POLAR CURVE SKETCHING

ELECTRONICS & TELECOMMUNICATION

(1ST Yr. B.E. PRESENTATION)

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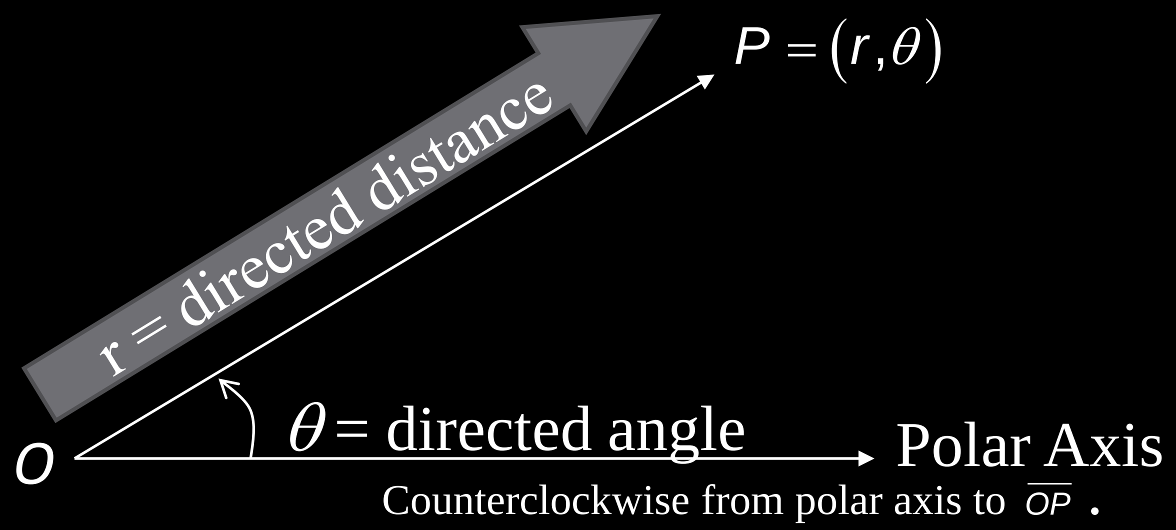
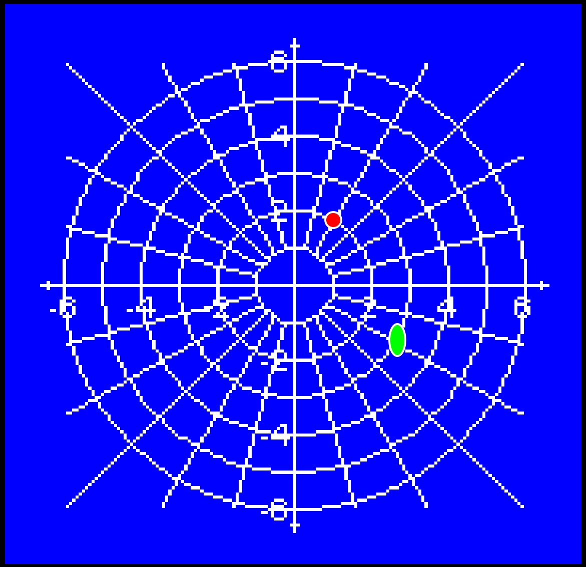
POLAR COORDINATES AND GRAPHING



PLOTTING POINTS IN THE POLAR COORDINATE SYSTEM

The point $(r,\theta) = \left(2, \frac{\pi}{3}\right)$ lies two units from the pole on the terminal side of the angle $\theta = \frac{\pi}{3}$.



The point $(r,\theta) = \left(3, -\frac{\pi}{6}\right)$ lies three units from the pole on the terminal side of the angle $\theta = -\frac{\pi}{6}$.

The point $(r,\theta) = \left(3, \frac{11\pi}{6}\right)$ coincides with the point $\left(3, -\frac{\pi}{6}\right)$.

