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CONSTRUCTION OF INLET CONDITIONS FOR LES OF TURBULENT CHANNEL FLOW.

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Abstract. *Implementation of boundary conditions for LES is a serious problem. Whilst a great deal of effort has been expended investigating wall boundary conditions, much less has been done on the equally problematic issue of inlet boundaries. On a mechanistic level this involves creating a time-varying inlet velocity with mathematical properies that ensure that it is compatible with the Navier-Stokes equations : on a physical level the inlet flow should behave like a turbulent flow. In this paper we concentrate on the simplest possible problem of turbulent channel flow between two infinite planes. Three different inlet conditions are tested : i. a Fourier synthesis technique, ii. a precomputed library technique and iii. a mapping technique to create an inlet section to the computational domain. Downstream of the inlet, profiles of the various moments of velocity are compared, and the structure of the turbulent flow throughout the computational domain is analysed. Technique iii. is the most straightforward to use, but technique i. provides the possibility of tailoring the inlet turbulence conditions.*

1 INTRODUCTION

Implementation of boundary conditions for RANS turbulence modelling is relatively straightforward. This is because of the implicit scale separation involved in RANS : all quantities being computed in a RANS simulation are steady, or if not, vary on a timescale much longer than the computational timestep, and do so in a deterministic rather than quasi-random manner. Hence we can specify the velocity of flow and turbulence levels at the inlet, and these are wel-defined, known quantities which are often constant. LES does not have such a scale separation, and in fact requires the time-dependent simulation of scales down to the mesh size. Wall boundaries in LES have been extensively investigated. Much less work however has been focussed on the equally troublesome issue of implementing practical and accurate inlet conditions. On a mechanistic level this involves creating a stochastically time-varying inlet velocity on time scales down to the shortest scales simulated in the flow, and with mathematical properties that ensure that it is compatible with the Navier-Stokes equations being solved within the flow domain. On a physical level we would wish to create an inlet boundary which 'looks' like turbulence, and in particular can be manipulated to possess particular turbulence properties (length/time scales, correlation coefficients, mean profiles). The simplest approach, often adopted, is to superimpose random noise on the inlet flow profile (eg.[8, 6]), but this cannot reproduce the physical structure of the turbulence. Some work has been carried out in this area [7, 10], but more work needs to be done in this area, which is extremely important as many flows are strongly dependent on inlet conditions[9].

In this paper, we concentrate on the simplest possible problem of turbulent channel flow between two infinite planes (see figure 1 for details of the geometry). Chung and Sung[2] have used a similar case to investigate LES inlets : in particular they concentrated on precomputation techniques, where a preliminary calculation is run on a reduced mesh and a limited set of the results (eg. a single timestep) used as a 'turbulence library' to play into the inlet. They compare several techniques for doing this, including methods for manipulating the inlet data to destroy any periodicity resulting from re-use of the limited data set, referred to as phase or amplitude jittering. They compare the results with their own precomputation technique, combining data from several timesteps to create an inlet with very good statistical properties. Whilst Chung and Sung compare variations on the same theme (a precomputed library), here we are comparing three very different inlet conditions, as follows :

1. A synthesis technique, based on a Fourier decomposition of the required mean inlet conditions, with a stochastic variation of the coefficients to generate the time dependence,

