DUT – RU International School
of Information Science & Engineering

Topic # 3

Electricity and Magnetism

(part 2: Magnetism)

Lecture Course (online)
Dalian, 2021

Contents

Magnetism Around Us

In terms of applications, **magnetism** is one of the most important fields in physics.

picking up heavy loads

loudspeakers

motors

storing computer data

magnetic resonance imaging (MRI)

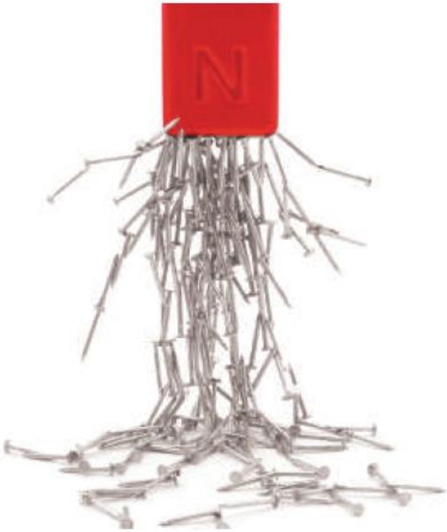
refrigerator magnets

synchrotrons and free-electron lasers

The Subject of Magnetism

Magnetic phenomena were first observed around 2500 years ago in the ancient city of Magnesia (Manisa, western Turkey) by means of fragments of magnetized iron ore which we now call **permanent magnets**.

Experimental facts:

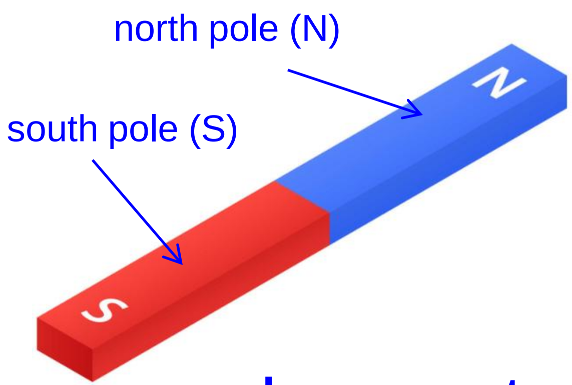


- permanent magnets exert forces on each other
- permanent magnets exert forces on pieces of initially non-magnetized iron
- after affecting the non-magnetized iron rod with a permanent magnet also becomes magnetized
- when floating or being suspended a magnetized rod tends to align itself in the north-south direction

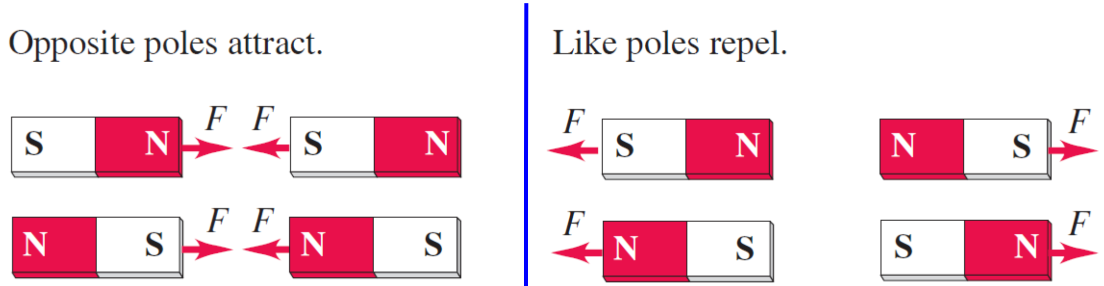
Note: the needle of an ordinary compass is just a piece of magnetized iron!

The Subject of Magnetism

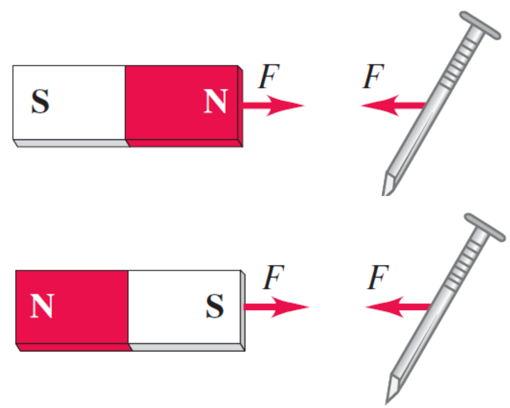
Initially the interactions of permanent magnets and compass needles were described in terms of **magnetic poles**.



bar magnet



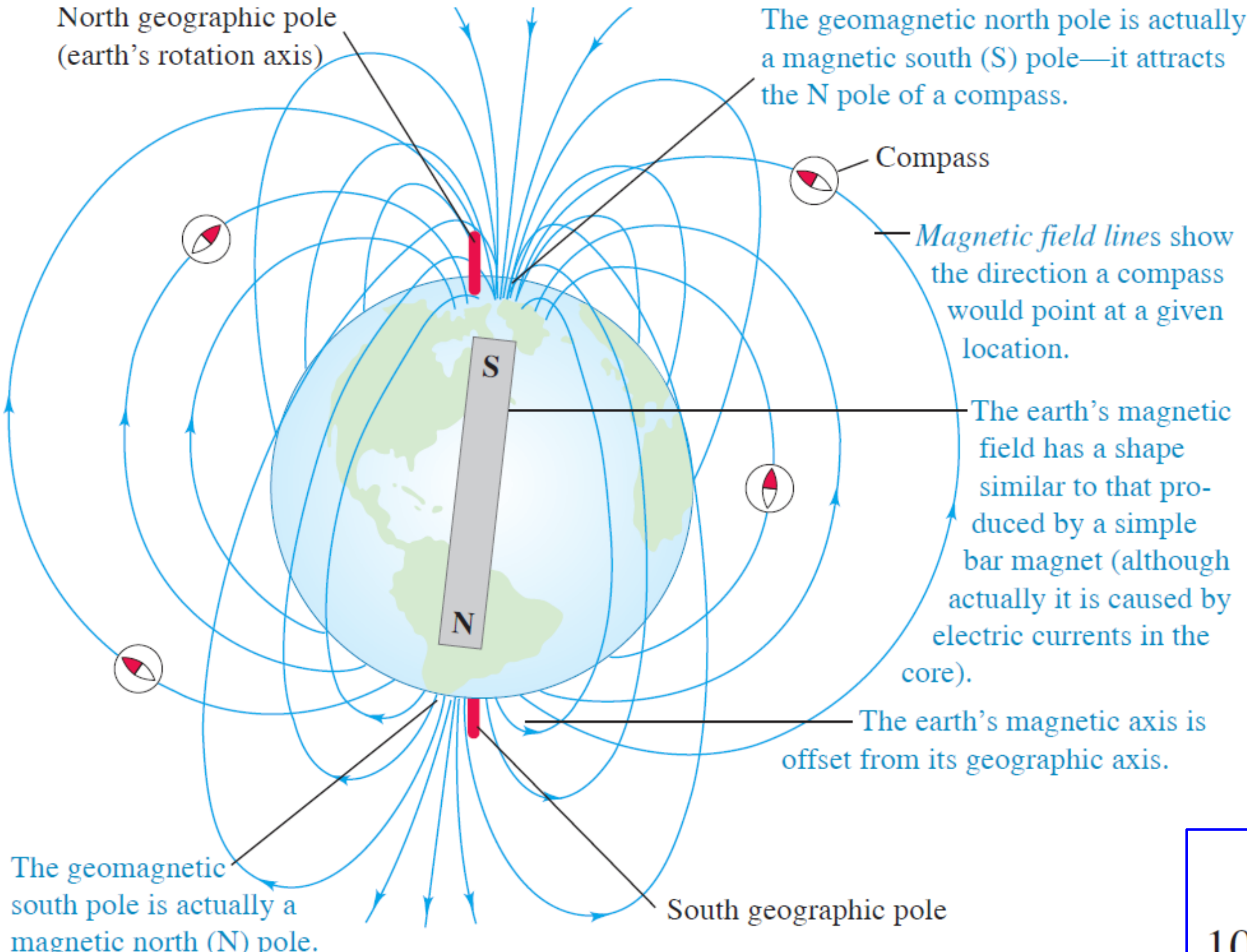
Either pole of a bar magnet attracts an unmagnetized object



by analogy to electric interactions, we describe these interactions by saying that a bar magnet sets up a **magnetic field** in the space around it and a second object **responds** to that field

The Subject of Magnetism

The Earth itself is a magnet !



switching time

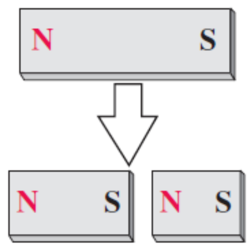
10^4 to 10^6 years

Magnetic Poles vs. Electric Charge

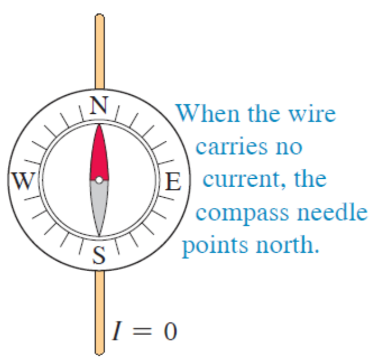
The concept of magnetic poles may appear similar to that of electric charge (N and S poles may seem to be analogous to "+" and "–" charges). However, this is **misleading**.

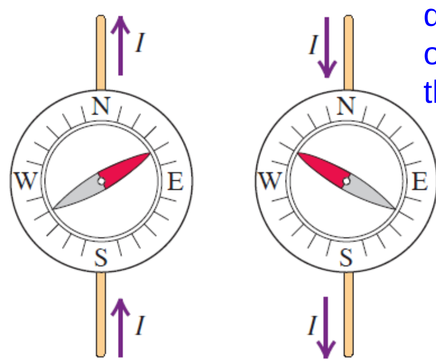
while isolated positive and negative charges exist, there is no experimental evidence that one isolated magnetic pole exists.

if a bar magnet is broken into two, each broken end becomes a pole



In 1820 Oersted discovered the first evidence of relationship of magnetism to **moving** charges: a compass needle was deflected by a current-carrying wire





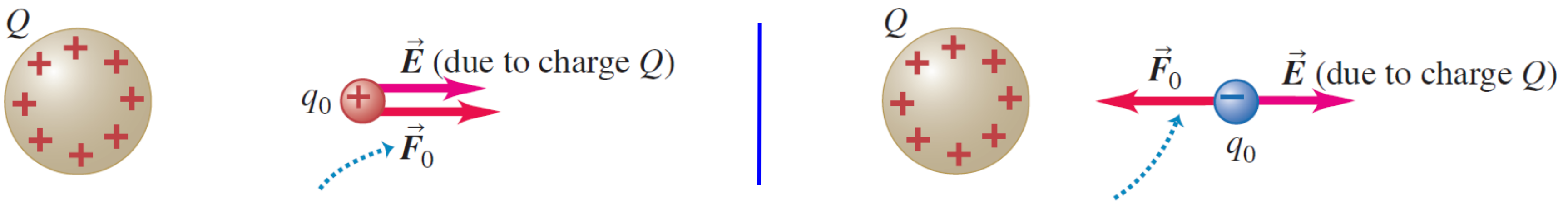
deflection depends on the direction of the current

Electric and magnetic interactions prove to be intimately connected – the summit has been reached in **Maxwell's equations**

Magnetic Fields

Our first goal is to define the **magnetic field**. In order to reach it, let us revise that

electric force arises in two stages: (1) a charge produces an electric field in the space around it, and (2) a second charge responds to this field.



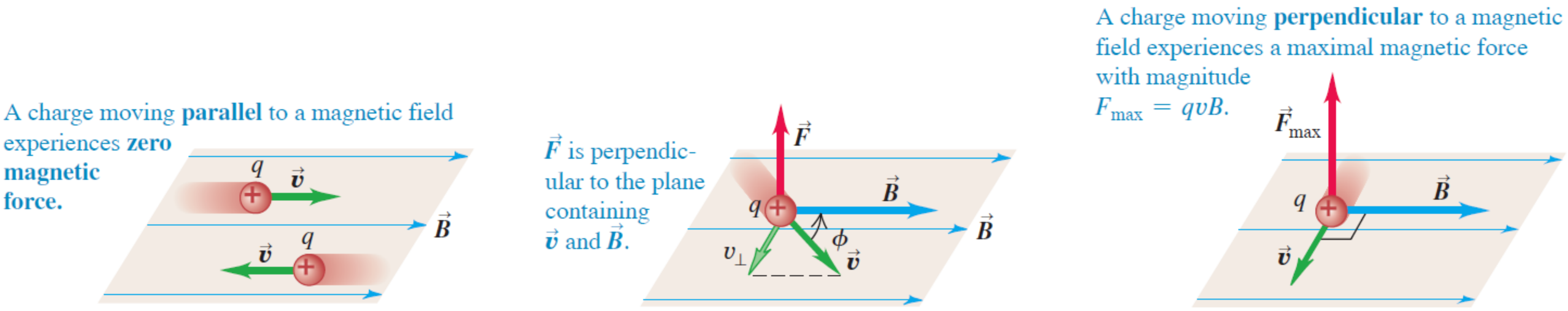
unlike electric forces, which act on electric charges despite they are moving or not, magnetic forces act only on **moving** charges

magnetic force arises in two stages: (1) a **moving** charge (or current) produces a **magnetic** field in the space around it (in addition to electric field), and (2) a second moving charge responds to this field.

Magnetic Fields

In order to define the **magnetic field**, we use the *experimental* fact that when a charged particle moves through a magnetic field ***B***, a magnetic force ***F*** acts on this particle.