MULTIPLE REPRESENTATIONS OF POINTS

In the polar coordinate system, each point does not have a unique representation. In addition to $\pm 2\pi$, we can use negative values for r. Because r is a directed distance, the coordinates (r, θ) and $(-r, \theta + \pi)$ represent the same point.

In general, the point (r, θ) can be represented as

$$(\mathrm{r}, \theta) = (r, \theta \pm 2n\pi) \text{ or } (\mathrm{r}, \theta) = \left(-r, \theta \pm (2n+1)\pi\right)$$

where n is any integer.

TRACING OF POLAR CURVES

The following points may be considered while tracing polar curves of the type $r = f(\theta)$:

1. Symmetry :
   1. The curve $r = f(\theta)$ is symmetrical about initial line $\theta = 0$, if the equation remains unchanged if θ is replaced by $-\theta$.
   2. The curve is symmetrical about the line $\theta = \frac{\pi}{2}$, if the equation remains unchanged when r is replaced by $-r$.
   3. The curve is symmetrical about the pole.

2. Pole : The curve passes through the pole if for $r = 0$, there corresponds a real value of θ.

3. Asymptotes : Find out the asymptotes to the curve if any.

TRACING OF POLAR CURVES

4. Region : Find the region in which the curve doesn't exist. If r is imaginary for some values of θ lying between $\theta 1$ & $\theta 2$, then there's no portion of the curve between the lines : $\theta = \theta 1$ & $\theta = \theta 2$.

   If the greatest & smallest values of r be a & b respectively, the curve lies entirely between the circles of radii a & b respectively. ($a > r > b\ ; a > 0, b > 0$).
5. Value of ϑ : Find ϑ using $tan\vartheta = r\frac{d\theta}{dr}$.
6. Special points : Trace the variations of r as θ varies.

   If $\frac{dr}{d\theta} > 0, r$ increases as θ increases.

   If $\frac{dr}{d\theta} < 0, r$ decreases as θ increases.

ANALYZING POLAR GRAPHS

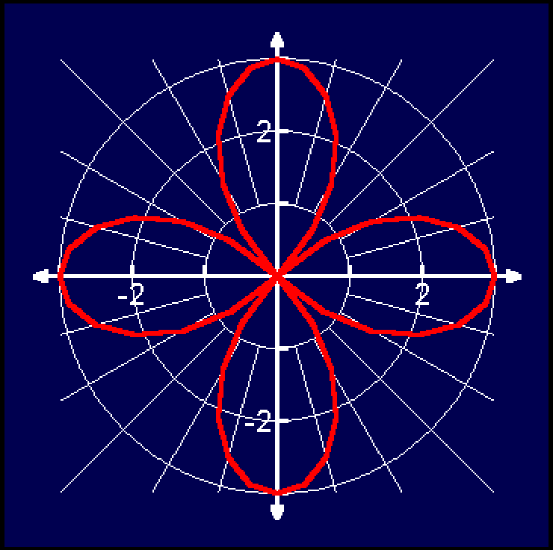
Analyze the basic features of $r = 3\cos 2\theta$.

Type of Curve: Rose Curve with 2b petals = 4 petals

Symmetry: Polar axis, pole, and $\theta = \frac{\pi}{2}$.

Maximum Value of | r |: | r | = 3 when $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

Zeros of r: *r = 0 when* $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$.



We can use this same process to analyze any polar graph.

