

RunningTrials writeup

Sunday, December 1, 2019 10:46 PM

(a) Describe the optimal substructure/recurrence that would lead to a recursive solution

- Our strategy would be to test the lowest speed up to the highest speed.
- First, base cases should be explained. Base case 1, is made up of two edge cases. If we have one speed or 0 speeds no matter how many days we can only test 1 or 0 speeds.
- Set a variable "minTest" to infinity as an upper bound. Also create variable "maxText".
Go through 1 to n possible speeds and check for two scenarios at each iteration.
- At each iteration make a recursive call to check for no injury an injury. If there is no injury check for the same day and the remaining speeds to be tested. If there is a injury check one speed and day lower.
- For example, 6 speeds 2 days. 1st iteration, i=1 no injury at speed one so minus speeds(6) and I and keep days the next recursive call would be 5 speeds 2 days. Same iteration, injury scenario means make recursive call to test a lower speed and lose one day due to injury.
- For each iteration find the maximum speed we can reach with no injury. Compare that to the other iterations and get the minimum. That is why we set "minTest" initially to infinity as an upper bound.
- Pseudo code provided below.
- Recurrence relation,
 - $T(n \leq 1, m)$ return n
 - $T(n, m = 1)$ return n
 - $T(n, m) = C + T(n - 1, m) + T(n - 1, m - 1)$

```
runTrialsRecur(int possibleSpeeds, int days) {
    int minTests = 0;

    int maxTest = 0;
    // Base case 1
    // if we have one speed no matter how many days I can only test one speed
    if(possibleSpeeds == 1 || possibleSpeeds == 0)
        return possibleSpeeds;

    // Base Case 2
    // if we have only one day at worst we have to test all speeds
    if(days == 1)
        return possibleSpeeds;

    minTests = infinity;
    for( i = 1 to possibleSpeeds){
        // Test for 2 possible scenarios, injury and no injury
        // No injury, test the same amount of days and remaining speeds
        // injury, try prev speed and lost a day
        // add a 1 every call is one possible trial
        maxTest = 1 + max(
            runTrialsRecur(possibleSpeeds - i, days), runTrialsRecur(i - 1, days - 1));

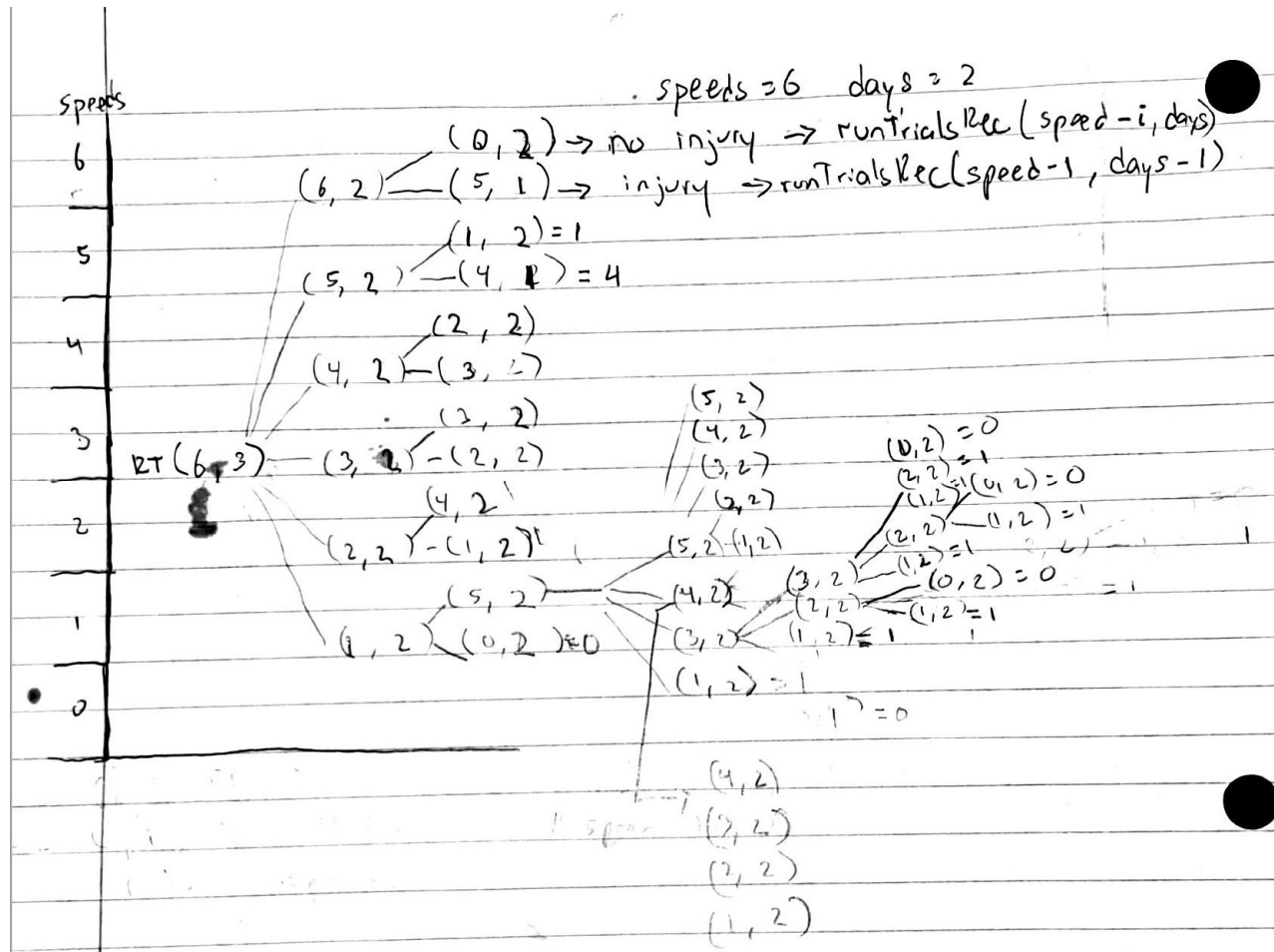
        minTests = min(maxTest, minTests);
    }
}
```

```

return minTests;
}

