Topic # 2

Mechanics

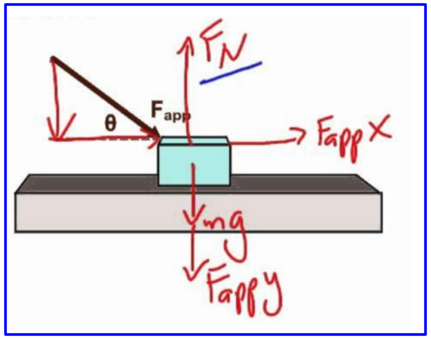
(part 2: Dynamics)

Lecture Course (online)
Dalian, 2021

Contents

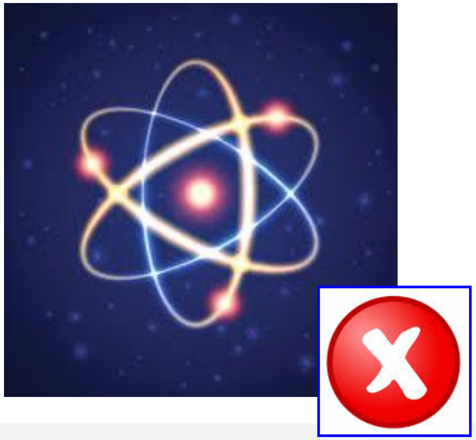
Basic Concepts of Dynamics

Up to now we have restricted ourselves to **describe** the motion of a point mass **without** investigating the **primary cause** of the motion.



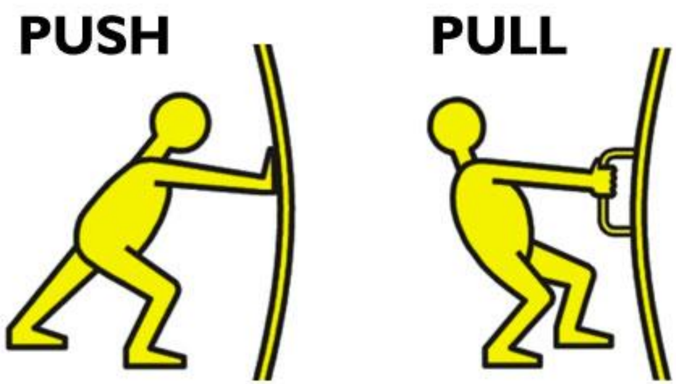
Dynamics is the study of laws (reasons) that determine the motion of material bodies.

Now the **goal** is to develop procedures by which one can derive the explicit movement of the mass point from a **known** driving **cause**. As long as the system under study doesn't involve objects comparable in size to an atom or travelling close to the speed of light, classical mechanics provides excellent description of nature.



Forces and Interactions

When formulating the fundamental laws of dynamics, we have to introduce the new term – **force**.



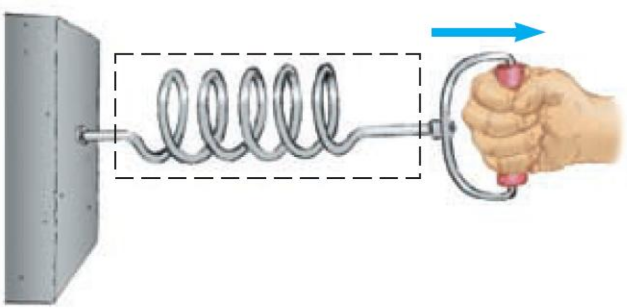
Can be defined **indirectly** via its effect:

- if we want to **change the state of motion** or **the shape of a body**, for instance, by exertion of our muscles, it needs an **effort**.
- this effort is called "**force**".

SI unit : (N)

Note: force is a vector quantity because you can push or pull an object in different directions.

Contact forces



Field forces

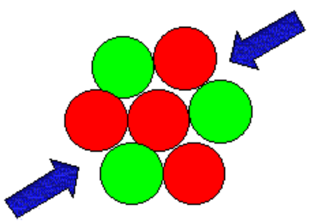
Forces and Interactions

The known **fundamental** forces in nature are all **field** forces!

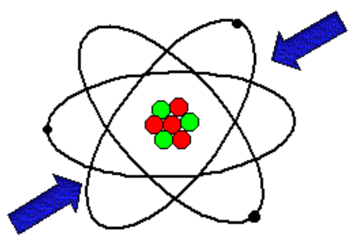
decreasing strength

- the **strong** nuclear force between the subatomic particles
- **electromagnetic** forces between the electric charges
- the **weak** nuclear force, which arises in some radioactive decay processes
- the **gravitational** force between the objects

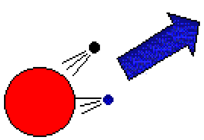
very short range $\sim 10^{-15}\,m$



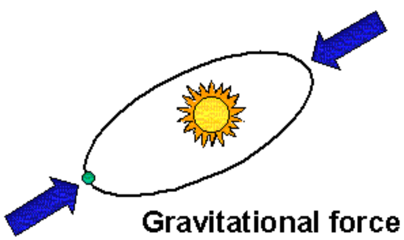
Strong force binds the nucleus



Electromagnetic force binds atoms



Weak force in radioactive decay



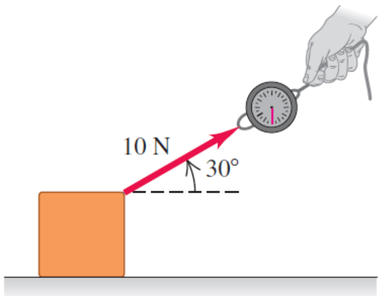
Gravitational force binds the solar system

Note: more precisely, a force is an interaction between to objects or between an object and its environment.

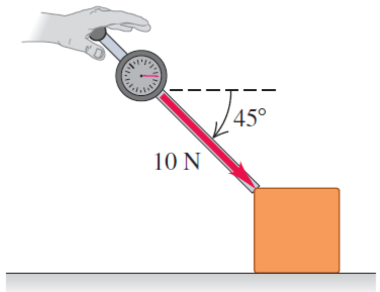
Principle of Superposition

A common instrument for measuring force magnitudes is the spring balance. It consists of a coil spring enclosed in a case with pointer attached to one end.

(a) A 10 N pull directed 30° above the horizontal



(b) A 10 N push directed 45° below the horizontal



- when forces are applied to the ends of the spring, it stretches by an amount that depends on the force
- one can make a scale for the pointer by using a number of identical objects

Experiments show that when two or more forces act at the same time at the same point on an object, the effect on the object's motion is the same as if a single force which is a vector sum, or resultant, were acting on that object.

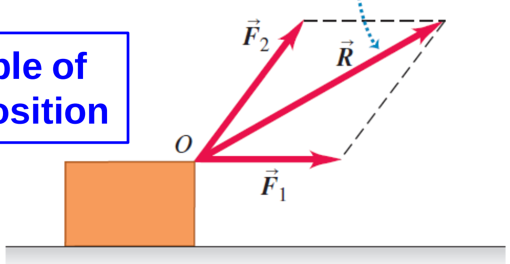
The net force acting on an object ...

$$\vec{R} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

... is the vector sum, or resultant, of all individual forces acting on that object.

Principle of superposition

Two forces $\vec{F}_1$ and $\vec{F}_2$ acting on an object at point O have the same effect as a single force $\vec{R}$ equal to their vector sum.



