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MATH 3503: Tashfeen's Discrete Mathematics
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Homework 4

Question 1. Please read chapter 5 of Chartrand et al. and write a couple sentences about a topic/example/concept that you found difficult or interesting and why?

Question 2. The following are relations on the set $\mathbb{R}$ of real numbers. Which of the properties reflexive, symmetric and transitive does each relation below possess?

(a) $x\ R_1\ y$ if $|x - y| \leq 1$.
  (a) Reflexive since $x - x = 0 \leq 1$
  (b) Symmetric since $|x - y| = |y - x|$
  (c) Not Transitive since it could be $|y - z| > 1$

(b) $x\ R_2\ y$ if $y \leq 2x + 1$.
  (a) Not Reflexive since x could be < -1
  (b) Not Symmetric since x could be $> 2y + 1$
  (c) Not Transitive since y could be $> 2z + 1$

(c) $x\ R_3\ y$ if $y = x^2$.
  (a) Not Reflexive since $x \neq x^2$
  (b) Not Symmetric since y may not equal x^2
  (c) Not Transitive since z may not equal y^2

(d) $x\ R_4\ y$ if $x^2 + y^2 = 9$.
  (a) Not Reflexive since $2x^2$ may not equal 9
  (b) Symmetric since order of addition does not matter
  (c) Not Transitive since $y^2 + z^2$ may not equal 9

Question 3. Let $S = \{1, 2, 3, 4, 5, 6, 7\}$. The relation

$$R = \{(1, 1), (1, 3), (1, 4), (2, 2), (3, 1), (3, 3), (3, 4), (4, 1), (4, 3), (4, 4), (5, 5), (5, 7), (6, 6), (7, 5), (7, 7)\}$$

on S is an equivalence relation. Determine the distinct equivalence classes.

$$[1] = \{1, 3, 4\}$$
$$[2] = \{2\}$$
$$[5] = \{5, 7\}$$
$$[6] = \{6\}$$

Question 4. Give an example of an equivalence relation R on the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ with $\mathcal{P}$ the set of equivalence classes such that the following four properties are satisfied:

1) $|\mathcal{P}| = 3$,

$$\{1, 2\}$$
$$\{3, 4\}$$
$$\{5, 6, 7\}$$

2) There exists no set $S \in \mathcal{P}$ such that $|S| = 3$,

$$\{1, 2\}$$
$$\{3, 4\}$$
$$\{5, 6\}$$
$$\{7\}$$

3) $3 \not R\ 4$ but $3\ R\ 5$,

$$\{1,2\}$$

$$\{3,5\}$$

$$\{4,6\}$$

$$\{7\}$$

4) There exists a set $T \in \mathcal{P}$ such that $1, 7 \in T$.

$$\{1,2,7\}$$

$$\{3,5\}$$

$$\{4,6\}$$

Question 5. Let $A = \{1,2,3\}$, $B = \{1,2,3,4,5\}$ and $C = \{1,2,3,4\}$.
Also let $f : A \to B$ and $g : B \to C$,
where $f = \{(1,4),(2,5),(3,1)\}$ and $g = \{(1,3),(2,3),(3,2),(4,4),(5,1)\}$,

(a) Determine $(g \circ f)(1), (g \circ f)(2)$ and $(g \circ f)(3)$.
  (a) $(g \circ f)(1) = g(4) = 4$
  (b) $(g \circ f)(2) = g(5) = 1$
  (c) $(g \circ f)(3) = g(1) = 3$

(b) Determine $g \circ f$. $g \circ f = \{(1,4),(2,1),(3,3)\}$

Question 6. Each of the following is a function from $\mathbb{N} \times \mathbb{Z}$ to $\mathbb{Z}$. Which of these are onto?

(a) $f(a,b) = 2a + b$ Onto
  $z \in \mathbb{Z}$
  $a = 1$
  $b = z - 2$
  $2a + b = z$

(b) $f(a,b) = b$ Onto since b is always an integer

(c) $f(a,b) = 2^a b$ Not onto since the output will always be a multiple of 2 so odd integers cannot be outputs

(d) $f(a,b) = |a| - |b|$ Onto. Negative integers can be reached by setting a to 0 and positive integers can be reached by setting b to 0

(e) $f(a,b) = a + 10$ Not onto since there are integers less than 10

Question 7. Let f, g and h be functions from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = e^x, g(x) = x^3$ and $h(x) = 3x$ for each $x \in \mathbb{R}$. Determine each of the following:

(a) $(g \circ f)(x)$.
(b) $(f \circ g)(x)$.
(c) $(h \circ f)(x)$.
(d) $(f \circ h)(x)$.
(e) a composition of functions that results in e^{3x^3}
(f) a composition of functions that results in $3e^{x^3}$

Question 8. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 5x + 3$.

1) Show that f is one-to-one.
2) Show that f is onto.
3) Find $f^{-1}(x)$ for $x \in \mathbb{R}$.

Question 9. Prove that the function $f : \mathbb{R} - \{3\} \to \mathbb{R} - \{1\}$ defined by $f(x) = \frac{x}{x-3}$ is bijective.

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