2. A precomputation technique, albeit without the jittering techniques analysed by Chung and Sung,

3. An internally computed inlet (referred to as a mapping technique), in which the

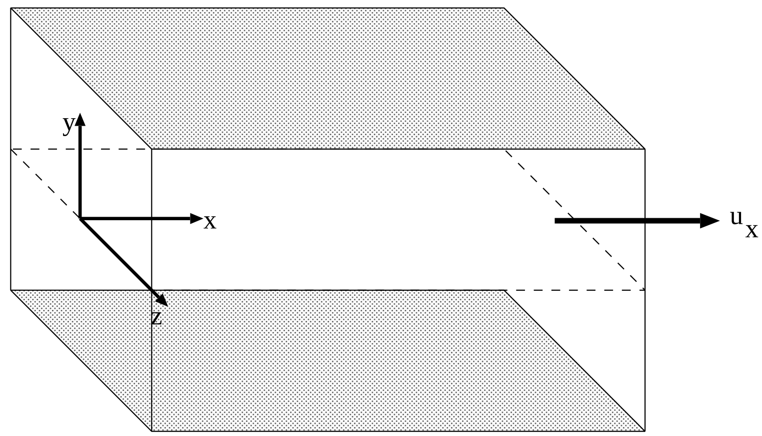

Figure 1: Geometry for a typical channel flow. Shaded planes are walls : the mesh is broken into 2 sections providing refinement towards each wall.

domain is extended upstream of the inlet, and a mapping technique used to create a cyclic section of flow in this region which attains fully developed conditions.

More detailed descriptions of each of these techniques follow.

1.1 Fourier synthesis technique

One of the most significant aspects of turbulence is its spatial and temporal coherence, quantified by the Taylor or Integral scales. The reason why a random noise inlet condition, described earlier, does not work is that it lacks correlation in both spatial and temporal directions, and so the Navier Stokes solver in the code works very hard to eliminate what to it is merely numerical error. Thus one of the most important requirements for a working inlet is some degree of spatial and temporal coherence in the turbulent fluctuations.

Any periodic function can be considered as a fourier series. Thus the expected mean velocity profile can be decomposed as follows :

$$\langle u\rangle_x = u_m\left(\frac{\langle a\rangle_0}{2} + \langle a\rangle_1\cos\kappa y + \langle a\rangle_2\cos 2\kappa y + \ldots + \langle b\rangle_1\sin\kappa y + \langle b\rangle_2\sin 2\kappa y\ldots\right) \tag{1}$$

for which we can calculate the coefficients $\langle a\rangle_i$, $\langle b\rangle_i$ in the usual manner. See figure (1) for details of the coordinate system and geometry being used. u_m is the maximum flow velocity, and the fourier series provides the shape of the profile. If instead of these coefficients being constants, we make them stochastic random processes, then we generate a new series

$$u_x = u_m\left(\frac{a_0}{2} + a_1\cos\kappa y + a_2\cos 2\kappa y + \ldots + b_1\sin\kappa y + b_2\sin 2\kappa y\ldots\right) \tag{2}$$

The coefficients will be Uhlenbeck-Ornstein processes with mean value the original mean value, and some specified variance

$$a_i = a_i(\mu = \langle a \rangle_i, \sigma^2) \tag{3}$$

Since the coefficients in this are coefficients of a fourier series, this generates spatial coherence – in fact rather too much, as the various fourier series terms are coherent to $\pm\infty$. Since they are varying according to a stochastic process they will exhibit a degree of temporal coherence as well. Also, time-averaging equation (1.1) averages the coefficients individually, and the time average of each coefficient

$$\langle a_i(\mu = \langle a \rangle_i, \sigma^2) \rangle = \mu = \langle a \rangle_i \tag{4}$$

so the fluctuating profile will average to give the desired mean flow profile (1.1).

From this we can evaluate the fluctuations around the mean :

$$\begin{aligned} u'_x = u_x - \langle u \rangle_x = u_m \Big[& \frac{a_0 - \langle a \rangle_0}{2} + (a_1 - \langle a \rangle_1) \cos \kappa y + (a_2 - \langle a \rangle_2) \cos 2\kappa y + \ldots \\ & + (b_1 - \langle b \rangle_1) \sin \kappa y + (b_2 - \langle b \rangle_2) \sin 2\kappa y \ldots \Big] \end{aligned}$$

which is itself a fourier series. At this point we can make some assumptions about the form of these fluctuations : we will assume that $a_0 = \langle a \rangle_0 = 0$ (i.e. that there is no channel-scale fluctuation) and that $u(-L) = u(L) = 0$ (velocity is zero at the walls). With these assumtions we can evaluate the fluctuating velocity component