Approximate Force Values

Typical Force Magnitudes

Sun's gravitational force on the earth	3.5×10^{22} N
Weight of a large blue whale	1.9×10^{6} N
Maximum pulling force of a locomotive	8.9×10^{5} N
Weight of a 250 lb linebacker	1.1×10^{3} N
Weight of a medium apple	1 N
Weight of the smallest insect eggs	2×10^{-6} N
Electric attraction between the proton and the electron in a hydrogen atom	8.2×10^{-8} N
Weight of a very small bacterium	1×10^{-18} N
Weight of a hydrogen atom	1.6×10^{-26} N
Weight of an electron	8.9×10^{-30} N
Gravitational attraction between the proton and the electron in a hydrogen atom	3.6×10^{-47} N

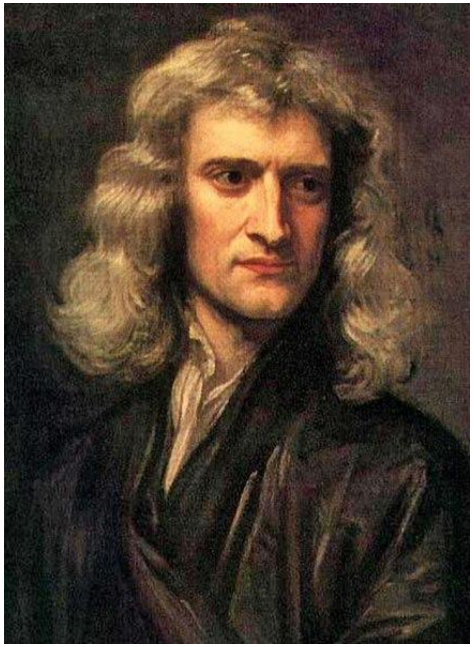
Newton's Laws of Motion

In 1687, Isaac Newton published one of the greatest scientific works of all time: *The Mathematical Principles of Natural Philosophy*. In this work he stated the three laws of motion that form the **basis of classical mechanics**.

The 1st law or Galilei's law of inertia.

The 2nd law or the equation of motion.

The 3rd law or action equals reaction.



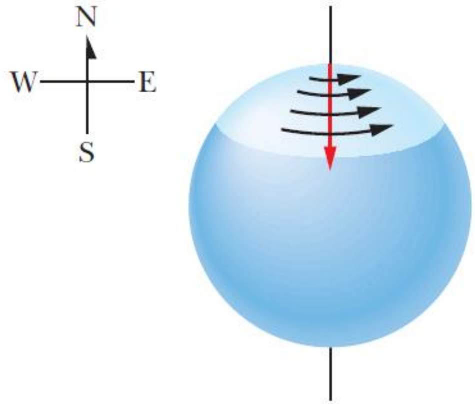
Isaac Newton
(1643-1727)

Newton's First Law

The 1st law or Galilei's law of inertia:
There exist frames of reference in which a force-free body (point particle) persists in the state of rest or in the state of uniform straight-line motion. Such reference frames shall be called **inertial** (IFR).

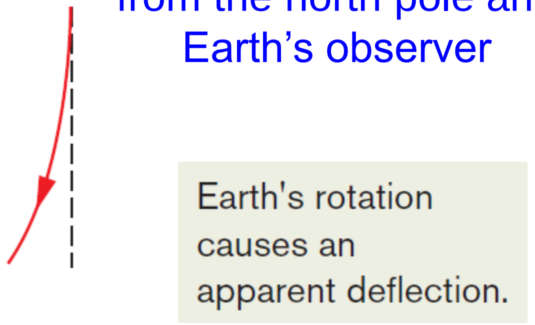
Example: we can approximately assume that the ground is an IFR provided we can neglect Earth's astronomical motions (such as its rotations).

works well for short strip of frictionless ice



ok for stationary frame in space…

… but does not work for long ice strip extending from the north pole and Earth's observer



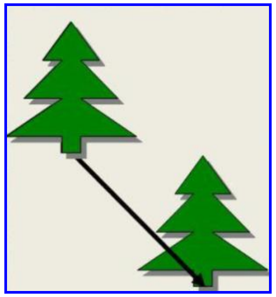
Inertial Frames of Reference

Why do we need the inertial frames of reference?

- kinematics describes the motion **irrespective** to its **causes** and makes **no** essential difference between various frames of reference considering them as **equivalent**
- dynamics is **investigating** the **causes** of motion and deals with the laws of motion. That’s why it is important to identify **advantages** of one class of frames over the others
- the laws of mechanics may have different form in different frames of reference, so that the laws governing simple phenomena may become be very complicated

Note: within IFR **time** and **space** possess definite **symmetry** properties.

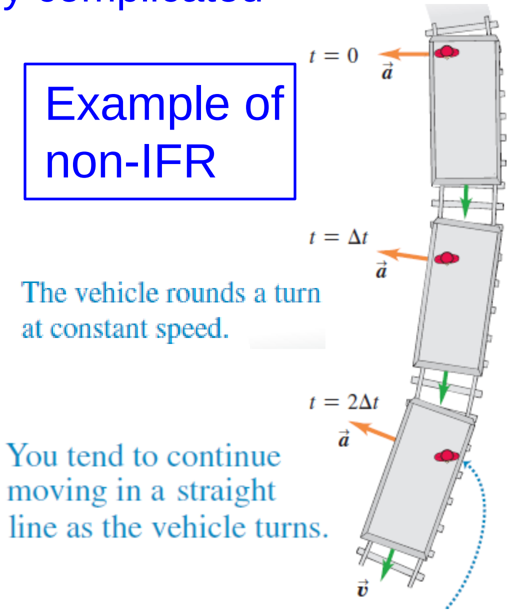
time is uniform



space is uniform and isotropic

Note: there is no single IFR that is preferred over all others.

Example of non-IFR



Inertial Frames of Reference

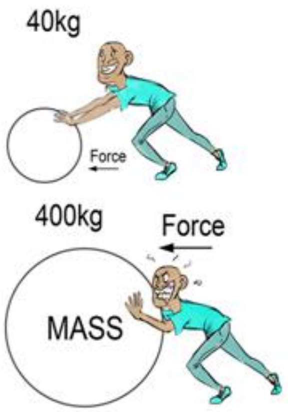
QUIZ

Check your understanding:

In which of the following situations is there zero net external force on the object?
(i) an airplane flying due north at a steady 120 m/s and at a constant altitude;
(ii) a car driving straight up a hill with a 3º slope at a constant 90 km/h;
(iii) a hawk circling at a constant 20 km/h at a constant height of 15 m above an open field;
(iv) a box with slick, frictionless surfaces in the back of a truck as the truck accelerates forward on a level road at 5 m/s^2.

Mass

Everyday experience: applying a **given** force to **different** bodies results in **different accelerations**; every body "**resists**" any effort to change its velocity, both in magnitude and direction.



- the tendency of an object to keep moving once it is set in motion is called **inertia**
- **mass** is a quantitative measure of inertia (a body with a greater mass is more inert)
- the **ratio** of masses of two different bodies is the **inverse** ratio of accelerations given to them by **equal** forces

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

Note: we do not require any preliminary measurements of forces!

- having adopted a certain body for a mass **standard**, we may compare the mass of **any** body against the standard
- (a) mass is an **additive** quantity; (b) mass is a **constant** quantity

Newton's Second Law

All the definitions, experiments, and observations so can be summarized in a one neat statement:

The 2nd law or the equation of motion:
The net force acting on a body is equal to the product of the body's mass and its acceleration

$$\boxed{\vec{F}_{\text{net}} = m\vec{a}}$$

$$1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2$$

One newton is the amount of net external force that gives an acceleration of 1 meter per second squared to an object with a mass of 1 kilogram.

this expression has the form of a **differential equation** from which it is possible to derive the law of motion of a particle if the net force is known

$$\vec{a} = \ddot{\vec{r}} = \frac{\vec{F}(\vec{r}, \dot{\vec{r}}, t)}{m} \equiv \vec{f}(\vec{r}, \dot{\vec{r}}, t) \quad \Rightarrow \quad \vec{r}(t)$$

Note: it is impossible for an object to affect its own motion by exerting a force on itself. The forces that affect an object's motion are **external** forces, i.e. the forces exerted on the object by other objects in its environment.

