#### Efficient Exact Verification of Binarized Neural Networks

July 24, 2022

- 1 Introduction
- 2 Efficient Exact Verification of Binarized Neural Networks
- 3 Training solver-friendly BNNs
- 4 Training verifiably robust BNNs
- 6 Other Experiments

- Introduction
- 2 Efficient Exact Verification of Binarized Neural Networks
- 3 Training solver-friendly BNNs
- 4 Training verifiably robust BNNs
- 6 Other Experiments

# Exact Real-valued NN Verification Challenges

- Unscalability
  - CPU inference: 2ms
  - Verification (Xiao et al., 2019): 420s
- Unverifiability
  - Floating point error makes real-valued NNs unverifiable
  - Adversarial inputs exist for verified NNs in practice Kai Jia and Martin Rinard. "Exploiting Verified Neural Networks via Floating Point Numerical Error." arXiv preprint arXiv:2003.03021 (2020).

# Binarized Neural Networks (BNNs)

- Weights & activations constrained to be binary
- Computational benefits:
  - Efficient inference:
    - BNN: 600 GOPS/W on FPGA
    - Floating point: 50 GFLOPS/W on GPU
    - 32x reduction of memory footprint
    - Comparable classification accuracy as fp32 networks

#### BNN Verification

- Binary values support
  - exact logical reasoning without floating point error
  - $\bullet \implies$  exact verification with correctness guarantees
- Previous state of the art BNN verifiers are unsatisfactory
  - Less scalable than real-valued verifiers: do not scale to CNNs on CIFAR10
  - No reported evaluation of verifiable robustness

- Introduction
- 2 Efficient Exact Verification of Binarized Neural Networks
- 3 Training solver-friendly BNNs
- 4 Training verifiably robust BNNs
- 6 Other Experiments

#### BNN structure

The basic building block of a BNN is a *linear-BatchNorm-binarize* module that maps an input tensor  $x \in \{0,1\}^n$  to an output tensor  $y \in \{0,1\}^m$  with a weight parameter  $W \in \mathbb{R}^{m \times n}$  and also trainable parameters  $\gamma \in \mathbb{R}^m$  and  $\beta \in \mathbb{R}^m$  in the batch normalization [30]:

$$y = bin_{act}(BatchNorm(bin_w(W)x))$$
 (1)

where:

$$\begin{aligned} \text{bin}_w(W) &= \text{sign}(W) \in \{-1,1\}^{m \times n} \\ \text{sign}(x) &= \left\{ \begin{array}{l} 1 & \text{if } x \geq 0 \\ -1 & \text{otherwise} \end{array} \right. \\ \text{BatchNorm}(x) &= \gamma \odot \frac{x - \mathbf{E}[x]}{\sqrt{\text{Var}[x] + \epsilon}} + \beta \end{aligned}$$

where  $\epsilon=1\mathrm{e}{-5}$ , and  $\odot$  denotes element-wise multiplication

$$bin_{act}(x) = (x \ge 0) = (sign(x) + 1)/2 \in \{0, 1\}^m$$

- First layer:  $x^{\mathbf{q}} = \left\lfloor \frac{x}{s} \right\rfloor \cdot s$
- Last layer: remove the bin<sub>act</sub>;no softmax;all channel use one mean and var

Encoding and Reified cardinality constraints arising in BNN encoding For

$$y = \mathrm{bin}_{act} \left( k^{\mathrm{BN}} \odot \left( W^{\mathrm{bin}} x \right) + b^{\mathrm{BN}} \right)$$

encoding as

$$y_i = \left(\sum_{j=1}^n l_{ij}(x_j) \geqslant \left[b_i\left(k^{\mathrm{BN}}, W^{\mathrm{bin}}, b^{\mathrm{BN}}\right)\right]\right)$$

the untargeted attack goal :  $\forall_{i\neq c} (y_i - y_c > 0)$ 

First layer: For 
$$v = \left\lfloor \frac{x}{s} \right\rceil \in \mathbb{Z} \cap [a, b], v = a + \sum_{i=1}^{b-a} t_i, t_i \vee \neg t_j \text{ for } 1 \leq i < j \leq b-a.$$

- Prior work: Encoding as CNF to be solved by an off-the shelf solver
- This work:natively solve such constraints by modifying a SAT solver
- Result: 100x 10,000x speedup compared to the unmodified solver

Extend a CDCL-based SAT from MiniSat2, 2 to MiniSatCS.

