

Recitation 3: Introductory Propositional Logic

Dead TA's Society

Discrete Mathematics
Habib University
Karachi, Pakistan

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Gödel must be proud

Question: Is the assertion “This statement is false” a proposition?

Gödel must be proud

Question: Is the assertion “This statement is false” a proposition?

Answer: **No.** A proposition is a statement that can have either be true or false but not both. This can not therefore this is not a proposition.

Question

What is the negation of each of these propositions?

Statement: Chika is the best girl.

Question

What is the negation of each of these propositions?

Statement: Chika is the best girl.

Negation: Chika is not the best girl.

Question

What is the negation of each of these propositions?

Statement: Everyone loves Speedwagon.

Question

What is the negation of each of these propositions?

Statement: Everyone loves Speedwagon.

Negation: There is someone who doesn't love speedwagon.

Question

What is the negation of each of these propositions?

Statement: There are 42 items in a Mujtaba's dozen.

Question

What is the negation of each of these propositions?

Statement: There are 42 items in a Mujtaba's dozen.

Negation: There aren't 42 items in a Mujtaba's dozen.

Question

What is the negation of each of these propositions?

Statement: Everyday, Blingblong sends more than 100 text messages to the guy who delivered him pizza 2 months ago.

Question

What is the negation of each of these propositions?

Statement: Everyday, Blingblong sends more than 100 text messages to the guy who delivered him pizza 2 months ago.

Negation: On someday, Blingblong doesn't send more than 100 text messages to the guy who delivered him pizza 2 months ago.

Question

What is the negation of each of these propositions?

Statement: 121 is a perfect square.

Question

What is the negation of each of these propositions?

Statement: 121 is a perfect square.

Negation: 121 is not a perfect square.

Question

What is the negation of each of these propositions?

Statement: Karen took the kids and the dog

Question

What is the negation of each of these propositions?

Statement: Karen took the kids and the dog

Negation: Either Karen didn't take the kids or Karen didn't take the dog, or Karen took neither.

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).

Proposition: We live in a society and I am an emo kid.

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
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Write these propositions using p and q and logical connectives (including negations).

Proposition: We live in a society and I am an emo kid.

Answer: $p \wedge q$

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).

Proposition: We live in a society but I am not an emo kid.

Question

Let p and q be the propositions

$p =$ We live in a society

$q =$ I am an emo kid

Write these propositions using p and q and logical connectives (including negations).

Proposition: We live in a society but I am not an emo kid.

Answer: $p \wedge \neg q$

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).

Proposition: Either we live in a society or I am an emo kid or both.

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).

Proposition: Either we live in a society or I am an emo kid or both.

Answer: $p \vee q$

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).

Proposition: If we live in a society, I am an emo kid.

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).

Proposition: If we live in a society, I am an emo kid.

Answer: $p \implies q$

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).

Proposition: Either we live in a society or I am an emo kid, but not both.

Question

Let p and q be the propositions

$p =$ We live in a society

$q =$ I am an emo kid

Write these propositions using p and q and logical connectives (including negations).

Proposition: Either we live in a society or I am an emo kid, but not both.

Answer: $p \oplus q$

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).

Proposition: We living in a society is necessary for me to be an emo kid.

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).

Proposition: We living in a society is necessary for me to be an emo kid.

Answer: $q \implies p$

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).

Proposition: We living in a society is sufficient for me to be an emo kid.

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).

Proposition: We living in a society is sufficient for me to be an emo kid.

Answer: $p \implies q$

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).

Proposition: We living in a society is necessary and sufficient for me to be an emo kid.

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).

Proposition: We living in a society is necessary and sufficient for me to be an emo kid.

Answer: $p \iff q$

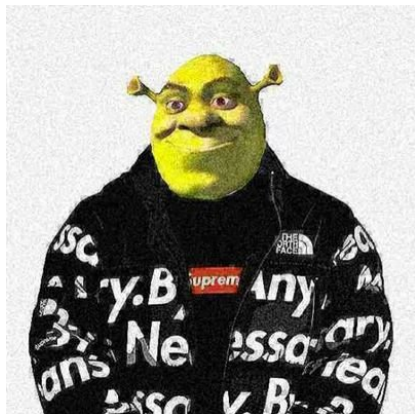
Question

Given

p : Shrek is adorable.

q : Kermit is in gamer rage.

r : The TAs get enough sleep



Best TAs Ever



Best TAs Ever

Question

Let

p : Shrek is adorable.

q : Kermit is in gamer rage.

r : The TAs get enough sleep

Express each of these compound propositions as an English sentence.

Compound propositions: $\neg r$

Question

Let

p : Shrek is adorable.

q : Kermit is in gamer rage.

r : The TAs get enough sleep

Express each of these compound propositions as an English sentence.