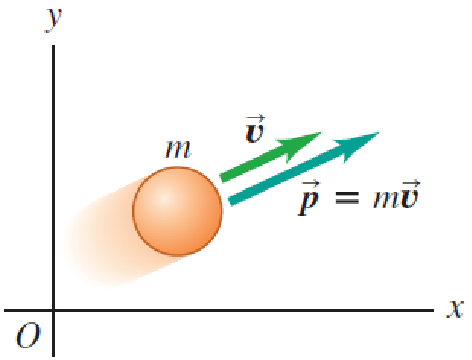
Linear Momentum

- generalization for the case of a varying mass can be done by writing Newton's 2nd law in terms of (linear) **momentum**:

$$\vec{F} = \dot{\vec{p}}, \quad \vec{p} \equiv m\vec{v}$$

Note: we introduce **momentum** as a product of particles's mass onto its velocity

- the net external force acting on a particle equals the rate of change of the particle's momentum



Momentum $\vec{p}$ is a vector quantity; a particle's momentum has the same direction as its velocity $\vec{v}$.

- obviously, if the net external force acting on a particle equals 0, the particle's momentum is conserved

$$\vec{F} = 0 \quad \Rightarrow \quad \dot{\vec{p}} = 0 \quad \Rightarrow \quad \vec{p} = const$$

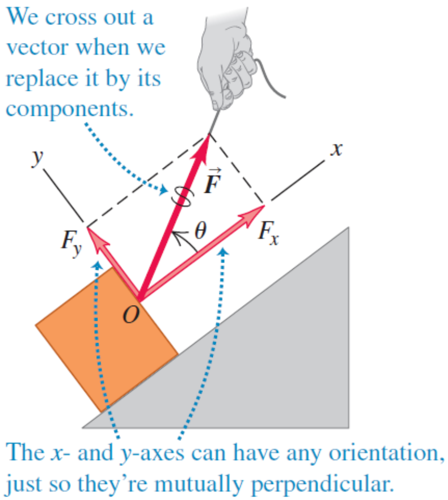
- another option – conservation of projection of momentum onto some direction: $|\vec{n}| = 1$

$$\vec{F} \perp \vec{n} \quad \Rightarrow \quad p_n = const \quad (p_n = \vec{p} \cdot \vec{n})$$

Newton's Second Law

QUIZ Check your understanding:

Figure shows a force **F** acting on a crate. (a) With the x- and y-axes shown in the figure, is the x-component of the gravitational force that the earth exerts on the crate (the crate's weight) positive, negative, or zero? (b) What about the y-component?



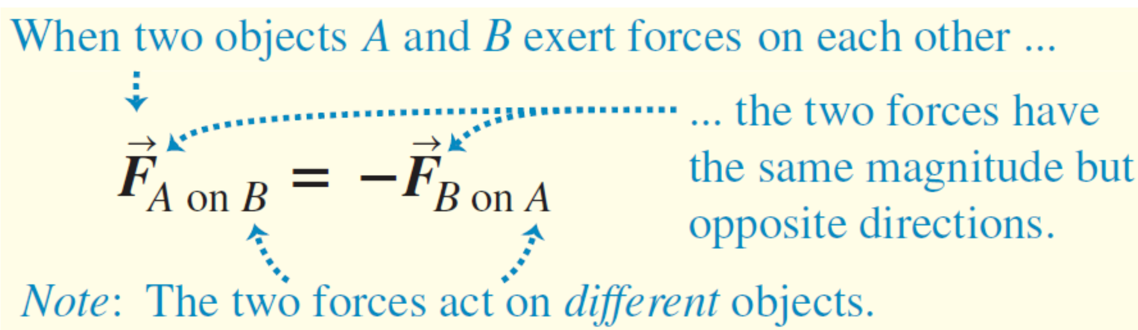
QUIZ Check your understanding:

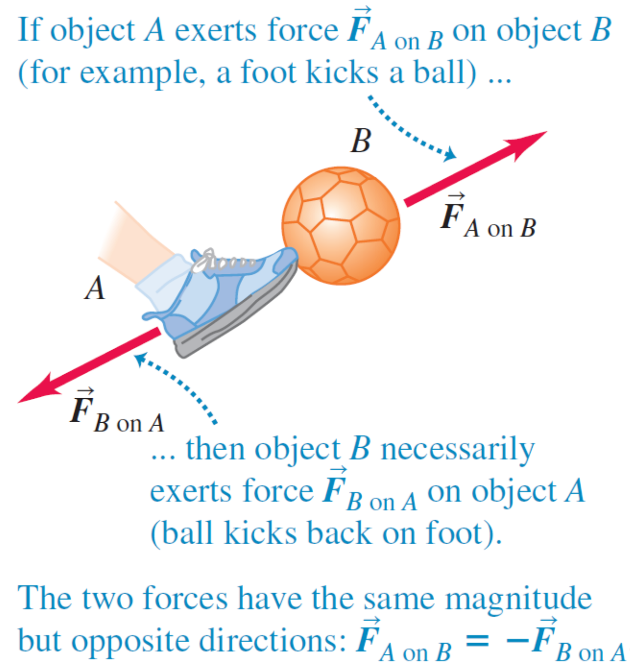
Rank the following situations in order of the magnitude of the object's acceleration, from lowest to highest. Are there any cases that have the same magnitude of acceleration? (i) A 2.0 kg object acted on by a 2.0 N net force; (ii) a 2.0 kg object acted on by an 8.0 N net force; (iii) an 8.0 kg object acted on by a 2.0 N net force; (iv) an 8.0 kg object acted on by a 8.0 N net force.

Newton's Third Law

In all experiments involving only two bodies *A* and *B*, if body *A* imparts acceleration to *B*, it turns out that *B* imparts acceleration to *A*.

The 3rd law or action equals reaction:
Action and reaction are equal in magnitude and opposite in direction.





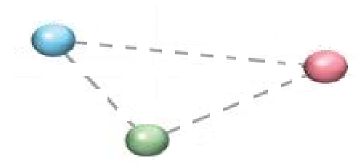
this law governs the dynamics of a system of two or more point particles

Note: it is important to stress that the two forces described in this law act on **different** objects.

Note: the **instantaneous** propagation of interactions (aka long-range action) is assumed.

Dynamics of a System of Point Particles

Motion of a system of *N* point masses interacting with each other and some external objects (not included in the system) obeys the following equations of motion:



$$m_\alpha \dot{\vec{v}}_\alpha = \vec{F}_{a,\mathrm{ext}} + \sum_{\beta\neq\alpha}^{N} \vec{F}_{\alpha\beta,\mathrm{int}} \qquad (\alpha = 1, 2, \ldots, N)$$

external force

internal forces

EXERCISE

$$\sum_{\alpha=1}^{N} m_\alpha \dot{\vec{v}}_\alpha \quad \Rightarrow \quad \boxed{\dot{\vec{P}} = \vec{F}_{\mathrm{ext}}}$$

$$\vec{P} \equiv \sum_{\alpha=1}^{N} \vec{p}_\alpha = \sum_{\alpha=1}^{N} m_\alpha \vec{v}_\alpha$$

total momentum of the system

$$\vec{F}_{\mathrm{ext}} \equiv \sum_{\alpha=1}^{N} \vec{F}_{a,\mathrm{ext}}$$

net external force

➡ isolated systems: $\vec{F}_{\mathrm{ext}} = 0$

$$\dot{\vec{P}} = 0 \quad \Rightarrow \quad \vec{P} = const$$

➡ the total momentum of an isolated system is conserved !

Newton's Third Law

QUIZ

Check your understanding:

You are driving a car on a country road when a mosquito splatters on the wind-shield. Which has the greater magnitude: the force that the car exerted on the mosquito or the force that the mosquito exerted on the car? Or are the magnitudes the same? If they are different, how can you reconcile this fact with Newton’s third law? If they are equal, why is the mosquito splattered while the car is undamaged?

