms, we recover the conventional RS equation for a compressible system[30].

We proceed here by reverting to considering incompressible systems. This will be appropriate for liquid/liquid systems, and is a fair approximation for gas/liquid systems. Under these circumstances, the individual phase densities are constant, Favré averaging becomes equivalent to Reynolds averaging, and terms of the form $\overline{\boldsymbol{u}''}=0$. In addition we want an equation for the turbulent kinetic energy $k_k=\mathrm{tr}(\overline{\boldsymbol{u}'\otimes\boldsymbol{u}'})$, so we will contract the RS equation. This gives us

$$\frac{\partial}{\partial t}\alpha_k\overline{\rho}_k k+\nabla.\alpha_k\overline{\rho}_k k\otimes\widetilde{\boldsymbol{u}}_k = -\nabla.\alpha_k D_k+\alpha P_k-\alpha\overline{\rho}_k\epsilon_k+\widehat{\varsigma}_k\Sigma \tag{24}$$

as the generic turbulent kinetic energy equation in phase k.

3.4 Modelling of the continuous phase

Considering first the continuum terms for phase b (the continuous phase), we have

$$P_b=\overline{(\rho\boldsymbol{u}'\otimes\boldsymbol{u}')}_b:\nabla\widetilde{\boldsymbol{u}}_b \tag{25}$$

for the production. In a single-phase flow, we could invoke Reynolds hypothesis to write $\overline{(\rho \boldsymbol{u}' \otimes \boldsymbol{u}')} = \mu^t \left(\nabla \overline{\boldsymbol{u}} + \nabla \overline{\boldsymbol{u}}^t\right)$, with $\mu^t = c_\mu k^2/\epsilon$. If we hypothesise that turbulence in the continuous phase is created and diffused in an analogous manner to turbulence in a single-phase system, with additional contributions from the dispersed phase, we can base the generation on $\nabla \overline{\boldsymbol{u}}_b$, ie. $\overline{(\rho \boldsymbol{u}' \otimes \boldsymbol{u}')}_b =$. This approach has been used in the past, and is certainly appropriate for low phase fractions. An alternative is to consider the system as a mixture with a phase-weighted velocity of the form $\overline{\boldsymbol{u}}_+ = \alpha \overline{\boldsymbol{u}}_a + \beta \overline{\boldsymbol{u}}_b$, and model the generation as

$$P_b = \mu^t \left(\nabla \overline{\boldsymbol{u}}_+ + \nabla \overline{\boldsymbol{u}}_+^t\right) : \nabla \widetilde{\boldsymbol{u}}_+ \tag{26}$$

This may be more appropriate for high phase fraction systems. (We must examine the implications of this choice on the momentum equation for the phase).

Following the usual modelling for single-phase flows, D_b can be written

$$\nabla.\alpha D_b \quad = \quad \nabla.\alpha_b \left(\mu + \frac{\mu^t}{\sigma_b}\right) \nabla k_b \tag{27}$$

This leaves the surface terms to be modelled. Contracting these gives

$$\widehat{\varsigma}_b = \overline{\boldsymbol{u}}_b.\widehat{\overline{\boldsymbol{\tau}_b.\boldsymbol{n}_b}} - \widehat{\overline{\boldsymbol{u}.\boldsymbol{\tau}.\boldsymbol{n}_b}} \tag{28}$$

The two stress terms represent the total and mean work done through interactions between the flow and the interface, just as the equation as a whole represents the difference between the total flow kinetic energy $\overline{u^2}$ and the mean flow kinetic energy $\overline{u}^2$. The term $\widehat{\overline{\boldsymbol{\tau}_b.\boldsymbol{n}_b}}$ is the interphase force which is modelled in the Reynolds averaged momentum equation in terms of the drag, lift and other forces, and so $\overline{\boldsymbol{u}}_b.\widehat{\overline{\boldsymbol{\tau}_b.\boldsymbol{n}_b}}$ is the work done by the mean flow velocity on the interface, and thus on the other phase. Similarly $\boldsymbol{u}.\boldsymbol{\tau}.\boldsymbol{n}_b$ is the work done by the complete flow velocity, and its surface average gives the value at the surface. If we can write $\boldsymbol{u} = \widehat{\boldsymbol{u}} + \boldsymbol{u}' + \boldsymbol{u}^\#$, and assuming that $\widehat{\boldsymbol{u}} \equiv \overline{\boldsymbol{u}}_a$ (see next section), then we find

$$\overline{\boldsymbol{u}}_b.\widehat{\overline{\boldsymbol{\tau}.\boldsymbol{n}_b}} - \widehat{\overline{\boldsymbol{u}.\boldsymbol{\tau}.\boldsymbol{n}_b}} \quad = \quad -\overline{\boldsymbol{u}}_r.\widehat{\overline{\boldsymbol{\tau}.\boldsymbol{n}_b}} - \widehat{\overline{\boldsymbol{u}'.\boldsymbol{\tau}.\boldsymbol{n}_b}} - \widehat{\overline{\boldsymbol{u}^\#.\boldsymbol{\tau}.\boldsymbol{n}}} \tag{29}$$

with $\overline{\boldsymbol{u}}_r = \overline{\boldsymbol{u}}_a - \overline{\boldsymbol{u}}_b$.