```

(c) Draw recurrence tree for given (# speeds = 6, # days = 2)



(d) How many distinct subproblems do you end up with given 6 speeds and 2 days?

$$O(mn^2) \Rightarrow 2 \cdot 4^2 = 32$$

(e) How many distinct subproblems for N speeds and M days?

$$O(mn^2),$$

(f) Describe how you would memoize `runTrialsRecur`.

Pseudo code provided below.

```

runTrialsRecurMemo(int possibleSpeeds, int days) {
    int minTests = 0;

    int maxTest = 0;

```

```

// 2D table is days + 1 x possibleSpeeds + 1
// the plus one is because we will use the previous table cells(sub problems) if they are solved already.
// This is how we use memoization
// fill from 2nd row and column to infinity. It can be done here or create a method for it
    int [][] table = new int[days + 1][possibleSpeeds+1];

// Base case 1
// if we have one speed no matter how many days I can only test one speed
    if(possibleSpeeds == 1 || possibleSpeeds == 0)
        return possibleSpeeds;

// Base Case 2
// if we have only one day at worst we have to test all speeds
    if(days == 1)
        return possibleSpeeds;

// Before going in to loop check if we have solved it already. If is not infinity
//we have solved it already return the value
    if(table[days][possibleSpeeds] != infinity)
        return table[days][possibleSpeeds];

// Test for 2 possible scenarios, injury and no injury
// No injury, test the same amount of days and remaining speeds
// injury, try prev speed and lost a day
// add a 1 every call is one possible trial
// store the min to the table and return table

    for( i = 1 to possibleSpeeds){
        maxTest = 1 + max(
            runTrialsRecurMemo(possibleSpeeds - i, days), runTrialsRecurMemo(i- 1, days - 1));

        table[days][possibleSpeeds] = min(maxTest, table[days][possibleSpeeds] );
    }
    return table;
}

```