Compound propositions: $\neg r$

English sentence: The TAs don't get enough sleep

Question

Let

p : Shrek is adorable.

q : Kermit is in gamer rage.

r : The TAs get enough sleep

Express each of these compound propositions as an English sentence.

Compound propositions: $q \wedge \neg r$

Question

Let

p : Shrek is adorable.

q : Kermit is in gamer rage.

r : The TAs get enough sleep

Express each of these compound propositions as an English sentence.

Compound propositions: $q \wedge \neg r$

English sentence: Kermit is in gamer rage and The TAs don't get enough sleep

Question

Let p, q, r be the propositions “Shrek is adorable”, “Kermit is in
gamer Let

p : Shrek is adorable.

q : Kermit is in gamer rage.

r : The TAs get enough sleep

Express each of these compound propositions as an English sentence.

Compound propositions: $p \Rightarrow \neg r$

Question

Let p, q, r be the propositions “Shrek is adorable”, “Kermit is in
gamer Let

p : Shrek is adorable.

q : Kermit is in gamer rage.

r : The TAs get enough sleep

Express each of these compound propositions as an English sentence.

Compound propositions: $p \Rightarrow \neg r$

English sentence: If Shrek is adorable then the TAs don't get
enough sleep

Question

Let

p : Shrek is adorable.

q : Kermit is in gamer rage.

r : The TAs get enough sleep

Express each of these compound propositions as an English sentence.

Compound propositions: $\neg p \vee \neg r$

Question

Let

p : Shrek is adorable.

q : Kermit is in gamer rage.

r : The TAs get enough sleep

Express each of these compound propositions as an English sentence.

Compound propositions: $\neg p \vee \neg r$

English sentence: Either the TAs don't get enough sleep or Shrek is not adorable or both.

Question

Let

p : Shrek is adorable.

q : Kermit is in gamer rage.

r : The TAs get enough sleep

Express each of these compound propositions as an English sentence.

Compound propositions: $\neg p \Leftrightarrow q$

Question

Let

p : Shrek is adorable.

q : Kermit is in gamer rage.

r : The TAs get enough sleep

Express each of these compound propositions as an English sentence.

Compound propositions: $\neg p \Leftrightarrow q$

English sentence: Kermit is in gamer rage if and only if Shrek is not adorable.

Context

"If I tell the truth about everything then I will offend someone."



Among scholars, this is known as the Youtube Commentator's Fallacy.

Question

“If I tell the truth about everything then I will offend someone.”

Write this statement in propositional logic and its converse, contrapositive, and inverse in English and propositional logic

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p : I tell the truth about everything

q : I will offend someone

$$p \implies q$$

Question

“If I tell the truth about everything then I will offend someone.”

Write this statement in propositional logic and its converse, contrapositive, and inverse in English and propositional logic

p : I tell the truth about everything

q : I will offend someone

$$p \implies q$$

Converse: $q \implies p$

Question

“If I tell the truth about everything then I will offend someone.”

Write this statement in propositional logic and its converse, contrapositive, and inverse in English and propositional logic

p : I tell the truth about everything

q : I will offend someone

$$p \implies q$$

Converse: $q \implies p$

If I will offend someone then I'm telling the truth about everything

Question

“If I tell the truth about everything then I will offend someone.”

Write this statement in propositional logic and its converse, contrapositive, and inverse in English and propositional logic

p : I tell the truth about everything

q : I will offend someone

$$p \implies q$$

Converse: $q \implies p$

If I will offend someone then I'm telling the truth about everything

Contrapositive: $\neg q \implies \neg p$

Question

“If I tell the truth about everything then I will offend someone.”

Write this statement in propositional logic and its converse, contrapositive, and inverse in English and propositional logic

p : I tell the truth about everything

q : I will offend someone

$$p \implies q$$

Converse: $q \implies p$

If I will offend someone then I'm telling the truth about everything

Contrapositive: $\neg q \implies \neg p$

If I will offend no one then I'm not telling the truth about something

Question

“If I tell the truth about everything then I will offend someone.”

Write this statement in propositional logic and its converse, contrapositive, and inverse in English and propositional logic

p : I tell the truth about everything

q : I will offend someone

$$p \implies q$$

Converse: $q \implies p$

If I will offend someone then I'm telling the truth about everything

Contrapositive: $\neg q \implies \neg p$

If I will offend no one then I'm not telling the truth about something

Inverse: $\neg p \implies \neg q$

Question

“If I tell the truth about everything then I will offend someone.”

