Topic 02. Mechanics

ANSWERS

1. (a) $x(t) = \frac{At^3}{6} - \frac{Bt^4}{12}$, $v_x(t) = \frac{At^2}{2} - \frac{Bt^3}{3}$; (b) $v_{\max} = \frac{A^3}{6B^2} \approx 39.1$ m/s

2. (a) 0.625 m/s^3; (b) 107 m

3. (a) $x = v_0 t\left(1 - \frac{t}{2\tau}\right)$; $x = 0.24,\, 0\,\text{and}\, -4.0\,\text{m}$;
   (b) $1.1,\, 9.0\,\text{and}\, 11\,\text{s}$;
   (c) $s = \begin{cases} \left(1 - \frac{t}{2\tau}\right) v_0 t, & t \le \tau \\ \left[1 + \left(1 - \frac{t}{\tau}\right)^2\right] \frac{v_0 t}{2}, & t \ge \tau \end{cases}$; 24 and 34 cm

4. (a) $\vec{v} = \vec{c}(1 - 2\alpha t)$, $\vec{a} = -2\alpha\vec{c} = \text{const}$;
   (b) $\Delta t = \frac{1}{\alpha}$, $s = \frac{c}{2\alpha}$

5. (a) $\vec{v}(t) = v_x\hat{i} + v_y\hat{j}$, $\vec{r}(t) = x\hat{i} + y\hat{j}$
   $v_x = v_{0x} + \frac{\alpha t^3}{3} = (1.00 + 0.833t^3)$ m/s, $v_y = v_{0y} + \beta t - \frac{\gamma t^2}{2} = (7.00 + 9.00t - 0.700t^2)$ m/s
   $x = v_{0x}t + \frac{\alpha t^4}{12} = (1.00t + 0.208t^4)$ m, $y = v_{oy}t + \frac{\beta t^2}{2} - \frac{\gamma t^3}{6} = (7.00t + 4.50t^2 - 0.233t^3)$ m
   (b) 341 m, (c) 3.85×10^4 m

6. (a) $x = \left(\frac{\alpha}{2v_0}\right) y^2$;
   (b) $a = \alpha v_0$, $a_\tau = \frac{\alpha^2 y}{\sqrt{1+\left(\frac{\alpha y}{v_0}\right)^2}}$, $a_n = \frac{\alpha v_0}{\sqrt{1+\left(\frac{\alpha y}{v_0}\right)^2}}$

7. (a) $F = \frac{(k_1 - k_2) m_1 m_2 g \cos\alpha}{m_1 + m_2}$;
   (b) $\tan\alpha_{\min} = \frac{k_1 m_1 + k_2 m_2}{m_1 + m_2}$

8. $\tan\beta = k$; $T_{\min} = \frac{mg(\sin\alpha + k\cos\alpha)}{\sqrt{1+k^2}}$

9. $W = \vec{F}\cdot(\vec{r}_2 - \vec{r}_1) = -17$ J

10. $W = \frac{3mg}{4\alpha}$; $\Delta U = \frac{mg}{2\alpha}$

11. $\vec{v} = -\vec{u}\ln\frac{m_0}{m}$

12. $F = \sqrt{-\frac{2\alpha U}{\sin 2\theta}} = 2.4$ N

13. 4.83

14. $\theta = \frac{\pi}{2} + \arcsin\frac{1}{n} = 120^\circ$

15. $F_D = m\sqrt{g^2 + (\omega^2 r)^2 + (2v'\omega)^2} \approx 7.9$ N