Logic

Predicate Logic

Limitations of propositional logic

- 命题逻辑对于反映在自然语言中的逻辑思维进行了精确的形式化描述，能够对一些比较复杂的逻辑推理用形式化方法进行分析。在命题逻辑中，把命题分解到原子命题为止，认为原子命题是不能再分解的，仅仅研究以原子命题为基本单位的复合命题之间的逻辑关系和推理。但这对科学中的演绎推理和数学中的推理是不够的，有些推理用命题逻辑就难以确切地表示出来

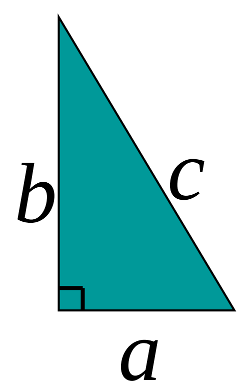
- 例如数学中常用的判断x＞3，x＋y=z等就无法用命题的形式表达出来，因为这两个数学判断中都含有变量。一般而言，我们不能判断x ＞ 3是真还是假。只有我们把变量x代之以具体的值时，如以5代替x的值时，这是一个真命题，而当x取值为2时．就成了一个假命题。因此，对于含有变量的数学判断通常不能用命题来描述

Limitations of propositional logic

Propositional logic – logic of simple statements

$$\neg, \wedge, \vee, \rightarrow, \leftrightarrow$$

How to formulate Pythagoreans' theorem using propositional logic?



How to formulate the statement that there are infinitely many primes?

This Lecture

Last time we talked about propositional logic, a logic on simple statements.

This time we will talk about **predicate logic (first order logic)**, a logic on **quantified statements**.

First order logic is much more expressive than propositional logic.

The topics on first order logic are:

- Predicates (谓词)
- Quantifiers (量词)
- Nested quantifiers (嵌套量词)

Predicates

Predicates are propositions (i.e. statements) with variables

- Can be used to express the meaning of a wide range of statements
- Allow us to reason and explore relationship between objects

Example: $P(x,y) ::= x + 2 = y$

$x = 1$ and $y = 3$: $P(1,3)$ is true

$x = 1$ and $y = 4$: $P(1,4)$ is false

$\neg P(1,4)$ is true

When there is a variable, we need to specify what to put in the variables.

The **domain (辖域)** of a variable is the set of all values that may be substituted in place of the variable.

Example: x > 3

- The variable x is the subject of the statement
- **Predicate** "is greater than 3" refers to a property that the subject of the statement can have
- Can denote the statement by P(x) where P denotes the predicate "is greater than 3" and x is the variable
- P(x): also called the value of the **propositional function** p at x
- Once a value is *assigned* to the variable x, P(x) becomes a proposition and has a truth value

Example

- Let $P(x)$ denote the statement "$x > 3$"
  - $P(4)$: setting $x=4$, thus $p(4)$ is true
  - $P(2)$: setting $x=2$, thus $p(2)$ is false
- Let $A(x)$ denote the statement "computer x is under attack by an intruder". Suppose that only CS2 and MATH1 are currently under attack
  - $A(CS1)$? : false
  - $A(CS2)$? : true
  - $A(MATH1)$?: true

So..

- Predicate logic uses the following new features:
  - Variables: x, y, z
  - Predicates: P(x), M(x)
  - Quantifiers (to be covered in a few slides):
- Propositional functions are a generalization of propositions.
  - They contain variables and a predicate, e.g., P(x)
  - Variables can be replaced by elements from their domain.

Propositional functions

- Propositional functions become propositions (and have truth values) when their variables are each replaced by a value from the domain (or bound by a quantifier, as we will see later).
- The statement $P(x)$ is said to be the value of the propositional function P at x.
- For example, let $P(x)$ denote “$x > 0$” and the domain be the integers. Then:

  $P(-3)$ is false.

  $P(0)$ is false.

  $P(3)$ is true.
- Often the domain is denoted by U. So in this example U is the integers.

N-ary predicate

- A statement involving n variables, x_1, x_2, …, x_n, can be denoted by $P(x_1, x_2, \ldots, x_n)$
- $P(x_1, x_2, \ldots, x_n)$ is the value of the **propositional function** P at the n-tuple $(x_1, x_2, \ldots, x_n)$
- P is also called **n-ary predicate**
- We **cannot** change the **order** of these variables!!

