Recitation 2: Sets (again but advanced)

Discrete Mathematics

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January 28, 2022

Question

Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find

- $A \cup B$
- $A \cap B$
- $A - B$
- $B - A$

The Solution of part A

This contains elements of both A and B therefore

$$A \cup B = \{a, b, c, d, e, f, g, h\}$$

The Solution of part B

This contains elements that are in both A and B therefore

$$A \cap B = \{a, b, c, d, e\}$$

The Solution of part C

This contains elements of A that are not in B therefore

$$A - B = \{\}$$

The Solution of part D

This contains elements of B that are not in A therefore

$$B - A = \{f, g, h\}$$

Question

Find the Cartesian product of $A = \{1, 2\}$ and $B = \{a, b, c\}$.

The Solution

Definition

The *Cartesian product* of two sets X and Y, written $X \times Y$, is the set $X \times Y = \{(x, y) \mid x \in X, y \in Y\}$.

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Since, $|A| = 2$ and $|B| = 3$, Cartesian Product of A and B will have

$$2 \times 3 = 6$$

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elements(*multiplication principle*).
The Cartesian Product $A \times B$ is

$$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$$

□

Question

Cartesian Product is not commutative: Given some set A and B show that $A \times B$ may not be equal to $B \times A$

Solution

Proof by counterexample: we show that there exits pair of sets A and B such that, $A \times B \neq B \times A$

Solution

Proof by counterexample:

Let $A = \{1, 2\}$ and $B = \{a, b, c\}$

Solution

Proof by counterexample:

Let $A = \{1, 2\}$ and $B = \{a, b, c\}$
The Cartesian product of $B \times A$ is

$$B \times A = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$$

Solution

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The Cartesian product of $B \times A$ is

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The Cartesian product of $A \times B$ is

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We can see $A \times B \neq B \times A$. $\square$

Question

Can you conclude that $A = B$ if A, B, and C are set such that

- $A \cup C = B \cup C$
- $A \cap C = B \cap C$

Solution to part A

Let $A = \{a\}$, $B = \{a, b\}$ and $C = \{a, b, c\}$.

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Let $A = \{a\}$, $B = \{a, b\}$ and $C = \{a, b, c\}$.

$$A \cup C = \{a, b, c\}$$

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$$A \cup C = B \cup C$$

$$A \neq B$$

So the answer is No! :(□

Solution to part B

Let $A = \{a\}$, $B = \{b\}$ and $C = \emptyset$.

Solution to part B

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$$A \cap C = \emptyset$$

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Solution to part B

Let $A = \{a\}$, $B = \{b\}$ and $C = \emptyset$.

$$A \cap C = \emptyset$$

$$B \cap C = \emptyset$$

$$A \cap C = B \cap C$$

$$A \neq B$$

Solution to part B

Let $A = \{a\}$, $B = \{b\}$ and $C = \emptyset$.

$$A \cap C = \emptyset$$

$$B \cap C = \emptyset$$

$$A \cap C = B \cap C$$

$$A \neq B$$

So the answer is No! :($\square$

Question

Suppose that $A \times B = \emptyset$, where A and B are set. What can you conclude?

Solution

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The other way around

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The other way around

If $A \neq \emptyset$ and $B \neq \emptyset$ then there is exists $a \in A$ and $b \in B$, thus $(a, b) \in A \times B$

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Definition

The *Cartesian product* of two sets X and Y, written $X \times Y$, is the set $X \times Y = \{(x, y) \mid x \in X, y \in Y\}$.

If $A = \emptyset$ or $B = \emptyset$ then there is no (a, b) such that $a \in A$ and $b \in B$

The other way around

If $A \neq \emptyset$ and $B \neq \emptyset$ then there is exists $a \in A$ and $b \in B$, thus $(a, b) \in A \times B$
So we can conclude that either A or either B is a null set.
This is similar to saying "multiplying by 0".

$$A \times B = \emptyset \Leftrightarrow A = \emptyset \vee B = \emptyset$$

$\square$

Question

Let A, B, C be sets. Show that

$$\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$$

Solution

Using set identities.

TABLE 1 Set Identities.	
Identity	***Name***
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws

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$$
\begin{aligned}
\overline{A \cup (B \cap C)} &= \overline{A} \cap (\overline{B \cap C}) && \text{by the first De Morgan law} \\
&= \overline{A} \cap (\overline{B} \cup \overline{C}) && \text{by the second De Morgan law} \\
&= (\overline{B} \cup \overline{C}) \cap \overline{A} && \text{by the commutative law for intersections} \\
&= (\overline{C} \cup \overline{B}) \cap \overline{A} && \text{by the commutative law for unions}
\end{aligned}
$$

$\square$

Question

For sets A and B, prove that $A \cup (B - A) = A \cup B$

Solution

Using set identities.

TABLE 1 Set Identities.

Identity	*Name*
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

Solution

$$
\begin{aligned}
A \cup (B - A) &= A \cup (B \cap \overline{A}) && \text{Using the definition of set complement} \\
&= A \cup (\overline{A} \cap B) && \text{Using commutative law for intersections} \\
&= (A \cup \overline{A}) \cap (A \cap B) && \text{Using Distributive Law} \\
&= U \cap (A \cup B) && \text{Using complement law } A \cup A = U \\
&= A \cup B && \text{Using Domination Law}
\end{aligned}
$$

Therefore shown that they are equal.

Solution

Using a Truth table.

Solution

Using a Truth table.

A	B	$B - A$	$A \cup (B - A)$	$A \cup B$
F	F	F	F	F
F	T	T	T	T
T	F	F	T	T
T	T	F	T	T

$\square$

Conclusion

That's all folks! Attendance time and don't forget to read the book and subscribe to our youtube channel!