The first term in eqn.(29) is the turbulence generation due to the mean relative velocity. In the momentum equation $\widehat{\overline{\boldsymbol{\tau}.\boldsymbol{n}_b}}$ is the drag term, which in the BRITE-II project is modelled in terms of two components : a mean drag $A_d \overline{\boldsymbol{u}}_r$ and a turbulent component

$$\boldsymbol{F}_{td} = A_d \frac{\nu_b^t}{\alpha \beta \sigma_\alpha} \nabla \alpha \tag{30}$$

This split arises from the double-averaging techniques used during that project, and does not arise directly from the conditional averaging techniques used here. However a similar dispersive term does arise from the general manipulation used here[31], and so we will retain this term for the moment. The mean component in this term may or may not be included. It is quite large, particularly at startup (this can cause problems), and potentially could swamp the other effects. On the other hand it is not obvious why it should be left out, and other authors do introduce it[32].

The second term is the correlation between the random component of the fluid particle flow field and the surface stresses. Once again, we can write $\boldsymbol{\tau}.\boldsymbol{n}_b = A_d(\overline{\boldsymbol{u}}_r + \boldsymbol{u}'_r)$ Resubstituting we find

$$\begin{aligned}
\widetilde{\boldsymbol{u}'.\boldsymbol{\tau}.\boldsymbol{n}_b} &= A_d\overline{\boldsymbol{u}}_r.\widetilde{\boldsymbol{u}'} + A_d\widetilde{\boldsymbol{u}'.\boldsymbol{u}_r} = A_d\overline{\boldsymbol{u}'_a.\boldsymbol{u}'_r} \\
&= A_d(\overline{\boldsymbol{u}'_a.\boldsymbol{u}'_a} - \overline{\boldsymbol{u}'_a.\boldsymbol{u}'_b})
\end{aligned} \tag{31}$$

since $\widetilde{\boldsymbol{u}'} = 0$. This is the difference between the velocity fluctuation autocorrelation and cross-correlation, and represents another form of coupling between the phases. We could formulate an equation for this cross-stress, by expanding eqn.(10) in a different manner :

$$\begin{aligned}
\overline{\chi_a\rho\boldsymbol{u}\otimes\boldsymbol{u}} &= \overline{\chi_a\rho(\widetilde{\boldsymbol{u}}_a + \boldsymbol{u}''_a)\otimes(\widetilde{\boldsymbol{u}}_b + \boldsymbol{u}''_b)} \\
&= \overline{\chi_a\rho\widetilde{\boldsymbol{u}}_a\otimes\widetilde{\boldsymbol{u}}_b} + \overline{\chi_a\rho\boldsymbol{u}''_a\otimes\boldsymbol{u}''_b} \\
&= \alpha_a\overline{\rho}_a\widetilde{\boldsymbol{u}}_a\otimes\widetilde{\boldsymbol{u}}_b + \alpha_a\overline{\rho}_a(\widetilde{\boldsymbol{u}''_a\otimes\boldsymbol{u}''_b})
\end{aligned} \tag{32}$$

However this is excessively complicated for the current formulation. Instead we regard the cross-stress as being related to one of the phase stresses. If we choose to relate it to the correlation in the b phase then $\overline{\boldsymbol{u}'_a.\boldsymbol{u}'_b} = C_\zeta\overline{\boldsymbol{u}'_b.\boldsymbol{u}'_b}$ then we have a coupling between the phases themselves, and can write this term as

$$A_d(k_a - C_\zeta k_b) \tag{33}$$

C_ζ is related to the C_t function postulated in the BRITE-II project, and will probably have some functional form itself. However we can probably regard it as an arbitary constant for the moment.

The third term is the interaction between the turbulence and the fluid particles. Contrary to [33], eqn.(24) also contains a term which is due to surface tension. This can be represented by a contribution to $\boldsymbol{\tau}$ of the form

$$\boldsymbol{\tau}_s = -\sigma_s(\boldsymbol{I} - \boldsymbol{n}\otimes\boldsymbol{n})\delta_s \tag{34}$$

where σ_s is the surface tension. In the mean this has no effect, since the surface tension forces on any individual fluid particle must sum to zero, and so this term does not show up in the mean momentum equation. However the term $\widetilde{\boldsymbol{u}^\#.\boldsymbol{\tau}_s.\boldsymbol{n}}$ is the rate of deformation of the fluid particle, which is non-zero when integrated around the fluid particle surface. We can thus write the surface term in the k_b equation

$$\widetilde{\varsigma}_k\Sigma = -\boldsymbol{F}_d.\overline{\boldsymbol{u}}_r - \widetilde{\boldsymbol{u}^\#.\boldsymbol{\tau}_d.\boldsymbol{n}_b}\Sigma - \widetilde{\boldsymbol{u}^\#.\boldsymbol{\tau}_s.\boldsymbol{n}_b}\Sigma \tag{35}$$

The second term here represents the advective coupling between turbulent eddies and fluid particles, and the last term the energy drain involved in distorting the particles.