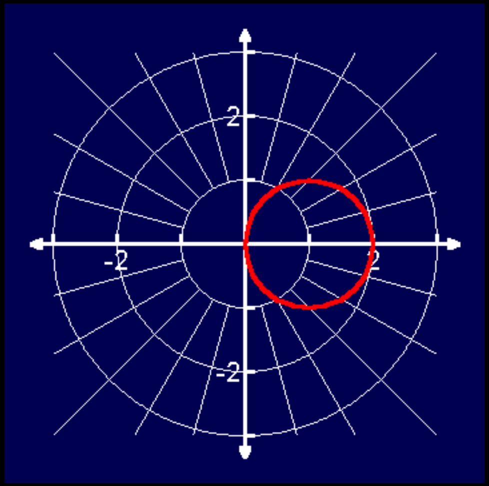
GRAPHING A POLAR EQUATION

1. Trace the curve : $r = 2a\ sin\theta, a > 0$.

OBSERVATIONS:

1. Curve is symmetrical about the initial line as well as through the perpendicular pole.
2. $r = 0 \rightarrow \theta = n\pi$. Hence,the curve passes through pole & these values of θ are the tangents at the pole.
3. There are no asymptotes to the curve.
4. $r_{max} = 2a \rightarrow$ Curve lies wholly in circle of radius a.
5. For $\theta\ vs\ r$, the following observations are made:

$\boldsymbol{\theta}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$
$\boldsymbol{r}$	$\boldsymbol{0}$	$\boldsymbol{2a}$	$\boldsymbol{0}$	$\boldsymbol{2a}$	$\boldsymbol{0}$	$\boldsymbol{-2a}$

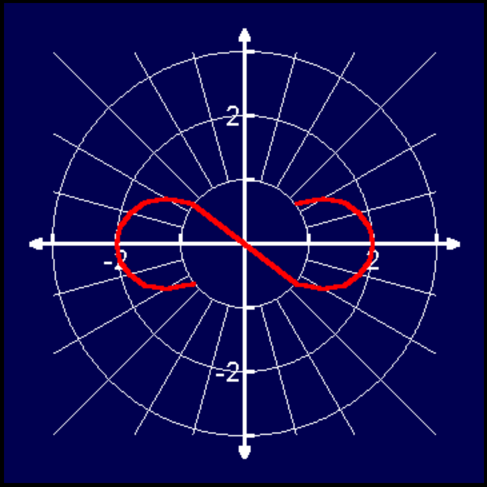


GRAPHING A POLAR EQUATION

2. Trace the curve : $r^2 = a^2 cos2\theta, a > 0$.

OBSERVATIONS:

1. Curve is symmetrical about the initial line as well as through the perpendicular pole.
2. $r = 0 \rightarrow \theta = \frac{(2n+1)\pi}{4}$. Hence, the curve passes through pole & these values of θ are the tangents at the pole.
3. There are no asymptotes to the curve.
4. $r_{max} = a \rightarrow$ Curve lies wholly in circle of radius a.
5. For θ vs r, the following observations are made:



$\boldsymbol{\theta}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$
$\boldsymbol{r}$	$\mp \boldsymbol{a}$	$\boldsymbol{0}$	$\mp \boldsymbol{a}$	$\boldsymbol{0}$	$\mp \boldsymbol{a}$	$\boldsymbol{0}$

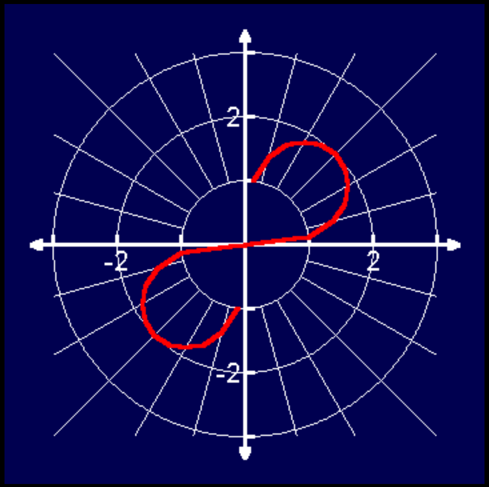
GRAPHING A POLAR EQUATION

3. Trace the curve : $r^2 = a^2 sin2\theta, a > 0$.

OBSERVATIONS:

1. Curve is not symmetrical about the initial line and symmetrical about the perpendicular pole.
2. $r = 0 \rightarrow \theta = \frac{n\pi}{2}$. Hence, the curve passes through pole & these values of θ are the tangents at the pole.
3. There are no asymptotes to the curve.
4. $r_{max} = a \rightarrow$ Curve lies wholly in circle of radius a.
5. For θ vs r, the following observations are made:

$\boldsymbol{\theta}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$
$\boldsymbol{r}$	$\boldsymbol{0}$	$\boldsymbol{\mp a}$	$\boldsymbol{0}$	$\boldsymbol{\mp a}$	$\boldsymbol{0}$	$\boldsymbol{\mp a}$



GRAPHING A POLAR EQUATION

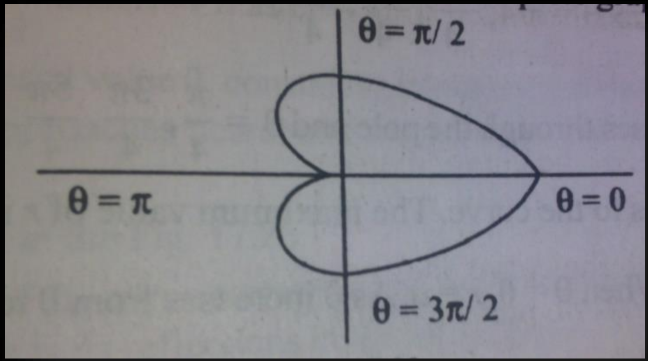
4. Trace the curve : $r = a(1 + cos\theta), a > 0.$

OBSERVATIONS:

1. Curve is symmetrical about the initial line .
2. $r = 0 \rightarrow \theta = \pi.$ Hence, the curve passes through pole & this value of θ is the tangent at the pole.
3. There are no asymptotes to the curve.
4. $r_{max} = 2a \rightarrow$ Curve lies wholly in circle of radius $2a$.
5. $\frac{dr}{d\theta} = -asin\theta \rightarrow tan\vartheta = \frac{rd\theta}{dr} = \cot\frac{\theta}{2} = \tan\frac{\pi}{2} + \frac{\theta}{2}. \quad \rightarrow \vartheta = \frac{\pi}{2} + \frac{\theta}{2}.$

   Hence tangent to curve at $(2a, 0)$ is perpendicular to initial line.
1. For $\theta\ vs\ r,$ the following observations are made.

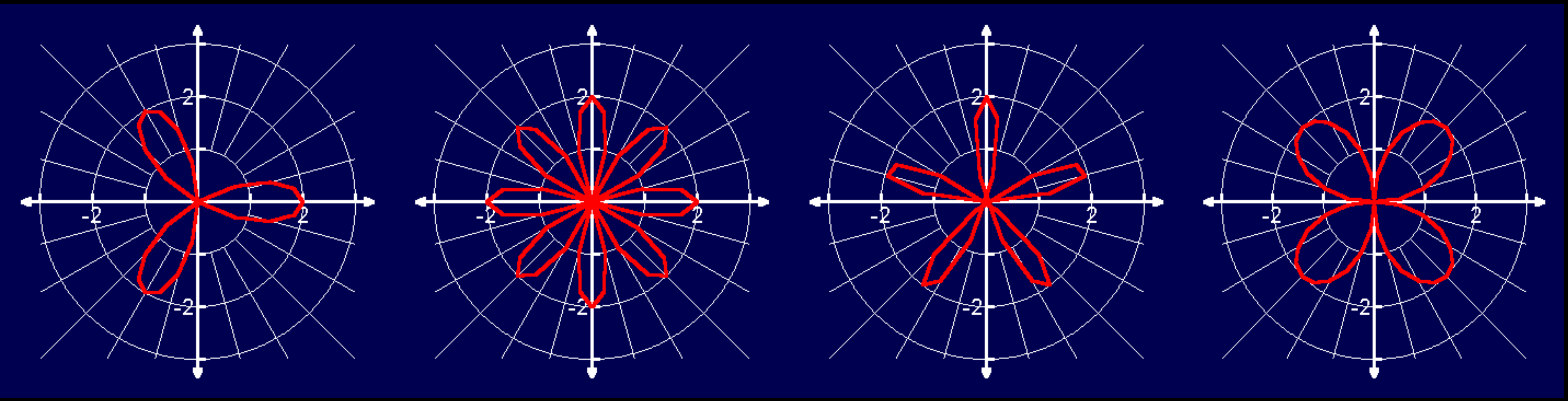
$\boldsymbol{\theta}$	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π
$\boldsymbol{r}$	$\boldsymbol{2a}$	$\boldsymbol{\frac{3a}{2}}$	$\boldsymbol{a}$	$\boldsymbol{\frac{a}{2}}$	*0*