Examples of propositional functions

- Let “x + y = z” be denoted by R(x, y, z) and U (for all three variables) be the integers. Find these truth values:

  R(2,-1,5)

  Solution: F

  R(3,4,7)

  Solution: T

  R(x, 3, z)

  Solution: Not a proposition
- Let “x - y = z” be denoted by Q(x, y, z), with U as the integers. Find these truth values:

  Q(2,-1,3)

  Solution: T

  Q(3,4,7)

  Solution: F

  Q(x, 3, z)

  Solution: Not a proposition

Compound expressions

- Connectives from propositional logic carry over to predicate logic.
- If $P(x)$ denotes "$x > 0$," find these truth values:

  $P(3) \vee P(-1)$ **Solution**: T

  $P(3) \wedge P(-1)$ **Solution**: F

  $P(3) \rightarrow P(-1)$ **Solution**: F

  $P(-3) \rightarrow P(-1)$ **Solution**: T
- Expressions with variables are not propositions and therefore do not have truth values. For example,

  $P(3) \wedge P(y)$

  $P(x) \rightarrow P(y)$
- When used with quantifiers (to be introduced next), these expressions (propositional functions) become propositions.

Quantifiers

- Express the extent to which a predicate is true
- In English, *all, some, many, none, few*
- Focus on two types:
  - **Universal**: a predicate is true for **every** element under consideration
  - **Existential**: a predicate is true for there is **one or more** elements under consideration
- **Predicate calculus**: the area of logic that deals with predicates and quantifiers

Quantifiers

- We need quantifiers to express the meaning of English words including all and some:
  - "All men are Mortal."
  - "Some cats do not have fur."
- The two most important quantifiers are:
  - universal quantifier, "For all," symbol: $\forall$
  - existential quantifier, "There exists," symbol: $\exists$
- We write as in $\forall x\ P(x)$ and $\exists x\ P(x)$.
- $\forall x\ P(x)$ asserts $P(x)$ is true for <u>every</u> x in the domain.
- $\exists x\ P(x)$ asserts $P(x)$ is true for <u>some</u> x in the domain.
- The quantifiers are said to bind the variable x in these expressions.

Universal quantifier $\forall$

- "P(x) for all values of x in the domain"

$$\forall x\ P(x)$$

- Read it as "for all x P(x)" or "for every x P(x)"
- A statement $\forall x\ P(x)$ is false if and only if P(x) is not always true
- An element for which P(x) is false is called a **counterexample**
- A single counterexample is all we need to establish that $\forall x\ P(x)$ is not true

Examples:

- If P(x) denotes "$x > 0$" and U is the integers, then $\forall x\ P(x)$ is false.
- If P(x) denotes "$x > 0$" and U is the positive integers, then $\forall x\ P(x)$ is true.
- If P(x) denotes "x is even" and U is the integers, then $\forall x\ P(x)$ is false.

Examples

- Let $P(x)$ be the statement "$x+1>x$". What is the truth value of $\forall x\ P(x)$?
  - Implicitly assume the domain of a predicate is not empty
  - Best to avoid "for any x" as it is ambiguous to whether it means "every" or "some"
- Let $Q(x)$ be the statement "$x<2$". What is the truth value of $\forall x\ Q(x)$ where the domain consists of all real numbers?

Examples

- Let P(x) be "$x^2>0$". To show that the statement $\forall x\, P(x)$ is false where the domain consists of all integers
  - Show a counterexample with $x=0$
- When all the elements can be listed, e.g., $x_1, x_2, \ldots, x_n$, it follows that the universal quantification $\forall x\, P(x)$ is the same as the conjunction $P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)$

Examples

- What is the truth value of $\forall x\, P(x)$ where $P(x)$ is the statement "$x^2 < 10$" and the domain consists of positive integers not exceeding 4?
  - $\forall x\, P(x)$ is the same as $p(1) \wedge p(2) \wedge p(3) \wedge p(4)$

Existential quantification $\exists$

- "There exists an element x in the domain such that $P(x)$ (is true)"
- Denote that as $\exists x\, P(x)$ where $\exists$ is the existential quantifier
- In English, "for some", "for at least one", or "there is"
- Read as "There is an x such that $P(x)$", "There is at least one x such that $P(x)$", "For some x, $P(x)$", or "For at least one x, $P(x)$"

Examples:

- If $P(x)$ denotes "$x > 0$" and U is the integers, then $\exists x\, P(x)$ is true. It is also true if U is the positive integers.
- If $P(x)$ denotes "$x < 0$" and U is the positive integers, then $\exists x\, P(x)$ is false.
- If $P(x)$ denotes "x is even" and U is the integers, then $\exists x\, P(x)$ is true.