$$u'_x = u_m \sum_{i=1}^{N} a'_i \cos i\kappa y \tag{5}$$

where a'_i is a stochastic random process centred on zero, and superimpose this on the desired flow profile $\langle u \rangle_x$. To evaluate the variances of the stochastic processes, we can apply Parseval's identity to this

$$\frac{1}{L} \int_{-L}^{L} \left[u'_x(y) \right]^2 \, dy = u_m^2 \sum_{i=1}^{N} (a'_i)^2 \tag{6}$$

Now we can time-average this. The l.h.s. is $2\times$ the tubulent kinetic energy in the x-direction flow, so

$$2E_{k_x} = u_m^2 \sum_{i=1}^{N} \langle (a'_i)^2 \rangle \tag{7}$$

On the r.h.s. each term is just the variance of the random process a'_i, and so the sum of the variances of the various terms equals the turbulent kinetic energy in this flow component. We are therefore able to specify the exact form of the turbulent energy spectrum for this component. If we note that each a'_i relates to a specific cosine term with a specific wavenumber : in fact we are representing our turbulent energy spectrum as a discrete number of wavenumber components, doubling the wavenumber each time. Thus we should relate each variance to the ammount of energy in the spectrum at that wavenumber. We can make the assumption that the energy spectrum is selfsimilar, with the standard $-5/3$ power law spectrum, i.e.

$$E_k = A\kappa^{-5/3} \tag{8}$$

Thus, the ratio

$$\frac{\sigma^2_{a'_i}}{\sigma^2_{a'_{i+1}}} = 2^{5/3} \tag{9}$$

which gives us a relation between successive variances. The overall level of turbulence can then be set by specifying a value for $i = 0$ (here we take $\sigma^2_{a'_i} = 0.3$, but this could be further tuned. The SGS turbulent kinetic energy k can be approximated by

$$k = \int_{\kappa_\Delta}^{\infty} A\kappa^{-5/3}\, d\kappa = \frac{3A}{2\kappa_\Delta^{2/3}} \tag{10}$$

where κ_Δ is the wavenumber of the filter scale for the LES). This is a fairly crude approximation to the spectrum, and it could be refined if necessary.

The methodology outlined above was for the u_x velocity, giving a time-averaged profile that can be specified : in the cases presented here we have used a 1/7 power law mean profile, but this could easilly be reworked for any required velocity profile. Of course we also have to provide similar data for the u_y and u_z components of velocity. The mean profiles for these are both zero, so the only contribution is the fluctuating component. In addition, the methodology has been outlined for a 1-d inlet. Clearly this needs to be extended to cover a 2-d inlet, and there are several possible ways of approaching this. Since the flow profile in the z-direction is uniform (the profile is basically 1-d, although we do need a 2d inlet for the computation) we will assume the solution can be expressed as the product of two independent fourier series. As the individual coefficients are statistically independent, the two series will decouple under time-averaging, and so we can treat them as entirely separate series.

1.2 Precomputation technique

This is our implementation of one of the most commonly used techniques. It is possible to link the inlet and outlet of a channel calculation so as to provide a cyclic channel with a prescribed flow velocity (or alternatively pressure drop). In this way we can compute fully developed turbulent flow in a channel of infinite length, using a much shorter section of channel [3]. Having performed such a calculation, a single timestep can be used to generate a data library, sweeping a slice through the data in the downstream direction, and applying Taylor's hypothesis that spatial variation of turbulence at a given instant of time is equivalent to temporal variation of turbulence at a given location (ie. the inlet). Since the channel is short, the amount of data being stored is quite small, and since the channel is cyclic, once the sweep has reached the end of the channel it can start again at the beginning. The turbulence being generated here will not be perfect due to the cyclic nature of the data, leading to an error in correlation on the order of $L/\langle U \rangle$, where L is the length of the precomputed channel, and U the mean flow velocity. However since the channel is at least 100 wall units long it seems unlikely that the correlation error will be significant for most important turbulence scales, and in fact Chung and Sung show very little difference between their various jittering techniques designed to eliminate this error. It might be a significant problem though if the time period of the library is close to some characteristic time period of interest within the domain, for example a vortex shedding timescale. Finally there is the issue of the rate at which the slices are played into the inlet. If we take each layer of cells (the mesh is cartesian) to provide a new set of inlet values, then the timescale between values is