- the magnitude of the force is proportional to the magnitude of the charge
- the magnitude of the force is proportional to the magnitude of the field
- the magnitude of the force depends on the particle's velocity
- the magnetic force is always perpendicular to both ***B*** and ***v***



Magnetic Fields

In order to define the **magnetic field**, we use the *experimental* fact that when a charged particle moves through a magnetic field ***B***, a magnetic force ***F*** acts on this particle.

$$F = |q| v_{\perp} B = |q| v B \sin\phi$$

magnitude is not the complete characteristic of the force as a vector quantity

Magnetic force on a moving charged particle → $\vec{F} = q\vec{v} \times \vec{B}$

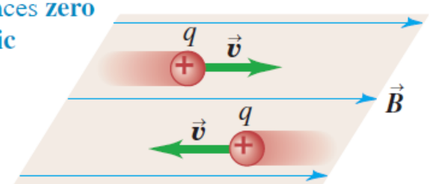
Particle's charge

Particle's velocity

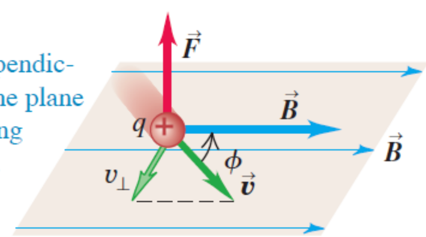
Magnetic field

<u>experimental</u>
deduction

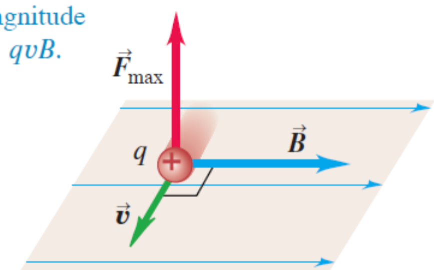
A charge moving **parallel** to a magnetic field experiences **zero magnetic force.**



$\vec{F}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.



A charge moving **perpendicular** to a magnetic field experiences a maximal magnetic force with magnitude $F_{max} = qvB$.



Magnetic Fields

In order to define the **magnetic field**, we use the *experimental* fact that when a charged particle moves through a magnetic field ***B***, a magnetic force ***F*** acts on this particle.

$$F = |q|v_{\perp}B = |q|vB\sin\phi$$

magnitude is not the complete characteristic of the force as a vector quantity

Magnetic force on a moving charged particle

$$\vec{F} = q\vec{v} \times \vec{B}$$

Particle's charge

Particle's velocity

Magnetic field

experimental deduction

because magnetic-field patterns are 3D, it's often necessary to draw magnetic field lines that point into or out of the plane of drawing

$(\bullet)$

towards us

$(\times)$

from us

Magnetic Fields

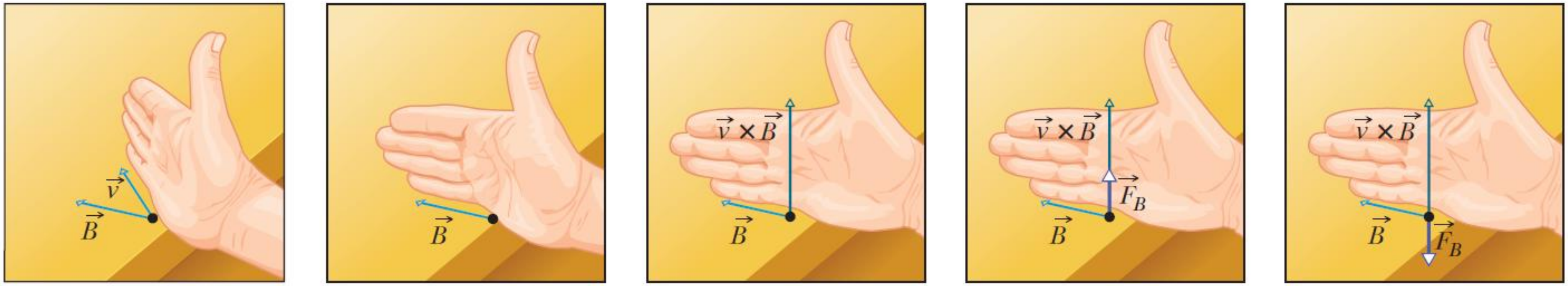
$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

Right-hand rule

Cross $\vec{v}$ into $\vec{B}$ to get the new vector $\vec{v} \times \vec{B}$.

Force on positive particle

Force on negative particle



Magnetic field SI units: tesla (T) = (N / (C m/s)) = (N / A m)

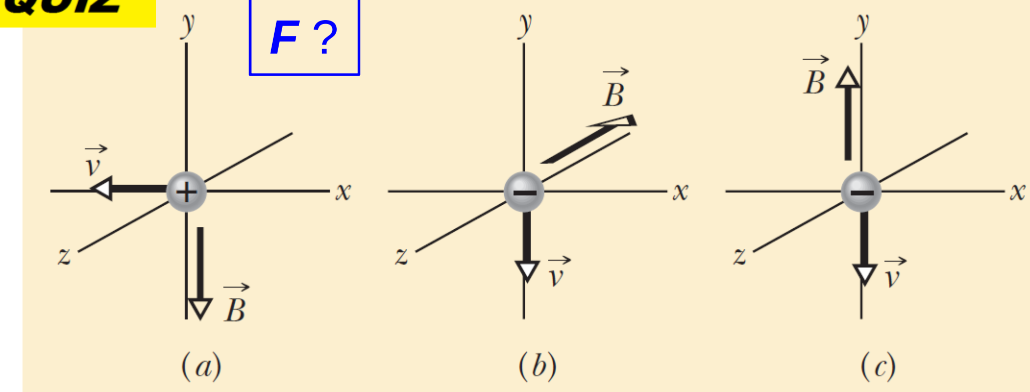
gauss (1 G = 10^{-4} T)

Check your understanding

At surface of neutron star	10^{8} T
Near big electromagnet	1.5 T
Near small bar magnet	10^{-2} T
At Earth's surface	10^{-4} T
In interstellar space	10^{-10} T
Smallest value in magnetically shielded room	10^{-14} T

QUIZ

F ?

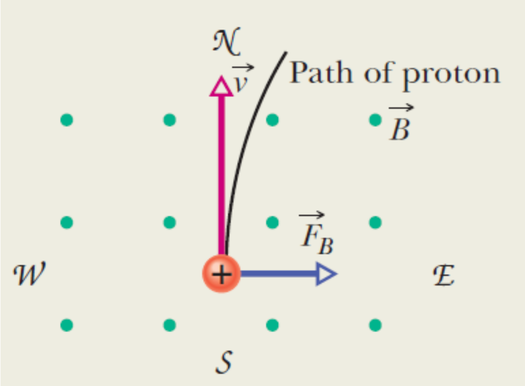


Magnetic Fields

EXERCISE

Task #1: A uniform magnetic field , with magnitude 1.2 mT, is directed vertically upward throughout the volume of a laboratory chamber. A proton with kinetic energy 5.3 MeV enters the chamber, moving horizontally from south to north. What magnetic deflecting force acts on the proton as it enters the chamber? The proton mass is 1.67×10^{-27} kg.

Solution:



In order to find the magnitude of the force, we need to calculate the speed of the proton first.

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(5.3\ \text{MeV})(1.60 \times 10^{-13}\ \text{J/MeV})}{1.67 \times 10^{-27}\ \text{kg}}}$$
$$= 3.2 \times 10^{7}\ \text{m/s.}$$

$$\begin{aligned} F_B &= |q|vB \sin\phi \\ &= (1.60 \times 10^{-19}\ \text{C})(3.2 \times 10^{7}\ \text{m/s}) \\ &\quad \times (1.2 \times 10^{-3}\ \text{T})(\sin 90^\circ) \\ &= 6.1 \times 10^{-15}\ \text{N.} \end{aligned}$$

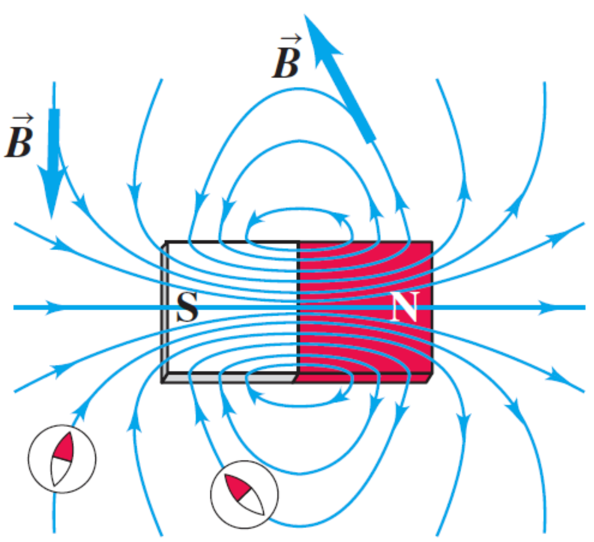
Note: the answer seems to be a small force, but it acts on a particle of small mass, producing a huge acceleration!

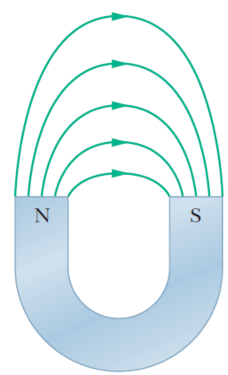
$$a = 3.7 \times 10^{12}\ \text{m/s}^2$$

Magnetic Field Lines

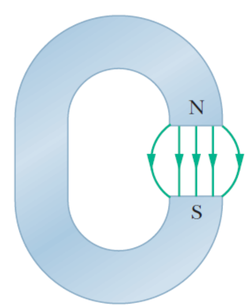
We can represent **magnetic** fields with **field lines**, as we did for electric fields.

- the direction of the tangent to a magnetic field line at any point gives the direction of ***B*** at that point.
- the spacing (density) of the lines represents the magnitude of ***B***: the magnetic field is stronger where the lines are closer together, and conversely.
- magnets have two **poles**: north pole where field lines emerge and south pole where they enter the magnet



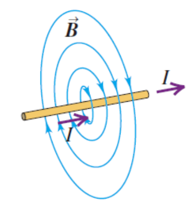


horseshoe magnet

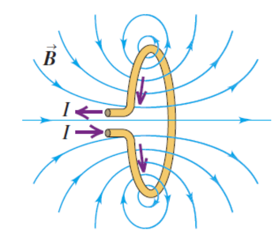


C-shaped magnet

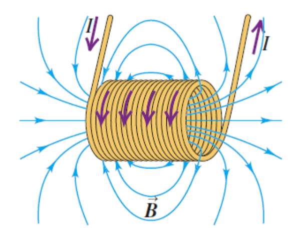
current-carrying wires



straight wire



loop



solenoid

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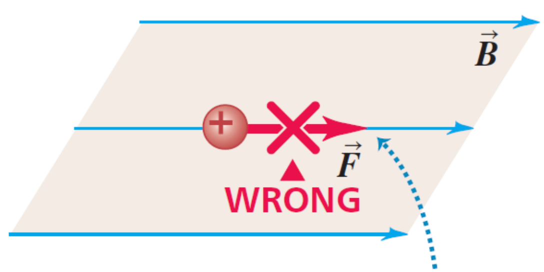
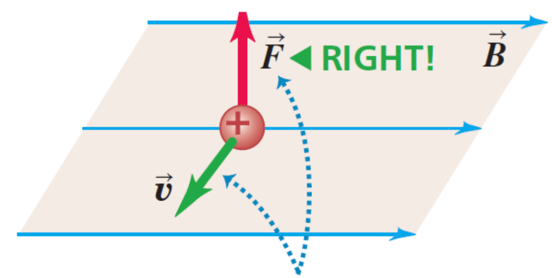
Magnetic field lines are **NOT** the “lines of force” !!!



Magnetic field lines are *not* “lines of force.” The force on a charged particle is not along the direction of a field line.



The direction of the magnetic force depends on the velocity $\vec{v}$, as expressed by the magnetic force law $\vec{F} = q\vec{v} \times \vec{B}$.

Magnetic Field Lines

We can represent **magnetic** fields with **field lines**, as we did for electric fields.