We can bound the energy drain through deformation of the interfaces by considering the energetics of turbulent breakup. If the turbulence is sufficiently strong the resulting deformation can lead to breakup of the fluid particles. If the breakup is into 2 equal sized particles, there is an energy barrier of size

$$\Delta E = \pi d^2 \sigma \left(2^{1/3} - 1\right)$$

to breakup. If the particle size distribution is stable, the energy supplied to the particles by deformation must be less than this per particle. Thus the overall energy contained in the particle distortion is

$$E_{st} < \frac{6\alpha\sigma}{d} \left(2^{1/3} - 1\right)$$

3.5 Modelling of the dispersed phase

For the dispersed phase we first have to interpret the velocity $\boldsymbol{u}_a$. As indicated earlier, the assumption is that motions within the fluid particle (particularly turbulent motions) are negligable, which is entirely reasonable if the particles are small. This implies that the velocities involved are those of the interface, thus allowing the identification $\widehat{\boldsymbol{u}} \equiv \overline{\boldsymbol{u}}_a$ used in the previous section. That being so, we can model $\nabla.\alpha_a \boldsymbol{D_a}$ as before, as a diffusive effect.

We will make the assumption that the dispersed phase turbulence is driven by the continuous phase shear, in other words that the turbulence production term in the dispersed phase equation is related to that in the continuous phase equation. Introducing a new function C_r,

$$P_a = C_r P_b \tag{36}$$

Again, C_r will eventually be a function related to C_t, but for the moment we will consider it as a constant $C_t = 1.0$.

This leaves the surface terms. As before we can expand $\boldsymbol{u} = \widehat{\boldsymbol{u}} + \boldsymbol{u}^{\#}$ on the interface, but this time since $\widehat{\boldsymbol{u}} \equiv \overline{\boldsymbol{u}}_a$,

$$\widehat{\varsigma}_a = \overline{\boldsymbol{u}}_a.\overbrace{\boldsymbol{\tau}.\boldsymbol{n}_a} - \overbrace{\boldsymbol{u}.\boldsymbol{\tau}.\boldsymbol{n}_a} = -\overbrace{\boldsymbol{u}^{\#}.\boldsymbol{\tau}.\boldsymbol{n}_a} \tag{37}$$

The identification of the interface velocity with the dispersed phase velocity also implies that there is no coupling between the interface distortion and the dispersed phase velocity. This means that the only interface term is that involving $\boldsymbol{\tau}_d$, which is identical to that term in the b-phase equation, but with opposite sign since $\boldsymbol{n}_b = -\boldsymbol{n}_a$, i.e.

$$A_d(C_\zeta k_b - k_a) \tag{38}$$

Again this is a coupling term between the two phases. This does raise the issue as to whether coupling should be present both in this term *and* in the production term. For the moment both terms will be present.

3.6 Dissipation equation.

For a full $k - \epsilon$ model derivation we need to derive a similar transport equation for the dissipation ϵ. For a compressible system this is exceptionally difficult, with the resulting equation (single phase case) having more than 20 terms. Single phase analyses therefore often take a modelling approach [30], recognising that $[\epsilon] = [k]/[T]$, and therefore scaling the k-equation by a suitable timescale to produce a modelled ϵ-equation. This has the advantage that the source term modelling in both equations is equivalent. Applying similar reasoning to eqn.(24) gives

$$\frac{\partial}{\partial t}\alpha_k\overline{\rho}_k\epsilon + \nabla.\alpha_k\overline{\rho}_k\epsilon \otimes \widetilde{\boldsymbol{u}}_k = \frac{\epsilon}{k}\left[-\nabla.\alpha_k D_k + \alpha P_k - \alpha\overline{\rho}_k\epsilon_k\right] + \widehat{\varsigma}_k\Sigma \tag{39}$$

Numerical coefficients introduced in modelling $\boldsymbol{D}$ and $\boldsymbol{P}$ will change in the normal way. The only outstanding issue here relates to the form of the surface terms. If we can regard the separate phases as individually incompressible, the dissipation can be written as

$$\begin{aligned}\epsilon_k &= \nu\overline{(\nabla\boldsymbol{u}' : \nabla\boldsymbol{u}')}_k \\ &= \nu\overline{(\nabla\boldsymbol{u} : \nabla\boldsymbol{u})}_k - \nu\nabla\overline{\boldsymbol{u}}_k : \nabla\overline{\boldsymbol{u}}_k\end{aligned}$$

An analysis following a similar route to the derivation of the k equation is possible by taking the gradient of the momentum equation and multiplying by $\nabla\boldsymbol{u}$. This suggests a form for the surface terms

$$\nu\left(\widehat{\boldsymbol{\omega} : \nabla\boldsymbol{\tau}.\boldsymbol{n}} - \overline{\boldsymbol{\omega}}_k : \widehat{\nabla\boldsymbol{\tau}.\boldsymbol{n}}\right)\Sigma$$

which differs from the term in the k-equation by a factor of dimension $[T^{-1}]$. Thus a similar scaling for the surface terms will be possible.

