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Q2. Given :

e is a unit vector which forms a basis for a vector space such that projecting x_i's on this space gives a projection of them that is their best approximation.

It turns out that e is an eigen vector corresponding to largest eigen-value 'λ_1' of covariance matrix 'C' of vectors x_i. by maximizing $e^T C e$.

To prove :

f which is a unit vector $\perp$ to e & forms another basis vector of vector space onto which x_i are projected for which $f^T C f$ is maximized, is the eigen vector of C with second largest eigen value assuming that all non-zero eigen values of C are distinct & $\text{rank}(C) > 2$

Proof :

To maximize $f^T C f$ with below two conditions

(i) $f^t f = 1$

(ii) $f^t e = 0$

We use method of Lagrange multipliers

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$$J(f) = f^T C f - \lambda(f^T f - 1) + \delta(f^T e)$$

Taking derivative of $J(f)$ w.r.t f and setting it zero

$$2Cf - \lambda 2f - \delta e = 0 \longrightarrow (1)$$

Finding λ & δ

We know

$$Ce = d_1 e$$

$$e^T C^T = d_1 e^T$$

... transpose on both sides

$$e^T C^T f = d_1 e^T f$$

... postmultiply by f

$$e^T C^T f = 0 \longrightarrow (2)$$

....

$$e^T f = 0$$

Premultiplying by e^T ~~& postmultiplying by~~ to (1)

$$2e^T C f - \lambda 2 e^T f - \delta e^T e = 0$$

$$0 - 0 - \delta = 0$$

.... from (2) & $e^T e = 1$ & $e^T f = 0$

$$\therefore \boxed{\delta = 0}$$

Substituting 's in eqⁿ (1)

$$2CF - \lambda 2F = 0$$

$$\therefore \quad CF = \lambda F$$

so 'λ' is an eigen value of C having 'F' as eigen vector.

$$\boxed{F^T C F = \lambda F^T F = \lambda}$$

but we have to maximize $F^T C F$; so λ must be second largest eigen value as the first largest i.e λ_1 corresponds to eigen vector e.

Hence F ~~corres~~ is an eigen vector of C ~~on~~ corresponding to second largest eigen value.