Harmonic Balance Method for Unsteady Periodic Flows

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Abstract

This work presents the Harmonic Balance method for incompressible non-linear periodic flows. The method is implemented and tested in foam-extend, a fork of the open source software OpenFOAM. Simulation results for passive scalar transport and NACA 2412 are presented and compared with a conventional transient simulation. Harmonic Balance for passive scalar transport is validated on four cases with simple harmonic and square waves. The Harmonic Balance Navier-Stokes solver is validated using NACA 2412 test case in 2-D and 3-D.

1. Introduction

Harmonic Balance method is a quasi-steady state method developed for simulating non-linear temporally periodic flows. It is based on assumption that each primitive variable can be accurately presented by a Fourier series in time, using first n harmonics and the mean value. Such assumptions are used to replace the time derivative term with coupled source terms, transforming the transient equations into a set of coupled steady state equations. The improvement over steady-state methods is that Harmonic Balance is able to describe the transient effects of a periodic flow without long time-domain simulations.

2. Mathematical Model

Primitive variables are expressed by a Fourier series in time, with n harmonics. Substituting the variables in transport equations with Fourier series, $2n+1$ coupled equations are obtained:

- Harmonic Balance momentum equation:

$$\nabla\!\bullet(\mathbf{u}_{t_j}\mathbf{u}_{t_j}) - \nabla\!\bullet(\gamma\nabla\mathbf{u}_{t_j}) = -\frac{2\omega}{2n+1}\left(\sum_{i=1}^{2n}\mathsf{P}_{(i-j)}\mathbf{u}_{t_i}\right),$$

- Harmonic Balance scalar transport equation:

$$\nabla\!\bullet(\mathbf{u}\mathsf{Q}_{t_j}) - \nabla\!\bullet(\gamma\nabla\mathsf{Q}_{t_j}) = -\frac{2\omega}{2n+1}\left(\sum_{i=1}^{2n}\mathsf{P}_{(i-j)}\mathsf{Q}_{t_i}\right),$$

- Harmonic Balance pressure equation:

$$\nabla\!\bullet\left(\frac{1}{a_P}\nabla p_{t_j}\right) = \nabla\!\bullet\left(\frac{\mathbf{H}(\mathbf{u}_{t_j})}{a_P}\right) = \sum_f \mathsf{S}.\left(\frac{\mathbf{H}(\mathbf{u}_{t_j})}{a_P}\right)_f,$$

- Harmonic Balance continuity equation:

$$\nabla\!\bullet\mathbf{u}_{t_j} = 0,$$

where

$$\mathsf{P}_i = \sum_{k=1}^{n} k\sin(k\omega i\Delta t), \qquad \text{for} \quad i = \{1{,}2n\}.$$

Corresponding to the Fourier series expansion, for n harmonics $2n+1$ equally spaced time steps within a period are obtained. Each of the $2n+1$ equations represents one time instant. Equations without the ddt term in its original form (pressure and continuity equation) remain the same, using variables corresponding to the time instant currently calculated.

3. Numerical Procedure

Second order accurate, polyhedral Finite Volume Method is used. Segregated SIMPLE solution algorithm is adopted. Each of the $2n+1$ time steps is resolved in its own SIMPLE loop.

4. Passive Scalar Transport

Passive scalar transport in 2D rectangular domain is simulated. Four test cases are presented, differing in signal imposed on inlet:

- Single sine wave
- Two harmonic waves
- Ramped square wave
- Complex square wave

Inlet velocity is 10 m/s, diffusion coefficient is $1.5\cdot10^{-5}$ m^2/s. This is valid for all the cases. Test cases are compared to transient simulation and data is extracted along the centreline of the domain.