4 Demonstration case

The preceeding section outlines the intended modelling. For an initial test, the shear layer case from the BRITE-II project (case 2.2) was computed. Various combinations of drag terms in the turbulence model were tried, the details of which are presented in the following table ;

Term	Model A	Model B	Model C
Continuous Phase			
Production		Based on $\overline{\boldsymbol{u}}_+$	
Mean Drag term	Present	Present in k_b equation	Absent
Turbulent drag term		$A_d \frac{\nu_b^t}{\alpha\beta\sigma_\alpha} \nabla\alpha$ (Eqn.(30))	
Interphase term		(Eqn.(33)) with $C_\zeta = 1.0$	
Dispersed Phase			
Production term		eqn.(36), $C_r = 1.0$	
Interphase term		eqn.(38)	

Figures 2, 3 and 4 compare the calculations for monitoring locations $20, 40, 50$ cm downstream of the inlet for this particular case. The experimental data is plotted as square points, results from the BRITE-II model as dot-dashed lines and models A-C as dotted, dashed and long dashed lines respectively. The following points are noteworthy :

- Mean flow data is much the same for all the models. Any difference is probably insignificant.
- On the α profiles, the only significant difference is in the complexity of the profiles in the mixing layer. None of the models performs well on this.
- The continuous phase turbulence seems to decay faster in the new models than in the BRITE-II model, and again there is less actual structure connected with the shear layer. For the $y = 20$ cm location this results in a better match with experiment than for the BRITE-II model, whilst further downstream the reverse is the case.
- The dispersed phase turbulence is only predicted by the current models, so no comparison with the BRITE-II modelling is possible. The levels for Models A, C seem to be around 30% too low, which could possibly be rectified by judicious choice of C_r. There seems little sign of structure within the profile, but there is little sign of any in the data either.
- Concerning variation between the models for this u'_a and u'_b, Model B (which has the mean drag term in the k_b equation but not in the ϵ_b equation) is clearly incorrect. The other two models give identical profiles, which plot one on top of the other. This seems implausible, and must be investigated.
- All the profiles show interesting behaviour near to the walls. Normally when this data has been displayed for publication only the central region has been displayed, and the walls are probably far enough away not to have too much of an effect on the flow. However it does suggest the necessity of developing wall models for the dispersed phase, and possibly modifying those used for the continuous phase as well.

With reference to the wall terms, currently the standard single-phase wall models are used, with source terms dependent on properties of the continuous phase alnone. These terms are included into both ϵ equations (multiplied by the appropriate phase fraction in each case). A number of possibilities could be suggested here, including using properties of both phases to construct the wall terms, and introducing separate modelling for the dispersed phase based on the known behaviour of bubbles near the wall. There is some literature on the effect of particles on near-wall turbulence, e.g. Pan and Banerjee[34], although in that case for solid particles at too low a phase fraction (10^{-4}) to be of over much use.

We can look separately at the various terms going into the k_b equation. The table below defines the nomenclature. Several of the terms will be the same in the k_a equation.

Name	Form	Origin
Gen_1	$\mu^t \left(\nabla\overline{\boldsymbol{u}}_+ + \nabla\overline{\boldsymbol{u}}_+^t\right) : \nabla\widetilde{\boldsymbol{u}}_+$	Eqn.(26)
Gen_2	$\mu^t \left(\nabla\overline{\boldsymbol{u}}_b + \nabla\overline{\boldsymbol{u}}_b^t\right) : \nabla\widetilde{\boldsymbol{u}}_b$	Eqn.(25)
$term_1$	$A_d\overline{\boldsymbol{u}}_r$	
$term_2$	$A_d\frac{\nu_b^t}{\alpha\beta\sigma_\alpha}\nabla\alpha$	Eqn.(30)
$term_3$	$A_d(k_a - C_\zeta k_b)$	Eqn.(33)

Calculation of these terms has been performed for the Model A case above. The results are shown in figure 5. The difference between the two forms of the source term is minimal, being basically a change in the peak level. Term 2 is very spatially homogeneous, which probably accounts for much of the lack of structure. Term 1, which is the mean drag term, shows some structure but is quite low in overall value, which might account for the similar turbulence profiles noted earlier. Some very interesting structure is apparent in Term 3, which is the interphase coupling term. Overall, except for the near-inlet region, there is very little structure away from the walls.

5 Conclusions

This report presents an outline of a new turbulence model for two-phase flow. The analysis begins from first principles, to determine what physical effects are likely to be of importance in multiphase flow, and then to investigate how these effects can be expressed in mathematical terms. Initial modelling has been proposed, and preliminary calculations have been performed on a shear layer case to check the consistency of the model. Results have been good but not yet spectacular, but there is much room for manouver to improve the model.