QUIZ

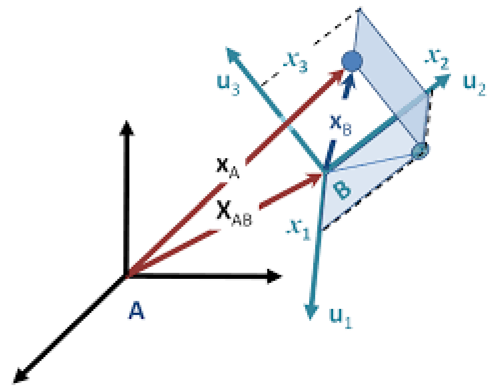
Check your understanding:

The buoyancy force is one half of an action-reaction pair. What force is the other half of this pair? (i) The weight of the swimmer; (ii) the forward thrust force; (iii) the backward drag force; (iv) the downward force that the swimmer exerts on the water; (v) the backward force that the swimmer exerts on the water bykicking.

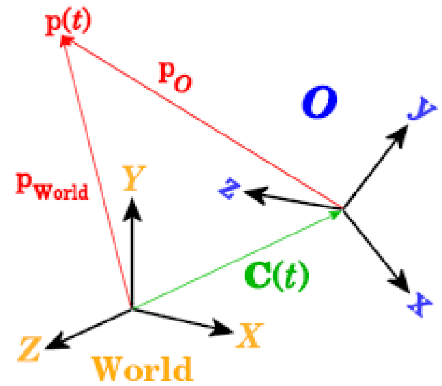
Galilean Transformations

Newton's laws deal with the **motion** of physical bodies. However, motion is a **relative** term: the motion of a body can be defined only **relative to a system** of coordinates.

In **choice** of such systems of coordinates there are **hardly any limits**.



The systems are **rigidly** shifted or **rigidly** inclined



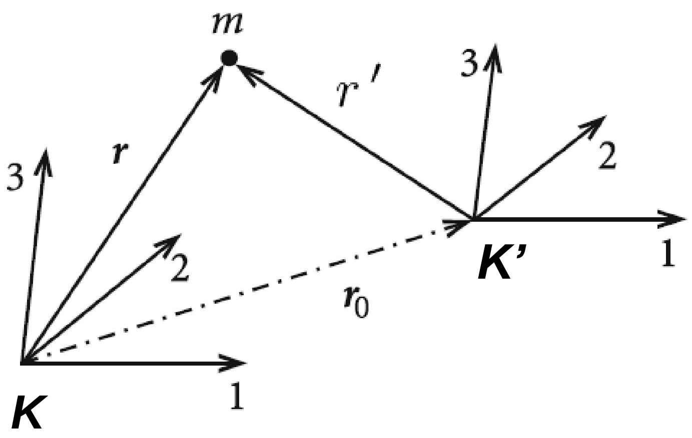
The systems are **moving relatively** to each other

- the systems are completely equivalent with respect to the dynamics of the point mass
- geometrical shape of the path or the temporal process of the particle motion are the same

- the situation with equivalence is not clear (example: rotation of one system of coordinates in respect to the other one)
- Newton's laws make sense only if they are referred to a definite class of system of coordinates (IRF) !

Galilean Transformations

Statement: There exists at least one inertial system, for instance that in which the fixed stars are at rest.



Let ***K*** and ***K'*** be two different coordinate systems where we assume ***K*** = ***K'*** at *t* = 0. Let ***K*** be an inertial system. We know that ***K'*** is also an inertial system only if:

$$m\ddot{\vec{r}} = 0 \quad \Rightarrow \quad m\ddot{\vec{r}}\,' = 0$$

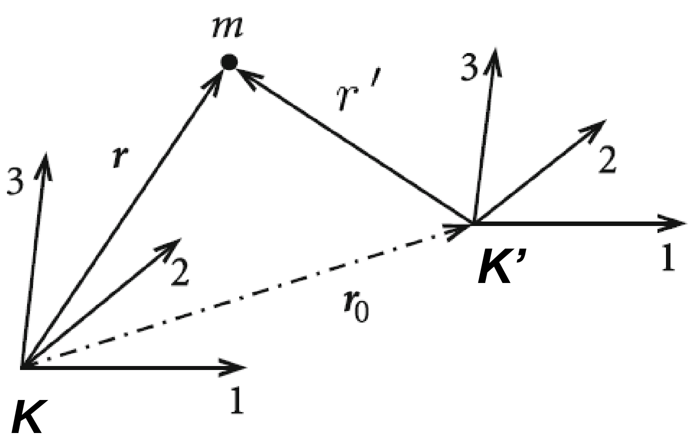
time-dependent **rotation** of ***K'*** relative to ***K*** is **excluded** from the very beginning because it always automatically generates an acceleration!

Note: a constant inclination (time-independent rotation) is allowed but this is not interesting here

we can restrict our consideration to systems moving relative to each other with parallel (Cartesian) axes

Galilean Transformations

Statement: There exists at least one inertial system, for instance that in which the fixed stars are at rest.



Let ***K*** and ***K'*** be two different coordinate systems where we assume ***K*** = ***K'*** at *t* = 0. Let ***K*** be an inertial system. We know that ***K'*** is also an inertial system only if:

$$m\ddot{\vec{r}} = 0 \quad \Rightarrow \quad m\ddot{\vec{r}}' = 0$$

EXERCISE

Position of a point mass at time *t*: $\vec{r}(t) = \vec{r}_0(t) + \vec{r}\,'(t)$

Its acceleration: $\ddot{\vec{r}} = \ddot{\vec{r}}_0 + \ddot{\vec{r}}'$

we satisfy the condition above if only $\ddot{\vec{r}}_0 = 0 \quad \Leftrightarrow \quad \boxed{\vec{r}_0(t) = \vec{v}_0 t}$

Galilean Transformations

Statement: There exists at least one inertial system, for instance that in which the fixed stars are at rest.

Galileo Galilei
(1564-1642)

Let ***K*** and ***K'*** be two different coordinate systems where we assume ***K*** = ***K'*** at *t* = 0. Let ***K*** be an inertial system. We know that ***K'*** is also an inertial system only if:

Galilean Transformation:

$$\begin{cases} \vec{r} = \vec{r}\,' + \vec{v}_0 t\,' \\ t = t\,' \end{cases}$$

Note: the concept of **absolute time** is valid only in the **non-relativistic** case!

Note: There are infinitely many inertial systems moving relatively to each other with constant velocities.

Laws of Forces

Now we can define the law of motion of a particle in **strictly mathematical terms** provided we know the laws of forces acting on this particle, i.e. the dependence of the force on the quantities determining it.

→ **gravitational** and **electrical** forces are the most **fundamental** forces underlying all mechanical phenomena

The gravitational force

$$F = G\frac{m_1 m_2}{r^2}$$

$G = 6.673 \times 10^{-11}\, kg^{-1} m^3 s^{-2}$

The Coulomb force

$$F = k\frac{q_1 q_2}{r^2}$$

$k = 8.9875 \times 10^9\, N\, m^2 C^{-2}$

The uniform force of gravity

$$F = mg$$

The elastic force

$$\vec{F} = -\kappa \vec{r}$$

The sliding friction force

$$F = \mu R$$

The resistance force

$$\vec{F} = -\alpha \vec{v}$$

The Fundamental Equation of Dynamics

The fundamental equation of dynamics of a mass point is nothing but a mathematical expression of Newton's 2nd law:

$$\boxed{m\frac{d\vec{v}}{dt} = \vec{F}}$$

this is a differential equation of motion of a mass point in vector form

Possible formulations of the problem:

(i) – to find the force acting on a point if the mass and the radius vector as a function of time are known

→ differentiation; considered by us in details within Topic 2, part 1

(ii) – to find the law of motion of a point (= radius vector as a function of time) if the mass and the force are known + initial conditions (position and velocity at t_0)

→ integration; can be solved either in **vector form** or in **coordinates** or in **projections** on the **tangent** and on the **normal** to the trajectory at a given point.

