Math 2565 - Tutorial 3

Review for Test 1

In preparation for Test 1, you will work in your groups via the Zoom breakout rooms to answer the following 5 questions. Try to use the collaboration spaces on OneNote, so your progress can be monitored throughout the session and everyone can contribute. Test 1 will be open book (make sure to use the textbook), and many of the questions will test your conceptual understand instead of simply being computational. Therefore you will be expected write sentences to accompany your working and explain your reasoning.

Solutions to the problems will be uploaded directly after the 2nd tutorial session.

1 (Integration by Parts) Use integration by parts to evaluate

$$\int x \cos x \, dx.$$

Explain your choices for u and $\frac{dv}{dx}$.

2 (Trigonometric Integrals) Given the integral

$$\int \sin^5 x \, dx.$$

Use a relevant trigonometric identity to transform the integral into a form which can be easily solved and explain how to solve it.

3 (Trigonometric Substitution) Choose a relevant trigonometric substitution for the integral

$$\int \frac{x^2}{\sqrt{16 - x^2}} \, dx.$$

Explain why you chose this substitution and rewrite the integral using this substitution.

Hint: Explain how the substitution simplifies $\sqrt{16 - x^2}$ and allows you to evaluate the integral.

4 (Partial Fractions) Given the functions

$$f(x) = \frac{x + 2}{(x^2 + 1)} \quad \text{and} \quad g(x) = \frac{x + 2}{x(x + 1)}.$$

Decompose both $f(x)$ and $g(x)$ into their partial fractions without solving for the resulting coefficients. Explain why the two decomposed forms differ and evaluate the integral $\int g(x) \, dx$ using the decomposed form.

5 (Improper Integrals) Evaluate the following integral

$$\int_0^\infty e^{ct} \, dt,$$

Determine the values of c for which the integral converges.