The main points of the model are that we are solving $k - \epsilon$ systems of equations for each phase separately. The turbulence production is due to shear in the phase-weighted

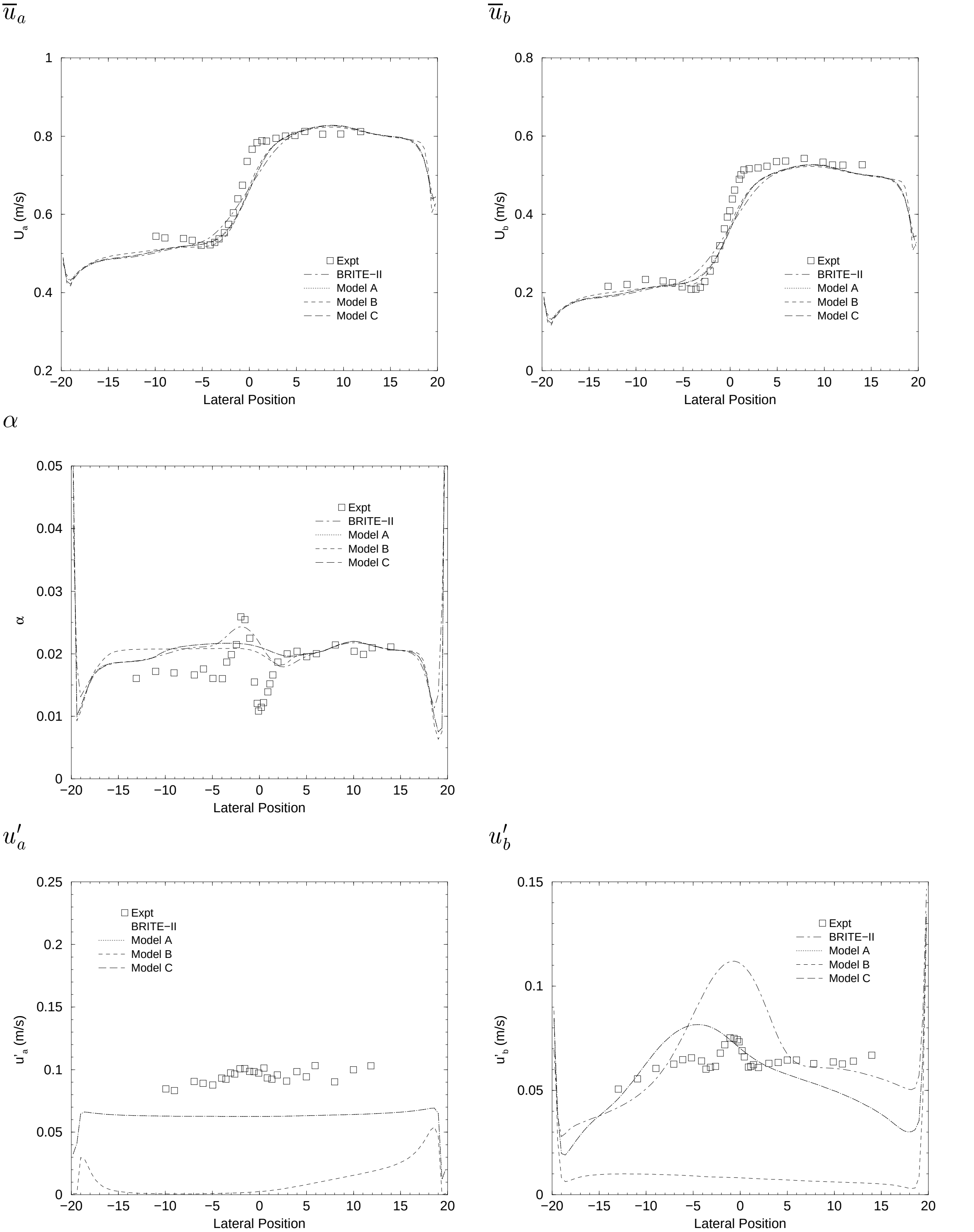

Figure 2: Comparison between current modelling, BRITE-II modelling and experimental results. Toulouse shear flow case 2.2, 20 cm from inlet. For more details see figure 4