- CDCL framework: can handle clauses not in the disjunctive form, as long as each clause permits inferring values of undecided variables.
- For  $y = (\sum_{i=1}^{n} l_i \leq b)$ 
  - Operand-inferring: If y is known and enough of the  $\{l_i\}$  are known, then the remaining  $\{l_i\}$  can be inferred. For example, if y is known to be true and there are already b literals in  $\{l_i\}$  known to be true, then the other literals must be false.
  - <u>Target-inferring</u>: If enough of the  $\{l_i\}$  are known, then y can be inferred. For example, if the number of false literals in  $\{l_i\}$  reaches n-b, then y can be inferred to be true.

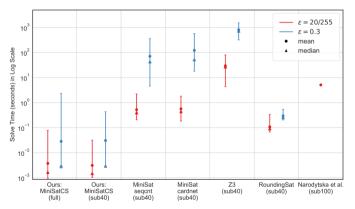


Figure 1: Performance comparison on searching adversarial inputs for an undefended MNIST-MLP network. The

- Introduction
- 2 Efficient Exact Verification of Binarized Neural Networks
- 3 Training solver-friendly BNNs
- 4 Training verifiably robust BNNs
- 6 Other Experiments

# Balanced layer-wise sparsity

- Prior work: ternary weights, sparsity concentrates in FC layers
- This work: reparameterization with sparse(W) = bin(W) \* bin(M)

$$bin_w(W) = sign(W) \circ \frac{sign(M_w) + 1}{2}$$

• Result: 100x - 100,000x verification speedup

# Balanced layer-wise sparsity

Table 1: Comparing BinMask and ternary quantization on undefended conv-small networks. While both methods produce similar total sparsity, the more balanced layer-wise sparsity of BinMask results in faster verification. Total sparsity is the proportion of zero parameters in the whole network, which is largely determined by the sparsity of the third layer — a fully connected layer with a large weight matrix. Solve time is measured by applying MiniSatCS on 40 randomly chosen test images with a one hour time limit.

|                         | MNIST           | $\epsilon = 0.1$ | CIFAR10 $\epsilon = 2/255$ |                 |  |  |
|-------------------------|-----------------|------------------|----------------------------|-----------------|--|--|
| Sparsifier              | Ternary         | BinMask          | Ternary                    | BinMask         |  |  |
| Total Sparsity          | 81%             | 84%              | 82%                        | 79%             |  |  |
| Layer-wise Sparsity     | 16% 40% 84% 36% | 91% 90% 83% 92%  | 16% 48% 84% 23%            | 84% 85% 79% 87% |  |  |
| Mean Solve Time (sub40) | 756.288         | 0.002            | 0.267                      | 0.003           |  |  |
| Max Solve Time (sub40)  | 3600.001        | 0.007            | 5.707                      | 0.005           |  |  |
| Natural Test Accuracy   | 97.59%          | 97.35%           | 54.78%                     | 55.22%          |  |  |

# Low cardinality bounds

- A regularizer to induce lower bounds in the cardinality constraints
- For  $y = (\sum_{i=1}^{n} l_i \le b)$ :
  - Converts the constraint into CNF needs O(nb) auxiliary variables and clauses. Thus smaller broduces a simpler encoding.
  - MiniSatCS can infer y to be false once the number of true literals in  $\{l_i\}$  exceeds b, and a smaller bincreases the likelihood of this inference.
  - If the literals  $\{l_i\}$  are drawn from independent symmetrical Bernoulli distributions, then the entropy of y is a symmetrical concave function with respect to b which is maximized when  $b = \frac{n}{2}$ . Therefore the further b deviates from  $\frac{n}{2}$ , the more predictable y becomes.
- by adding an  $L_1$  penalty on the bias terms in cardinality constraints that exceed a threshold  $\tau$ :

$$L^{\mathrm{CBD}} = \eta \max \left( b \left( k^{\mathrm{BN}}, W^{\mathrm{bin}}, b^{\mathrm{BN}} \right) - au, 0 \right)$$

