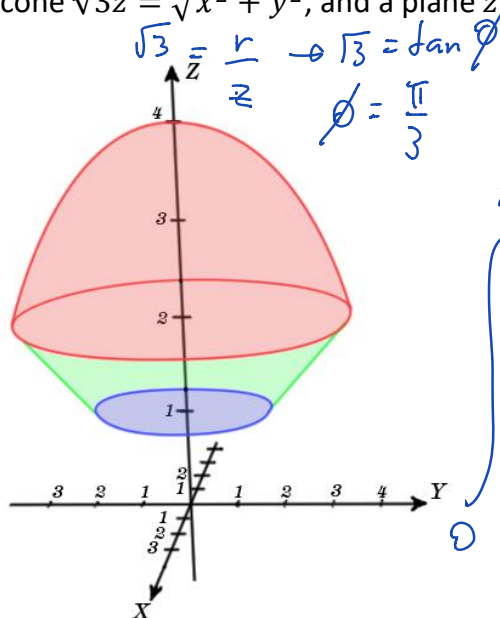


Post-test : Triple integrals in spherical coordinates

$$r^2 = 16 \rightarrow r = 4$$

1. Find a volume of a solid covered by a sphere $x^2 + y^2 + z^2 = 16$,

a cone $\sqrt{3}z = \sqrt{x^2 + y^2}$, and a plane $z = 1$.



$$\int_0^{2\pi} \int_0^{\pi/3} \int_1^4 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

unit³

$$= \int_0^{2\pi} \int_0^{\pi/3} \left[\frac{\rho^3}{3} \sin \phi \right]_1^4 \, d\phi \, d\theta = \int_0^{2\pi} \int_0^{\pi/3} \sin \phi \left[\frac{64}{3} - \frac{1}{3} \right] \, d\phi \, d\theta$$

$$= \frac{63}{3} \int_0^{2\pi} \left[-\cos \phi \right]_0^{\pi/3} \, d\theta = \frac{63}{3} \int_0^{2\pi} \left[-\frac{1}{2} - (-1) \right] \, d\theta$$

$$= \frac{63}{6} \int_0^{2\pi} 1 \, d\theta = \frac{63}{6} \left[\theta \right]_0^{2\pi} = \frac{63}{6} [2\pi]$$

$$= 21\pi \text{ unit}^3$$