CS113/DISCRETE MATHEMATICS-SPRING 2024

Worksheet 3

Topic: Predicates And Quantifiers

Now, you will explore additional rules of equivalences involving quantifiers. Utilize these rules along with the previous rules to establish the validity of quantified statements. Happy Learning!

Student's Name and ID: ____________________

Instructor's name: ____________________

1 Table 1: Quantifiers.

Statement	When True?	When False?
$\forall x P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

2 Table 2: De Morgan's Laws for Quantifiers.

Negation	Equivalent Statement	When is Negation True?
$\neg\exists x P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.
$\neg\forall x P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is true.

1. Suppose that the domain of the propositional function P(x) consists of the integers 0, 1, 2, 3, and 4. Write out each of these propositions using disjunctions, conjunctions, and negations.
   (a) $\exists x P(x)$

   (b) $\forall x P(x)$

(c) $\exists x \neg P(x)$

(d) $\forall x \neg P(x)$

(e) $\neg \exists x P(x)$

(f) $\neg \forall x P(x)$

2. . Express the negation of these propositions using quantifiers, and then express the negation in English.
   (a) Some drivers do not obey the speed limit.

   (b) All Swedish movies are serious.

   (c) No one can keep a secret

(d) There is someone in this class who does not have a good attitude.

3. Determine whether $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall x P(x) \leftrightarrow \forall x Q(x)$ are logically equivalent. Justify your answer.

4. Show that $\forall x P(x) \vee \forall x Q(x)$ and $\forall x(P(x) \vee Q(x))$ are not logically equivalent.