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Q2. a.

$$g(x) = (h * f)(x)$$

$$G(u) = H(u)\, F(u)$$

.... taking Fourier Transform and using convolution thm.

here
$$G(u) = \mathcal{F}(g(x))$$
$$H(u) = \mathcal{F}(h(x))$$
$$F(u) = \mathcal{F}(f(x))$$

$$F(u) = \frac{G(u)}{H(u)}$$

$$f(x) = \mathcal{F}^{-1}\left(\frac{G(u)}{H(u)}\right)$$

..... taking Inverse Fourier Transform

Note: $h(x)$ is a gradient kernel thus $H(u)$ is high pass filter.

Difficulties:

① For low frequencies $H(u) \to 0$ and if $H(u) = 0$ then low frequency component of $F(u)$ cannot be calculated

② Even if $H(u) \neq 0$, a small noise along with $H(u) \to 0$ will blow up value of $F(u)$ amplifying ~~error~~ noise in $f(x)$

Q2. b. Consider below conventions

g_x : gradient of 2d image along x-axis,
g_y : gradient of 2d image along y-axis
h_x : kernel for gradient along 'x' axis
h_y : kernel for gradient along 'y' axis

$$g_x(x,y) = (h_x * f)(x,y)$$
$$g_y(x,y) = (h_y * f)(x,y)$$

.... Fourier Transform of above eqⁿ along with convolution thm.

$$G_x(u,v) = H_x(u,v)\ F(u,v)$$
$$G_y(u,v) = H_y(u,v)\ F(u,v)$$

$$\therefore \quad F(u,v) = \frac{G_x(u,v)}{H_x(u,v)} \longrightarrow (1)$$

$$F(u,v) = \frac{G_y(u,v)}{H_y(u,v)} \longrightarrow (2)$$

..... taking Inverse Fourier Transform

$$f(x,y) = \mathcal{F}^{-1}\left(\frac{G_x(u,v)}{H_x(u,v)}\right) \longrightarrow (3)$$

$$f(x,y) = \mathcal{F}^{-1}\left(\frac{G_y(u,v)}{H_y(u,v)}\right) \longrightarrow (4)$$

Note: $h_x(x,y)$ & $h_y(x,y)$ are gradient kernels thus H_x & H_y are high pass filters in 'u' & 'v' respectively

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Useful facts discovered.

① When u is small $H_x(u,v)$ will be small and calculating $F(u,v)$ from eqn ① will not be appropriate as its value would blow up but equn ① can be used for large u and small or large v.

② Similarly as above equn ② can be used for large v and small or large u.

③ Thus when both u & v are large we can use either of equn ① or ②; when u is large & v is small we will use equn ① and when v is large & u is small make use of equn ②

④ Problem arises when both u & v are small & both of the equn (① & ②) blow up (attain very very large values)

Difficulties:
① When we have low frequencies in both u & v and if both $H_x(u,v)$ and $H_y(u,v)$ become 0 then low frequency components of $F(u,v)$ can not be extracted.
② Even if any one of H_x or H_y doesn't become 0 then too a small amount of noise gets amplified as whichever non-zero $H \to 0$ leading to blow-up of $F(u,v)$.