Report for question 3

3 (a)

Histogram h(I) is split into $h_1(I)$ and $h_2(I)$ where:

i. $h_1(I)$ is over the domain [0, a]
ii. $h_2(I)$ is over the domain (a, 1]

For some arbitrary a $\in$ (0, 1). The histogram mass within [0, a] is $\alpha \in (0, 1)$.

Probability density of $h_1(I)$ after histogram equalization:

Initially we choose a transformation function which is monotonically increasing function in the interval [0, a].

$$e = T(I) \qquad\qquad 0 \leq I \leq a$$

Here the random variables e and I take values that are in the interval [0, a]. The histogram $h_{1e}(I)$ is the probability density function of the transformed variable 'e'. The histogram $h_1(I)$ can be considered as the probability density function of the given image as its mass is 1. From fundamental probability theory we can say that if $h_1(I)$ and T(I) are known and T(I) is continuous and differentiable over the range, then probability density function of transformed variable 'e' can be obtained as

$$h_{1e}(I) = h_1(I)\left|\frac{dI}{de}\right| \tag{1}$$

The transformation function here would be the *cumulative distribution function (CDF)* of random variable I. As PDF's are always positive so CDF will always be monotonically increasing in the given range. Here when the upper limit in the equation is 'a' then the function T(I) should evaluate to 'a'. So, here we divide the integral with α as the integral of the function $h_1(I)$ over the interval [0, a] will be α.

$$e = T(I) = \frac{a}{\alpha}\int_0^I h_1(w)dw \tag{2}$$

Here 'w' is a dummy variable for integration.

From Leibniz's rule for derivative of a definite integral with respect to its upper limit is the integral evaluated at the limit itself.

$$\frac{de}{dI} = \frac{dT(I)}{de}$$

$$= \frac{a}{\alpha} \frac{d}{de} \left[\int_0^I h_1 (w) dw \right]$$

$$= \frac{a}{\alpha} h_1(I) \qquad (3)$$

Substitute the result obtained in 3 to equation 1 we get

$$h_{1e}I = h_1(I) \left| \frac{dI}{de} \right|$$

$$= h_1(I) \left| \frac{1}{\left(\frac{a}{\alpha}\right) h_1 (I)} \right|$$

$$= \frac{\alpha}{a}$$

Probability density of $h_2(I)$ after histogram equalization:

Initially we choose a transformation function which is monotonically increasing function in the interval (a, 1].

$$e = T(I) \qquad a < I \leq 1$$

Here the random variables e and I take values that are in the interval (a, 1]. The histogram $h_{2e}(I)$ is the probability density function of the transformed variable 'e'. The histogram $h_2(I)$ can be considered as the probability density function of the given image as its mass is 1. From fundamental probability theory we can say that if $h_2(I)$ and T(I) are known and T(I) is continuous and differentiable over the range, then probability density function of transformed variable 'e' can be obtained as

$$h_{2e}(I) = h_2(I) \left| \frac{dI}{de} \right| \qquad (1)$$

The transformation function here would be the *cumulative distribution function (CDF)* of random variable I. As PDF's are always positive so CDF will always be monotonically increasing in the given range. Here when the upper limit in the equation is 1 and the lower limit in the equation is 'a' then the function T(I) should evaluate to '1-a'. So, here we divide the function with '1-α' as the integral of the function $h_2(I)$ over the interval (a, 1] will be '1-α' (as the CDF for h_1 is α and total CDF is 1)

$$e = T(I) = \frac{1-a}{1-\alpha} \int_a^1 h_2(w) dw \qquad (2)$$

Here 'w' is a dummy variable for integration.

From Leibniz's rule for derivative of a definite integral with respect to its upper limit is the integral evaluated at the limit itself.

$$\frac{de}{dI} = \frac{dT(I)}{de}$$

$$= \frac{1-a}{1-\alpha}\frac{d}{de}\left[\int_a^1 h_2\,(w)dw\right]$$

$$= \frac{1-a}{1-\alpha}\,h_2(I) \qquad (3)$$

Substitute the result obtained in 3 to equation 1 we get

$$h_{2e}I = h_2(I)\left|\frac{dI}{de}\right|$$

$$= h_2(I)\left|\frac{1}{\left(\frac{1-a}{1-\alpha}\right)h_2(I)}\right|$$

$$= \frac{1-\alpha}{1-a}$$

Calculating the mean of the resulting function:

For calculating the mean of probability of the Probability Distribution Function

$$f(x) = \begin{cases} \dfrac{\alpha}{a}, & x \in [0, a] \\ \dfrac{1-\alpha}{1-a}, & x \in (a, 1] \end{cases}$$

$$\text{Mean} = \int_{-\infty}^{\infty} xf(x)dx$$

$$= \int_0^a x\frac{\alpha}{a}dx + \int_a^1 x\frac{1-\alpha}{1-a}dx$$

Substituting values, we get

$$\text{Mean} = \frac{\alpha.a}{2} + \frac{1}{2}(1-\alpha)(1+a)$$

3 (b)

If median intensity occurs at 'a' then it means that h(0<I<a)=0.5. And mean intensity occurring at 'a' means that on an average the value of I would be 'a'.