The Fundamental Equation of Dynamics

The fundamental equation of dynamics of a mass point is nothing but a mathematical expression of Newton's 2^{nd} law:

$$\boxed{m\frac{d\vec{v}}{dt} = \vec{F}}$$

this is a differential equation of motion of a mass point in vector form

In projections on the Cartesian coordinate axes:

$$m\frac{dv_x}{dt} = F_x, \quad m\frac{dv_y}{dt} = F_y, \quad m\frac{dv_z}{dt} = F_z$$

⇒ the components of acceleration vector along a given axis is caused **only by** the sum of the force components along **same** axis, and not by the force components along any other axis.

The Fundamental Equation of Dynamics

EXERCISE

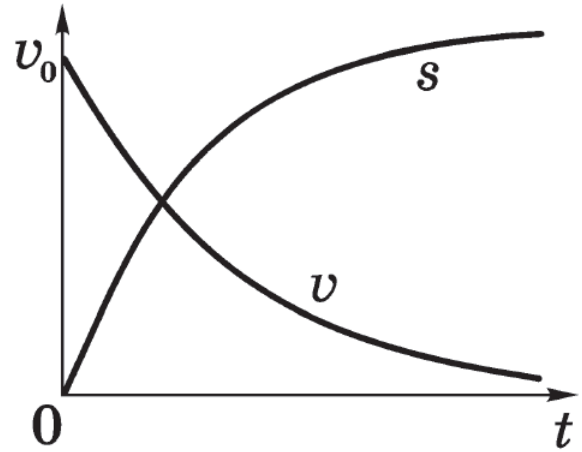
Task #1: A particle of mass *m* moves due to the action of the force ***F*** = - *k**v***. Find the position of the particle as a function of time if ***r***(0) = 0 and ***v***(0) = $\boldsymbol{v}_0$.

Solution:

$$\frac{d\vec{v}}{dt} = -\left(\frac{k}{m}\right)\vec{v} \rightarrow \frac{dv}{dt} = -\left(\frac{k}{m}\right)v \rightarrow \ln\left(\frac{v}{v_0}\right) = -\left(\frac{k}{m}\right)t$$

$$\vec{v} = \vec{v_0}e^{-\frac{kt}{m}}$$

$$\vec{r} = \int_0^t \vec{v}d\tau = \left(1 - e^{-\frac{kt}{m}}\right)\frac{m}{k}\vec{v_0}$$



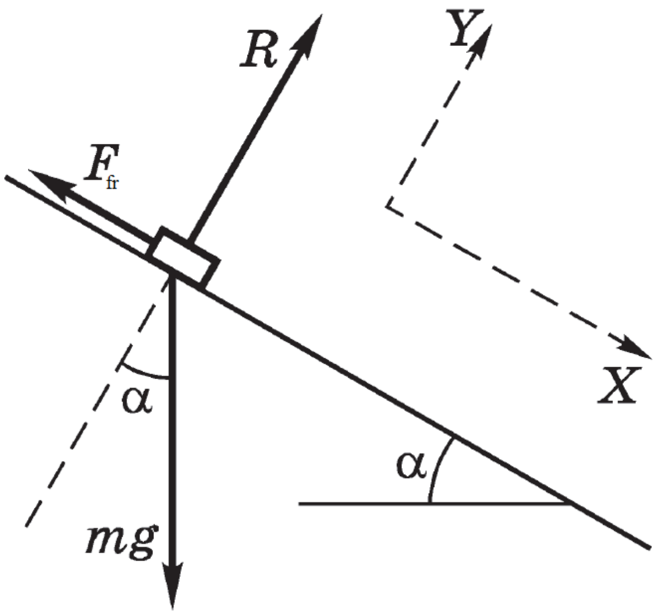
$s = r$

The Fundamental Equation of Dynamics

EXERCISE

Task #2: A small bar of mass *m* slides down an inclined plane forming an angle *α* with the horizontal. The friction coefficient is equal to *μ*. Find the acceleration of the bar relative to the plane (this reference frame is assumed to be inertial).

Solution:



Free-body diagram

$$ma_x = mg_x + R_x + F_{fr,x} =$$
$$= mg\sin\alpha - F_{fr}$$

$$ma_y = 0 = R - mg\cos\alpha$$

$$F_{fr} = \mu R = \mu mg\cos\alpha$$

$$a_x = g\left(\sin\alpha - \mu\cos\alpha\right)$$

The Fundamental Equation of Dynamics

EXERCISE

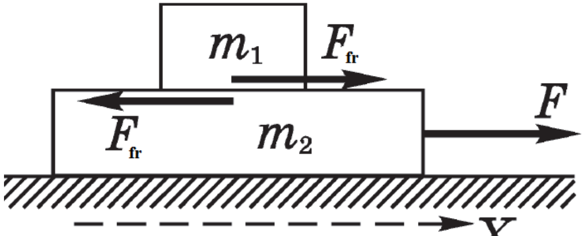
Task #3: A bar of mass m_1 is placed on the plank of mass m_2, which rests on a smooth horizontal plane. The coefficient of friction between the surfaces of the bar and the plank is equal to k. The plank is subjected to the horizontal force F depending on time as $F = at$ (a = const). Find (a) the moment of time t_0 at which the plank starts sliding from under the bar; (b) the accelerations of the bar and the plank in the process of motion.

Solution:

(a)

$$m_1 a_1 = F_{fr} \qquad m_2 a_2 = F - F_{fr}$$

$$F_{fr,max} = k m_1 g \Rightarrow a_2 \geq a_1$$



Free-body diagram

$$\frac{\alpha t - k m_1 g}{m_2} \geq kg \Rightarrow t_0 = \frac{(m_1 + m_2) kg}{\alpha}$$

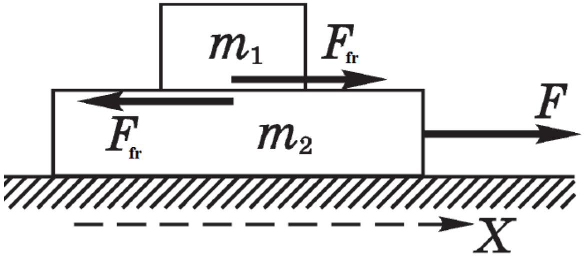
The Fundamental Equation of Dynamics

EXERCISE

Task #3: A bar of mass m_1 is placed on the plank of mass m_2, which rests on a smooth horizontal plane. The coefficient of friction between the surfaces of the bar and the plank is equal to k. The plank is subjected to the horizontal force F depending on time as $F = at$ (a = const). Find (a) the moment of time t_0 at which the plank starts sliding from under the bar; (b) the accelerations of the bar and the plank in the process of motion.