Example

- Let $P(x)$ be the statement "$x>3$". Is $\exists x\ P(x)$ true for the domain of all real numbers?
- Let $Q(x)$ be the statement "$x=x+1$". Is $\exists x\ Q(x)$ true for the domain of all real numbers?
- When all elements of the domain can be listed, , e.g., $x_1, x_2, \ldots, x_n$, it follows that the existential quantification is the same as disjunction $P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)$

Example

- What is the truth value of $\exists x\, P(x)$ where $p(x)$ is the statement "$x^2 > 10$" and the domain consists of positive integers not exceeding 4?
  - $\exists x\, P(x)$ is the same as $p(1) \vee p(2) \vee p(3) \vee p(4)$

Uniqueness Quantifier $\exists!$ (optional)

- $\exists! x\, P(x)$ means that $P(x)$ is true for one and only one x in the universe of discourse.
- This is commonly expressed in English in the following equivalent ways:
  - "There is a unique x such that $P(x)$."
  - "There is one and only one x such that $P(x)$"

Examples:
- if $P(x)$ denotes "$x + 1 = 0$" and U is the integers, then $\exists! x\, P(x)$ is true.
- but if $P(x)$ denotes "$x > 0$," then $\exists! x\, P(x)$ is false.

The uniqueness quantifier is not really needed as the restriction that there is a unique x such that $P(x)$ can be expressed as:

$$\exists x\, (P(x) \land \forall y\, (P(y) \rightarrow y = x))$$

Quantifiers with restricted domains

- What do the following statements mean for the domain of real numbers?

$$\forall x < 0, x^2 > 0 \quad \text{same as} \quad \forall x(x < 0 \rightarrow x^2 > 0)$$

$$\forall y \neq 0, y^3 \neq 0 \quad \text{same as} \quad \forall y(y \neq 0 \rightarrow y^3 \neq 0)$$

$$\exists z > 0, z^2 = 2 \quad \text{same as} \quad \exists z(z > 0 \wedge z^2 = 2)$$

Be careful about → and ∧ in these statements

Quantifiers with restricted domains

What do the statements $\forall x < 0\ (x^2 > 0)$, $\forall y = 0\ (y^3 = 0)$, and $\exists z > 0\ (z^2 = 2)$ mean, where the domain in each case consists of the real numbers?

Solution:

- The statement $\forall x < 0\ (x^2 > 0)$ states that for every real number x with $x < 0$, $x^2 > 0$.
- That is, it states "The square of a negative real number is positive." This statement is the same as $\forall x(x < 0 \rightarrow x^2 > 0)$.
- The statement $\forall y = 0\ (y^3 = 0)$ states that for every real number y with $y = 0$, we have $y^3 = 0$.
- That is, it states "The cube of every nonzero real number is nonzero." Note that this statement is equivalent to $\forall y(y = 0 \rightarrow y^3 = 0)$.
- Finally, the statement $\exists z > 0\ (z^2 = 2)$ states that there exists a real number z with $z > 0$ such that $z^2 = 2$. That is, it states "There is a positive square root of 2." This statement is equivalent to $\exists z(z > 0 \wedge z^2 = 2)$.

Quantifiers with restricted domains

- The restriction of a universal quantification is the same as the universal quantification of a conditional statement.
- For instance, $\forall x < 0\ (x^2 > 0)$ is another way of expressing $\forall x(x < 0 \rightarrow x^2 > 0)$.
- The restriction of an existential quantification is the same as the existential quantification of a conjunction.
- For instance, $\exists z > 0\ (z^2 = 2)$ is another way of expressing $\exists z(z > 0 \wedge z^2 = 2)$.

Thinking about Quantifiers

- When the domain of discourse is finite, we can think of quantification as looping through the elements of the domain.
- To evaluate $\forall x\, P(x)$ loop through all x in the domain.
  - If at every step $P(x)$ is true, then $\forall x\, P(x)$ is true.
  - If at a step $P(x)$ is false, then $\forall x\, P(x)$ is false and the loop terminates.
- To evaluate $\exists x\, P(x)$ loop through all x in the domain.
  - If at some step, $P(x)$ is true, then $\exists x\, P(x)$ is true and the loop terminates.
  - If the loop ends without finding an x for which $P(x)$ is true, then $\exists x\, P(x)$ is false.
- Even if the domains are infinite, we can still think of the quantifiers this fashion, but the loops will not terminate in some cases.