POLAR CURVE EXAMPLES

ROSE CURVE : $r = a\ cos b\theta, \& \ r = a\ sin b\theta\ ;\ a > 0.$

$r = a\cos b\theta$ $\quad$ $r = a\cos b\theta$ $\quad$ $r = a\sin b\theta$ $\quad$ $r = a\sin b\theta$

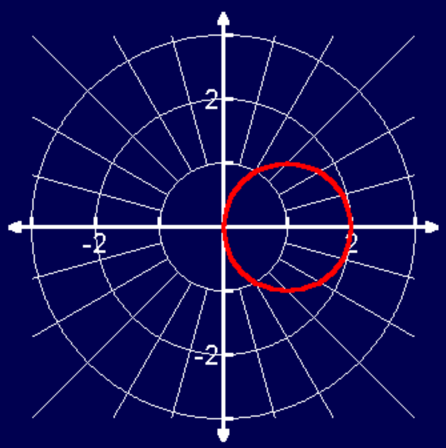


- No. of petals = b, if b is odd
  2b, if b is even

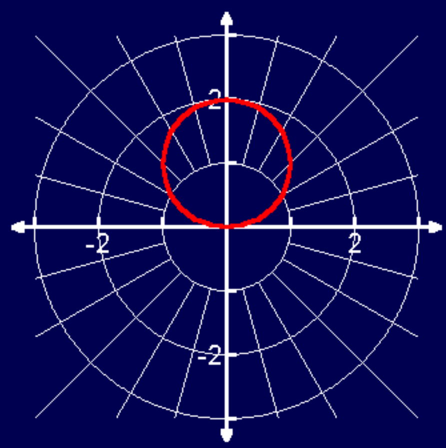
POLAR CURVE EXAMPLES

CIRCLE : $r = a\ cos\theta, \&\ r = a\ sin\theta\ ;\ a > 0.$
LEMNISCATE : $r^2 = a^2 sin2\theta\ \&\ r^2 = a^2 cos2\theta; a > 0.$