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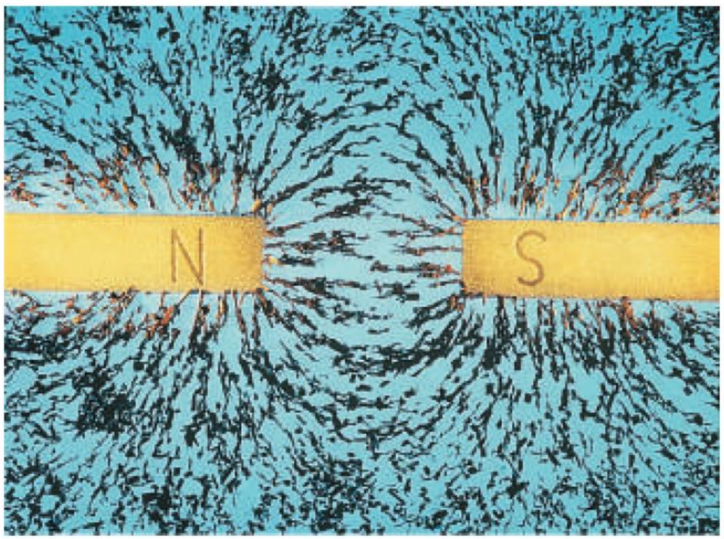
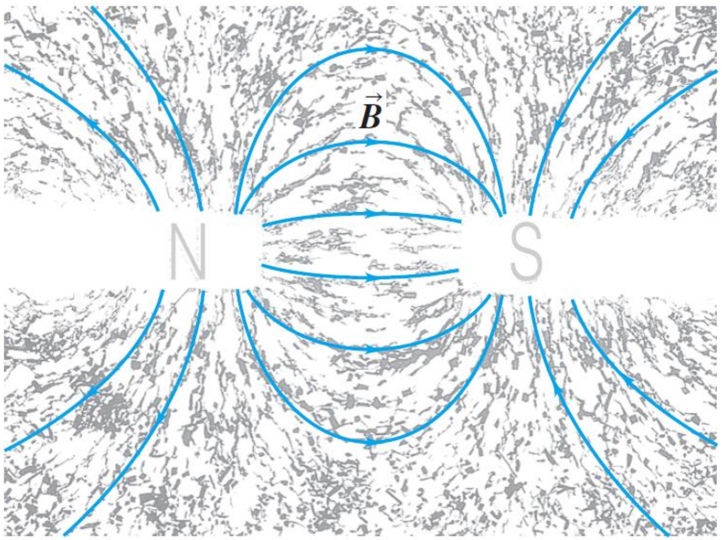
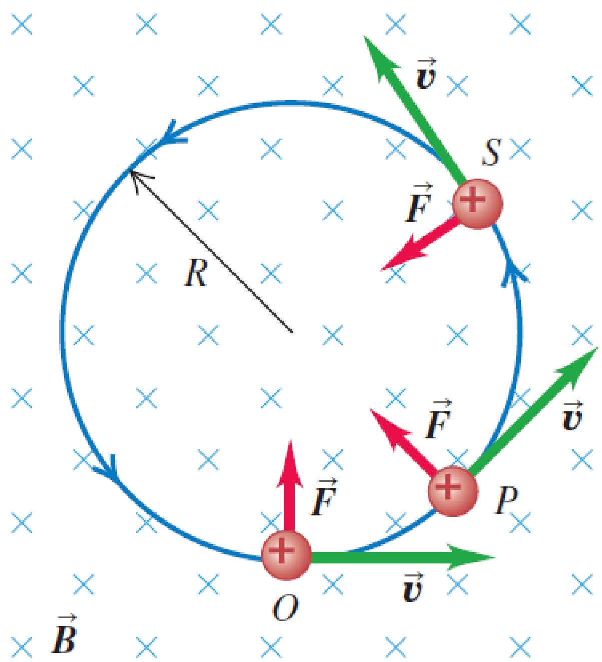




Iron filings arrange themselves along the magnetic field lines

A Circulating Charged Particle

A **charged** particle moving with velocity **perpendicular** to a uniform magnetic field will travel in a **circle**.



2nd Newton's law applied to uniform circular motion:

$$F = m\frac{v^2}{r}$$

$$|q|vB = \frac{mv^2}{r}$$

$$r = \frac{mv}{|q|B} \quad \text{(radius)}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{|q|B} = \frac{2\pi m}{|q|B} \quad \text{(period)}$$

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m} \quad \text{(frequency)} \qquad \omega = 2\pi f = \frac{|q|B}{m} \quad \text{(angular frequency)}$$

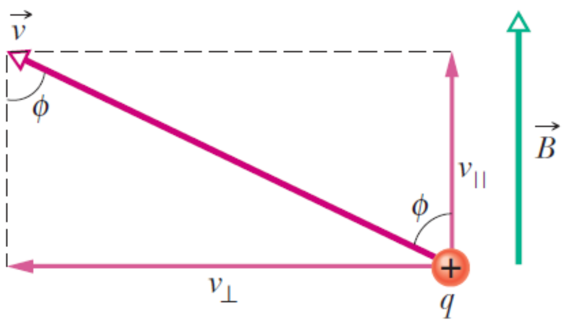
cyclotron frequency (independent of *r*)

A Circulating Charged Particle

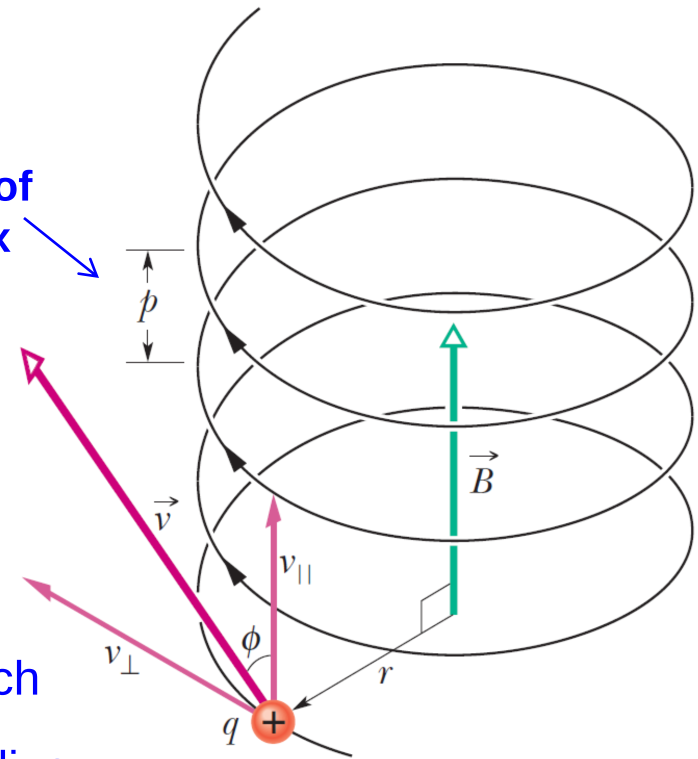
Helical paths

If the velocity of a charged particle has a component parallel to the magnetic field, the particle will move in a helical path about the direction of the magnetic field vector.

The velocity component perpendicular to the field causes circling, which is stretched upward by the parallel component.



pitch of a helix

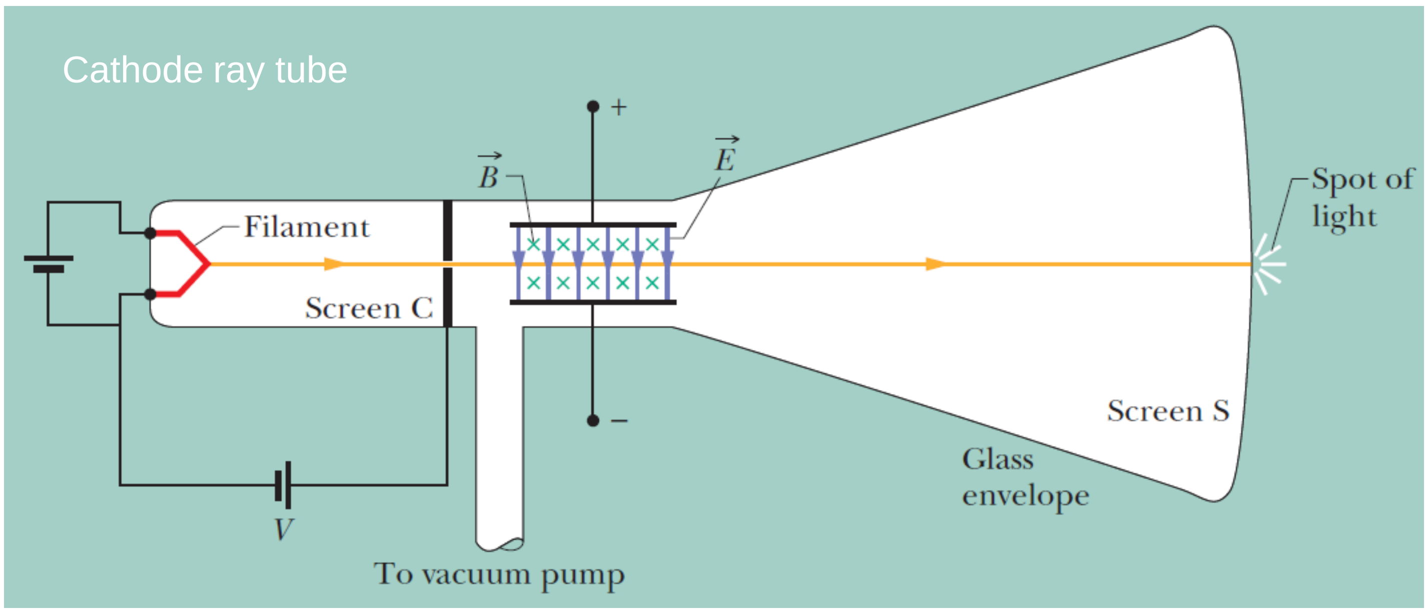


$v_{||} = v\cos\phi$ → determines the pitch

$v_{\perp} = v\sin\phi$ → determines the radius

Crossed Fields

If a charged particle moves through a region containing **both** an **electric** field and a **magnetic** field, it can be affected by both an electric force and a magnetic force. If the fields are perpendicular to each other, they are said to be **crossed fields**.



$$\vec{E} \qquad B = 0$$

EXERCISE

$$y = \frac{|q|\,EL^2}{2mv^2}$$

vertical deflection due to ***E*** only

Then we switch on B and try to compensate this deflection

$$|q|E = |q|vB\sin(90^\circ) = |q|vB$$

(opposite forces canceling)

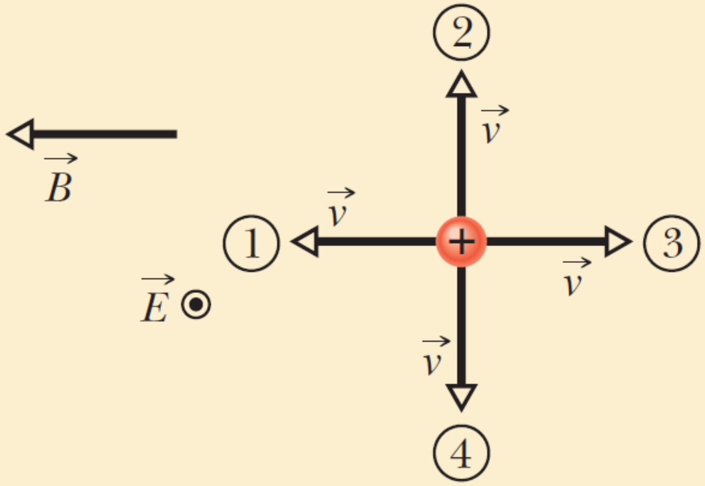
$$\boxed{v = \frac{E}{B}}$$

$$\boxed{\frac{m}{|q|} = \frac{B^2L^2}{2yE}}$$

Crossed Fields

QUIZ Check your understanding:

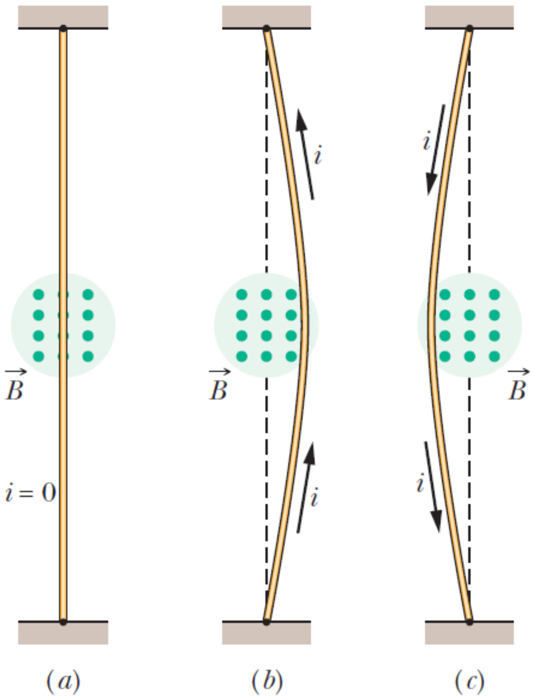
The figure shows four directions for the velocity vector $\vec{v}$ of a positively charged particle moving through a uniform electric field $\vec{E}$ (directed out of the page and represented with an encircled dot) and a uniform magnetic field $\vec{B}$. (a) Rank directions 1, 2, and 3 according to the magnitude of the net force on the particle, greatest first. (b) Of all four directions, which might result in a net force of zero?



Magnetic Force on a Current-Carrying Wire

Magnetic field affects the motion of any charged particles.

➡ **electrons** moving in a **wire** should also be affected by the external magnetic field. As a result, the **force** which is exerted on the electrons should be **transmitted** to the wire itself.



A force acts on a current through a B field.

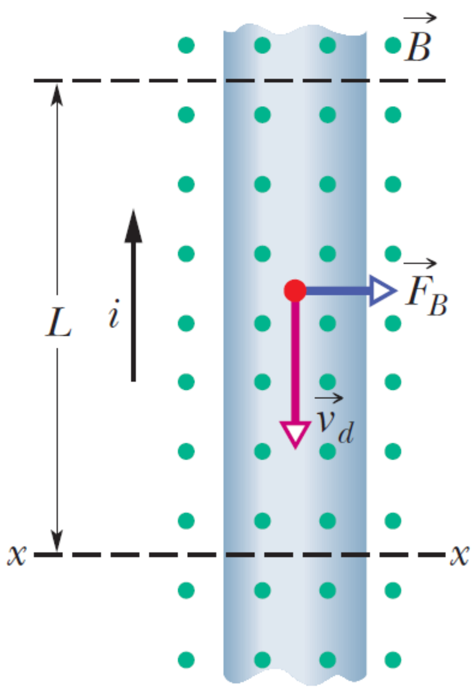
$$q = it = i\frac{L}{v_d}$$

$$F_B = qv_dB\sin\phi = \frac{iL}{v_d}v_dB\sin 90^\circ$$

$$F_B = iLB$$

magnetic force that acts on a straight current-carrying wire in case when a uniform magnetic field is **perpendicular** to the wire

electrons are shown



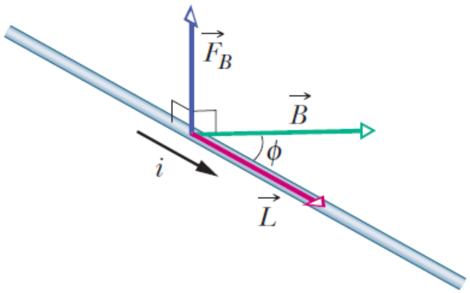
Note: it does not matter whether we consider negative charges drifting downward or positive charges drifting upward → the direction of the deflecting force on the wire is the same.