$$\tau = \frac{\Delta x}{\langle U \rangle} \tag{11}$$

However since the Courant number is around 0.2 for the simulation, $\tau > \Delta t$, so we introduce a linear interpolation between the slices to equilise the timescales.

1.3 Mapping technique

One concern with library lookup techniques is that they require one to perform a preliminary computation and store the data. This involves either a large storage of data or the frequent repeating of the library data. It also requires the setting up of an appropriate reduced case to generate the library of data, which must be repeated for each new case. It would be much more convenient to combine the computation of the inlet with the computation of the main case. The same cyclic mapping technique used to create an infinite channel flow [3] can be used to map velocity data (and other data such as the SGS k as appropriate) from an internal plane in the computation back to the inlet. This in effect creates a preliminary cyclic computation section coupled into the main calculation. The mapping plane must be far enough upstream of significant features to prevent corruption of the inlet by downstream effects. If the location of the physical inlet is critical then the computational domain can be extended backwards to provide an appropriate inlet section to develop the flow (as shown in figure 2). The mapping plane should also be a reasonable distance downstream of the computational inlet so as to provide a reasonable repeat distance for the developing flow. In our case the channel was kept the same length and the first 4 m section (out of a total channel length of 20m) was used to develop the flow.

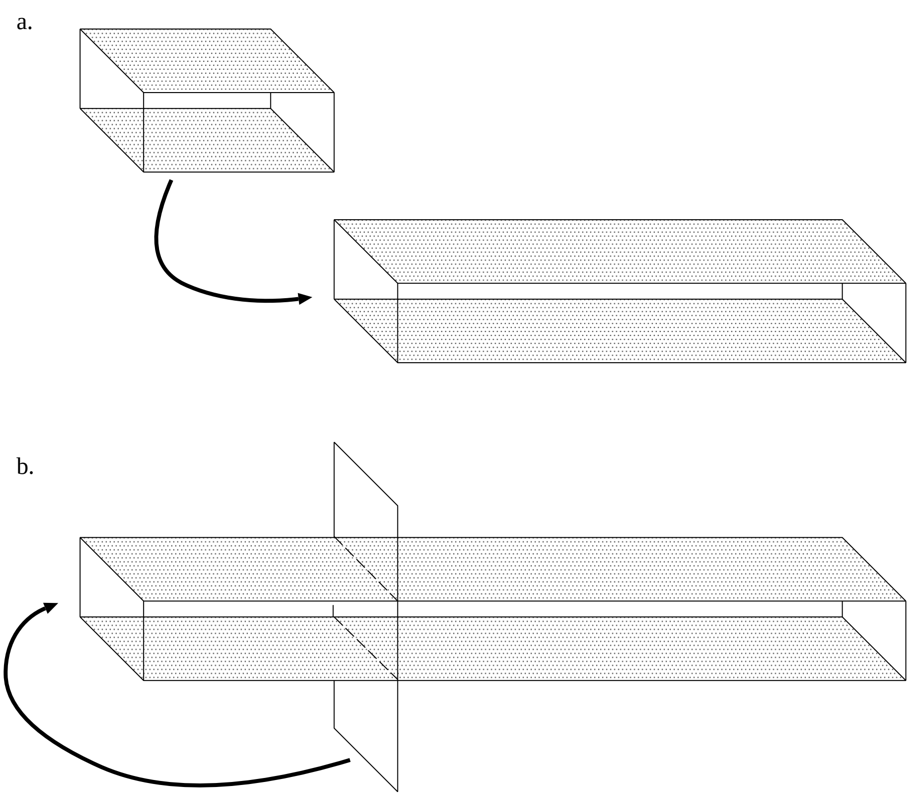

Figure 2: Diagram illustrating a library lookup technique (a.) in which a previous computation is used to provide inlet data, and a mapping technique (b.) in which data from an interior plane within the simulation is mapped backwards to the inlet.