Properties of Quantifiers

- The truth value of $\exists x\, P(x)$ and $\forall\, x\, P(x)$ depend on both the propositional function $P(x)$ and on the domain U.

Examples:

- If U is the positive integers and $P(x)$ is the statement "$x < 2$", then $\exists x\, P(x)$ is true, but $\forall\, x\, P(x)$ is false.
- If U is the negative integers and $P(x)$ is the statement "$x < 2$", then both $\exists x\, P(x)$ and $\forall\, x\, P(x)$ are true.
- If U consists of 3, 4, and 5, and $P(x)$ is the statement "$x > 2$", then both $\exists x\, P(x)$ and $\forall\, x\, P(x)$ are true. But if $P(x)$ is the statement "$x < 2$", then both $\exists x\, P(x)$ and $\forall\, x\, P(x)$ are false.

Precedence of Quantifiers

- The quantifiers $\forall$ and $\exists$ have higher precedence than all the logical operators.
- For example, $\forall x\, P(x) \vee Q(x)$ means $(\forall x\, P(x)) \vee Q(x)$
- $\forall x\, (P(x) \vee Q(x))$ means something different.
- Unfortunately, often people write $\forall x\, P(x) \vee Q(x)$ when they mean $\forall x\, (P(x) \vee Q(x))$.

$$\forall x\ P(x) \vee Q(x) \equiv (\forall x\ P(x)) \vee Q(x) \text{ rather than } \forall x\ (P(x) \vee Q(x))$$

Binding variables (绑定变量)

- When a quantifier is used on the variable x, this occurrence of variable is **bound**
- If a variable is not bound, then it is **free**
- All variables occur in propositional function of predicate calculus must be bound or set to a particular value to turn it into a proposition
- The part of a logical expression to which a quantifier is applied is the **scope** of this quantifier

Example

- What are the scope of these expressions?
- Are all the variables bound?

$$\exists x(x + y = 1)$$

$$\exists x(p(x) \wedge q(x)) \vee \forall x \mathrm{R}(x)$$

$$\exists x(p(x) \wedge q(x)) \vee \forall y R(y)$$

The same letter is often used to represent variables bound by different quantifiers with scopes that do not overlap

Equivalences in predicate logic

- Statements involving predicates and quantifiers are logically equivalent if and only if they have the same truth value
  - for every predicate substituted into these statements
  - for every domain of discourse used for the variables in the expressions.
- The notation $S \equiv T$ indicates that S and T are logically equivalent.
- **Example**: $\forall x \neg\neg S(x) \equiv \forall x\, S(x)$

Thinking about Quantifiers as Conjunctions and Disjunctions

- If the domain is finite, a universally quantified proposition is equivalent to a conjunction of propositions without quantifiers and an existentially quantified proposition is equivalent to a disjunction of propositions without quantifiers.
- If U consists of the integers 1,2, and 3:

$$\forall x P(x) \equiv P(1) \wedge P(2) \wedge P(3)$$

$$\exists x P(x) \equiv P(1) \vee P(2) \vee P(3)$$

- Even if the domains are infinite, you can still think of the quantifiers in this fashion, but the equivalent expressions without quantifiers will be infinitely long.

Negating quantified expressions

- Consider $\forall x\ J(x)$
  "Every student in your class has taken a course in Java."
  Here $J(x)$ is "x has taken a course in calculus" and
  the domain is students in your class.
- Negating the original statement gives "It is not the case that every student in your class has taken Java." This implies that "There is a student in your class who has not taken calculus."
  Symbolically $\neg\forall x\ J(x)$ and $\exists x\ \neg J(x)$ are equivalent

Negating quantified expressions (cont.)

- Now Consider $\exists x\, J(x)$

  "There is a student in this class who has taken a course in Java."

  Where $J(x)$ is "x has taken a course in Java."
- Negating the original statement gives "It is not the case that there is a student in this class who has taken Java." This implies that "Every student in this class has not taken Java"

  Symbolically $\neg\exists x\, J(x)$ and $\forall x\, \neg J(x)$ are equivalent

De Morgan's laws for quantifiers

- The rules for negating quantifiers are:

TABLE 2 De Morgan's Laws for Quantifiers.