Magnetic Force on a Current-Carrying Wire

Magnetic field affects the motion of any charged particles.

electrons moving in a **wire** should also be affected by the external magnetic field. As a result, the **force** which is exerted on the electrons should be **transmitted** to the wire itself.

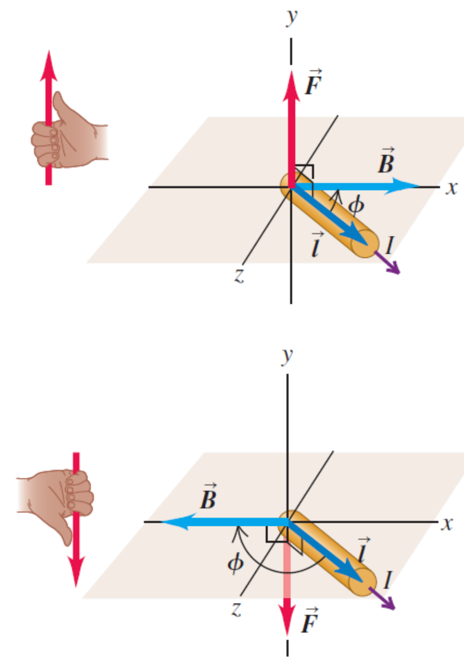
The force is perpendicular to both the field and the length.



Generalization:

$$\boxed{\vec{F}_B = i\vec{L} \times \vec{B}}$$

$$F_B = iLB\sin\phi$$



Case of a crooked (non-straight) wire:

$$d\vec{F}_B = i\, d\vec{L} \times \vec{B}$$

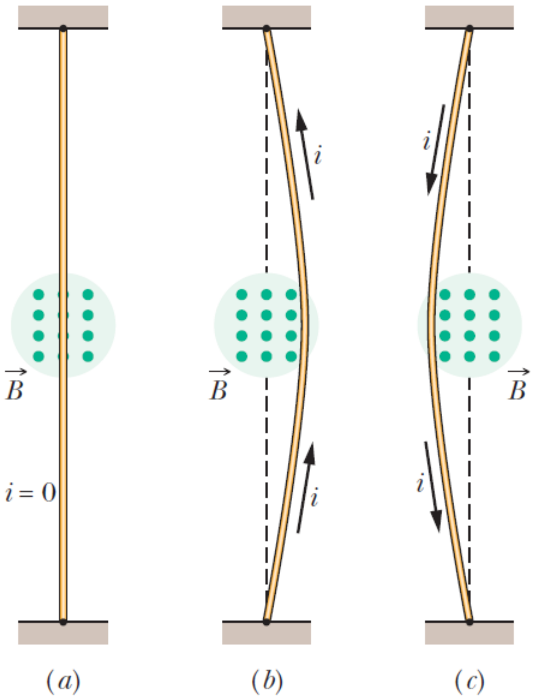
to find the net force one should integrate along the wire

Note: current is NOT a vector quantity!

Magnetic Force on a Current-Carrying Wire

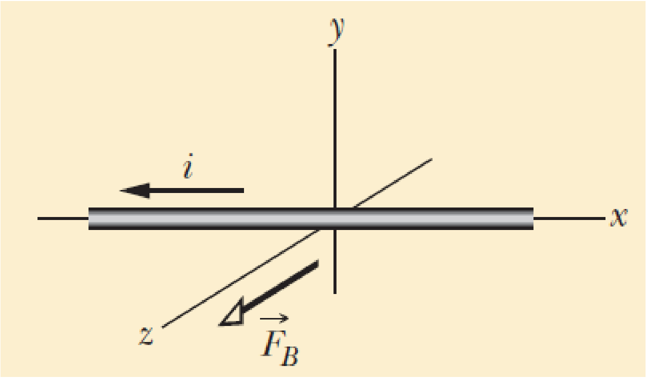
Magnetic field affects the motion of any charged particles.

electrons moving in a **wire** should also be affected by the external magnetic field. As a result, the **force** which is exerted on the electrons should be **transmitted** to the wire itself.



QUIZ Check your understanding:

The figure shows a current i through a wire in a uniform magnetic field $\vec{B}$, as well as the magnetic force $\vec{F}_B$ acting on the wire. The field is oriented so that the force is maximum. In what direction is the field?



Note: it does not matter whether we consider negative charges drifting downward or positive charges drifting upward → the direction of the deflecting force on the wire is the same.

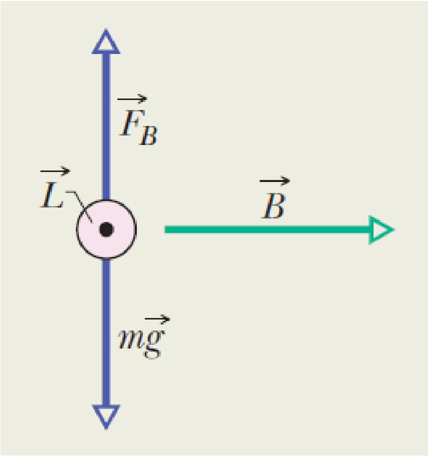
Magnetic Force on a Current-Carrying Wire

EXERCISE

Task #2: A straight, horizontal length of copper wire has a current ***i*** = 28 A through it. What are the magnitude and direction of the minimum magnetic field needed to suspend the wire – that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 46.6 g/m.

Solution:

Because ***L*** is directed horizontally, the right-hand rule for the cross products tells us that ***B*** must be horizontal and rightward in order to give the required upward $\boldsymbol{F}_B$



$$iLB \sin \phi = mg$$

$$B = \frac{mg}{iL \sin \phi} = \frac{(m/L)g}{i}$$

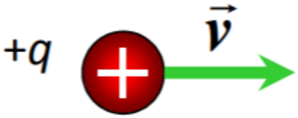
$$B = \frac{(46.6 \times 10^{-3}\ \text{kg/m})(9.8\ \text{m/s}^2)}{28\ \text{A}}$$
$$= 1.6 \times 10^{-2}\ \text{T}$$

Note: this is about 160 times the strength of Earth's magnetic field.

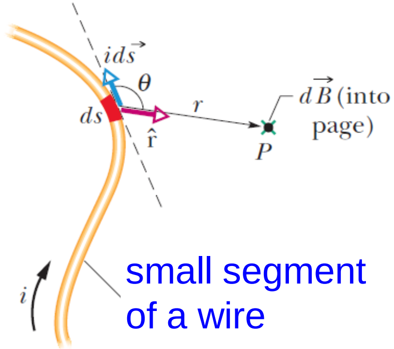
Sources of Magnetic Field

Up to the moment we just took the existence of a magnetic field as a given fact and did not worry about the details of how the magnetic field was created.

- now it's time consider this in details
- electric force arises in two stages: (1) a charge produces an electric field in the space around it, and (2) a second charge responds to this field.
- magnetic force arises in two stages: (1) a **moving** charge (or current) produces a **magnetic** field in the space around it (in addition to electric field), and (2) a second moving charge responds to this field.



single charge



small segment of a wire

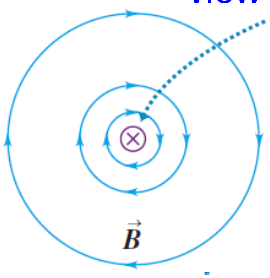
conductor of any shape

Magnetic Field of a Moving Charge

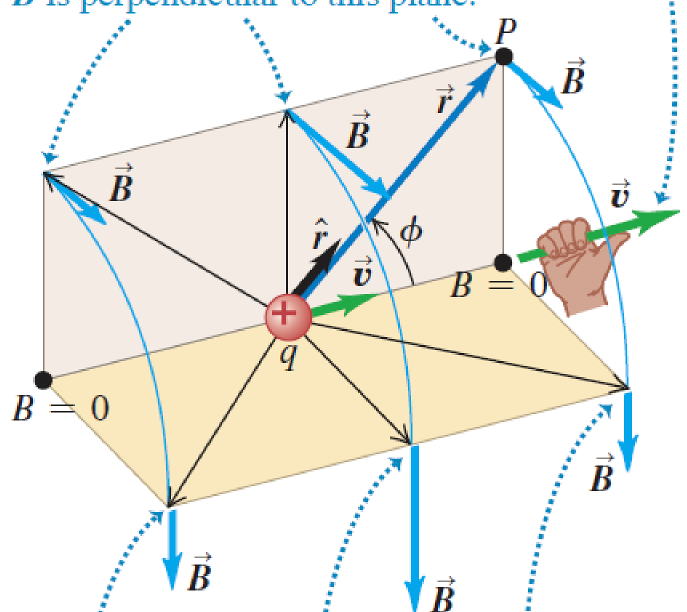
Let's consider a single point charge q moving with some constant velocity $\boldsymbol{v}$.

unlike to electric field $\boldsymbol{E}$, the direction of magnetic field $\boldsymbol{B}$ is **not** along the line from source point to field point. Moreover, $\boldsymbol{B}$ is perpendicular to the plane of $\boldsymbol{r}$-$\boldsymbol{v}$

side view



For these field points, $\vec{r}$ and $\vec{v}$ both lie in the beige plane, and $\vec{B}$ is perpendicular to this plane.



For these field points, $\vec{r}$ and $\vec{v}$ both lie in the gold plane, and $\vec{B}$ is perpendicular to this plane.

Experimental fact: $$B = \frac{\mu_0}{4\pi}\frac{|q|v\sin\phi}{r^2}$$

Magnetic constant, Charge, Velocity, Unit vector from point charge toward where field is measured

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

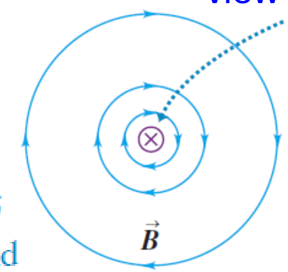
Distance from point charge to where field is measured

Magnetic field due to a point charge with constant velocity

$$\boxed{\mu_0 \cong 4\pi \times 10^{-7}\ \mathrm{T\cdot m/A}}$$

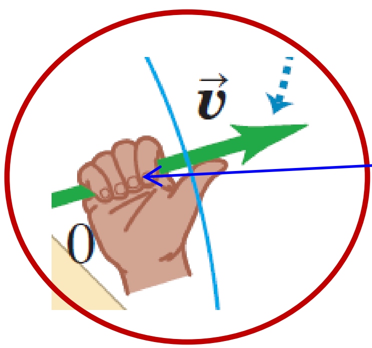
Magnetic Field of a Moving Charge

Let's consider a single point charge *q* moving with some constant velocity ***v***.

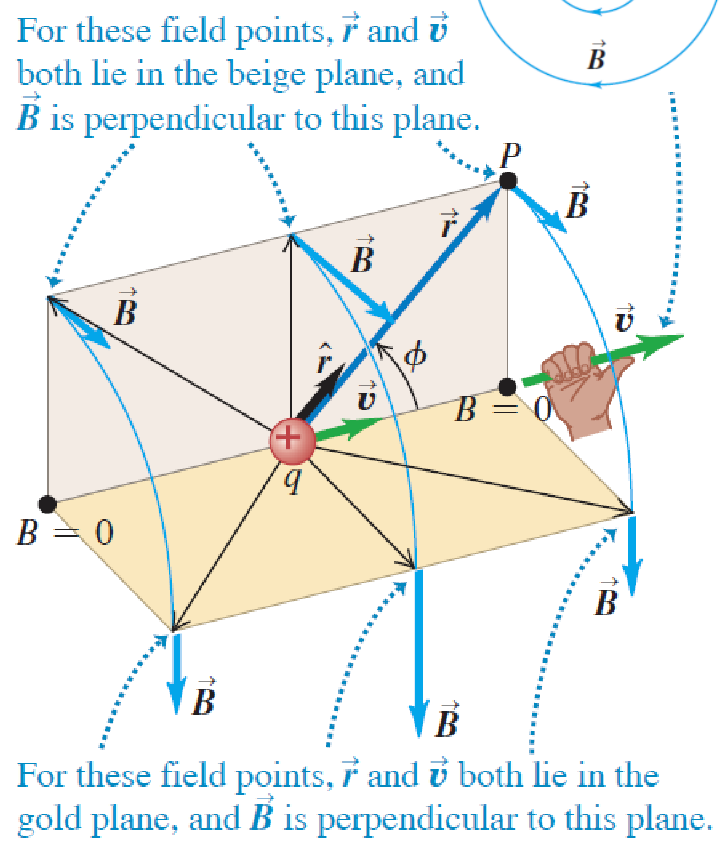


- unlike to electric field ***E***, the direction of magnetic field ***B*** is **not** along the line from source point to field point. Moreover, ***B*** is perpendicular to the plane of ***r***-***v***

- the field-line directions for a positive charge can be obtained by means of the right-hand rule



the fingers curl around the velocity line in the same way as the magnetic field lines do



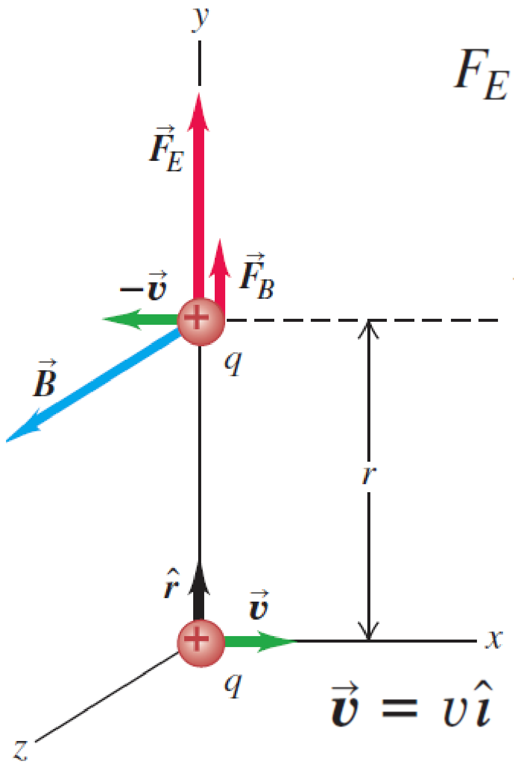
- the expression for ***B*** is valid for constant velocities only! If the charge accelerates, the field can be much more complicated!

