CS113/DISCRETE MATHEMATICS-SPRING 2024

Worksheet 2

Topic: Logic And Proofs

Use the given tables of laws/Truth tables to construct equivalences between compound propositions. Happy Learning!

Student's Name and ID: ____________________

Instructor's name: ____________________

1 Laws Of Logical Equivalences:

Equivalence	Name
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	Negation laws

Equivalence	Rule
Conditional Law 1	$p \to q \equiv \neg p \lor q$
Conditional Law 2	$p \to q \equiv \neg q \to \neg p$
Conditional Law 3	$p \lor q \equiv \neg p \to q$
Conditional Law 4	$p \land q \equiv \neg(p \to \neg q)$
Conditional Law 5	$\neg(p \to q) \equiv p \land \neg q$
Conditional Law 6	$(p \to q) \land (p \to r) \equiv p \to (q \land r)$
Conditional Law 7	$(p \to r) \land (q \to r) \equiv (p \lor q) \to r$
Conditional Law 8	$(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$
Conditional Law 9	$(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

Equivalence	Rule
Biconditional Law 1	$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$
Biconditional Law 2	$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
Biconditional Law 3	$p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$
Biconditional Law 4	$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

1. Show that following conditional statements are tautologies. (use laws of equivalences and not the truth table to prove it.)

   (a)

$$(\neg p \land (p \lor q)) \to q$$

   (b)

$$((p \to q) \land (q \to r)) \to (p \to r)$$

(c)

$$(p \wedge (p \to q)) \to q$$

(d)

$$((p \vee q) \wedge (p \to r) \wedge (q \to r)) \to r$$

2. Show using truth table that $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent.

3. Show using truth table that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is tautology.