Negation	*Equivalent Statement*	*When Is Negation True?*	*When False?*
$\neg\exists x\, P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg\forall x\, P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

- The reasoning in the table shows that:

$$\neg\forall x P(x) \equiv \exists x \neg P(x)$$

- These are important. You will use these.

$$\neg\exists x P(x) \equiv \forall x \neg P(x)$$

Why call "De Morgan's laws"

When the domain has n elements $x_1, x_2, \ldots, x_n$, it follows that $\neg\forall xP(x)$ is the same as $\neg(P(x_1) \wedge P(x_2) \wedge \cdots \wedge P(x_n))$, which is equivalent to $\neg P(x_1) \vee \neg P(x_2) \vee \cdots \vee \neg P(x_n)$ by De Morgan's laws

$\neg\exists xP(x)$ is the same as $\neg(P(x_1) \vee P(x_2) \vee \cdots \vee P(x_n))$, which by De Morgan's laws is equivalent to $\neg P(x_1) \wedge \neg P(x_2) \wedge \cdots \wedge \neg P(x_n)$, and this is the same as $\forall x\neg P(x)$.

Examples

What is the negations of the statement "There is an honest politician"?

Solution:

- Let $H(x)$ denote "x is honest."
- "There is an honest politician" is represented by $\exists x H(x)$, where the domain consists of all politicians.
- The negation of this statement is $\neg \exists x H(x)$, which is equivalent to $\forall x \neg H(x)$.
- This negation can be expressed as "Every politician is dishonest."

- (Note: In English, the statement "All politicians are not honest" is ambiguous. In common usage, this statement often means "Not all politicians are honest." Consequently, we do not use this statement to express this negation.)

Examples

What is the negation of the statement "All Americans eat cheeseburgers"?

Solution:

- Let $C(x)$ denote "x eats cheeseburgers."
- "All Americans eat cheeseburgers" is represented by $\forall x C(x)$, where the domain consists of all Americans.
- The negation of this statement is $\neg \forall x C(x)$, which is equivalent to $\exists x \neg C(x)$.
- This negation can be expressed in several different ways, including "Some American does not eat cheeseburgers" and "There is an American who does not eat cheeseburgers."

Translation from English to logic

Examples:

1. "Some student in this class has visited Mexico."

   Solution: Let $M(x)$ denote "x has visited Mexico" and $S(x)$ denote "x is a student in this class," and U be all people.

   $$\exists x \ (S(x) \land M(x))$$

2. "Every student in this class has visited Canada or Mexico."

   Solution: Add $C(x)$ denoting "x has visited Canada."

   $$\forall x \ (S(x) \rightarrow (M(x) \lor C(x)))$$

Examples

What are the negations of the statements $\forall x(x^2 > x)$ and $\exists x(x^2 = 2)$?

Solution:

- The negation of $\forall x(x^2 > x)$ is the statement $\neg\forall x(x^2 > x)$, which is equivalent to $\exists x\neg(x^2 > x)$. This can be rewritten as $\exists x(x^2 \leq x)$.
- The negation of $\exists x(x^2 = 2)$ is the statement $\neg\exists x(x^2 = 2)$, which is equivalent to $\forall x\neg(x^2 = 2)$. This can be rewritten as $\forall x(x^2 != 2)$.
- The truth values of these statements depend on the domain.

Examples

Show that $\neg\forall x(P(x) \rightarrow Q(x))$ and $\exists x(P(x) \wedge \neg Q(x))$ are logically equivalent.

Solution:

- By De Morgan's law for universal quantifiers, we know that $\neg\forall x(P(x) \rightarrow Q(x))$ and $\exists x(\neg(P(x) \rightarrow Q(x)))$ are logically equivalent.
- It is also known that $\neg(P(x) \rightarrow Q(x))$ and $P(x) \wedge \neg Q(x)$ are logically equivalent for every x.
- Because we can substitute one logically equivalent expression for another in a logical equivalence, it follows that $\neg\forall x(P(x) \rightarrow Q(x))$ and $\exists x(P(x) \wedge \neg Q(x))$ are logically equivalent.

Translation from English to logic

Express the statement "Every student in this class has studied calculus" using predicates and quantifiers.

Solution:

- First, we rewrite the statement so that we can clearly identify the appropriate quantifiers to use. Doing so, we obtain:
  - "For every student in this class, that student has studied calculus."
- Next, we introduce a variable x so that our statement becomes
  - "For every student x in this class, x has studied calculus."
- We introduce $C(x)$, which is the statement "x has studied calculus."
- If the domain for x consists of the students in the class, we can translate our statement as $\forall x C(x)$.