Magnetic Field of a Moving Charge

EXERCISE

Task #3: Two protons move parallel to the *x*-axis in opposite directions at the same speed (small compared to the speed of light). At the instant shown, find the electric and magnetic forces on the upper proton and compare their magnitudes.

Solution:



$$F_E = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q(v\hat{\imath})\times\hat{\jmath}}{r^2} = \frac{\mu_0}{4\pi}\frac{qv}{r^2}\hat{k}$$

$$\vec{F}_B = q(-\vec{v})\times\vec{B} = q(-v\hat{\imath})\times\frac{\mu_0}{4\pi}\frac{qv}{r^2}\hat{k} = \frac{\mu_0}{4\pi}\frac{q^2v^2}{r^2}\hat{\jmath}$$

$$\frac{F_B}{F_E} = \frac{\mu_0 q^2v^2/4\pi r^2}{q^2/4\pi\epsilon_0 r^2} = \frac{\mu_0 v^2}{1/\epsilon_0} = \epsilon_0\mu_0 v^2$$

$$\frac{F_B}{F_E} = \frac{v^2}{c^2}$$

for small speeds magnetic force is much smaller than the electric force!

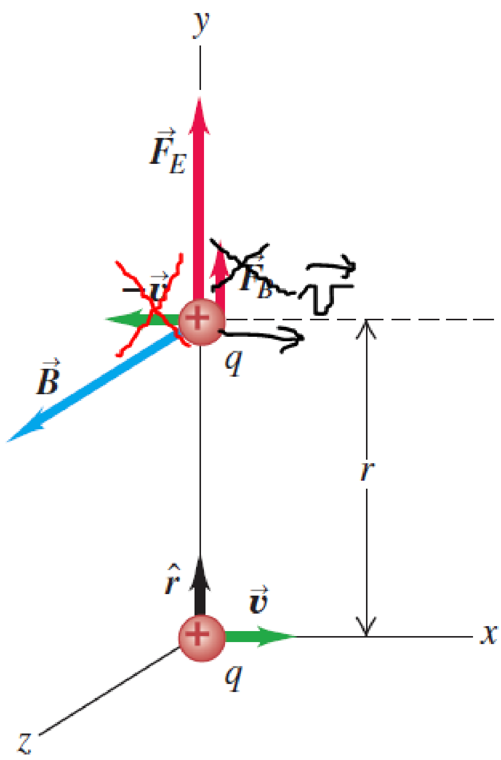
$$\boxed{c = \frac{1}{\sqrt{\epsilon_0\mu_0}}}$$

Magnetic Field of a Moving Charge

EXERCISE

Task #3: Two protons move parallel to the *x*-axis in opposite directions at the same speed (small compared to the speed of light). At the instant shown, find the electric and magnetic forces on the upper proton and compare their magnitudes.

Solution:



QUIZ Check your understanding:

If both protons travel in the same direction, is the magnetic force between them
(i) attractive or (ii) repulsive?

Magnetic Field of a Current Element

PRINCIPLE OF SUPERPOSITION OF MAGNETIC FIELDS **The total magnetic field caused by several moving charges is the vector sum of the fields caused by the individual charges.**

- based on this principle, one can find the magnetic field produced by a current
- let us consider a short segment *dl* of a current-carrying conductor

EXERCISE

Magnetic constant

Current

Vector length of element (points in current direction)

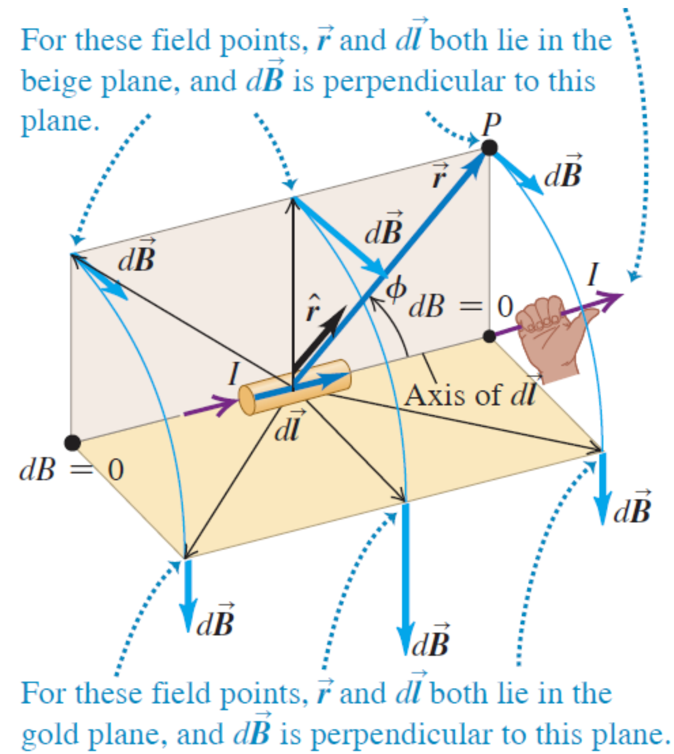
Unit vector from element toward where field is measured

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

Distance from element to where field is measured

Magnetic field due to an infinitesimal current element

Biot-Savart's law



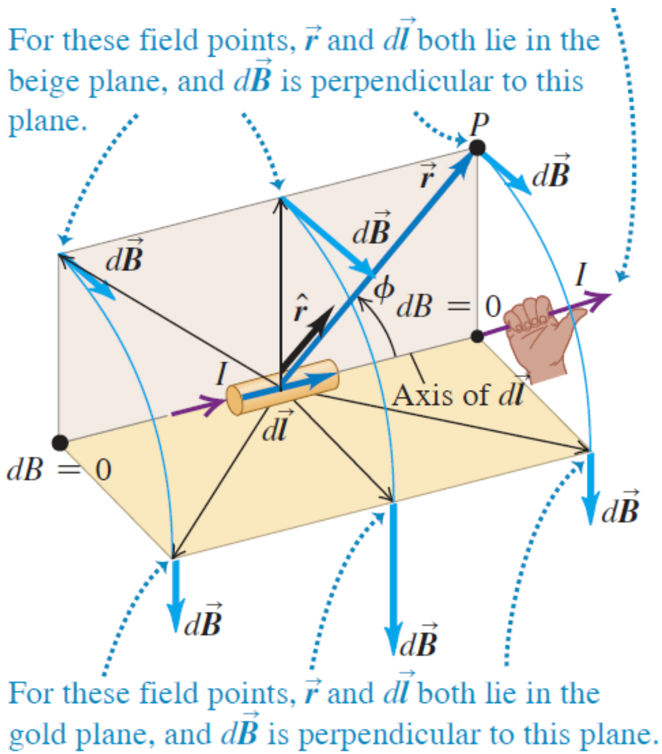
Magnetic Field of a Current Element

PRINCIPLE OF SUPERPOSITION OF MAGNETIC FIELDS **The total magnetic field caused by several moving charges is the vector sum of the fields caused by the individual charges.**

- based on this principle, one can find the magnetic field produced by a current

- to find the total magnetic field ***B*** at any point in space due to the current in a complete circuit, one should integrate the expression over all segments that carry current

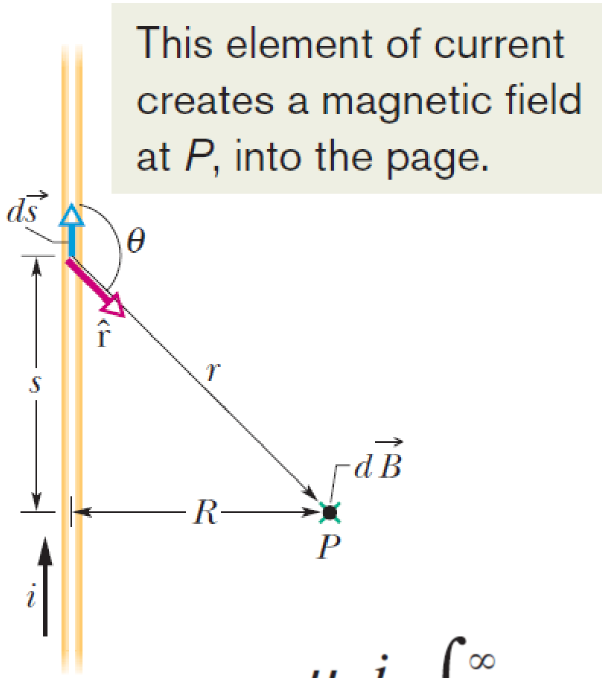
$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$



Magnetic Field due to a Current in a Long Straight Wire

Consider the magnetic field produced at some arbitrary point by a current in a long straight wire.

EXERCISE



$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2} \qquad r = \sqrt{s^2 + R^2}$$

$$B = 2\int_0^\infty dB = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{\sin\theta\,ds}{r^2}$$

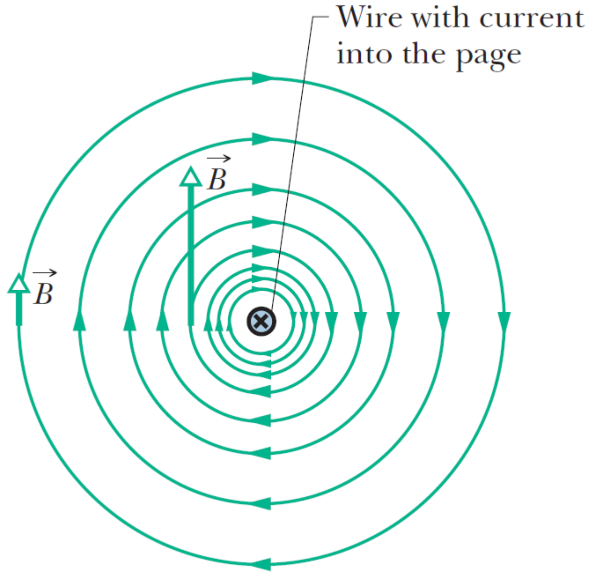
$$\sin\theta = \sin(\pi - \theta) = \frac{R}{\sqrt{s^2 + R^2}}$$

$$B = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{R\,ds}{(s^2 + R^2)^{3/2}} = \frac{\mu_0 i}{2\pi R}\left[\frac{s}{(s^2 + R^2)^{1/2}}\right]_0^\infty \boxed{= \frac{\mu_0 i}{2\pi R}}$$

Magnetic Field due to a Current in a Long Straight Wire

Consider the magnetic field produced at some arbitrary point by a current in a long straight wire.

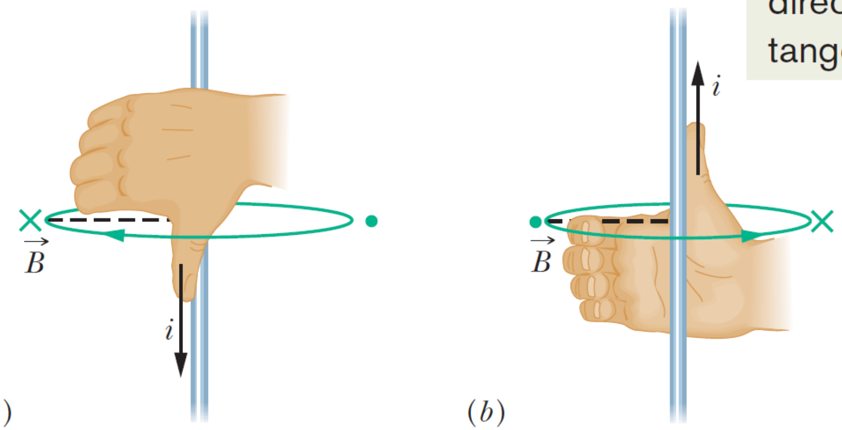
EXERCISE



The magnetic field vector at any point is tangent to a circle.

$$B = \frac{\mu_0 i}{2\pi R}$$

The thumb is in the current's direction. The fingers reveal the field vector's direction, which is tangent to a circle.

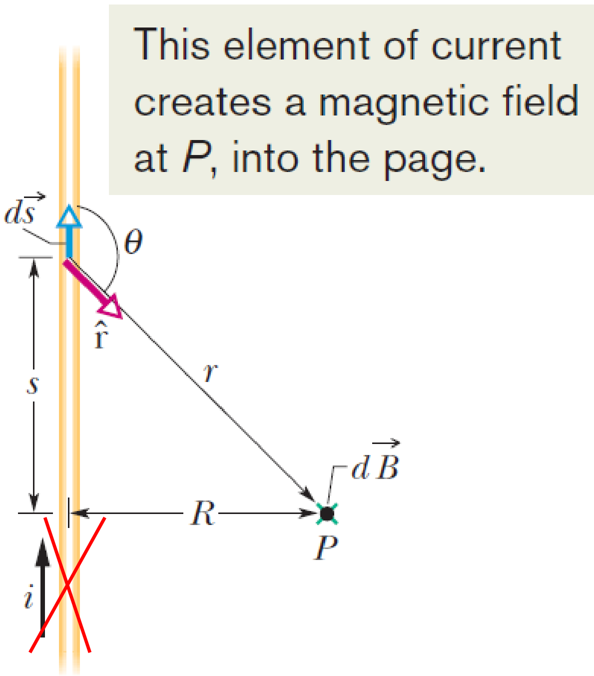


Curled–straight right-hand rule: Grasp the element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element.

