CS113/DISCRETE MATHEMATICS-SPRING 2024

Worksheet 4

Topic:Nested Quantifiers

Get ready to learn nested quantifiers, where one quantifier occurs within the scope of another quantifier. We will further explore why the order of nested quantifiers is important.
Happy Learning!

Student's Name and ID: ____________________

Instructor's name: ____________________

1 Nested Quantifiers.

Statement	When True	When False
$\forall x \forall y P(x, y)$	For every pair x, y $P(x, y)$ is true	There is a pair x, y for which $P(x, y)$ is false
$\forall y \forall x P(x, y)$	For every pair x, y $P(x, y)$ is true	There is a pair x, y for which $P(x, y)$ is false
$\forall x \exists y P(x, y)$	For every x there is a y	There is an x such that $P(x, y)$ is false
$\exists x \forall y P(x, y)$	There is an x for which $P(x, y)$ is true for every y	For every x there is a y such that $P(x, y)$ is false
$\exists x \exists y P(x, y)$	There is a pair x, y for which $P(x, y)$ is true	$P(x, y)$ is false for every pair x, y
$\exists y \exists x P(x, y)$	There is a pair x, y for which $P(x, y)$ is true	$P(x, y)$ is false for every pair x, y

1. Express the negations of each of these statements so that all negation symbols immediately precede predicates.
   (a) $\exists z \forall y \forall x T(x, y, z)$

(b) $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$

(c) $\exists x \exists y (Q(x, y) \leftrightarrow Q(y, x))$

(d) $\forall y \exists x \exists z (T(x, y, z) \vee Q(x, y))$

2. Show that $\forall x P(x) \wedge \exists x Q(x)$ is logically equivalent to $\forall x \exists y (P(x) \wedge Q(y))$, where all quantifiers have the same nonempty domain.

3. Show that $\neg\exists x \forall y P(x, y)$ and $\forall x \exists y \neg P(x, y)$ are logically equivalent, where both quantifiers over the first variable in $P(x, y)$ have the same domain, and both quantifiers over the second variable in $P(x, y)$ have the same domain.