Single sine wave is resolved using one harmonic:

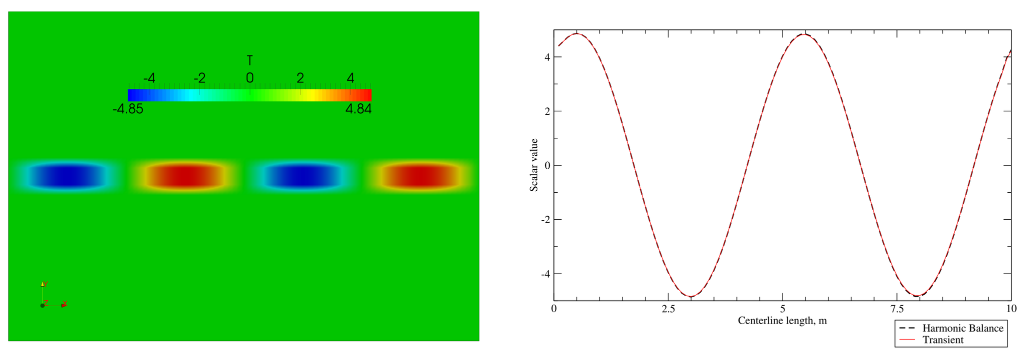

Figure 1: Left: scalar field visualisation at $t = T$. Right: scalar field comparison in $t = T$.

Two harmonic waves resolved using two harmonics:

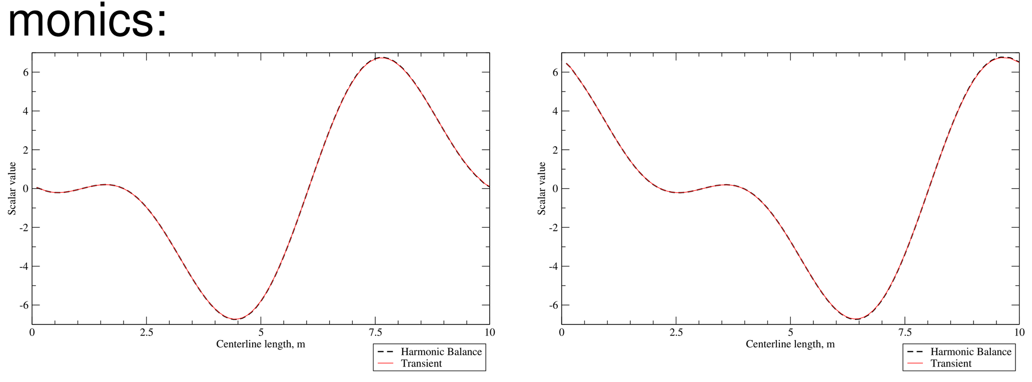

Figure 2: Scalar field comparison in $t = T/5$, $t = 2T/5$.

Ramped square wave: solution converges using higher number of harmonics:

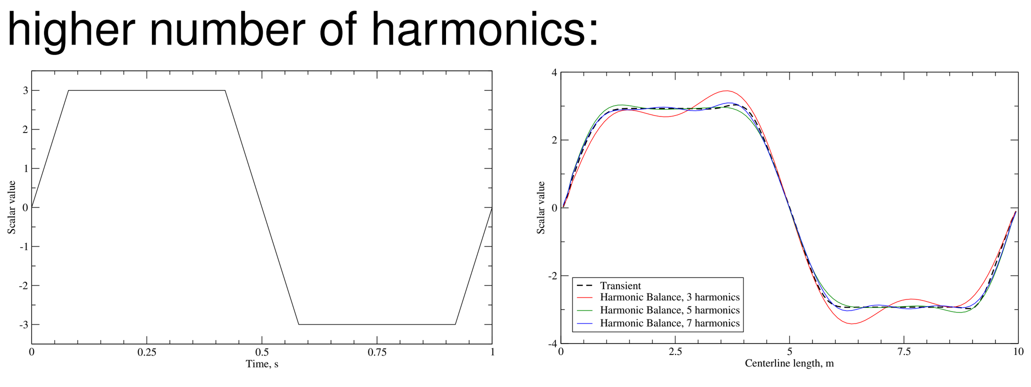

Figure 3: Left: imposed signal. Right: solution convergence.