# Low cardinality bounds

Table 2: Effect of Cardinality Bound Decay (CBD) on adversarially trained conv-large networks. The CBD loss effectively reduces cardinality bounds, resulting in significant verification speedup. The results suggest that it also improves robustness, perhaps by regularizing model capacity. Solve time is measured by applying MiniSatCS on 40 randomly chosen test images with a one hour time limit. Verifiable accuracy is evaluated on the complete test set without time limit for networks that can be verified within one second per case on average.

|                              |               | MNIST $\epsilon = 0.3$ |           |           |               | CIFAR10   | $\epsilon = 8/255$ |           |
|------------------------------|---------------|------------------------|-----------|-----------|---------------|-----------|--------------------|-----------|
| CBD Loss Penalty $(\eta)$    | 0             | 1e-5                   | 1e-4      | 5e-4      | 0             | 1e-5      | 1e-4               | 5e-4      |
| Mean / Max Card Bound        | 148.3 / 364.0 | 4.3 / 15.1             | 3.2 / 9.0 | 2.7 / 6.8 | 123.8 / 312.0 | 3.1 / 7.4 | 2.7 / 5.9          | 2.1 / 6.4 |
| Mean Solve Time (sub40)      | 2442.698      | 32.776                 | 0.009     | 0.005     | 206.037       | 0.010     | 0.009              | 0.009     |
| Max Solve Time (sub40)       | 3600.006      | 1287.739               | 0.040     | 0.012     | 3600.001      | 0.019     | 0.013              | 0.014     |
| Verifiable Accuracy          | -             | -                      | 69.04%    | 72.48%    | -             | 19.28%    | 18.81%             | 20.08%    |
| Natural Test Accuracy        | 98.88%        | 97.37%                 | 96.97%    | 96.26%    | 53.91%        | 40.80%    | 38.75%             | 35.17%    |
| PGD Accuracy                 | 89.23%        | 87.60%                 | 87.82%    | 87.82%    | 15.32%        | 27.06%    | 26.22%             | 24.69%    |
| First Layer / Total Sparsity | 85% / 82%     | 86% / 89%              | 83% / 89% | 82% / 87% | 95% / 88%     | 95% / 94% | 94% / 87%          | 89% / 90% |

- 1 Introduction
- 2 Efficient Exact Verification of Binarized Neural Networks
- 3 Training solver-friendly BNNs
- 4 Training verifiably robust BNNs
- 6 Other Experiments

# Training verifiably robust BNNs

- Prior work:robust training for real-valued networks (e.g., PGD)
- Challenge: A BNN directly trained with PGD has high PGD accuracy, low verifiable accuracy  $\implies$  PGD attack becomes ineffective.
- This work: adaptive gradient cancelling:  $htanh'(x) \to tanh'(x) \to 1 tanh^2(ax)$ , a is tuned by improving PGD attack success rate

# Training verifiably robust BNNs

Table 3: Comparing gradient computing methods for adversarial training on CIFAR10 with the conv-large network and  $\epsilon=8/255$ . The PGD accuracy is evaluated on the test set with the same gradient computing as in training. The verifiable accuracy is the exact adversarial robustness. Tanh gradient cancelling improves all the metrics, and adaptive gradient cancelling reduces the gap between PGD accuracy and verifiable accuracy.

|                       | hard tanh | tanh   | adaptive | adaptive + verifier adv |
|-----------------------|-----------|--------|----------|-------------------------|
| Natural Test Accuracy | 35.42%    | 38.79% | 35.17%   | 35.00%                  |
| PGD Accuracy          | 22.79%    | 24.98% | 24.69%   | 26.41%                  |
| Verifiable Accuracy   | 11.13%    | 14.70% | 20.08%   | 22.55%                  |

- Introduction
- 2 Efficient Exact Verification of Binarized Neural Networks
- 3 Training solver-friendly BNNs
- 4 Training verifiably robust BNNs
- 6 Other Experiments

#### Experiment

Table 4: Results of verifying adversarial robustness. EEV verifies BNNs significantly faster with comparable verifiable accuracy in most cases.