2 RESULTS

2.1 Computational details

The case being computed here is that of a channel of width 2 m, computed at $\mathcal{R}e_\tau = 395$ (equivalent to $\mathcal{R}e = 27,500$ based on the channel width). This is a case which has been examined previously [3] and for which we know our LES code works well. A high resolution cyclic channel was first calculated to act as a point of comparison (referred to as the reference channel) : this was a domain 2 m in length, with cyclic inlet/outlet to provide for a fully developed flow, and mesh grading towards the walls. The mesh was then coarsened in all three directions and a further calculation performed with cyclic inlet/outlet conditions, to generate a data library for the precomputation technique. Finally the coarse mesh was extended in the x direction to 20 m in length. This represents a channel of length 15800 wall units, insufficient for the flow to develop naturally but long enough to investigate the flow evolution downstream of our inlets. The individual calculations were run at a maximum courant number around 0.2 for 7500 timesteps, representing 11

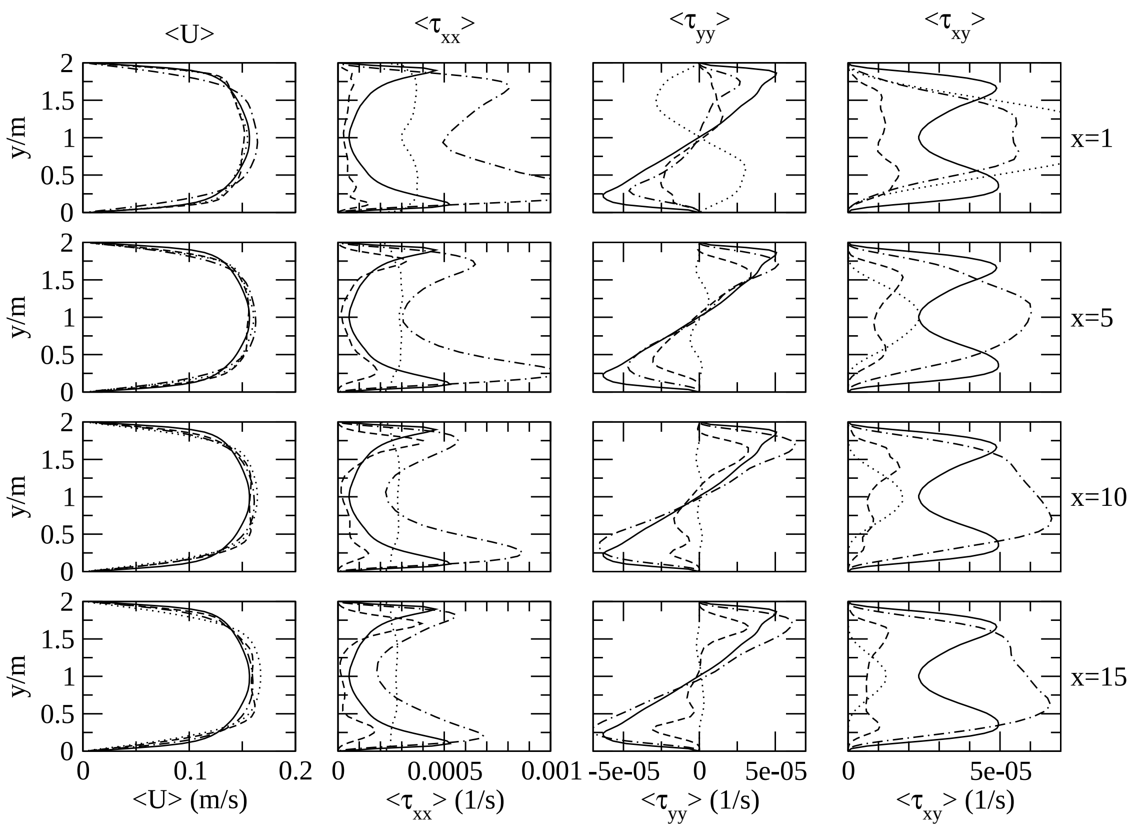

Figure 3: Profiles of mean velocity and stress components across the channel at $x = 1, 5, 10, 15$ m. Full line ≡ reference channel, dotted line ≡ fourier synthesis inlet, dashed line ≡ precomputation technique, dot-dash ≡ mapping inlet.

complete passes through the channel, as an initialisation step, followed by averaging for $25,000$ timesteps (38 complete passes) to generate the final results. The calculations were performed with a finite-volume representation of LES using second-order accurate differencing and the PISO scheme, with the 1-equation eddy viscosity model used to represent subgrid turbulence : details of the code have been provided elsewhere [4, 3, 5].

2.2 Velocity profiles

As a comparison, figure 3 compares the profiles of time-averaged grid scale mean velocity $\langle u \rangle_x$ and three components of the stress, $\langle \tau \rangle_{xx}$, $\langle \tau \rangle_{yy}$, and $\langle \tau \rangle_{xy}$, where $\tau = \nabla \vec{u}$. The