Solution:

(b)



Free-body diagram

$$t \leq t_0 : \quad a_1 = a_2 = \frac{\alpha t}{m_1 + m_2}$$

$$t \geq t_0 : \quad \begin{cases} a_1 = kg = const \\ a_2 = \frac{\alpha t - k m_1 g}{m_2} \end{cases}$$

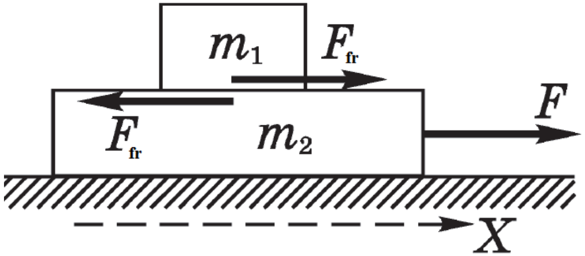
The Fundamental Equation of Dynamics

EXERCISE

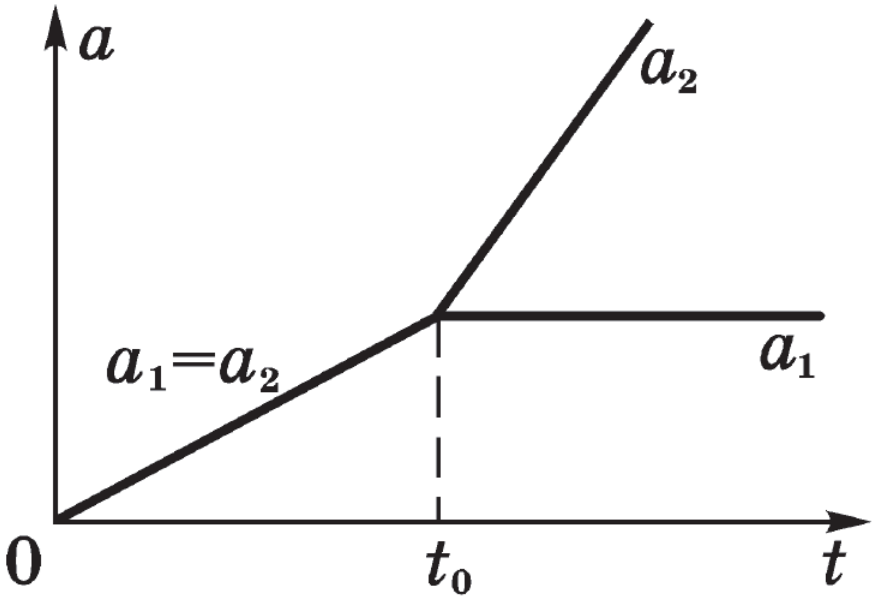
Task #3: A bar of mass m_1 is placed on the plank of mass m_2, which rests on a smooth horizontal plane. The coefficient of friction between the surfaces of the bar and the plank is equal to k. The plank is subjected to the horizontal force F depending on time as $F = at$ (a = const). Find (a) the moment of time t_0 at which the plank starts sliding from under the bar; (b) the accelerations of the bar and the plank in the process of motion.

Solution:

(b)



Free-body diagram



The Fundamental Equation of Dynamics

The fundamental equation of dynamics of a mass point is nothing but a mathematical expression of Newton's 2nd law:

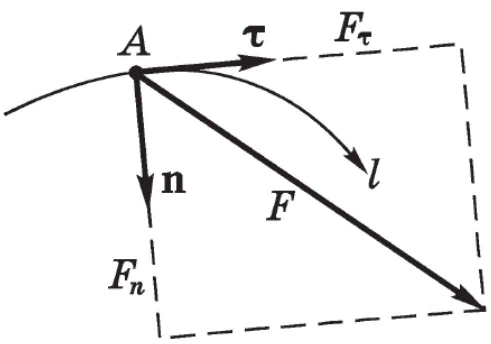
$$\boxed{m\frac{d\vec{v}}{dt} = \vec{F}}$$

this is a differential equation of motion of a mass point in vector form

In projections on the **tangent** and the **normal** to the trajectory at a given point:
(via the natural method)

$$m\frac{dv_\tau}{dt} = F_\tau, \quad m\frac{v^2}{\rho} = F_n$$

exploiting this method is convenient if the trajectory of motion of a point particle is known in advance



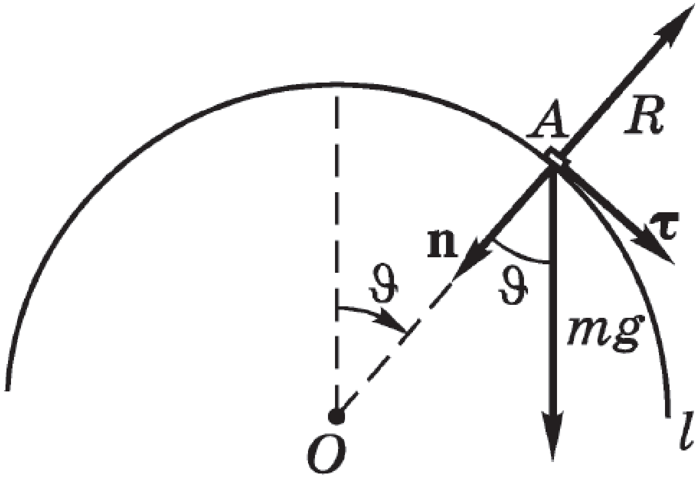
The Fundamental Equation of Dynamics

EXERCISE

Task #4: A small body *A* slides of the top of a smooth sphere of radius *r*. Find the velocity of the body at the moment when it loses contact with the surface of the sphere if its initial velocity is negligible.

Solution:

$$m\frac{dv}{dt} = mg\sin\vartheta \qquad m\frac{v^2}{r} = mg\cos\vartheta - R$$



$$dt = \frac{dl}{v} = \frac{r\,d\vartheta}{v}$$

$$v\,dv = gr\sin\vartheta\, d\vartheta$$

$$v^2 = 2gr(1-\cos\vartheta)$$

$R = 0$

loss of contact

$$v^2 = gr\cos\vartheta$$

$$v = \sqrt{\frac{2gr}{3}}$$

The Fundamental Equation of Dynamics

EXERCISE

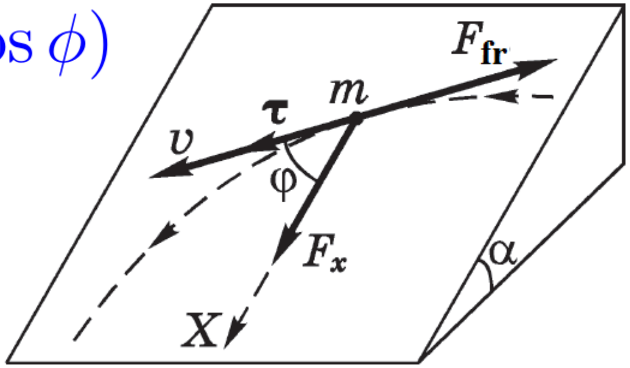
Task #5: A puck moves along an inclined plane which friction coefficient $k = \tan \alpha$, where α is the angle which the plane forms with the horizontal. Find the dependence of the velocity of the puck on the φ angle (angle between the velocity vector and x-axis) if at the initial moment of time $v = v_0$ and $\varphi_0 = \pi/2$.

Solution:

$$F_x = mg\sin\alpha \qquad F_{fr} = kmg\cos\alpha$$

$$ma_\tau = F_x\cos\phi - F_{fr} = mg\sin\alpha(\cos\phi - 1)$$

$$ma_x = F_x - F_{fr}\cos\phi = mg\sin\alpha(1 - \cos\phi)$$



$$a_\tau = -a_x \quad \Rightarrow \quad v = \frac{v_0}{1+\cos\phi}$$

$$v = -v_x + C$$

$$v_x = v\cos\phi \qquad \phi \to 0 \quad v \to \frac{v_0}{2}$$ with $t \to +\infty$

Angular Momentum and Torque

Forces acting on an object can affect not only its translational motion but also cause or change its rotational motion.

