A PROOFS

A.1 Proof of Lemma 1

Proof. In Algorithm 1, the $\lfloor \frac{\rho}{2} \rfloor$ nodes that hold the fewest replicas are included when selecting $\mathbf{n}_\alpha$ to ensure storage balance. Thus, generating $\frac{n}{\lfloor \frac{\rho}{2} \rfloor}$ good placement schemes ensures that each node is included in a good placement scheme at least once. □

A.2 Proof of Lemma 2 and 3

A.2.1 Proof of Lemma 2.

Proof. Let $g_{\alpha^*} = \sum_{n_\gamma \in \mathbf{n}_\alpha^*} g\left(n_\gamma, \mathcal{N}_i \setminus \mathbf{n}_\alpha^*, \mathcal{A}\right)$, which is equal to the sum of complement degrees from $\mathbf{n}_\alpha^*$ to the remainder of nodes in $\mathcal{N}_i$. The initial complement degree for any $n_\gamma \in \mathbf{n}_\alpha^*$ under $\mathcal{A}$ is $|\mathcal{N}_i| - \lfloor \frac{\rho}{2} \rfloor$. Additionally, each time a vertex is included in $\mathbf{n}_\alpha$, its complement degree under $\mathcal{A}$ decreases by at most $\lfloor \frac{\rho}{2} \rfloor$, and it can be selected into $\mathbf{n}_\alpha$ at most ω times. Thus the lower bound is $g_{\alpha^*} \geq \lfloor \frac{\rho}{2} \rfloor \left(|\mathcal{N}_i| - \lfloor \frac{\rho}{2} \rfloor (1+\omega)\right)$. According to the pigeonhole principle, when $g_{\alpha^*} > \left(\lfloor \frac{\rho}{2} \rfloor - 1\right)\left(|\mathcal{N}_i| - \lfloor \frac{\rho}{2} \rfloor\right)$, there must exist a $n_\iota \in \mathcal{N}_i$ s.t. $g(n_\iota, \mathbf{n}_\alpha^*, \mathcal{A}) = \lfloor \frac{\rho}{2} \rfloor$. Finally, this Lemma is proven by combining the two inequations above. □

A.2.2 Proof of Lemma 3.

Proof. Let $g_\alpha = \sum_{n_\gamma \in \mathbf{n}_\alpha} g\left(n_\gamma, \mathcal{N}_j, \mathcal{A}\right)$, which is equal to the sum of complement degrees from $\mathbf{n}_\alpha$ to all nodes in $\mathcal{N}_j$. Following the same processes as Lemma 2, we obtain an inequation for the lower bound $g_\alpha \geq \left(\lfloor \frac{\rho}{2} \rfloor + 1\right)\left(|\mathcal{N}_j| - \left(\lfloor \frac{\rho}{2} \rfloor + 1\right)\omega\right)$ and another inequation for the pigeonhole principle $g_\alpha > \lfloor \frac{\rho}{2} \rfloor |\mathcal{N}_j|$. This lemma is proven by combining the two inequations above. □

A.3 Proof of Lemma 4

Proof. In the worst case, generating ω schemes fills ρ nodes, as though all ω schemes select the same nodes. Since Algorithm 1 always avoids selecting duplicated node combinations to maximize the scatter width, the rate at which the number of filled nodes increases will not be faster than the aforementioned worst case. □

A.4 Proof of Lemma 5

Proof. According to Hall's marriage theorem [17], if for any subset of the shard set $\mathcal{R}$, the sum of the number of leader replicas that can be held by its associated nodes exceeds its size, each shard can be matched with a node. Since the commonly configured replication factor $\rho > 1$, which means for any shard, the sum of the number of leader replicas that can be held by its associated nodes is $\rho > 1$, the above condition is satisfied. □