CS113/DISCRETE MATHEMATICS-SPRING 2024

Worksheet 6

Topic: Laws Of Inference For Quantified Statements

Building upon your understanding of the laws of inference, we will now learn these laws for quantified statements.Happy Learning!

Student's Name and ID: ____________________

Instructor's name: ____________________

Table 1: Rules of Inference for Quantified Statements

Rule of Inference	Name
$\forall x P(x)$ $\therefore P(c)$	Universal instantiation
$P(c)$ for an arbitrary c $\therefore \forall x P(x)$	Universal generalization
$\exists x P(x)$ $\therefore P(c)$ for some element c	Existential instantiation
$P(c)$ for some element c $\therefore \exists x P(x)$	Existential generalization

1. Identify the error or errors in this argument that supposedly shows that if $\exists x P(x) \wedge \exists x Q(x)$ is true, then $\exists x(P(x) \wedge Q(x))$ is true.

    1. $\exists x P(x) \vee \exists x Q(x)$ **Premise**
    2. $\exists x P(x)$ **Simplification from (1)**
    3. $P(c)$ **Existential instantiation from (2)**
    4. $\exists x Q(x)$ **Simplification from (1)**

5. $Q(c)$ **Existential instantiation from (4)**
6. $P(c) \land Q(c)$ **Conjunction from (3) and (5)**
7. $\exists x(P(x) \land Q(x))$ **Existential generalization**

2. Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is also true, where the domains of all quantifiers are the same.

3. Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\neg Q(x) \vee S(x))$, $\forall x(R(x) \rightarrow \neg S(x))$, and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.