

Homework 4

Question 1. Please read chapter 5 of Chartrand et al. and write a couple sentences about a topic/example/concept that you found difficult or interesting and why?

Question 2. The following are relations on the set \mathbb{R} of real numbers. Which of the properties reflexive, symmetric and transitive does each relation below possess?

- (a) $x R_1 y$ if $|x - y| \leq 1$.
- (b) $x R_2 y$ if $y \leq 2x + 1$.
- (c) $x R_3 y$ if $y = x^2$.
- (d) $x R_4 y$ if $x^2 + y^2 = 9$.

Question 3. Let $S = \{1, 2, 3, 4, 5, 6, 7\}$. The relation

$$R = \{(1, 1), (1, 3), (1, 4), (2, 2), (3, 1), (3, 3), (3, 4), (4, 1), (4, 3), (4, 4), (5, 5), (5, 7), (6, 6), (7, 5), (7, 7)\}$$

on S is an equivalence relation. Determine the distinct equivalence classes.

Question 4. Give an example of an equivalence relation R on the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ with \mathcal{P} the set of equivalence classes such that the following four properties are satisfied:

- 1) $|\mathcal{P}| = 3$,
- 2) There exists no set $S \in \mathcal{P}$ such that $|S| = 3$,
- 3) $3 \not R 4$ but $3 R 5$,
- 4) There exists a set $T \in \mathcal{P}$ such that $1, 7 \in T$.

Question 5. Let $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$ and $C = \{1, 2, 3, 4\}$. Also let $f : A \rightarrow B$ and $g : B \rightarrow C$, where $f = \{(1, 4), (2, 5), (3, 1)\}$ and $g = \{(1, 3), (2, 3), (3, 2), (4, 4), (5, 1)\}$,

- (a) Determine $(g \circ f)(1)$, $(g \circ f)(2)$ and $(g \circ f)(3)$.
- (b) Determine $g \circ f$.

Question 6. Each of the following is a function from $\mathbb{N} \times \mathbb{Z}$ to \mathbb{Z} . Which of these are onto?

- (a) $f(a, b) = 2a + b$
- (b) $f(a, b) = b$
- (c) $f(a, b) = 2^a b$
- (d) $f(a, b) = |a| - |b|$
- (e) $f(a, b) = a + 10$

Question 7. Let f, g and h be functions from \mathbb{R} to \mathbb{R} defined by $f(x) = e^x$, $g(x) = x^3$ and $h(x) = 3x$ for each $x \in \mathbb{R}$. Determine each of the following:

- (a) $(g \circ f)(x)$.
- (b) $(f \circ g)(x)$.
- (c) $(h \circ f)(x)$.
- (d) $(f \circ h)(x)$.
- (e) a composition of functions that results in e^{3x^3}
- (f) a composition of functions that results in $3e^{x^3}$

Question 8. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 5x + 3$.

- 1) Show that f is one-to-one.
- 2) Show that f is onto.
- 3) Find $f^{-1}(x)$ for $x \in \mathbb{R}$.

Question 9. Prove that the function $f : \mathbb{R} - \{3\} \rightarrow \mathbb{R} - \{1\}$ defined by $f(x) = \frac{x}{x-3}$ is bijective.

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