Magnetic Field due to a Current in a Long Straight Wire

Consider the magnetic field produced at some arbitrary point by a current in a long straight wire.

EXERCISE



Case of a semi-infinite straight wire (with *P* being located in a perpendicular plane at one of its ends):

$$dB = \frac{\mu_0}{4\pi} \frac{i\, ds \sin\theta}{r^2} \qquad r = \sqrt{s^2 + R^2}$$

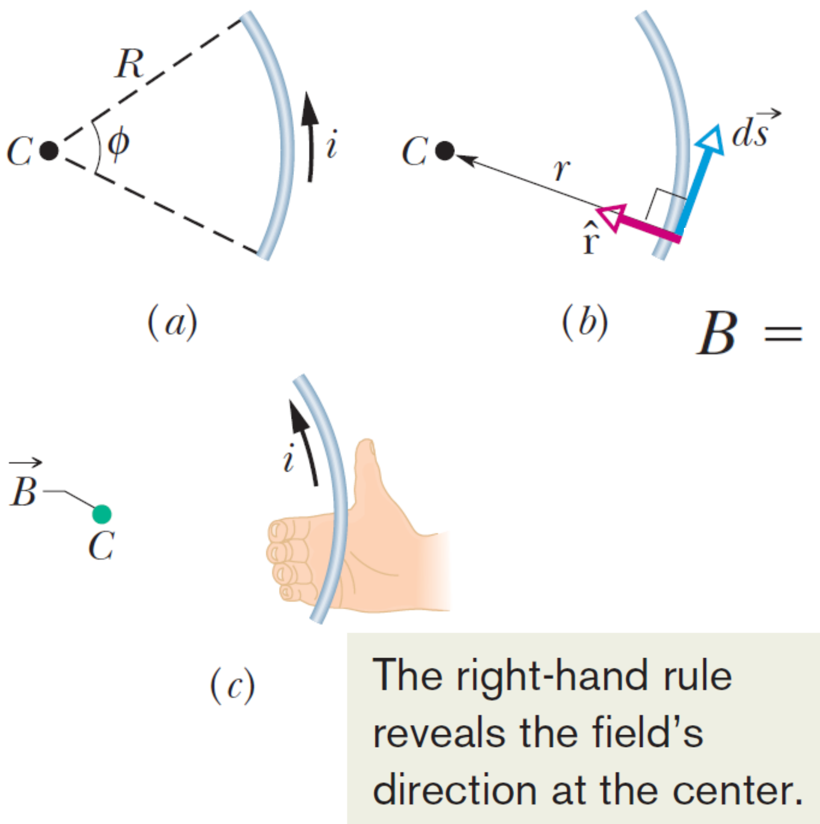
$$B = \cancel{2}\int_0^\infty dB = \frac{\mu_0 i}{2*2\pi} \int_0^\infty \frac{\sin\theta\, ds}{r^2}$$

$$\Rightarrow \boxed{B = \frac{\mu_0 i}{4\pi R}}$$

Magnetic Field due to a Current in a Circular Arc of Wire

Consider the magnetic field produced at the center *C* of an arc-shaped wire with the central angle *φ*.

EXERCISE



$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi}\frac{i\,ds}{R^2}$$

$$B = \int dB = \int_0^{\phi}\frac{\mu_0}{4\pi}\frac{iR\,d\phi}{R^2} = \frac{\mu_0 i}{4\pi R}\int_0^{\phi} d\phi$$

$$\boxed{B = \frac{\mu_0 i \phi}{4\pi R}}$$

at center of the circular arc

$$\boxed{B = \frac{\mu_0 i(2\pi)}{4\pi R} = \frac{\mu_0 i}{2R}}$$

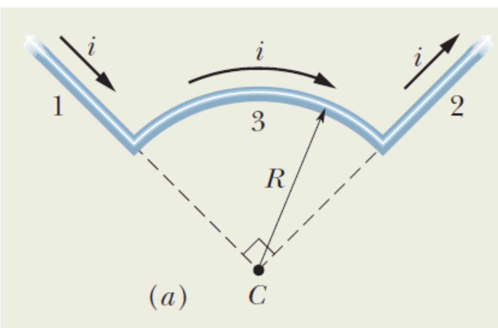
at center of full circle

Magnetic Field due to a Current in a Circular Arc of Wire

EXERCISE

Task #4: The wire shown on the figure below carries a current *i* and consists of a circular arc of radius *R* and central angle π/2 rad, and two straight sections whose extensions intersect the center *C* of the arc. What magnetic field (magnitude and direction) does the current produce at *C*?

Solution:

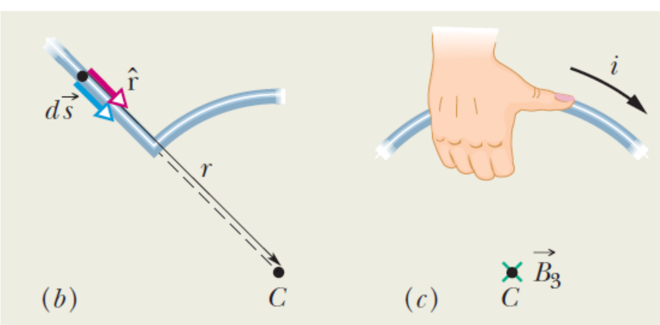


$$dB_1 = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 0}{r^2} = 0$$

Current directly toward or away from C does not create any field there.

$$B_1 = 0 \qquad B_2 = 0$$

$$B_3 = \frac{\mu_0 i(\pi/2)}{4\pi R} = \frac{\mu_0 i}{8R}$$

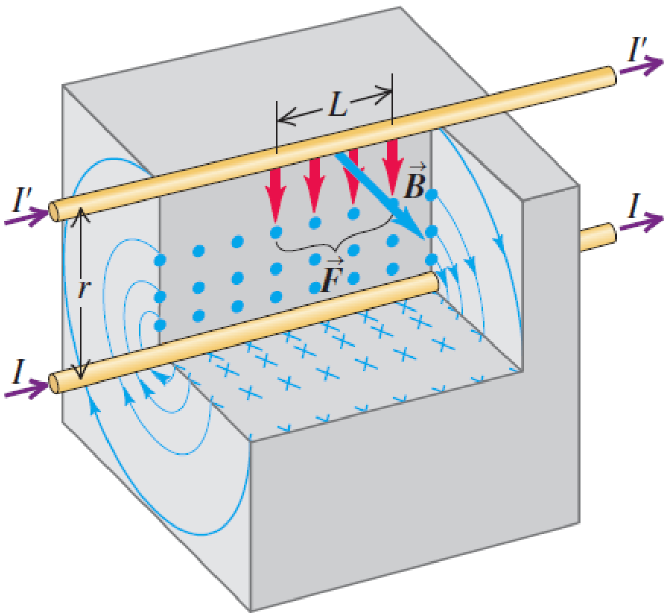


$$B = B_1 + B_2 + B_3 = 0 + 0 + \frac{\mu_0 i}{8R} = \frac{\mu_0 i}{8R}$$

The direction of $\vec{B}$ is the direction of $\vec{B}_3$

Force Between Two Parallel Currents

Two long **parallel** wires carrying **currents** exert forces on each other. This is due to magnetic field that one of the wires produces at the points of the other wire.



$$B = \frac{\mu_0 I}{2\pi r} \qquad \vec{F} = I'\vec{L} \times \vec{B}$$

Magnetic constant

Current in first conductor

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$$

Current in second conductor

Distance between conductors

magnetic force per unit length between two parallel long straight conductors

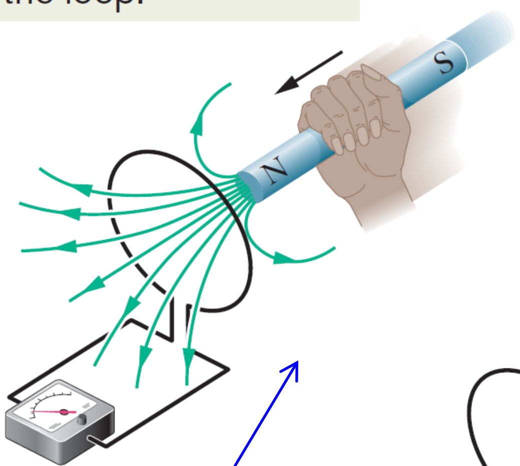
To find the force on a current-carrying wire due to a second current-carrying wire, first find the field due to the second wire at the site of the first wire. Then find the force on the first wire due to that field.

Parallel currents attract each other, and antiparallel currents repel each other.

Faraday's Law of Induction

We already know that a current produces a magnetic field. However, there exists a **reverse** effect: a magnetic field can produce an electric field that can drive a current.

The magnet's motion creates a current in the loop.



Experiment 1

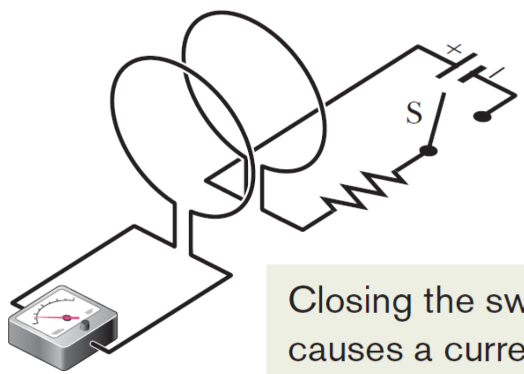
Faraday's law of induction: an emf is induced in the loop at the left in both figures when the number of magnetic field lines that pass through the loop is changing.

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$ (magnetic flux through area A)

SI units: (Wb) = (T m^2)

weber

Experiment 2



Closing the switch causes a current in the left-hand loop.

Special case:

$$\Phi_B = BA \quad (\vec{B} \perp \text{area } A, \vec{B} \text{ uniform})$$

$$\mathscr{E} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law)}$$

Faraday's Law of Induction

The magnitude of the emf $\mathscr{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.

single loop: $$\mathscr{E} = -\frac{d\Phi_B}{dt}$$

coil of *N* turns: $$\mathscr{E} = -N\frac{d\Phi_B}{dt}$$

The **magnetic flux** through a coil can be **changed** by:

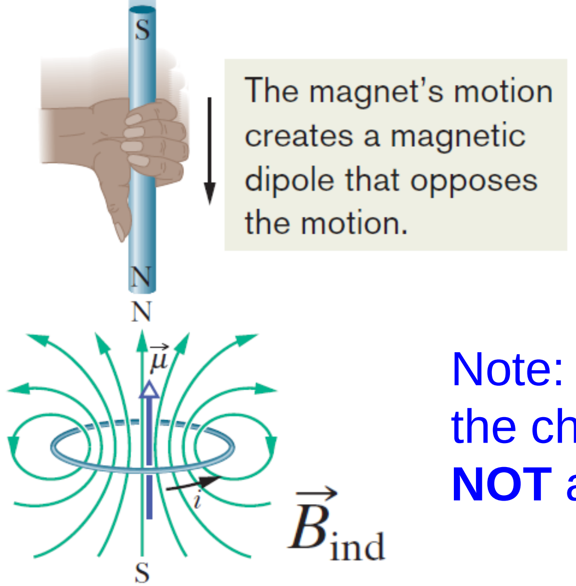
- changing the magnitude ***B*** of the magnetic field within the coil
- changing either the total area of the coil or the portion of that area that lies within the magnetic field (for example, by expanding the coil or sliding it into or out of the field)
- changing the angle between the direction of the magnetic field and the plane of the coil (for example, by rotating the coil so that field is first perpendicular to the plane of the coil and then is along that plane)

Lenz's Law

Soon after Faraday proposed his law of induction, Heinrich Friedrich Lenz devised a rule for determining the **direction** of an induced current in a loop.

> An induced current has a direction such that the magnetic field due to *the current* opposes the change in the magnetic flux that induces the current.

Note: the direction of an induced emf is that of the induced current.