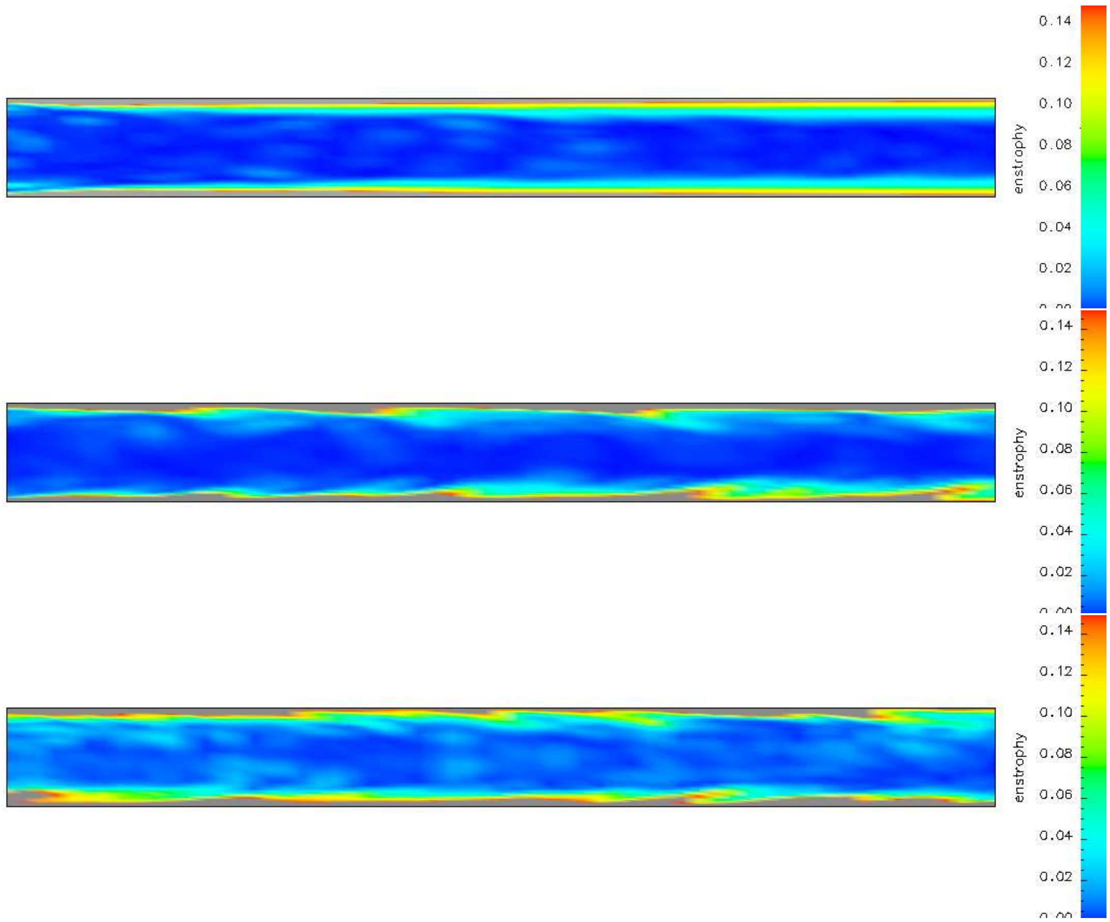

Figure 4: Enstrophy plotted on the channel centre plane. Top : fourier series, centre : inlet library, bottom : mapping method.

reference channel, which has been calibrated in previous work against experimental and DNS data [3] is used as a comparison at positions $1, 5, 10, 15$ m downstream of the inlet in each case. The solid line is the reference in each case, whilst the dotted, dashed and dash-dot lines refer to the fourier synthesis, precomputation and mapping technique in each case. In all cases the mean flow profiles are very