Complex square wave: solution converges using higher number of harmonics:

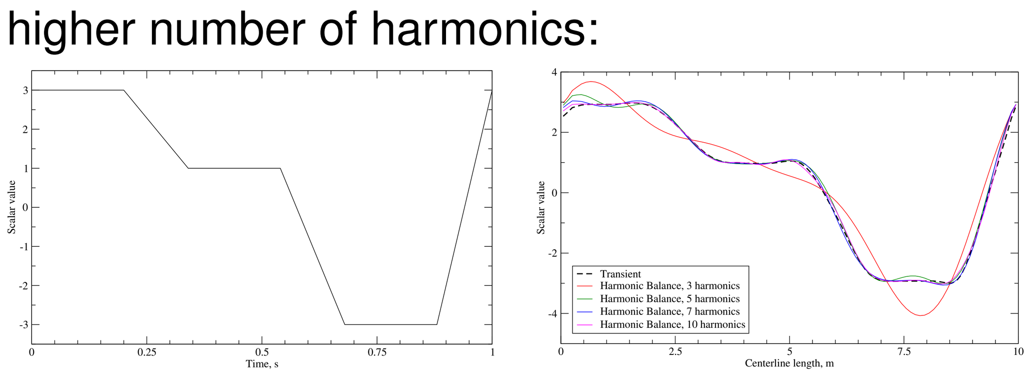

Figure 4: Left: imposed signal. Right: solution convergence.

5. NACA 2412 test case

Periodic airfoil pitching is simulated using 1, 3 and 6 harmonics and compared to a transient simulation. Two cases are presented: 2D and 3D case at $Re = 1695$. Comparison of pressure contours around the airfoil is given: 0–50 presents the lower camber, 50–100 presents the upper camber.

2D, low Re case:

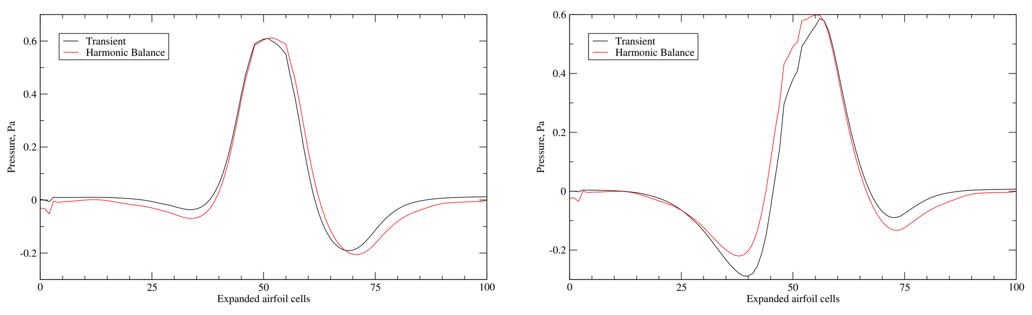

Figure 5: One harmonic comparison at $t = T/3$ and $t = 2T/3$.

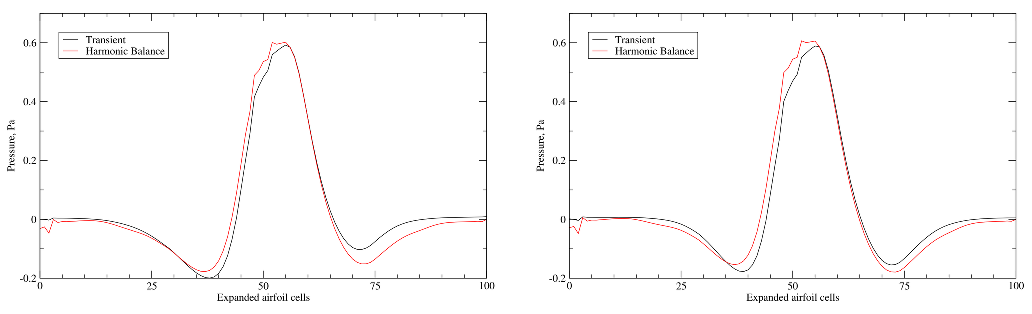

Figure 6: Three harmonics comparison at $t = 4T/7$ and $t = T$.