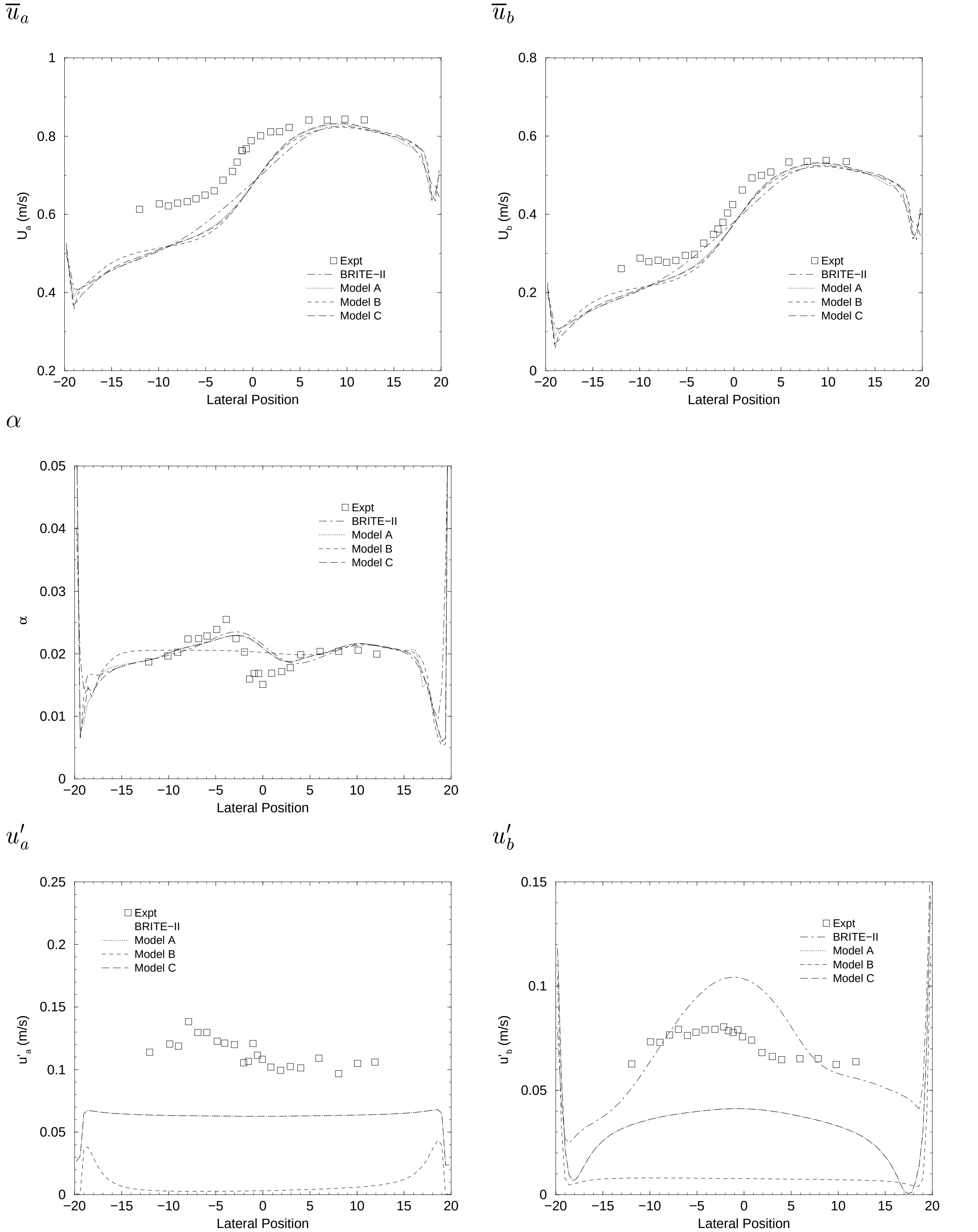

Figure 3: Comparison between current modelling, BRITE-II modelling and experimental results. Toulouse shear flow case 2.2, 40 cm from inlet. For more details see figure 4

