# Spring force direction constraints

Let’s look at the analytical form of the spring force, where is the mass of interest – the one on which the force is exerted – and is the mass at the other end of the spring:

The gradient wrt is:

With some algebra, we can identify the eigenvector-value pairs of this Jacobian as:

1. Input vectors along :
2. Input vectors :

Finally – and here is where some uncertainty creeps in – we can try to force to have this eigen-decomposition by applying the following constraints:

If points [1-2] hold, then [3-4] must be true; but does the converse apply?

# Energy conservation constraint

It’s difficult to apply an energy conservation constraint directly with our chosen network architecture for two reasons:

1. We operate on positions and forces; velocity information is missing, and the energies are “one derivative up” from the forces ()
2. Our “three-at-a-time” method of parsing the masses is strictly ‘local’, and we’d have to aggregate the networks’ outputs over the full mass chain to get global energy information

This begs the question: can we impose energy conservation locally, and using the forces only? It turns out we can, using our knowledge that all the forces at play (springs, gravity) are **conservative forces**. By definition, conservative forces are force fields which are expressible as the gradient of a scalar potential (). As a consequence, necessarily has the property that it is also **irrotational**, i.e. .To demonstrate, consider the sum of forces acting on the center mass:

We therefore have a new constraint! for conservative vector fields, or . Note that this is the same thing as saying the force Jacobian, , should be a symmetric matrix[[1]](#footnote-2). The constraint is thus:

On the other hand, it’s not clear that we can use the divergence information (diagonal entries of ) so easily. The gravity field is divergence-free because gravity is an inverse-square force and subject to Gauss’ law + divergence theorem. OTOH, spring forces are *linear*, and their divergence is instead a constant, uniform value everywhere. Using

For a spherical volume,

If we know that the divergence of the spring forces is a constant, as shown below:

Then we could possibly stipulate that the *second* derivatives of are zero. This simply says that the spring forces must be linear in , i.e. is not a possible solution for the force law. However, this constraint isn’t generalizable to other types of forces, unlike the previous curl constraint.

1. To be precise, the Jacobian wrt the center mass’s position, so . [↑](#footnote-ref-2)