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01/28/2022

HW 0

1. Sums and Products

a. 1. $\sum_{i=0}^{N} 1 = N$

2. $\sum_{k=1}^{K} \sum_{t=1}^{T} 1 = KT$

3. $\sum_{k=1}^{K} \sum_{t=1}^{T} 0.5^k = T(0.5 + 0.5^2 + 0.5^3 + \cdots + 0.5^k)$

$$= T\left(\frac{0.5(1-0.5^k)}{0.5}\right)$$

$$= T - T\,0.5^k$$

4. $\sum_{k=1}^{\infty} \sum_{t=1}^{T} 0.5^k = T(0.5 + 0.5^2 + \cdots + 0.5^\infty)$

$$= T\left(\frac{0.5}{1-0.5}\right)$$

$$= T$$

$S = \frac{a_1}{1-r} \quad (|r|<1)$

b. 1. $\prod_{i=1}^{m} \frac{1}{\theta} = \frac{1}{\theta^m}$

2. $\prod_{k=1}^{K} \frac{k}{k+1} = \frac{\prod_{k=1}^{k} k}{\prod_{k=1}^{k} k+1} = \frac{k!}{(k+1)!} = \frac{1}{k+1}$

3. $\ln\left(\prod_{k=1}^{K} e^k\right) = \sum_{k=1}^{K} \ln e^k = \frac{(1+K)K}{2}$

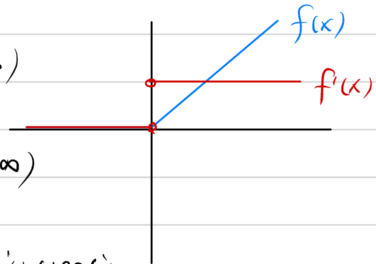
2. Asymptotics and Trends

1. Δ $\lim_{x \to -\infty} f(x) = 0$ $\quad \lim_{x \to \infty} f(x) = \infty$

Δ $f(x)$ is differentiable on $(-\infty, 0), (0, +\infty)$

Δ $f'(x)$ is differentiable on $(-\infty, 0), (0, +\infty)$

Δ on $(0, +\infty)$ $f(x)$ is strictly increasing

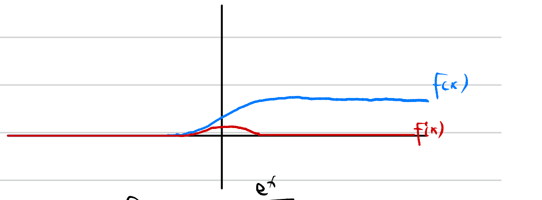


2. △ $\lim_{x \to -\infty} f(x) = 0$ $\quad \lim_{x \to \infty} f(x) = 1$

△ $f(x)$ is differentiable on $(-\infty, +\infty)$

△ $f(x)$ is differentiable on $(-\infty, +\infty)$

△ on $(-\infty, +\infty)$ $f(x)$ is strictly increasing.



$$f'(x) = \frac{e^x}{(e^x+1)^2} > 0 \text{ on } (-\infty, +\infty)$$

3. Points and Vectors

1. ∘ $\sqrt{0.4^2 + 0.3^2} = 0.5$

∘ $\sqrt{(-0.15)^2 + (0.2)^2} = 0.25$

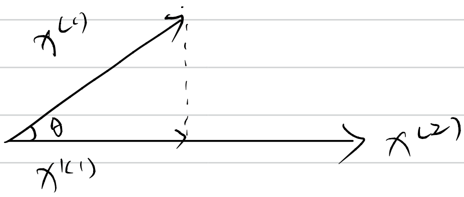
∘ $\cos\theta = \dfrac{a \cdot b}{\|a\|\|b\|} = \dfrac{(0.4 \times -0.15) + (0.3 \times 0.2)}{0.5 \times 0.25} = 0$

$\therefore \theta = \frac{\pi}{2}$

2. when $x^{(1)} \cdot x^{(2)} = 0$, $x^{(1)}$ is orthogonal to $x^{(2)}$

3. $\dfrac{x}{\|x\|}$

4. $x^{(2)}$



5. let $x'^{(1)}$ be the projection of $x^{(1)}$ onto $x^{(2)}$

$$\begin{aligned} x'^{(1)} &= \|x'^{(1)}\| u \\ &= \|x^{(1)}\| \cos\theta \, u \\ &= \|x^{(1)}\| \frac{x^{(1)} \cdot x^{(2)}}{\|x^{(1)}\| \|x^{(2)}\|} u \\ &= \frac{x^{(1)} \cdot x^{(2)}}{\|x^{(2)}\|} u \end{aligned}$$

$$\therefore c = \frac{x^{(1)} \cdot x^{(2)}}{\|x^{(2)}\|}$$

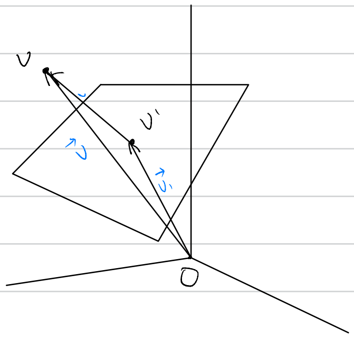
4. Planes

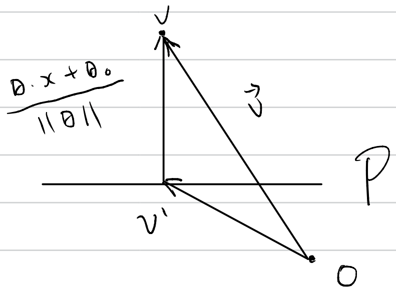
1. ∞

2. if $x \parallel \theta \Rightarrow \exists \lambda,\ x = \lambda\theta$, x is orthogonal to P

3. signed dis $= \dfrac{\theta \cdot x + \theta_0}{\|\theta\|}$

4. $v' = \dfrac{\theta \cdot v + \theta_0}{\|\theta\|} - v$





5. 1D Optimization

1. $f'(x) = x^2 - 2x - 3$

$= (x+1)(x-3)$ and $x_1 < x_2$

$\therefore\ x_1 = -1 \quad x_2 = 3$

$f''(x) = 2x - 2 \qquad f''(x_1) = -4 \qquad f''(x_2) = 4$

2&3 $f(x) \uparrow$ on $[-4, x_1)$

$f(x) \downarrow$ on (x_1, x_2)

$f(x) \uparrow$ on $(x_2, 4]$

4. $f(x)$ min on $[-4, 4]$ is $f(-4) = -\frac{46}{3}$

5. $f(x)$ max on $[-4, 4]$ is $f(-1) = \frac{35}{3}$

6. $f_1''(x) = 0$ on R not strictly concave

$f_2''(x) = -e^{-x} < 0$ on R strictly concave

$f_3''(x) = -\dfrac{99}{10000 x^{\frac{101}{100}}} < 0$ on $(0, \infty)$ strictly concave

$f_4''(x) = 2 > 0$ on R not strictly concave