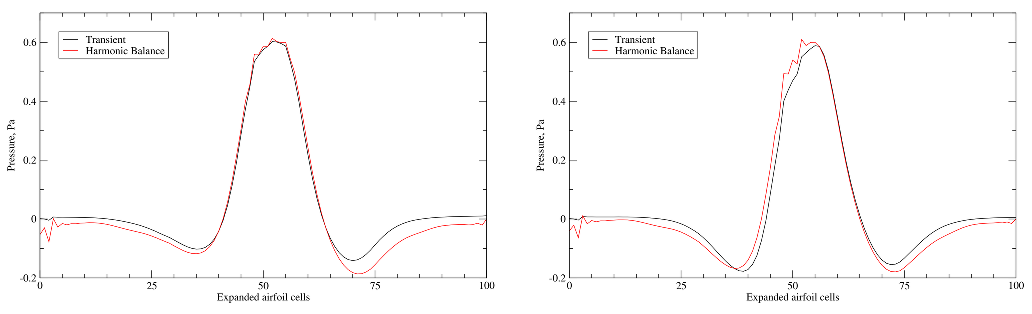

Figure 7: Six harmonics comparison at $t = 6T/13$ and $t = T$.

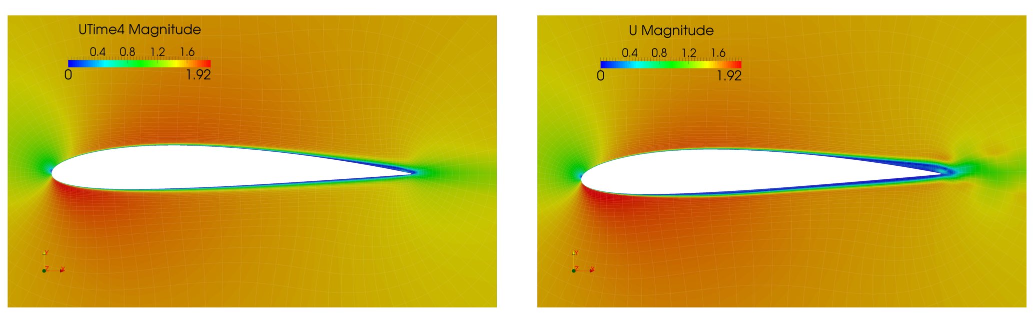

Figure 8: Flow field comparison at $t = T$.

3D, low Re case:

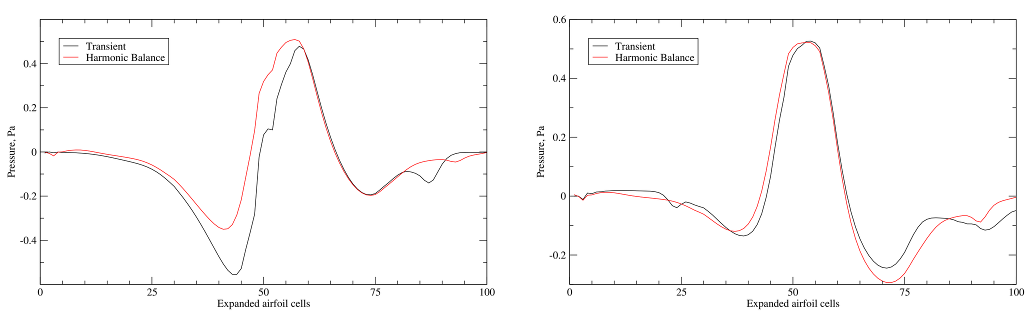

Figure 9: One harmonic comparison at $t = T/3$ and $t = 2T/3$.

6. Conclusion

The Harmonic Balance method is presented for passive scalar transport and Navier-Stokes equations. Scalar transport is validated using four types of periodic impulses resembling sine wave, complex harmonic wave and two square waves. Harmonic Balance for Navier-Stokes equation is validated on NACA 2412 test case both for 2D and 3D application.

It is shown that the Harmonic Balance method is a valuable tool for tackling periodic problems in computational fluid dynamics.