accurate, with little sign of further flow development. However the stress components are not produced as accurately. The fourier technique is producing profiles that are highly inaccurate and which are changing rapidly downstream, whilst the other techniques are producing flows that continue to evolve, in the case of the mapping technique seriously overpredicting τ_{xy} towards the end of the channel.

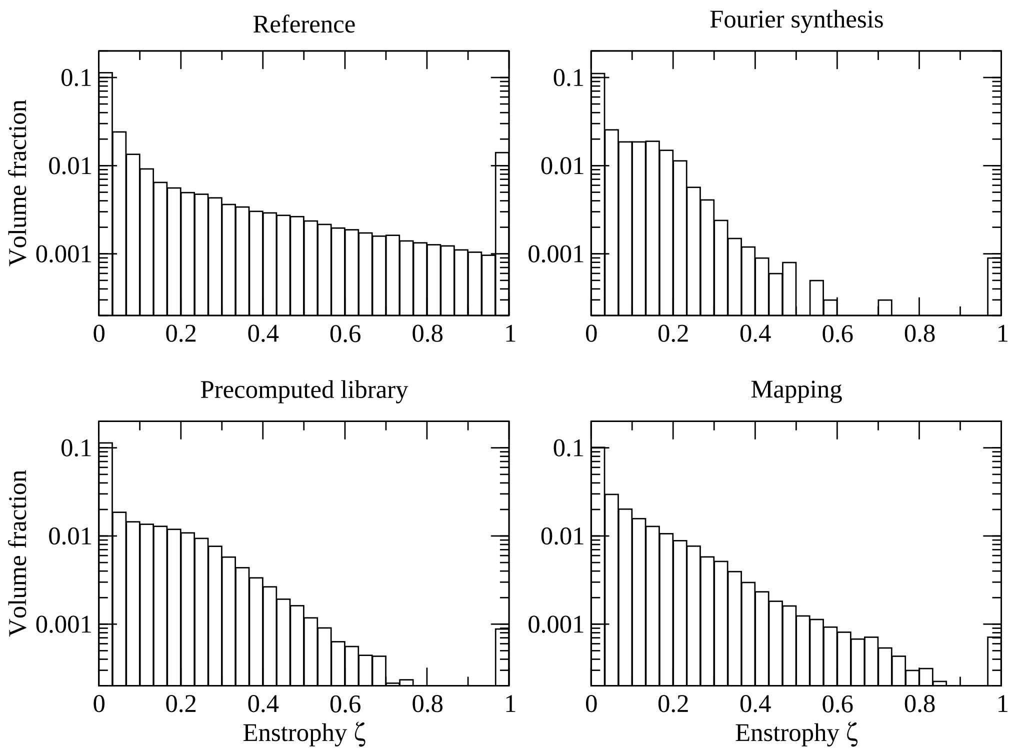

Figure 5: Volume-weighted histograms showing the distribution of enstrophy in each calculation, compared with the reference channel (top left).