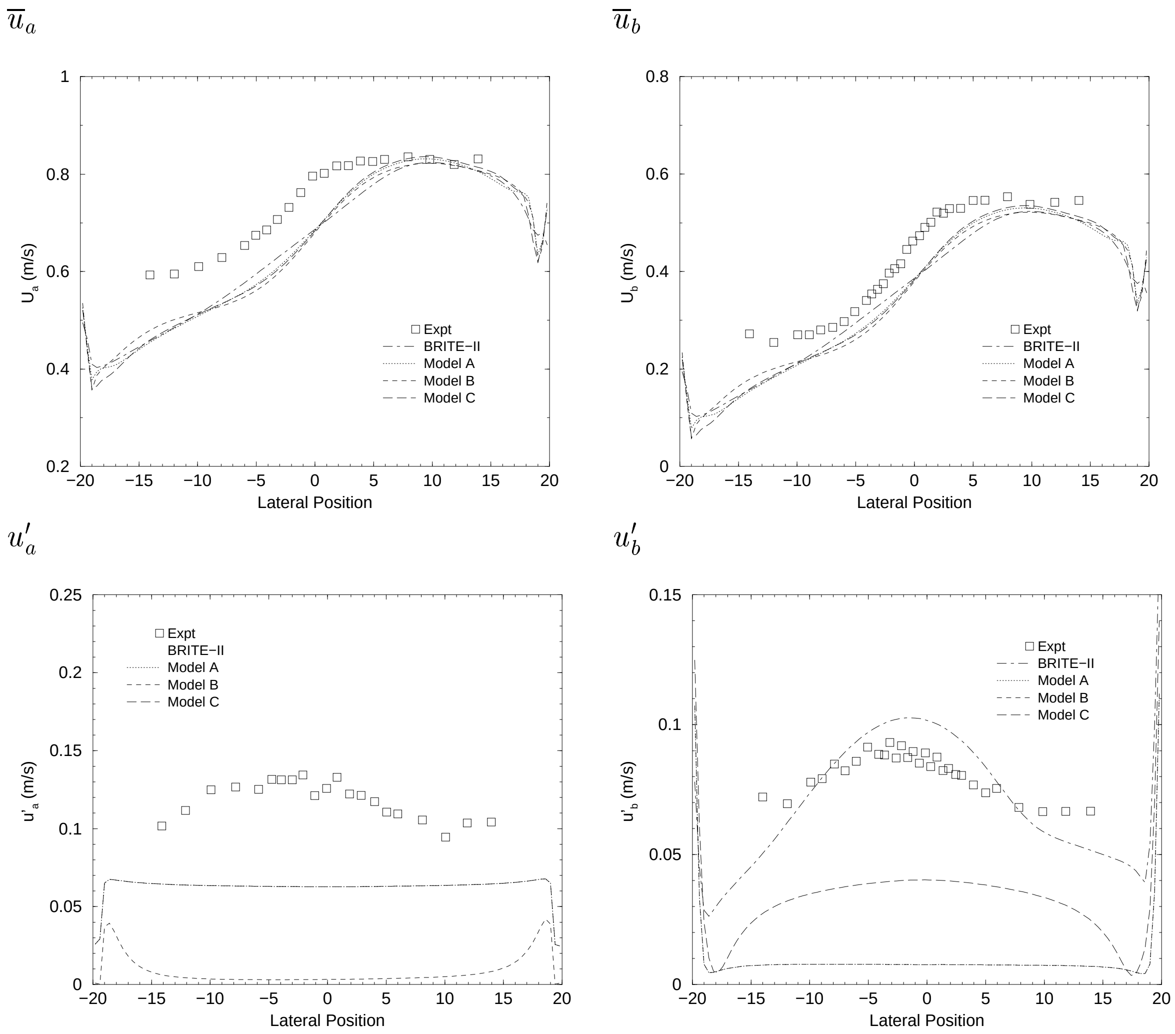

Figure 4: Comparison between suggested modelling, BRITE-II modelling and experimental results. Toulouse shear flow case 2.2, 50 cm from inlet. Square points are the experimental data points ; the long dashed line is the original BRITE-II model, and the other models are explained in the text. Models A and C have very similar behaviour on the turbulence profiles, and so their profiles are indistinguishable.

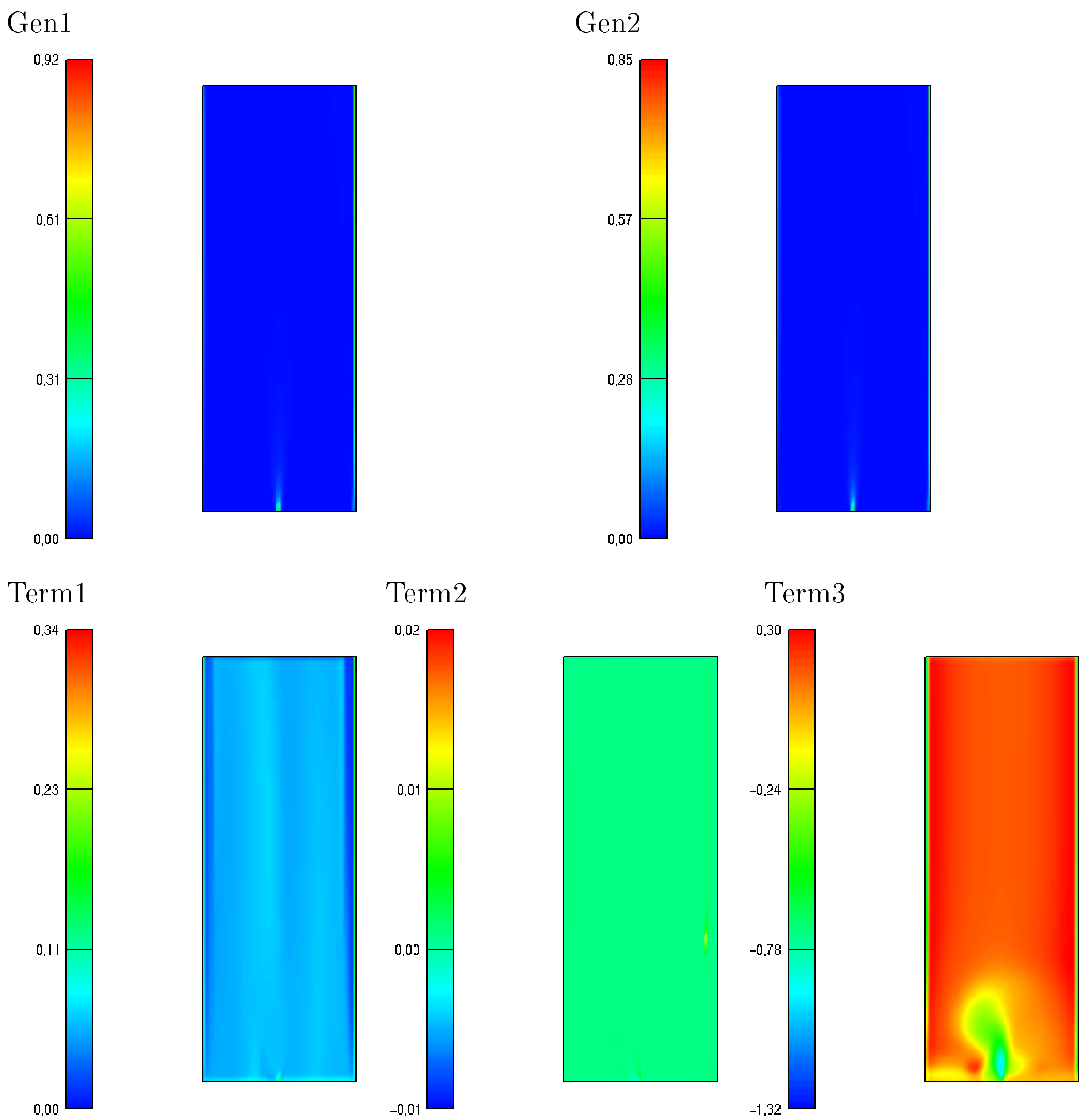

Figure 5: Source terms for k_b equation. The nomenclature used is defined in the text.

mean velocity, and we include other terms relating to particle-turbulence interaction in the model, specifically turbulence production due to drag and a turbulence transfer term between phases. This model has two free parameters at the moment, C_ζ and C_r, which represent the strength of linkage between the phases, and so are related to the C_t modelling of the previous project.