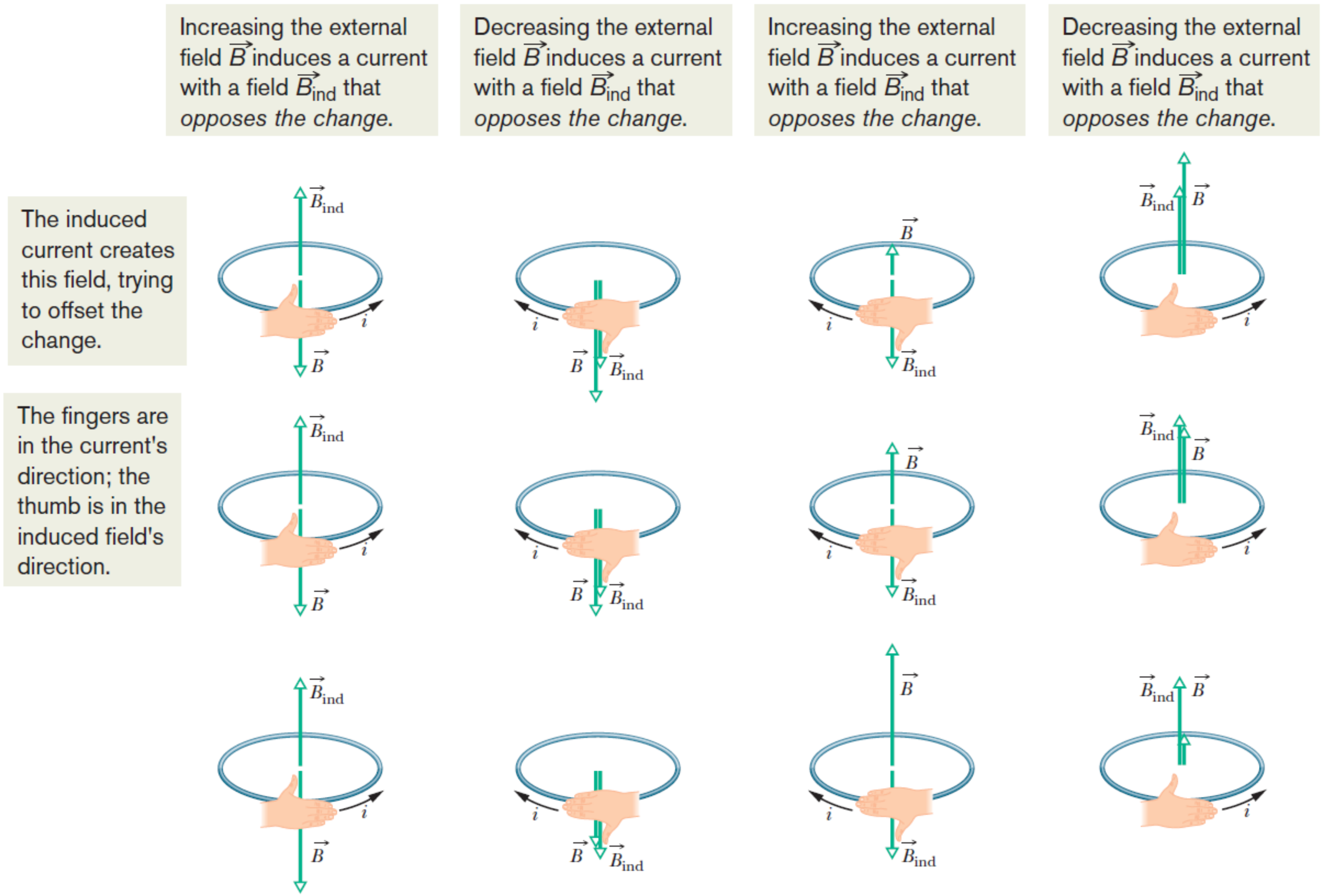


Key word: "opposition"

Note: the flux of $\boldsymbol{B}_{\text{ind}}$ always opposes the change in the flux of $\boldsymbol{B}$, but $\boldsymbol{B}_{\text{ind}}$ is **NOT** always opposite to $\boldsymbol{B}$.

Heinrich Friedrich Lenz
(1804-1865)

Lenz's Law

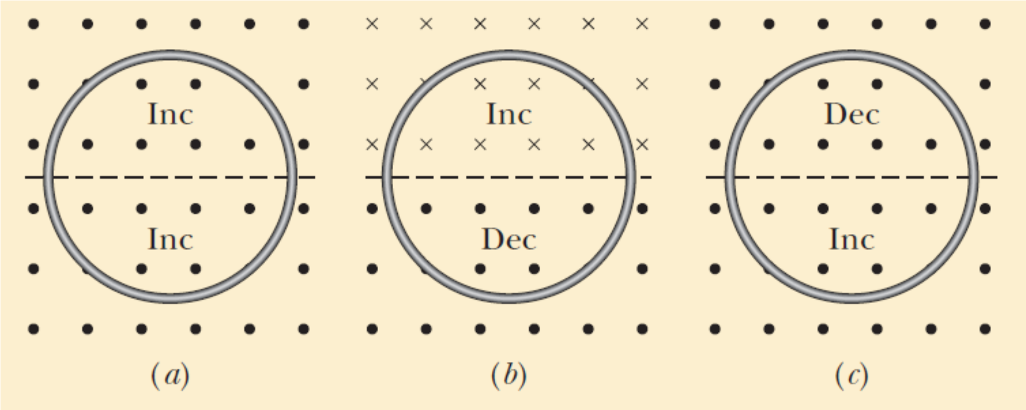


Increasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that *opposes the change.*
Decreasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that *opposes the change.*
Increasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that *opposes the change.*
Decreasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that *opposes the change.*
The induced current creates this field, trying to offset the change.
The fingers are in the current's direction; the thumb is in the induced field's direction.
$\vec{B}_{ind}$
$\vec{B}$
i

Lenz's Law

QUIZ Check your understanding:

The figure shows three situations in which identical circular conducting loops are in uniform magnetic fields that are either increasing (Inc) or decreasing (Dec) in magnitude at identical rates. In each, the dashed line coincides with a diameter. Rank the situations according to the magnitude of the current induced in the loops, greatest first.



Faraday's Law and Lenz's Law

EXERCISE

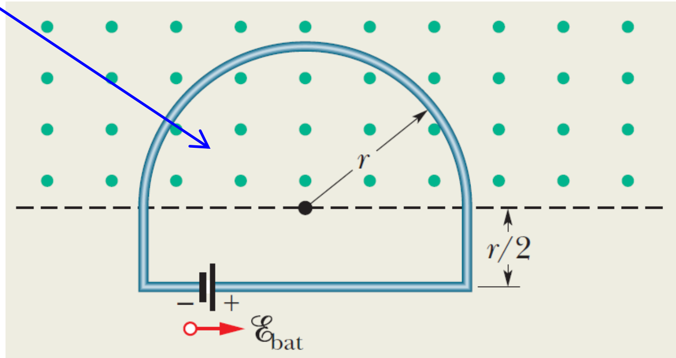
Task #5: Consider a conducting loop consisting of a half-circle of radius 0.20 m and three straight sections. The halfcircle lies in a uniform magnetic field that is directed out of the page; the field magnitude is given by $B = 4.0t^2 + 2.0t + 3.0$, with B in teslas and t in seconds. An ideal battery with emf 2.0 V is connected to the loop.The resistance of the loop is 2.0 Ω. (a) What are the magnitude and direction of the emf induced around the loop by field at t = 10 s? (b) What is the current in the loop at t = 10 s?

$\frac{1}{2}\pi r^2$

Solution:

(a) The flux changes only due to the time-dependent magnetic field

$$\mathscr{E}_{\text{ind}} = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt} = \frac{\pi r^2}{2}(8.0t + 2.0)$$



At $t = 10$ s

$$\mathscr{E}_{\text{ind}} = \frac{\pi(0.20\text{ m})^2}{2}[8.0(10) + 2.0] = 5.152\text{ V} \approx 5.2\text{ V}$$

Direction: clockwise

(because the induced field must oppose the increase of the initial field → it must go into the page)

Faraday's Law and Lenz's Law

EXERCISE

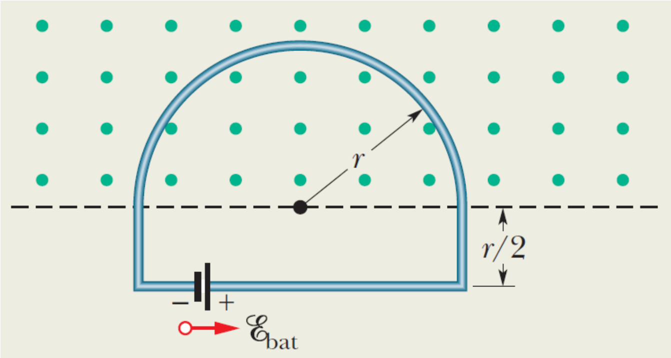
Task #5: Consider a conducting loop consisting of a half-circle of radius 0.20 m and three straight sections. The halfcircle lies in a uniform magnetic field that is directed out of the page; the field magnitude is given by $B = 4.0t^2 + 2.0t + 3.0$, with B in teslas and t in seconds. An ideal battery with emf 2.0 V is connected to the loop.The resistance of the loop is 2.0 Ω. (a) What are the magnitude and direction of the emf induced around the loop by field at t = 10 s? (b) What is the current in the loop at t = 10 s?

Solution:

(b) The induced emf (which has greater values) tends to drive a current **clockwise** around the loop; the battery's emf tends to drive a current counter-clockwise. Thus so is the current.

$$i = \frac{\mathscr{E}_{\text{net}}}{R} = \frac{\mathscr{E}_{\text{ind}} - \mathscr{E}_{\text{bat}}}{R}$$

$$= \frac{5.152\text{ V} - 2.0\text{ V}}{2.0\ \Omega} = 1.58\text{ A} \approx 1.6\text{ A}$$



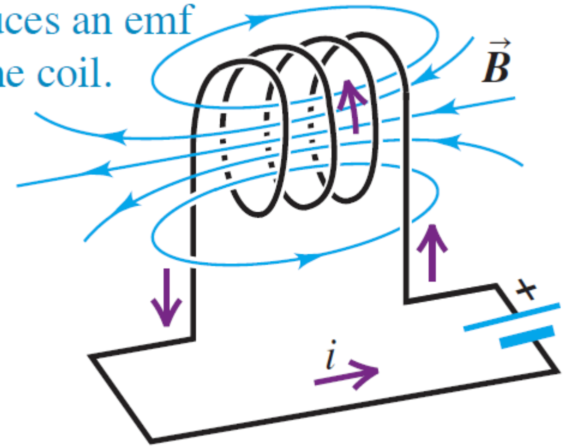
Direction of the resulting current: clockwise

Self-Inductance

An important related effect occurs in a **single** isolated circuit – **self-induced** emf.

- a current in a circuit sets up a magnetic field that causes magnetic flux though the same circuit. This flux changes when the current changes
- by Lenz's law, a self-induced emf opposes the change in the current that caused the emf and so tends to make it more difficult for variations in current to occur
- the effect is greatly enhanced if the circuit includes a coil with *N* turns of wire

Self-inductance: If the current i in the coil is changing, the changing flux through the coil induces an emf in the coil.



Self-inductance (or **inductance**) of a coil

Number of turns in coil

$$L = \frac{N\Phi_B}{i}$$

Flux due to current through each turn of coil

Current in coil

SI unit: (H) = (T m^2 / A)

$$N\frac{d\Phi_B}{dt} = L\frac{di}{dt}$$

Self-Inductance

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- by Lenz's law, a self-induced emf opposes the change in the current that caused the emf and so tends to make it more difficult for variations in current to occur
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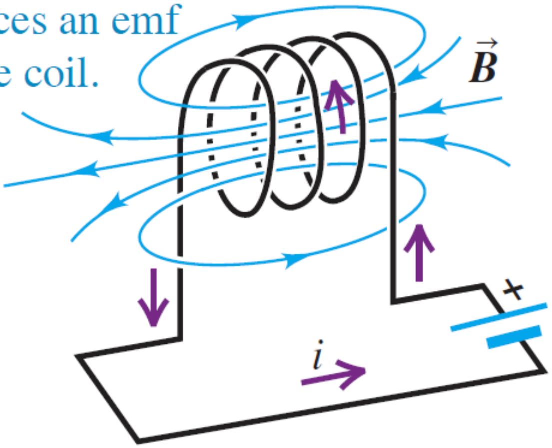


Inductance of circuit

Self-induced emf in a circuit

$$\mathcal{E} = -L\frac{di}{dt}$$

Rate of change of current in circuit

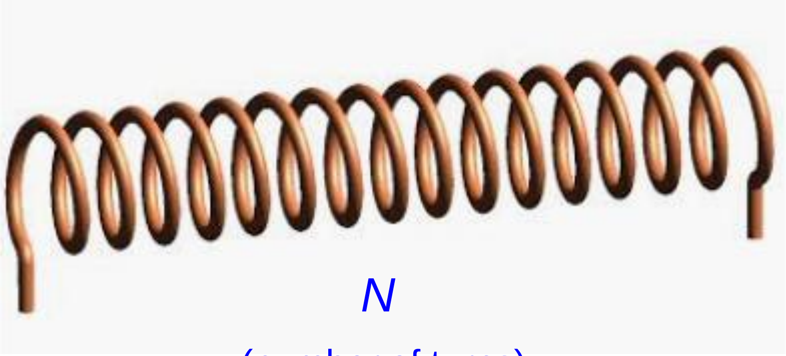
Inductors

Earlier we considered the parallel-plate arrangement as a basic type of a capacitor. Similarly, the **inductor** can be used to produce the desired **magnetic field**.

solenoid is the basic type of inductor

electric scheme symbol: 

SI unit: (H) = (T m^2 / A)



Definition of **inductance**

$$\boxed{L = \frac{N\Phi_B}{i}}$$

$N\Phi_B$ – magnetic flux **linkage**

the inductance is a measure of the flux linkage produced by the inductor per unit of current

Inductance of a solenoid

$$N\Phi_B = (nl)(BA)$$

$$B = \mu_0 i n$$

(ideal solenoid)

number of turns per unit length

$$L = \frac{N\Phi_B}{i} = \frac{(nl)(BA)}{i} = \frac{(nl)(\mu_0 i n)(A)}{i} = \mu_0 n^2 l A$$

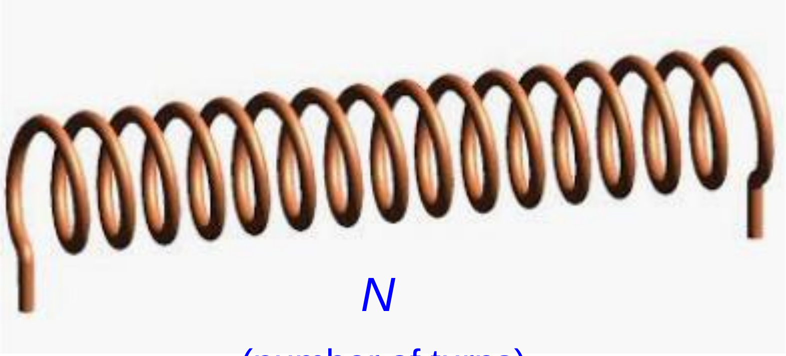
$$\boxed{\frac{L}{l} = \mu_0 n^2 A}$$

Inductors

Earlier we considered the parallel-plate arrangement as a basic type of a capacitor. Similarly, the **inductor** can be used to produce the desired **magnetic field**.

