Topic 02. Mechanics

1. The acceleration of a motorcycle is given by $a_x(t) = At - Bt^2$, where $A = 1.50$ m/s^3 and $B = 0.120$ m/s^4. The motorcycle is at rest at the origin at time $t = 0$. (a) Find its position and velocity as functions of time. (b) Calculate the maximum velocity it attains.

2. An object is moving along the x-axis. At $t = 0$ it has velocity $v_{0x} = 20.0$ m/s. Starting at time $t = 0$ it has acceleration $a_x = -Ct$, where C has units of m/s^3. (a) What is the value of C if the object stops in 8.00 s after t = 0? (b) For the value of C calculated in part (a), how far does the object travel during the 8.00 s?

3. At the instant $t = 0$ a particle leaves the origin and moves in the positive direction of x-axis. Its velocity varies in time as $\vec{v} = \vec{v}_0(1 - \frac{t}{\tau})$, where $\vec{v}_0$ is the initial velocity of a particle (its magnitude is $v_0 = 10.0$ cm/s) and $\tau = 5.0$ s. Find (a) the x coordinate of the particle at the instants 6.0, 10, and 20 s; (b) the instants of time when the particle is at the distance 10.0 cm from the origin; (c) the distance s covered by the particle during the first 4.0 and 8.0 s; draw the approximate plot $s(t)$.

4. A radius vector of a particle varies in time as $\vec{r} = t(1 - \alpha t)\vec{c}$, where $\vec{c}$ is a constant vector and α is a constant positive factor. Find (a) the velocity $\vec{v}$ and acceleration $\vec{a}$ of the particle as the functions of time; (b) the time interval Δt needed to return to the initial position (we start the motion at $t = 0$), and the distance s covered during this time.

5. A faulty model rocket moves in the xy-plane (the positive y-direction is vertically upward). The rockets acceleration has components $a_x(t) = \alpha t^2$ and $a_y(t) = \beta - \gamma t$, where $\alpha = 2.50$ m/s^4, $\beta = 9.00$ m/s^2, and $\gamma = 1.40$ m/s^3. At t = 0 the rocket is at the origin and has velocity $\vec{v}_0 = v_{0x}\hat{\imath} + v_{0y}\hat{\jmath}$ with $v_{0x} = 1.00$ m/s and $v0y = 7.00$ m/s. (a) Calculate the velocity and position vectors as functions of time. (b) What is the maximum height reached by the rocket? (c) What is the horizontal displacement of the rocket when it returns to $y = 0$?

6. A ballon starts rising from the surface of the Earth. The ascending rate is constant and equal to v_0. Due to the wind the balloon acquires the horizontal velocity component $v_x = \alpha y$ ($\alpha > 0$ is constant), where y is the height above the ground. Find (a) the horizontal displacement of the balloon $x(y)$ as a function of height; (b) the total, tangential, and normal accelerations of the balloon as a function of height.

7. Two bars are placed next to each other on the inclined plane (with the angle of inclination α), as shown on Fig.1. The masses of the bars are m_1 and m_2, and the friction coefficients are k_1 and k_2 ($k_1 > k_2$), respectively. Find (a) the reaction forces which the bars exert on each other in the process of motion; (b) the minimal value of the angle α at which the bars start sliding down.

8. A bar of mass m is pulled up along the inclined plane (with the angle of inclination α) by means of a thread (see Fig.2). The coefficient of kinetic friction is equal to k. Find the angle β (in respect to the inclined plane surface) so that the tenstion of the thread is minimal.

9. A particle moves along some trajectory from the initial point with $\vec{r}_1 = \hat{\imath} + 2\hat{\jmath}$ to another point with $\vec{r}_2 = 2\hat{\imath} - 3\hat{\jmath}$ under the action of the force $\vec{F} = 3\hat{\imath} + 4\hat{\jmath}$. Calculate the work done by the force $\vec{F}$, assuming that all numerical values have appropriate SI units.

10. A body of mass m is lifted up above the ground by means of a force $\vec{F} = 2(\alpha y - 1)m\vec{g}$, where α is a positive constant. Find the work done by this force and the increase of the potential energy of the body after the first half of the ascent.

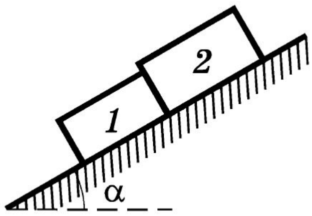
11. A rocket moves in the absence of any external force by means of a steady jet with constant velocity $\vec{u}$ relative to the rocket. Find the velocity $\vec{v}$ of the rocket at the instant when its mass is equal to m, if at the initial moment it possessed the mass m_0 and its velocity was equal to zero.

12. A patricle is placed into a planar force field with the potential energy $U(x, y) = -\alpha xy$ ($\alpha = 6.0$ J/m^2). Calculate the magnitude of the force acting on the particle at the point where $U = -0.24$ J and the force is directed at $\theta = 15°$ in respect to y-axis.

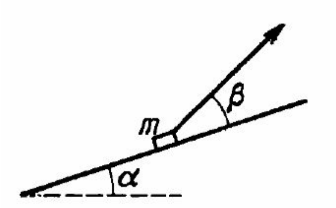
13. (ADVANCED) In the vertical jump, an athlete starts from a crouch and jumps upward as high as possible. Even the best athletes spend little more than 1.00 s in the air (their hang time). Treat the athlete as a particle and let y_{max} be his maximum height above the floor. To explain why he seems to hang in the air, calculate the ratio of the time he is above $y_{max}/2$ to the time it takes him to go from the floor to that height. Ignore air resistance.

14. (ADVANCED) A boat crosses the river with the velocity (relative to the water) which is n times less than the river flow velocity. At what angle to the stream direction should the boat move in order to minimize drifting? What is the answer in case $n = 2.0$?

15. (ADVANCED) A horizontal disc rotates with a constant angular velocity $\omega = 6.0$ rad/s around a vertical axis passing through its center. A small body of mass $m = 0.50$ kg moves along a diameter of the disc with a velocity $v' = 50$ cm/s which is constant relative to the disc. Find the force that the disc exerts on the body at the instant when it is located at the distacne $r = 30$ cm from the axis of rotation.



(a) Figure 1 (problem 7)



(b) Figure 2 (problem 8)