|                            |                   | N      | Mean Time (s) |        |            | Accuracy |        |       |  |
|----------------------------|-------------------|--------|---------------|--------|------------|----------|--------|-------|--|
|                            |                   | Build  | Solve         | Total  | Verifiable | Natural  | PGD    |       |  |
|                            | EEV S             | 0.0158 | 0.0004        | 0.0162 | 89.29%     | 97.44%   | 93.47% | 0     |  |
| MNIST                      | EEV L             | 0.1090 | 0.0025        | 0.1115 | 91.68%     | 97.46%   | 95.47% | 0     |  |
| $\epsilon = 0.1$           | Xiao et al. 63 S  | 4.98   | 0.49          | 5.47   | 94.33%     | 98.68%   | 95.13% | 0.05% |  |
|                            | Xiao et al. 63 L° | 156.74 | 0.27          | 157.01 | 95.6%      | 98.95%   | 96.58% | 0     |  |
|                            | EEV S             | 0.0140 | 0.0006        | 0.0146 | 66.42%     | 94.31%   | 80.70% | 0     |  |
| MNIST                      | EEV L             | 0.1140 | 0.0039        | 0.1179 | 77.59%     | 96.36%   | 87.90% | 0     |  |
| $\epsilon = 0.3$           | Xiao et al. 63 S  | 4.34   | 2.78          | 7.12   | 80.68%     | 97.33%   | 92.05% | 1.02% |  |
|                            | Xiao et al. 63 L° | 166.39 | 37.45         | 203.84 | 59.6%      | 97.54%   | 93.25% | 24.1% |  |
|                            | EEV S             | 0.0258 | 0.0013        | 0.0271 | 26.13%     | 46.58%   | 33.70% | 0     |  |
| CIFAR10                    | EEV L             | 0.1653 | 0.0097        | 0.1750 | 30.49%     | 47.35%   | 38.22% | 0     |  |
| $\epsilon = \frac{2}{255}$ | Xiao et al. 63 S  | 52.58  | 13.50         | 66.08  | 45.93%     | 61.12%   | 49.92% | 1.86% |  |
| 200                        | Xiao et al. 63 L* | 335.97 | 29.88         | 365.85 | 41.4%      | 61.41%   | 50.61% | 9.6%  |  |
|                            | EEV S             | 0.0313 | 0.0014        | 0.0327 | 18.93%     | 37.75%   | 24.60% | 0     |  |
| CIFAR10                    | EEV L             | 0.1691 | 0.0090        | 0.1781 | 22.55%     | 35.00%   | 26.41% | 0     |  |
| $\epsilon = \frac{8}{255}$ | Xiao et al. 63 S  | 38.34  | 22.33         | 60.67  | 20.27%     | 40.45%   | 26.78% | 2.47% |  |
|                            | Xiao et al. 63 L° | 401.72 | 20.14         | 421.86 | 19.8%      | 42.81%   | 28.69% | 5.4%  |  |

EEV is exact verification for BNNs with the proposed EEV system. Xiao et al. [63] is exact verification for real-valued networks with data taken from [63]. The S and L suffix indicates conv-small or conv-large architectures. The build time is the time required to energe the SAT or MILP formulation from the network weights the input.



#### Experiment

Table 8: Comparison of methods on 40 randomly chosen MNIST test images with solving time limit of 3600 seconds.

