Math 2565 - Tutorial 7

Sequences and Series

You will work in your groups via the Zoom breakout rooms to answer the following 3 questions. Try to use the collaboration spaces on OneNote, so your progress can be monitored throughout the session and everyone can contribute. You will only need to submit your answers your answers to questions 1, 2 and 3. You may need to use the fundamental limit $\lim_{a\to 0} \frac{\sin a}{a} = 1$ somewhere in your answers.

Solutions to the problems will be uploaded directly after the 2nd tutorial session.

1 (Limits of Sequences)

Find the limit of the following sequences.

(a) $\{a_n\} = \left\{\frac{(n^2+1)(6n-1)}{2n^3+5}\right\}$

(b) $\{b_n\} = \left\{\frac{(\ln n)^2}{n}\right\}$ [Hint: Use l'Hôpital's rule]

2 (Convergence of Infinite Series)

Determine whether the following series converge or diverge. For (c) and (d) use the nth term divergence test, and if it fails use the limit comparison test.

(a) $S_n = 96 + 48 + 24 + \cdots$

(b) $\sum_{k=2}^{\infty} \left(\frac{1}{k^2-1}\right) = \sum_{k=2}^{\infty} \frac{1}{2}\left(\frac{1}{k-1} - \frac{1}{k+1}\right)$

(c) $\sum_{j=1}^{\infty} \left(\frac{j(j+1)}{2j^2-1}\right)$

(d) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

3 (Comparison and Ratio Tests)

Use either the direct comparison, limit comparison or ratio test to determine whether the following series converge or diverge.

(a) $\sum_{n=1}^{\infty} \frac{1+\cos n}{n^3}$

(b) $\sum_{n=1}^{\infty} ne^{-n}$

(c) $\sum_{n=1}^{\infty} \frac{4^n}{2^n+3^n}$

4 (Revision for Test 2)

Test 2 will cover both Chapters 7 and 8. Use the following resources for revision.

(a) Tutorial 4 - Applications of Integrals

(b) Tutorial 5 - Review Questions and Differential Equations

(c) Tutorial 6 - Differential Equations