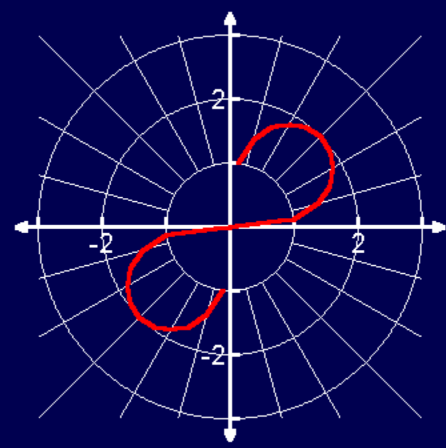
$r = a\cos\theta$



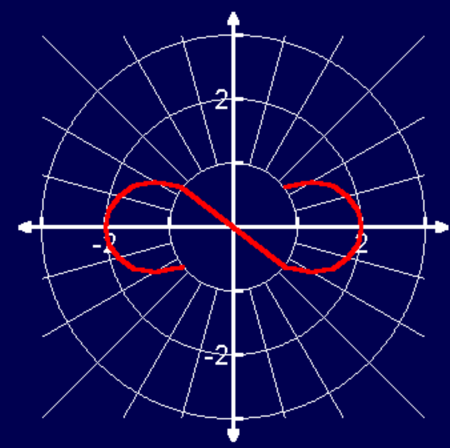
$r = a\sin\theta$



$r^2 = a^2\sin 2\theta$



$r^2 = a^2\cos 2\theta$



SYMMETRY TEST FAILS

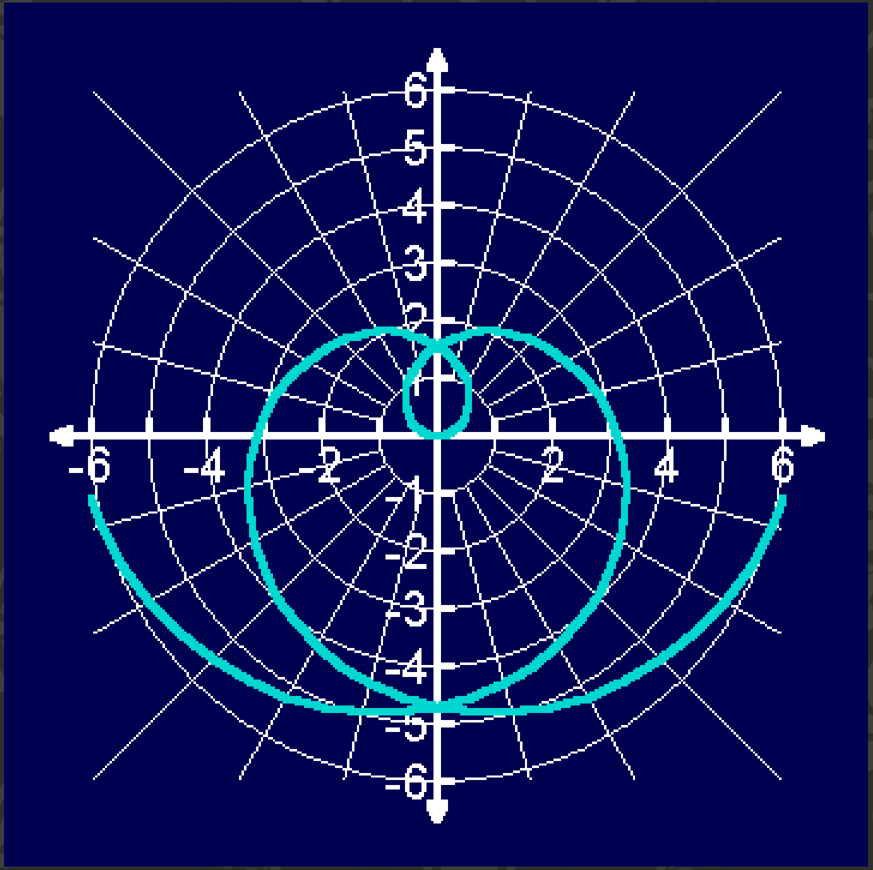
SPIRAL OF ARCHIMEDES : $r = \theta + 2\pi$

Original Equation	*Replacement*	*New Equation*	
$r = \theta + 2\pi$	(r, θ) with $(r, \pi - \theta)$	$r = -\theta + 3\pi$	→ Not symmetric about the line $\theta = \pi/2$.
$r = \theta + 2\pi$	(r, θ) with $(r, -\theta)$	$r = -\theta + 2\pi$	→ Not symmetric about the polar axis.
$r = \theta + 2\pi$	(r, θ) with $(-r, \theta)$	$-r = \theta + 2\pi$	→ Not symmetric about the pole.

All of the tests indicate that no symmetry exists. Now, let's look at the graph.

You can see that the graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

$$r = \theta + 2\pi$$



THANK YOU