From the above statements we can conclude that α=0.5 at 'a'. So, considering the result from part (a)

$$\frac{\alpha. a}{2} + \frac{1}{2}(1-\alpha)(1+a)$$

Substituting value for α as 0.5 we get the mean intensity as

$$= \frac{a}{4} + \frac{1}{4}(1+a)$$

$$= \frac{a}{4} + \frac{1}{4} + \frac{a}{4}$$

$$= \frac{a}{2} + \frac{1}{4}$$

3 (c)

Scenario where the described histogram-based intensity transformation is better than normal histogram equalization method would be in dark images where most of the pixels are very dark. This method also does not amplify noise too much in the image after the transformation. In dark images the image is amplified just enough such that it does not

As, *a* is chosen as the median of the distribution it splits the intensity distribution into two halves this results in the much more uniform distribution of intensities for the first half. And for the second half average intensity in this range decrease in general.

The mean intensity for histogram equalization is ideally $\frac{1}{2}$ so comparing this mean intensity with the given intensity transformation we get that the average brightness of the image will be reduced until the median 'a' reaches $\frac{1}{2}$ of the max value it will make the image on an average darker. But if the median image intensity is much higher than $\frac{1}{2}$ of the max value then meaning that the image is much brighter and has a lower contrast this will tend to increase the mean intensity of the image which will lead to a much more incomprehensible image. One of the main reasons why this happens is because of calculation of different cumulative distribution function for both the histograms.

3 (d)

The code for this has been included in *myMainScript.m* file.

Original Image:

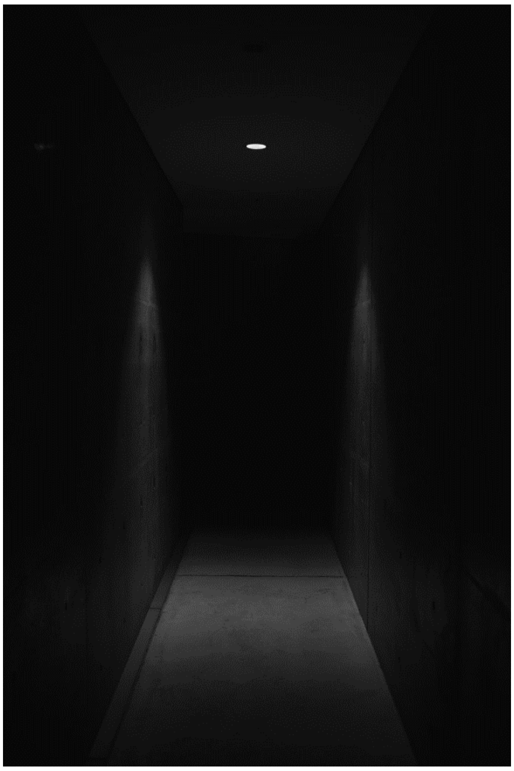
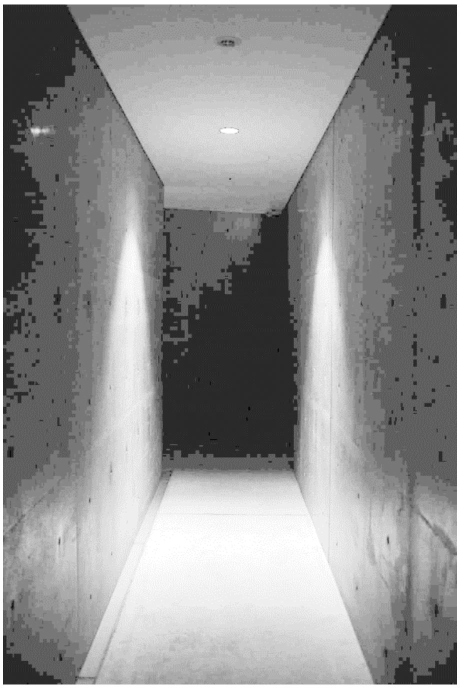
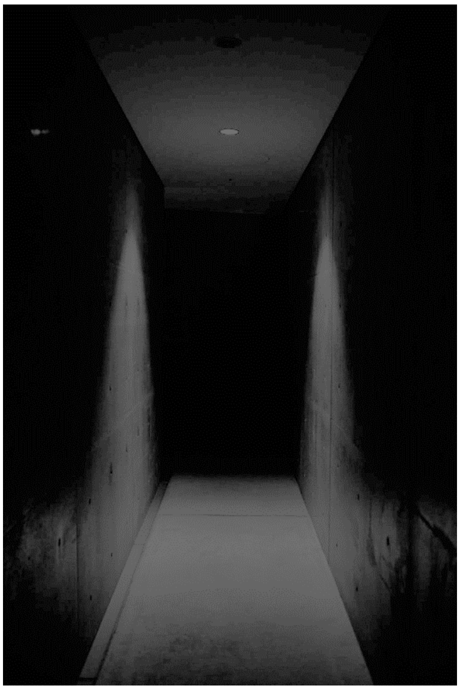


Image transformed using histogram equalization:



Here the image has a lot of noise compared to the original image and the image has been transformed to a different type of image altogether.

Image transformed using the modified intensity transformation method:



Here we can see the lighting in the hall is much better and the hall is visible much clearer than the origin or the transformed image using histogram equalization.