2.3 Flow enstrophy

The aim of this work is to develop an inlet condition which produces a good approximation to turbulence in the channel. We are interested not solely in the mean profile but also in the creation of turbulent structures within the flow. Turbulence is known to exhibit structure in the form of 'fat worms' – coherent vortical structures of considerable diameter, and this structure is observable in LES of isotropic turbulence [1]. The best way to visualise these structures is via the enstrophy $\zeta = \frac{1}{2}|\nabla \times \vec{u}|^2$. Figure 4 shows plots of enstrophy along the centre plane of the simulation. The mapping method gives the best result with highly complex turbulent structures. The precomputation method produces reasonable results, although there is a distinct downstream periodicity undoubtably generated by the cyclic nature of the inlet data. The fourier synthesis method produces the least structure, understandable since the method has not been developed to create the correct spatial structure.

As a further test, cell-volume-weighted histograms of the enstrophy have been plotted (figure 5) and compared with the results from the reference channel. The reference channel shows a fairly even decrease in volume fraction with increasing enstrophy up to a peak enstrophy value of around 5 s^{-2}. All of the inlet conditions produce flows that are too smooth, i.e. too biassed towards low enstrophy, with the Fourier inlet being particularly biassed, although strangely this simulation also provided the highest peak enstrophy (20 s^{-2}). The technique is probably producing sharp spatial gradients near the inlet due to the highest frequency components of the series, which are then being smoothed out as the flow proceeds downstream. Even the mapping technique is under-predicting the peaks of enstrophy, i.e. underpredicting the strongest turbulent features.

3 CONCLUSIONS

The aim of this work has been to investigate and compare different methodologies for producing inlet conditions for LES. The test case, flow in a channel between two infinite plates, is a simple one for which LES has been extensively validated. Any inlet condition imposed on a simulation should mimic the properties of turbulence, both physical and mathematical (and computational). In addition, it should if possible be simple to implement and to use. All the techniques compared here have generated good mean flow profiles, however they are less good when it comes to generating higher order moments. In the case of the Fourier synthesis technique, this is probably because the technique is not producing the correct long-range correlation : it is producing a flow field with an infinite corelation length, at least in one direction (across the duct). Despite this it is worth further investigation, as a synthesis technique like this would provide the possibility of generating a precisely tuned inlet turbulence, which would be valuable if the inlet flow is not fully developed. It may be possible to improve the higher order structural errors by using a basis set for the expansion which posesses compact support, e.g. a wavelet basis set. Of the other two techniques, the precomputation technique proves the least satisfactory. The cyclic nature of the inlet library of data produces a cyclicity in the turbulence which may be a problem in certain cases, for example in computing shedding problems. It is possible to break this periodicity by introducing random transformations into the data, but this makes the procedure more complex. It is already complex enough in that it requires the precomputation of an appropriate inlet section, which would be cumbersome if the inlet conditions had to be changed repeatedly. It does however generate better inlet profiles of the higher order moments. However the mapping technique also produces reasonable higher order moments (and it may be that the inaccuracies here are due to the low mesh resolution for the calculation), and is significantly easier to use as it computes the inlet data on the fly, as it were. It should be possible to include particular modifications to the mapped data to simulate, for example, swirl at the inlet. However it would be difficult to create non-fully developed flow using this technique, hence our continued interest in synthetic methods.

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