Inductance of a solenoid

$$\boxed{\frac{L}{l} = \mu_0 n^2 A}$$



(number of turns)

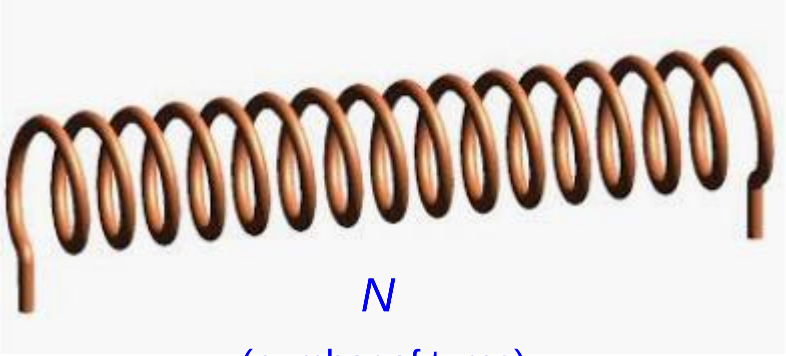
- Inductance – like capacitance – depends only on the geometry of the device (assuming that no non-linear magnetic effects are included and no magnetic material is present inside)

- The dependence on the square of the number of turns per unit length is to be expected (for example, if n = 3, we not only triple the number of turns but also triple the flux through each turn)

Inductors

Earlier we considered the parallel-plate arrangement as a basic type of a capacitor. Similarly, the **inductor** can be used to produce the desired **magnetic field**.

Inductance of a solenoid ⇒ $$\frac{L}{l} = \mu_0 n^2 A$$

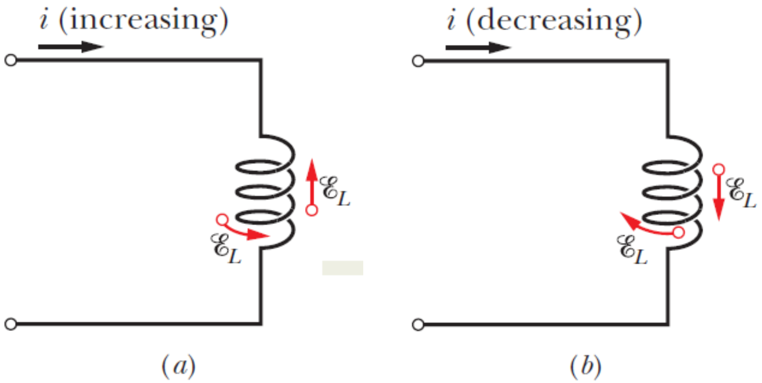


A

N

(number of turns)

Direction of the self-induced emf can be found from Lenz's law.

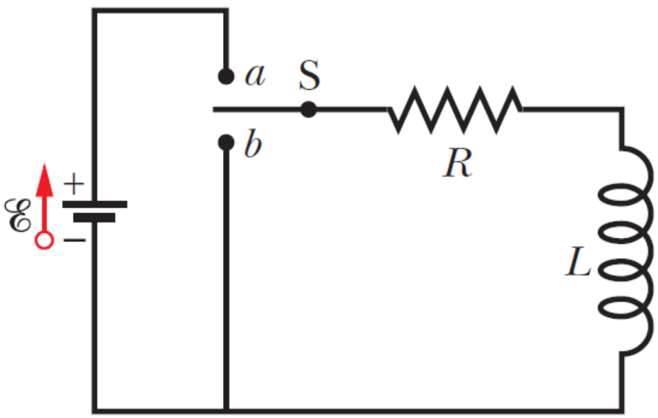


The changing current changes the flux, which creates an emf that opposes the change.

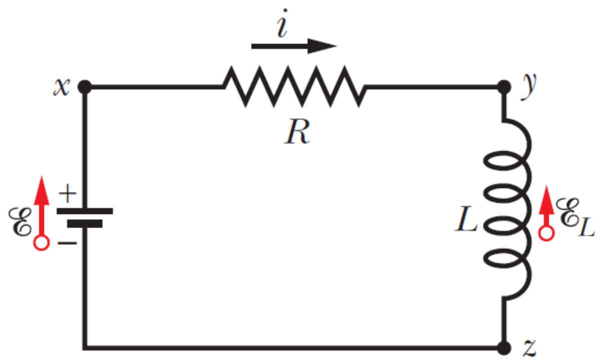
$$\mathcal{E}_L = -L\frac{di}{dt}$$

RL Circuits

Let's consider some examples of the circuit behavior of an inductor. As it is clear, an inductor in a circuit makes it difficult for rapid changes in current to occur. The greater is the rate of change of the current, the greater the self-induced emf between the inductor terminals.



after switching on "a"



LOOP RULE → $$-iR - L\frac{di}{dt} + \mathcal{E} = 0$$ or $$\boxed{L\frac{di}{dt} + Ri = \mathcal{E}}$$

Initially, an inductor acts to oppose changes in the current through it. A long time later, it acts like ordinary connecting wire.

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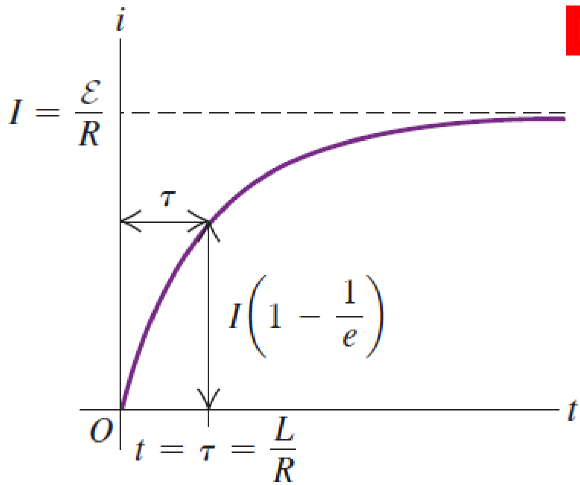


EXERCISE

$$L\frac{di}{dt} + Ri = \mathcal{E}$$

initial condition:

$$i(0) = 0$$

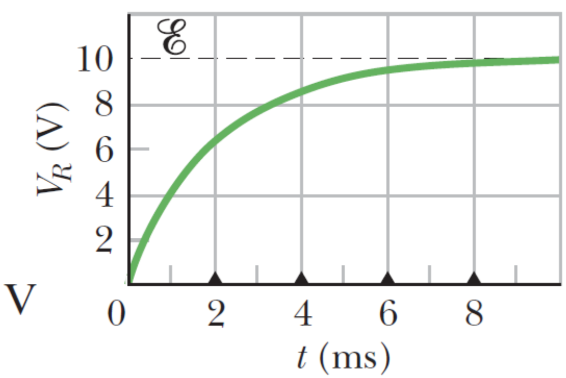
$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)$$

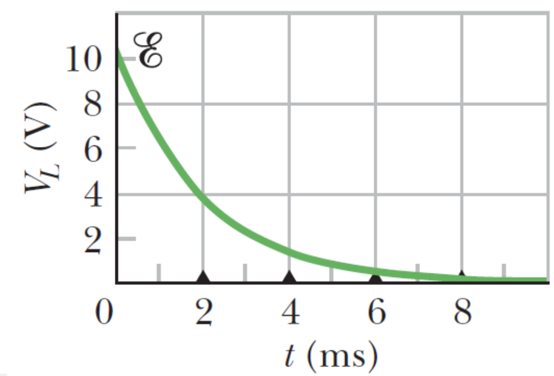
rise of current

$$\tau_L = \frac{L}{R}$$

inductive time constant

The resistor’s potential difference turns on. The inductor’s potential difference turns off.





$R = 2000\ \Omega \qquad L = 4.0\ \text{H} \qquad \mathcal{E} = 10\ \text{V}$

RL Circuits

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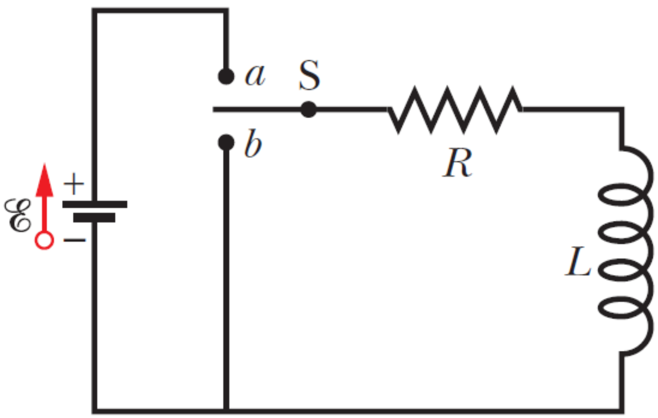


after switching on "b"

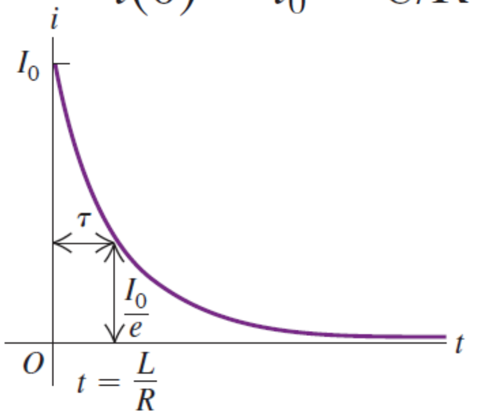
$$L\frac{di}{dt} + iR = 0$$

initial condition:

$$i(0) = i_0 = \mathcal{E}/R$$

$$i = \frac{\mathcal{E}}{R}e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$$

decay of current

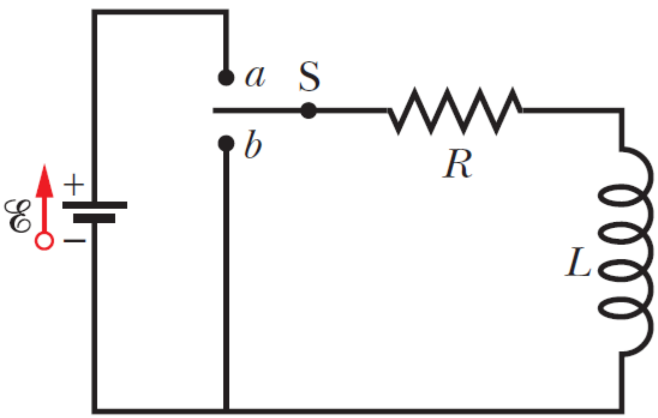


Setting the switch to "b", the effect will be to remove the battery from the circuit.

Note: the current can not drop immediately to zero but must decay to zero over time.

Energy Stored in a Magnetic Field

When we pull two charged particles of opposite signs away from each other, we say that the resulting electric potential energy is stored in the electric field of the particles. We get it back from the field by letting the particles move closer together again. In the same way we say **energy** is stored in a **magnetic field**, but now we deal with **current** instead of electric charges.



$$\mathscr{E} = L\frac{di}{dt} + iR \Rightarrow \mathscr{E}i = Li\frac{di}{dt} + i^2R$$

energy delivered by emf device
=
???
+
thermal energy in the resistor

⇒ ??? = energy stored in the magnetic field of the inductor

magnetic energy

$$\frac{dU_B}{dt} = Li\frac{di}{dt} \rightarrow dU_B = Li\,di \rightarrow \int_0^{U_B} dU_B = \int_0^i Li\,di \rightarrow \boxed{U_B = \tfrac{1}{2}Li^2}$$

Energy Stored in a Magnetic Field

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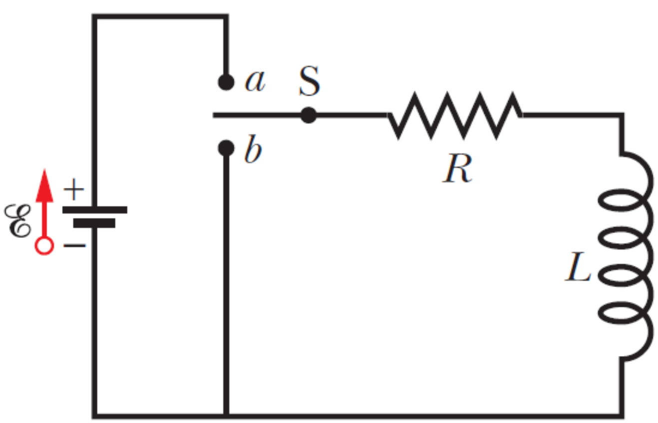

$$\mathscr{E} = L\frac{di}{dt} + iR \quad \Rightarrow \quad \mathscr{E}i = Li\frac{di}{dt} + i^2R$$

energy delivered by emf device
=
???
+
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⇒ ??? = energy stored in the magnetic field of the inductor

Energy density:

$$u_B = \frac{U_B}{Al} = \frac{Li^2}{2Al} = \frac{L}{l}\frac{i^2}{2A} \rightarrow u_B = \tfrac{1}{2}\mu_0 n^2 i^2 \rightarrow \boxed{u_B = \frac{B^2}{2\mu_0}}$$

Conclusions

- magnetic fields can affect the charged particles which **move** through the field (except for some certain directions). In order to obtain direction of this force, one can use the **right-hand rule**

- in order to visualize magnetic fields, we introduce **magnetic field lines**. Magnetic field (like electric field) obeys the **superposition** principle

- magnetic fields also affect the wires with current (through affecting the electrons moving inside them)

- **moving** charged particles and wires with **currents** produce the magnetic field around themselves. Due to this fact the two parallel wires with current interact with each other

- **Faraday's** law and **Lenz's** law describe how a magnetic field can produce an electric field that can drive the current

- **inductors** are the devices that can produce the desired magnetic field. **Solenoid** is the basic type of the inductor

- **magnetic** fields (like electric fields) can also **store energy**

- inductor as a part of a circuit fight **against** the rapid **changes in current**. Assembling it together with a resistor, or a capacitor (or both at the same time) leads to the peculiar dynamics of current in a circuit