In addition to these two adjustable parameters, there is some leaway for altering the modelling within the framework established in this report. In particular, at the moment the ϵ equations have identical terms to the corresponding k equations, scaled by ϵ/k to get the correct dimensions. This implies that the turbulence is being generated at the integral length scale. This is correct for the bulk generation terms, but is probably not for the interface terms, which should generate turbulence at a length scale linked to the size of the fluid particles. This will alter the dissipation rate which may address the problem of the too-rapid decay of turbulence down the duct. Other areas include exactly which terms to include. As noted earlier, currently there are two linkages between the phases : shear in the mean flow generating turbulence in both phases, and the $k_b - C_r k_a$ coupling term. It is not completely clear that both are necessary.

Finally, there is the issue of wall functions. This is quite a large issue in its own right. The work presented here utilises the standard single-phase wall model with minimal alteration to make it work with two phases. This definitely needs improving.

To conclude, the following issues need to be addressed (no particular order).

- Sorting out the exact modelling to match the shear flow data better.
- Investigating the impact of changing C_ζ and C_r.
- Introducing functional models for these quantities.
- Wall functions for the model.
- Validating the model on other flows, particularly other building-block flows and wall-constrained flows.

Appendix : Conditional Averaging

We introduce an indicator function

$$\begin{aligned} \chi_k &= 1 \qquad \text{if phase } k \text{ present} \\ &= 0 \qquad \text{otherwise} \end{aligned}$$

The spatial and temporal derivatives of this indicator function are

$$\begin{aligned} \nabla \chi_k &= \delta \boldsymbol{n}_k \\ \frac{\partial \chi_k}{\partial t} &= -\boldsymbol{v}_i . \nabla \chi_k \\ &= -\delta \boldsymbol{v}_i . \boldsymbol{n}_k \end{aligned} \tag{40}$$

We define the ensemble average of any quantity in the normal manner. Using this, we can define a conditioned average as

$$\overline{\chi_k \phi} = \lim_{N\to\infty} \frac{1}{N} \sum_N \chi_k \phi$$

From this we can show

$$\overline{\chi_k \phi} = \gamma_k \overline{\phi}_k \tag{41}$$

where $\overline{\phi}_k$ is the ensemble average of ϕ just in the phase k, and γ_k is the phase fraction. From this,

$$\overline{\chi_k} = \gamma_k \tag{42}$$

is trivially true.

Finally in this section we evaluate conditional averages of various types of derivative. If ϕ is any tensor quantity, then we can write

$$\begin{aligned}\overline{\chi_k \nabla \circ \phi} &= \overline{\nabla \circ \chi_k \phi} - \overline{\phi \circ \nabla \chi_k} \\ &= \nabla \circ \overline{\chi_k \phi} - \overline{\phi \circ \nabla \chi_k}\end{aligned}$$

where $\circ$ is some tensor composition operator (inner product, outer product, gradient, etc). Noting that we can write

$$\nabla \circ = \lim_{\delta V \to 0} \frac{1}{\delta V} \oint_{\delta S} d\boldsymbol{S} \circ$$

(see [35]). Following [36] we define the surface average of a property ϕ as

$$\widetilde{\phi} = \frac{1}{\Sigma} \lim_{\delta V\to 0} \overline{\frac{1}{\delta V} \oint_{\delta S} \phi dS}$$

where

$$\Sigma = \lim_{\delta V \to 0} \overline{\frac{1}{\delta V} \oint_{\delta S} dS}$$

is the surface area per unit volume of the surface. Thus

$$\overline{\chi_k \nabla \circ \phi} = \nabla \circ (\gamma_k \overline{\phi}) - \widetilde{\phi \circ \boldsymbol{n}_k} \Sigma$$

Similarly we can show

$$\overline{\chi_k \frac{\partial \phi}{\partial t}} = \frac{\partial \gamma_k \overline{\phi}}{\partial t} + \widetilde{\phi \boldsymbol{v}_i . \boldsymbol{n}_k} \Sigma$$

Substituting $\phi = 1$ into this gives

$$\begin{aligned} \nabla \gamma_k &= \widetilde{\boldsymbol{n}_k} \Sigma \\ \frac{\partial \gamma_k}{\partial t} &= -\widetilde{\boldsymbol{v}_i . \boldsymbol{n}_k} \Sigma \end{aligned} \tag{43}$$

Note that equation 40. can be written as

$$\frac{D\chi_k}{Dt} = 0 \tag{44}$$

indicating that the interface travels with the interface velocity $\boldsymbol{v}_i$. This is sometimes called the topological equation. Averaging this equation gives equation 43.

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