A PROOFS

A.1 Proof of Lemma 1

Proof. In Algorithm 1, the $\lfloor \frac{\rho}{2} \rfloor$ nodes that hold the fewest replicas are included when selecting $\mathbf{n}_\alpha$ to ensure storage balance. Thus, generating $\frac{n}{\lfloor \frac{\rho}{2} \rfloor}$ good placement schemes ensures that each node is included in a good placement scheme at least once. □

A.2 Proof of Lemma 2 and 3

While the degree of a node indicates its current scatter width, the complement degree—the number of nodes not adjacent to the specified node—implies if this node is capable of being involved in a good scheme. The more complement degree a node has, the more likely the scatter width increment will meet expectations if the node is included.

$$g(n_\gamma, \mathcal{V}, \mathcal{E}) = \sum_{n_\theta \in \mathcal{V}} [(n_\gamma, n_\theta) \notin \mathcal{E}] \tag{8}$$

We define the complement degree function in Equation 8, where the function [] is defined the same as in Equation 4. The input is a vertex n_γ, a vertices set $\mathcal{V}$ and an edges set $\mathcal{E}$. The output is a positive integer, which equals the number of nodes in $\mathcal{V}$ that are non-adjacent to the node n_γ under $\mathcal{E}$.

A.2.1 *Proof of Lemma 2.*

Proof. Let $g_{\alpha^*} = \sum_{n_\gamma \in \mathbf{n}^*_\alpha} g\left(n_\gamma, V_i \setminus \mathbf{n}^*_\alpha, E_i\right)$, which is equal to the sum of complement degrees from $\mathbf{n}^*_\alpha$ to the remainder of nodes in V_i. The initial complement degree for any $n_\gamma \in \mathbf{n}^*_\alpha$ under E_i is $|V_i| - \lfloor \frac{\rho}{2} \rfloor$. Additionally, each time a vertex is included in in $\mathbf{n}_\alpha$, its complement degree under E_i decreases by at most $\lfloor \frac{\rho}{2} \rfloor$, and it can be selected into $\mathbf{n}_\alpha$ at most ω times. Thus the lower bound is $g_{\alpha^*} \geq \lfloor \frac{\rho}{2} \rfloor \left(|V_i| - \lfloor \frac{\rho}{2} \rfloor (1+\omega)\right)$. According to the pigeonhole principle, when $g_{\alpha^*} > \left(\lfloor \frac{\rho}{2} \rfloor - 1\right)\left(|V_i| - \lfloor \frac{\rho}{2} \rfloor\right)$, there must exist a $n_\iota \in V_i$ s.t. $g(n_\iota, \mathbf{n}^*_\alpha, E_i) = \lfloor \frac{\rho}{2} \rfloor$. Finally, this Lemma is proven by combining the two inequations above. □

A.2.2 *Proof of Lemma 3.*

Proof. Let $g_\alpha = \sum_{n_\gamma \in \mathbf{n}_\alpha} g\left(n_\gamma, V_j, E'\right)$, which is equal to the sum of complement degrees from $\mathbf{n}_\alpha$ to all nodes in V_j. Following the same processes as Lemma 2, we obtain an inequation for the lower bound $g_\alpha \geq \left(\lfloor \frac{\rho}{2} \rfloor + 1\right)\left(|V_j| - \left(\lfloor \frac{\rho}{2} \rfloor + 1\right)\omega\right)$ and another inequation for the pigeonhole principle $g_\alpha > \lfloor \frac{\rho}{2} \rfloor |V_j|$. This lemma is proven by combining the two inequations above. □

A.3 Proof of Lemma 4

Proof. In the worst case, generating ω schemes fills ρ nodes, as though all ω schemes select the same nodes. Since Algorithm 1 always avoids selecting duplicated node combinations to maximize the scatter width, the rate at which the number of filled nodes increases will not be faster than the aforementioned worst case. □

A.4 Proof of Lemma 5

Proof. According to Hall's marriage theorem [16], if for any subset of the shards set R, the sum of the number of leader replicas that can be held by its associated nodes exceeds its size, each shard can be matched with a node. Since the commonly configured replication factor $\rho > 1$, which means for any shard, the sum of the number of leader replicas that can be held by its associated nodes is $\rho > 1$, the above condition is satisfied. □