| $\epsilon_{ m train}$ | Network<br>Architecture | Training Method | Test<br>Accuracy | Sparsity | Solver      | Mean Solve<br>Time | Median<br>Solve Time | Timeout | Verifiable<br>Accuracy |
|-----------------------|-------------------------|-----------------|------------------|----------|-------------|--------------------|----------------------|---------|------------------------|
|                       |                         | Ternary         | 97.59%           | 81%      | MiniSatCS   | 756.288            | 4.281                | 15%     | 0%                     |
|                       | conv-small              |                 |                  |          | MiniSatCS   | 0.002              | 0.002                | 0       | 52%                    |
|                       | CONV-BINALL             | BinMask         | 97.35%           | 84%      | MiniSat     | 2.249              | 1.142                | 0       | 52%                    |
|                       |                         | DIIIIVIASK      | 97.3370          | 0470     | Z3          | 0.089              | 0.089                | 0       | 52%                    |
|                       |                         |                 |                  |          | RoundingSat | 0.048              | 0.042                | 0       | 52%                    |
| 0                     | conv-large              | Ternary         | 99.07%           | 86%      | MiniSatCS   | 2522.082           | 3600.002             | 68%     | 0%                     |
|                       |                         | Ternary+CBD     | 95.58%           | 87%      | MiniSatCS   | 886.007            | 21.711               | 20%     | 0%                     |
|                       |                         | Ternary+10xCBD  | 92.91%           | 78%      | MiniSatCS   | 342.097            | 4.742                | 5%      | 2%                     |
|                       |                         | BinMask         | 98.94%           | 86%      | MiniSatCS   | 2595.032           | 3600.001             | 70%     | 2%                     |
|                       |                         | BinMask+CBD     | 96.88%           | 89%      | MiniSatCS   | 0.664              | 0.028                | 0       | 70%                    |
|                       |                         |                 |                  |          | MiniSat     | 225.861            | 18.761               | 0       | 70%                    |
|                       |                         |                 |                  |          | Z3          | 146.567            | 0.997                | 0       | 70%                    |
|                       |                         |                 |                  |          | RoundingSat | 33.922             | 0.702                | 0       | 70%                    |
|                       |                         | Ternary (wd0)   | 94.72%           | 80%      | MiniSatCS   | 186.935            | 0.105                | 5%      | 30%                    |
|                       |                         | Ternary (wd1)   | 89.53%           | 93%      | MiniSatCS   | 0.005              | 0.002                | 0       | 35%                    |
|                       | conv-small              |                 |                  |          | MiniSatCS   | 0.001              | 0.001                | 0       | 52%                    |
|                       |                         |                 |                  |          | MiniSat     | 0.060              | 0.024                | 0       | 52%                    |

# Experiment

Table 9: Comparison of methods on 40 randomly chosen CIFAR10 test images with solving time limit of 3600 seconds.

| $\epsilon_{ m train}$ | Network<br>Architecture | Training Method | Test<br>Accuracy | Sparsity | Solver                                    | Mean Solve<br>Time                     | Median<br>Solve Time                 | Timeout            | Verifiable<br>Accuracy   |
|-----------------------|-------------------------|-----------------|------------------|----------|---|--|--------------------------------------|--------------------|--------------------------|
|                       | conv-small              | Ternary         | 54.78%           | 82%      | MiniSatCS<br>MiniSat<br>Z3<br>RoundingSat | 0.267<br>327.303<br>411.638<br>0.361   | 0.006<br>50.036<br>117.604<br>0.098  | 0<br>7%<br>5%<br>0 | 0%<br>0%<br>0%<br>0%     |
| 0                     |                         | BinMask         | 55.22%           | 79%      | MiniSatCS<br>MiniSat<br>Z3<br>RoundingSat | 0.003<br>3.981<br>0.590<br>0.081       | 0.003<br>3.577<br>0.376<br>0.077     | 0<br>0<br>0        | 0%<br>0%<br>0%<br>0%     |
|                       |                         | Ternary         | 69.25%           | 89%      | MiniSatCS                                 | 823.370                                | 1.860                                | 20%                | 0%                       |
|                       |                         | BinMask         | 67.46%           | 94%      | MiniSatCS                                 | 300.404                                | 3.201                                | 5%                 | 0%                       |
|                       | conv-large              | BinMask+CBD     | 63.18%           | 88%      | MiniSatCS<br>MiniSat<br>Z3<br>RoundingSat | 1.415<br>168.079<br>3515.386<br>19.162 | 0.048<br>69.471<br>3600.121<br>0.858 | 0<br>0<br>92%<br>0 | 0%<br>0%<br>0%<br>0%     |
|                       |                         | Ternary         | 32.59%           | 95%      | MiniSatCS                                 | 0.002                                  | 0.002                                | 0                  | 15%                      |
|                       | conv-small              | BinMask         | 37.75%           | 96%      | MiniSatCS<br>MiniSat<br>Z3<br>RoundingSat | 0.001<br>0.070<br>0.050<br>0.033       | 0.002<br>0.082<br>0.052<br>0.043     | 0<br>0<br>0        | 18%<br>18%<br>18%<br>18% |

Introduction
Efficient Exact Verification of Binarized Neural Networks
Training solver-friendly BNNs
Training verifiably robust BNNs
Other Experiments

Thank you