# Recitation 1: Sets

Discrete Mathematics

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# Question

Check if the two sets
$A = \{x : x$ is a letter in the word 'ASSASSINATION'$\}$ and
$B = \{x : x$ is a letter in the word 'STATION'$\}$ are equal.
*Show your working as well.*

# The Solution

Sets have unique elements hence all redundant elements are removed.

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$$A = \{\text{‘A’}, \text{‘S’}, \text{‘I’}, \text{‘T’}, \text{‘O’}, \text{‘N’}\}$$

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$$A = \{\text{‘A’}, \text{‘S’}, \text{‘I’}, \text{‘T’}, \text{‘O’}, \text{‘N’}\}$$

$$B = \{\text{‘S’}, \text{‘A’}, \text{‘I’}, \text{‘T’}, \text{‘O’}, \text{‘N’}\}$$

# The Solution

Sets have unique elements hence all redundant elements are removed.

$$A = \{\text{'A'},\text{'S'},\text{'I'},\text{'T'},\text{'O'},\text{'N'}\}$$

$$B = \{\text{'S'},\text{'A'},\text{'I'},\text{'T'},\text{'O'},\text{'N'}\}$$

As sets have no order, bijection of elements corresponding to themselves is True $\square$

# Question

Find the cardinality of the following sets.

- $S = \{x \mid x \text{ is a positive integer less than } 10\}$
- $S = \mathbb{Z}^+$
- $S = \emptyset$
- $S = \{\emptyset\}$

# Solution

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# Solution

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This is the set of all positive numbers (same as the natural numbers). Since the set is countably finite, the cardinality is $\aleph_0$.

**Definition**

What is $\aleph_0$? $\aleph_0$ refers to the cardinality of the set of all natural numbers. (We use a symbol since it is infinity. There are more types of infinity so that is why aleph-nought has a specific name.

# Solution

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Since the set is empty, the cardinality is $0$.

# Solution

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This set contains one element, which is the empty set, therefore the cardinality is 1. You can think of this set as the power set of the set above.

# Question

Consider three sets $A$, $B$ and $U$ where $U$ is a universal set containing $A$ and $B$.
If $|(\overline{A \cup B})| = 3$ and $|U| = 10$ then, find $|A \cup B|$?

# Solution

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$$|A \cup B| = 10 - 3$$

$$|A \cup B| = 7$$

# Question

List down the size and elements of the following power sets:

- $X = \{a, b, c\}$
- $\mathcal{P}(X)$ if $X = \{a, b, \{a, b\}\}$
- $\mathcal{P}(\mathcal{P}(\emptyset))$

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$$P(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a.b, c\}\}$$

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Since there are 8 elements in $P(X)$, the cardinality of $P(X)$ is 8.

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$$P(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a.b, c\}\}$$

Since there are 8 elements in $P(X)$, the cardinality of $P(X)$ is 8. We can also use the fact that $|X| = 3$ and $|P(A)|$ for any arbitrary set $A$ is $2^{|A|}$. This will also give us $\mathbf{8}$.

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$$P(X) = \{\emptyset, \{a\}, \{b\}, \{\{a, b\}\}, \{a, b\}, \{b, \{a, b\}\}, \\ \{a, \{a, b\}\}, \{a, b, \{a, b\}\}\}$$

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$$P(X) = \{\emptyset, \{a\}, \{b\}, \{\{a, b\}\}, \{a, b\}, \{b, \{a, b\}\}, \{a, \{a, b\}\}, \{a, b, \{a, b\}\}\}$$

Since there are 8 elements in $P(X)$, the cardinality of $P(X)$ is 8.Alternatively, $P(X) = 2^3 = \mathbf{8}$

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Since there are 2 elements in $P(P(\emptyset))$, the cardinality of $P(P(\emptyset))$ is 2.

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Since there are 2 elements in $P(P(\emptyset))$, the cardinality of $P(P(\emptyset))$ is 2. Alternatively, $P(P(\emptyset)) = 2^{2^0} = 2^1 = \mathbf{2}$

# Question

Describe each of the following sets in set builder notation, or the format $\{x : \text{property of } x\}$.

1. $A = \{0, 2, 4, 6, 8, ...\}$
2. $D = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, ...\}$

# Solution

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We know that A is a subset of $\mathbb{N}$ with even numbers.

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Therefore,

$$\{x \; : \; x \in \mathbb{N} \land x \text{ is divisible by } 2\}$$

$$\{x \mid x = 2n \text{ where } n \in \mathbb{N}\}$$

$$\{2x \mid x \in \mathbb{N}\}$$

# Solution

$$D = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\right\}$$

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We know that $D$ contains all numbers of the form $\frac{1}{n}$ where $n \in \mathbb{Z}^+$(Positive Integers).

# Solution

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We know that $D$ contains all numbers of the form $\frac{1}{n}$ where $n \in \mathbb{Z}^+$(Positive Integers).
Therefore,

$$D = \left\{x |\ x = \frac{1}{n} \text{ where } n \in \mathbb{Z}^+\right\}$$

.

$$\left\{\frac{1}{x}\ |x \in \mathbb{Z}^+\right\}$$

# Question

Let $P(X)$ the power set of the set $X$: its elements are the subsets of $X$. Show that if $A \subseteq B$, then the $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

# Answer

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An element $S \in P(A)$ is a subset of $A$,

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Suppose $A \subseteq B$.
An element $S \in P(A)$ is a subset of $A$, and $S \subseteq A \subseteq B$ shows that $S$ is also a subset of $B$. Thus $S \in P(B)$. Since every element of $P(A)$ is also an element of $P(B)$ we conclude that $P(A) \subseteq P(B)$. $\square$

# Conclusion

That's all folks! Attendance time