EXERCISE

$$m\ddot{\vec{r}} = \vec{F} \quad \Rightarrow \quad \boxed{\frac{d}{dt}\vec{L} = \vec{\tau}}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

angular momentum

$$\vec{\tau} = \vec{r} \times \vec{F}$$

torque (moment)

- the rate of change of angular momentum of a particle equals the torque of the net force acting on it
- when the torque acting on a particle is zero, its total angular momentum is conserved:

$$\vec{\tau} = 0 \quad \Rightarrow \quad \frac{d}{dt}\vec{L} = 0 \quad \Rightarrow \quad \vec{L} = const$$

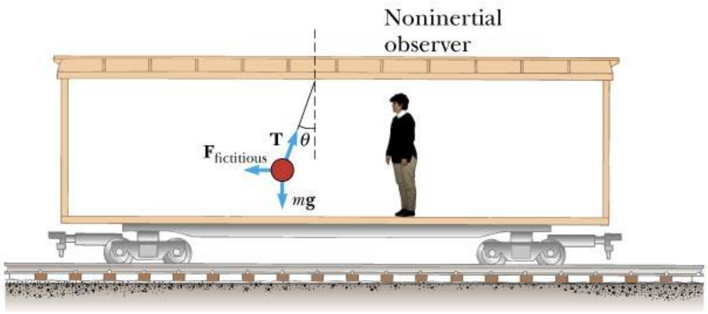
There are two possibilities to satisfy this condition:

1) $\vec{F} \equiv 0$ (trivial case)

2) $\vec{F} \parallel \vec{r}$ (**central** field)

Non-Inertial Frames of Reference

The fundamental equation of dynamics holds true only in the inertial frames of reference. Still there are many cases when a specific problem needs to be solved in **non-inertial** frame.



Example: motion of a simple pendulum in a carriage moving with an acceleration

how one should modify the fundamental equation of dynamics to make it valid in non-inertial frames as well ?

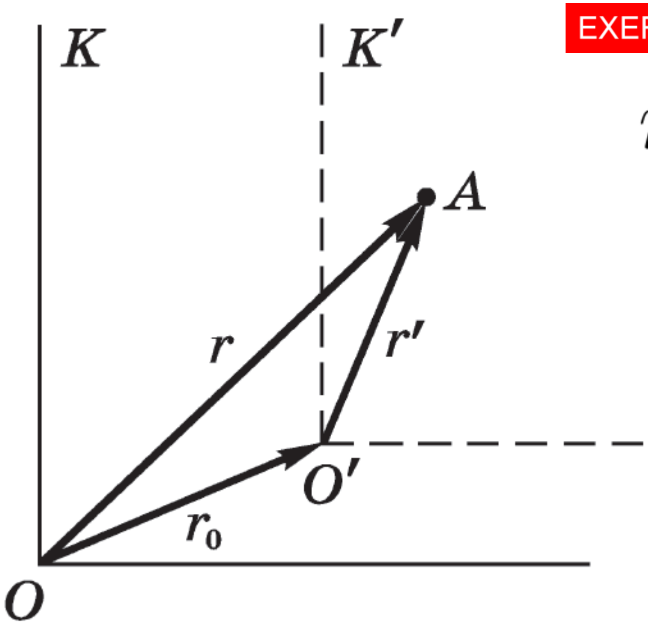
$$\vec{F}_{\text{net}} = m\vec{a}$$

in order to answer this question, one should check what happens with the acceleration vector when switching from IFR to non-IFR

Non-Inertial Frames of Reference

Let ***K*** be inertial frame and ***K'*** non-inertial one. Suppose we know the mass of a particle, the force exerted on this particle by surrounding bodies and the character of motion of ***K'*** relative to ***K***.

Case #1: The ***K'*** translates relative to the ***K*** frame



EXERCISE

$$\vec{r} = \vec{r}_0 + \vec{r}' \quad \Rightarrow \quad \vec{v} = \vec{v}_0 + \vec{v}'$$

$$\boxed{\vec{a} = \vec{a}_0 + \vec{a}'}$$

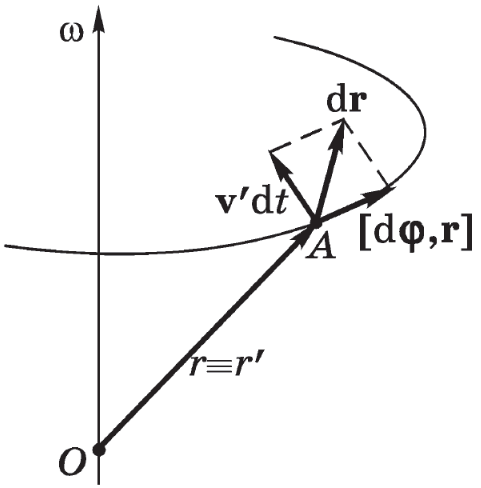
Note: in case of ***K'*** being an IFR

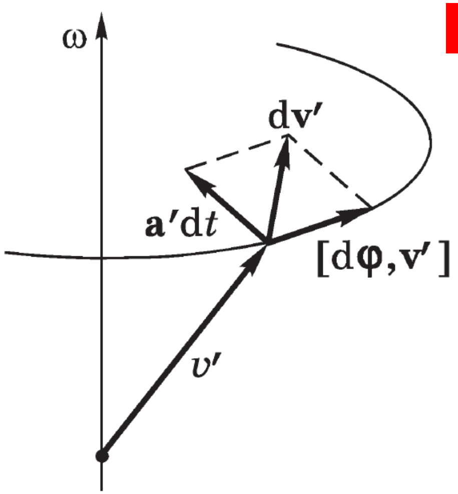
$$\vec{a}_0 = 0 \quad \Rightarrow \quad \vec{a} = \vec{a}'$$

Non-Inertial Frames of Reference

Let ***K*** be inertial frame and ***K'*** non-inertial one. Suppose we know the mass of a particle, the force exerted on this particle by surrounding bodies and the character of motion of ***K'*** relative to ***K***.

Case #2: The ***K'*** rotates with constant angular velocity *ω* around an axis which is stationary in ***K*** frame.





EXERCISE

$$d\vec{r} = \vec{v}'dt + [d\vec{\phi} \times \vec{r}]$$

$$\vec{v} = \vec{v}' + [\vec{\omega} \times \vec{r}]$$

$$d\vec{v} = d\vec{v}' + [\vec{\omega} \times d\vec{r}]$$

$$d\vec{v}' = \vec{a}'dt + [d\vec{\phi} \times \vec{v}']$$

$$\boxed{\vec{a} = \vec{a}' + 2[\vec{\omega} \times \vec{v'}] + [\vec{\omega} \times [\vec{\omega} \times \vec{r}]]}$$

Non-Inertial Frames of Reference

Let ***K*** be inertial frame and ***K'*** non-inertial one. Suppose we know the mass of a particle, the force exerted on this particle by surrounding bodies and the character of motion of ***K'*** relative to ***K***.

Case #3: The ***K'*** rotates with constant angular velocity *ω* around an axis which is translating with the velocity $\boldsymbol{v_0}$ and acceleration $\boldsymbol{a_0}$ relative to the ***K*** frame.

let us introduce auxiliary frame ***S*** which is rigidly fixed to the rotation axis of the ***K'*** frame and translates in the ***K*** frame

EXERCISE

$$\vec{v} = \vec{v}_0 + \vec{v}_S$$
$$\vec{v}_S = \vec{v}' + [\vec{\omega} \times \vec{r}] \quad \Rightarrow \quad \vec{v} = \vec{v}' + \vec{v}_0 + [\vec{\omega} \times \vec{r}]$$

$$\boxed{\vec{a} = \vec{a}' + \vec{a}_0 + 2[\vec{\omega} \times \vec{v}'] - \omega^2 \vec{\rho}}$$

Non-Inertial Frames of Reference

Fundamental equation of dynamics in a non-inertial frame of reference

$$m\vec{a}' = \vec{F} - m\vec{a}_0 + m\omega^2\vec{\rho} + 2m[\vec{v}' \times \vec{\omega}]$$

inertial forces

- even when ***F*** = 0, the particle will **move** in this frame **with** an **acceleration**, which in the general case is not equal to 0
- introducing the inertial forces makes it possible to **keep the format** of the fundamental equation of dynamics in non-inertial frame of reference the same (the left-hand side is the product of the mass of a particle and its acceleration)
- **inertial** forces are **caused** not by the interaction of bodies, but **by** the **properties** of non-inertial frames themselves → they do not obey Newton's 3rd law
- inertial forces **exist only** in non-inertial frames of reference. In the inertial frames there are NO inertial forces at all
- all inertial forces are **proportional to** the **mass** of a body → in a uniform field of inertial forces all bodies move with the same acceleration regardless of their masses