Write this statement in propositional logic and its converse, contrapositive, and inverse in English and propositional logic

p : I tell the truth about everything

q : I will offend someone

$$p \implies q$$

Converse: $q \implies p$

If I will offend someone then I'm telling the truth about everything

Contrapositive: $\neg q \implies \neg p$

If I will offend no one then I'm not telling the truth about something

Inverse: $\neg p \implies \neg q$

If I'm not telling the truth about something then I will offend no one

Question

Construct a truth table for the following compound proposition

$$(p \Leftrightarrow q) \oplus (p \Leftrightarrow \neg q)$$

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p	q	$p \Leftrightarrow q$	$p \Leftrightarrow \neg q$	$p \Leftrightarrow q \oplus p \Leftrightarrow \neg q$
T	T	T	F	T

Question

Construct a truth table for the following compound proposition

$$(p \Leftrightarrow q) \oplus (p \Leftrightarrow \neg q)$$

p	q	$p \Leftrightarrow q$	$p \Leftrightarrow \neg q$	$p \Leftrightarrow q \oplus p \Leftrightarrow \neg q$
T	T	T	F	T
T	F	F	T	T

Question

Construct a truth table for the following compound proposition

$$(p \Leftrightarrow q) \oplus (p \Leftrightarrow \neg q)$$

p	q	$p \Leftrightarrow q$	$p \Leftrightarrow \neg q$	$p \Leftrightarrow q \oplus p \Leftrightarrow \neg q$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T

Question

Construct a truth table for the following compound proposition

$$(p \Leftrightarrow q) \oplus (p \Leftrightarrow \neg q)$$

p	q	$p \Leftrightarrow q$	$p \Leftrightarrow \neg q$	$p \Leftrightarrow q \oplus p \Leftrightarrow \neg q$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Question

"Only a Sith deals in Absolutes" - Obi-Wan Kenobi

Translate the above statement in propositional logic. Is the statement above an absolute statement? If the statement above is true, what does this say about Obi-Wan Kenobi.

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Translate the above statement in propositional logic. Is the statement above an absolute statement? If the statement above is true, what does this say about Obi-Wan Kenobi.

p = The person is a Sith

q = The person deals in absolutes

Question

"Only a Sith deals in Absolutes" - Obi-Wan Kenobi

Translate the above statement in propositional logic. Is the statement above an absolute statement? If the statement above is true, what does this say about Obi-Wan Kenobi.

p = The person is a Sith

q = The person deals in absolutes

$$q \implies p$$

Question

This statement is an absolute statement since it uses the word **Only**. If Obi-wan is telling the truth, since he is saying an absolute statement he is a sith and should be removed from the council to be replaced by Anakin.

Question

This statement is an absolute statement since it uses the word **Only**. If Obi-wan is telling the truth, since he is saying an absolute statement he is a sith and should be removed from the council to be replaced by Anakin.

Note that the statement isn't bidirectional. Bidirectional refers to if and only if. It is also not $p \implies q$ since it is not necessary for a Sith to deal in absolutes. A dead sith can not deal in anything for example. If a person deals in absolutes then we are sure he is a sith therefore $q \implies p$

Question

Is the following statement valid?

Binary search runs in $O(\log n)$ if and only if π is irrational

Question

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Binary search runs in $O(\log n)$ if and only if π is irrational

Let $P : \pi$ is irrational.

Question

Is the following statement valid?

Binary search runs in $O(\log n)$ if and only if π is irrational

Let $P : \pi$ is irrational.

Let $B : \text{Binary search runs in } O(\log n).$

Question

Is the following statement valid?

Binary search runs in $O(\log n)$ if and only if π is irrational

Let P : π is irrational.

Let B : Binary search runs in $O(\log n)$.

Statement

$$B \Leftrightarrow P$$

Question

Is the following statement valid?

Binary search runs in $O(\log n)$ if and only if π is irrational

Let P : π is irrational.

Let B : Binary search runs in $O(\log n)$.

Statement

$$B \Leftrightarrow P$$

As P is always true:

$$B \Rightarrow P \text{ is true}$$

Question

Is the following statement valid?

Binary search runs in $O(\log n)$ if and only if π is irrational

Let P : π is irrational.

Let B : Binary search runs in $O(\log n)$.

Statement

$$B \Leftrightarrow P$$

As P is always true:

$$B \Rightarrow P \text{ is true}$$

As B is always true:

$$P \Rightarrow B \text{ is true}$$

Question

Is the following statement valid?

Binary search runs in $O(\log n)$ if and only if π is irrational

Let P : π is irrational.

Let B : Binary search runs in $O(\log n)$.

Statement

$$B \Leftrightarrow P$$

As P is always true:

$$B \Rightarrow P \text{ is true}$$

As B is always true:

$$P \Rightarrow B \text{ is true}$$

Thus

$$B \Leftrightarrow P \text{ is true}$$

Its True, as both statements are true.

Conclusion

That's all folks! Attendance time.

- 1 Read the book!
- 2 Practice more!
- 3 Don't forget to hit the like button
- 4 Remember that the TA's hours can be seen on canvas and TAs can be found in their hours on EHSAS Group (MS Teams)