Translation from English to logic

- However, there are other correct approaches; different domains of discourse and other predicates can be used. The approach we select depends on the subsequent reasoning we want to carry out.

- For example, we may be interested in a wider group of people than only those in this class. If we change the domain to consist of all people, we will need to express our statement as “For every person x, if person x is a student in this class then x has studied calculus.”

Translation from English to logic

- If $S(x)$ represents the statement that person x is in this class, we see that our statement can be expressed as $\forall x(S(x) \rightarrow C(x))$.

- [**Caution**! Our statement **cannot** be expressed as $\forall x(S(x) \wedge C(x))$ because this statement says that all people are students in this class and have studied calculus!]

- Finally, when we are interested in the background of people in subjects besides calculus, we may prefer to use **the two-variable quantifier** $Q(x, y)$ for the statement "student x has studied subject y." Then we would replace $C(x)$ by $Q(x, \text{calculus})$ in both approaches to obtain $\forall x Q(x, \text{calculus})$ or $\forall x(S(x) \rightarrow Q(x, \text{calculus}))$.

Translating from English to logic

Example 1: Translate the following sentence into predicate logic: "Every student in this class has taken a course in Java."

Solution:

First decide on the domain U.

Solution 1: If U is all students in this class, define a propositional function J(x) denoting "x has taken a course in Java" and translate as $\forall x\, J(x)$.

Solution 2: But if U is all people, also define a propositional function S(x) denoting "x is a student in this class" and translate as $\forall x\, (S(x) \rightarrow J(x))$.

$\forall x\, (S(x) \wedge J(x))$ is not correct. What does it mean?

Translating from English to logic

Example 2: Translate the following sentence into predicate logic: "Some student in this class has taken a course in Java."

Solution:

First decide on the domain U.

Solution 1: If U is all students in this class, translate as

$$\exists x\ J(x)$$

Solution 2: But if U is all people, then translate as

$$\exists x\ (S(x) \land J(x))$$

$\exists x\ (S(x) \rightarrow J(x))$ is not correct. What does it mean?

System Specification Example

- Predicate logic is used for specifying properties that systems must satisfy.
- For example, translate into predicate logic:
  - "Every mail message larger than one megabyte (MB) will be compressed."
  - "If a user is active, at least one network link will be available."
- Decide on predicates and domains (left implicit here) for the variables:
  - Let $L(m, y)$ be "Mail message m is larger than y megabytes."
  - Let $C(m)$ denote "Mail message m will be compressed."
  - Let $A(u)$ represent "User u is active."
  - Let $S(n, x)$ represent "Network link n is state x.
- Now we have:

$$\forall m(L(m, 1) \rightarrow C(m))$$

$$\exists u A(u) \rightarrow \exists n S(n, \text{available})$$

Lewis Carroll Example (I)

Charles Lutwidge Dodgson
(AKA Lewis Caroll)
(1832-1898)

- The first two are called *premises* and the third is called the *conclusion*.
  1. "All lions are fierce."
  2. "Some lions do not drink coffee."
  3. "Some fierce creatures do not drink coffee."
- Here is one way to translate these statements to predicate logic. Let P(x), Q(x), and R(x) be the propositional functions "x is a lion," "x is fierce," and "x drinks coffee," respectively.
  1. $\forall x\,(P(x) \rightarrow Q(x))$
  2. $\exists x\,(P(x) \land \neg R(x))$
  3. $\exists x\,(Q(x) \land \neg R(x))$
- Later we will see how to prove that the conclusion follows from the premises.

Lewis Carroll Example (II)

- Consider these statements, of which the first three are premises and the fourth is a valid conclusion.
  1. "All hummingbirds are richly colored."
  2. "No large birds live on honey."
  3. "Birds that do not live on honey are dull in color."
  4. "Hummingbirds are small."
- Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements "x is a hummingbird," "x is large," "x lives on honey," and "x is richly colored," respectively. Assuming that the domain consists of all birds, express the statements in the argument using quantifiers and $P(x)$, $Q(x)$, $R(x)$, and $S(x)$.
- **Solution**: We can express the statements in the argument as
  1. $\forall x(P(x) \rightarrow S(x))$.
  2. $\neg\exists x(Q(x) \land R(x))$.
  3. $\forall x(\neg R(x) \rightarrow \neg S(x))$.
  4. $\forall x(P(x) \rightarrow \neg Q(x))$.

The End