Inertial Forces

$$\boxed{m\vec{a}' = \vec{F} - m\vec{a}_0 + m\omega^2\vec{\rho} + 2m[\vec{v}' \times \vec{\omega}]}$$

$$\Downarrow$$

$$\boxed{m\vec{a}' = \vec{F} + \vec{F}_{\text{in}} + \vec{F}_{\text{cf}} + \vec{F}_{\text{Cor}}}$$

$\vec{F}_{\text{in}} = -m\vec{a}_0$ – inertial force caused by the **translational** motion of non-IFR

$\vec{F}_{\text{cf}} = m\omega^2\vec{\rho}$ – centrifugal force of inertia

$\vec{F}_{\text{Cor}} = 2m[\vec{v}' \times \vec{\omega}]$ – Coriolis force

caused by **rotation** of non-IRF

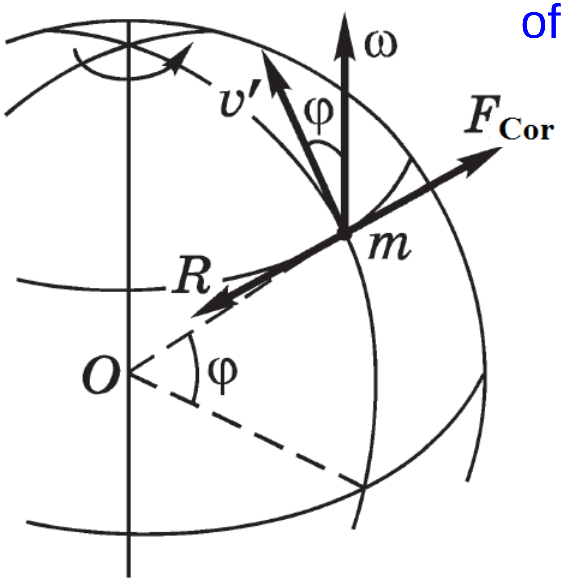
Note: the inertial forces depend on both the characteristics of the non-IFR ($\boldsymbol{a}_0$, $\boldsymbol{\omega}$) and the distance and the velocity of a particle in that frame of reference.

The Fundamental Equation of Dynamics in non-IFR

EXERCISE

Task #6: A train of mass *m* moves along a meridian at the latitude *φ* and the velocity ***v'***. Find the lateral force which the train exerts on the rails.

Solution:



In the reference frame fixed to Earth the normal component of acceleration in respect to the meridian plane is equal 0.

$$\Rightarrow \quad \vec{F}_{Cor} + \vec{R} = 0$$

$$\vec{R}' = -\vec{R} = \vec{F}_{Cor} = 2m[\vec{v}' \times \vec{\omega}]$$

$$R' = 2mv'\omega \sin\phi$$

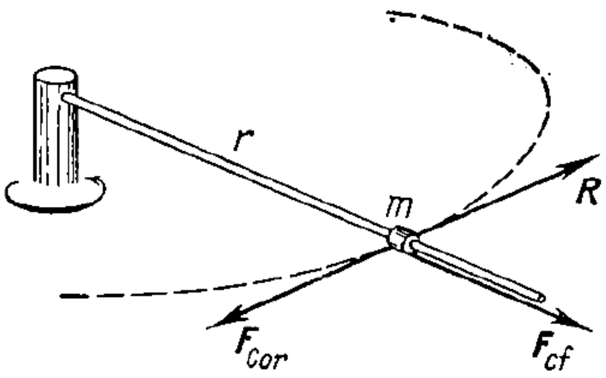
The Fundamental Equation of Dynamics in non-IFR

EXERCISE

Task #7: A small sleeve of mass *m* slides freely along a smooth horizontal shaft which rotates with the constant angular velocity *ω* around a fixed vertical axis passing through one of the shaft's ends. Find the horizontal component of the force which the shaft exerts on the sleeve when it is at the distance *r* from the axis (at the initial moment of time the sleeve was next to the axis and possessed a negligible velocity).

Solution:

Consider the rotating reference frame fixed to a shaft → motion of a sleeve is rectilinear



$$\vec{R} = -\vec{F}_{Cor} = 2m[\vec{\omega} \times \vec{v}']$$

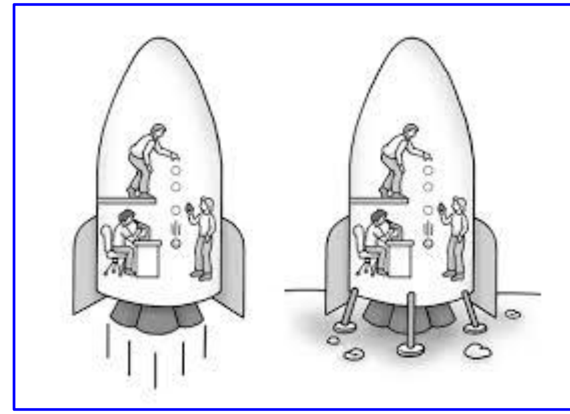
$$\frac{dv'}{dt} = \frac{F_{cf}}{m} = \omega^2 r$$

$$dt = \frac{dr}{v'} \Rightarrow v'dv' = \omega^2 r dr \qquad v' = \omega r$$

$$\vec{v}' = \omega \vec{r} \quad \Rightarrow \quad \vec{R} = 2m\omega[\vec{\omega} \times \vec{r}]$$

The Principle of Equivalence

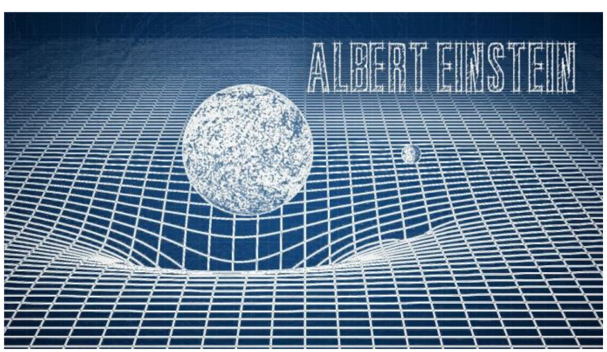
Since inertial forces are proportional to the masses of the bodies (as gravitational ones), a very **important conclusion** can be made:



No physical experiments of any kind can be used to distinguish the uniform field of gravitation from the uniform field of inertial forces.

or

Einstein’s principle of equivalence (of gravitational and inertial forces):
All physical phenomena proceed in the uniform field of gravitation in exactly the same way as in the corresponding uniform field of inertial forces



This analogy between gravitational and inertial forces was used as a starting point of developing the **general theory of relativity**

$$R_{ab} - \frac{1}{2} R g_{ab} = \frac{8\pi G}{c^4} T_{ab}.$$

Albert Einstein's General Theory of Relativity, 1916

Conclusions

• **forces** are **vector** quantities, i.e. they have both magnitude and direction

• the **velocity** of an object can **change** when one of more **forces** are exerted on this object from the side of the other objects / environment

• frames of reference in which Newtonian mechanics **is valid** are called **inertial**

• the inertial properties of an object are characterized by its **mass**

• Newton's 2nd law relates acceleration of an object and the net force acting on it serving a bridging point between the kinematics and dynamics

• in order to consider the same form of the fundamental equation of motion as in Newtonian mechanics in respect to the non-inertial frames of reference, one has to introduce **inertial forces** (these are fictious forces due to the motion of non-IFR)

• Einstein's **principle of equivalence** states that all physical phenomena proceed the same way either you consider them in the uniform field of gravitation or in the corresponding uniform field of inertial forces