Math 2565 - Tutorial 6

Differential Equations

You will work in your groups via the Zoom breakout rooms to answer the first 3 questions. You do not need to submit the challenge question! Try to use the collaboration spaces on OneNote, so your progress can be monitored throughout the session and everyone can contribute.

1 (Separable Differential Equations)

Given the first order differential equation $\frac{dy}{dx} = e^{x+4y}$, then use separation of variables to:

(a) Find the general solution for y.

(b) Find the potential solution for which $y(0) = 0$.

2 (First Order Linear Differential Equation)

Given the first order differential equation $2\frac{dy}{dx} - 4xy - 2x = 0$, then use the integrating factor to:

(a) Find the general solution for y.

(b) Find the potential solution for which $y(0) = 0$.

3 (Modelling using Differential Equations)

Connor's favourite brand of tea is *Yorkshire Tea* which should be brewed at $100°$. The temperature of the room is $20°$, so the constant C has already been calculated to be -80. After 7 minutes the tea has cooled to $80°$. If the tea cools according to *Newton's Law of Cooling* then how long after brewing up before the temperature of the tea falls below $40°$ and the cup of tea should be consigned to the sink?

4 (Challenge: Modelling using Differential Equations)

A spherical pill with volume V and surface area S is swallowed and slowly dissolves in the stomach, releasing an active component. In one model it is assumed that the capsule dissolves in the stomach acids such that the rate of change in volume, $\frac{dV}{dt}$, is directly proportional to the pill's surface area.

(a) Show that $\frac{dV}{dt} = -kV^{2/3}$ where k is a positive real constant and solve this given that $V = V_0$ at $t = 0$.

(b) Experimental measurements indicate that for a 4 mm pill, half of the volume has dissolved after 3 hours. Find the rate constant k (m s^{-1}).

(c) Estimate the time required for 95% of the pill to dissolve.

5 (Optional Further Reading)

There also exists further applications of first order differential equations including both carbon dating of fossils and economics. See Chapter 3 of `http://www1.maths.leeds.ac.uk/~amtyt/1400/doc/notes1400.pdf` for more details and examples.