Topic # 3

Electricity and Magnetism

(part 1: Electricity)

Contents

The Subject of Electromagnetism

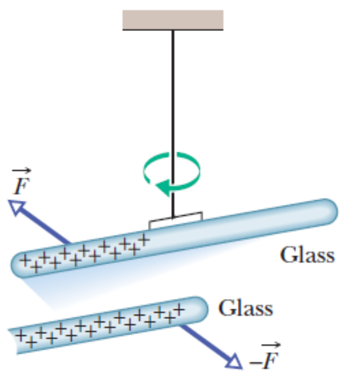
Electromagnetism is the lifeblood of technological civilization and modern society. Without it no telephones, no television, none of the household appliances that we take for granted could ever exist. Moreover, modern medicine would be a fantasy.

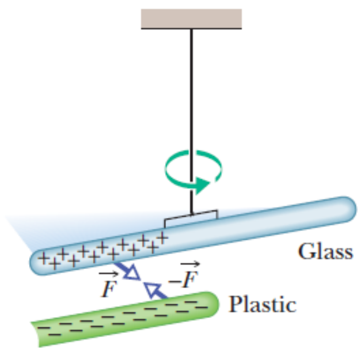
The 1st study of EM: early Greek philosophers (attraction of the rubbed amber and bits of straw; magnetic properties of iron).

The peak of development: ~1862 James Clerk Maxwell put electromagnetism on a sound theoretical basis by introducing the Maxwell's equations which are the fundamental basis of all electromagnetic phenomena occurring in nature.

Electric Charges and Coulomb's Law

There exist **two** types of **electric charges**: positive and negative.





Particles with the same sign of electrical charge repel each other, and particles with opposite signs attract each other.

SI unit of charge: (C) = (A s)

$$i = \frac{dq}{dt}$$

$$q = ne, \qquad n = \pm 1, \pm 2, \pm 3, \ldots,$$

→ electric charge is **quantized**: $e = 1.602 \times 10^{-19}\,\mathrm{C}$

→ the net electric charge of any isolated system is always **conserved**

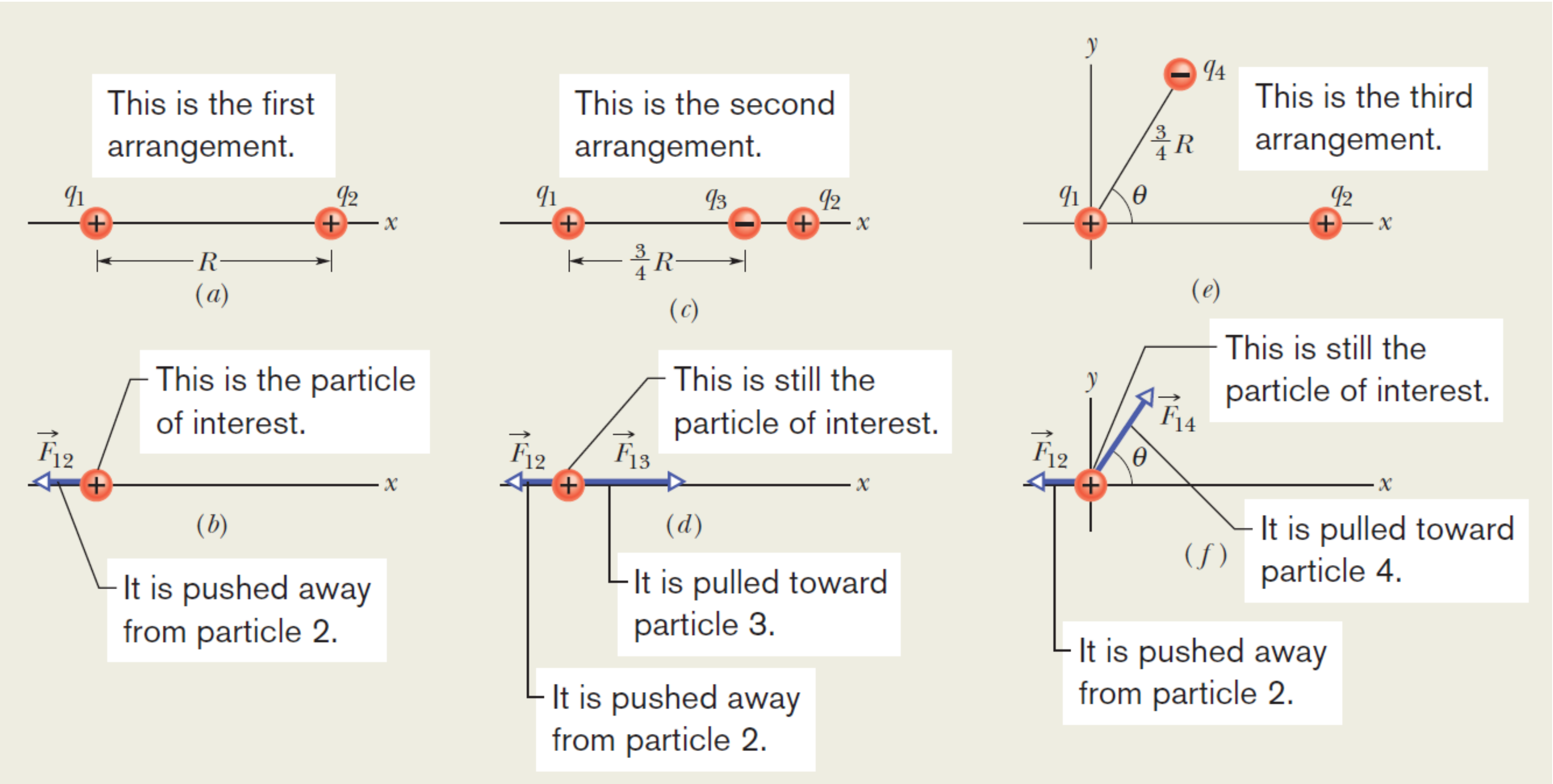
Coulomb's law: The electric force F between point charges q_1 and q_2 separated by a distance r is given by

$$\boxed{\vec{F} = k\,\frac{q_1 q_2}{r^2}\,\hat{\mathrm{r}}}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^{9}\,\mathrm{N \cdot m^2/C^2}$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$$

Electric Charges and Coulomb's Law



(a) Two charged particles of charges q_1 and q_2 are fixed in place on an x axis. (b) The free-body diagram for particle 1, showing the electrostatic force on it from particle 2. (c) Particle 3 included. (d) Free-body diagram for particle 1. (e) Particle 4 included. (f) Free-body diagram for particle 1.

Conductors and Insulators

We can classify materials generally according to the **ability** of charge **to move** through them.

- **Conductors** are materials through which charge can move rather freely; examples include metals (such as copper in common lamp wire), the human body, and tap water.

- **Nonconductors** (also called **insulators**) are materials through which charge cannot move freely; examples include rubber (such as the insulation on common lamp wire), plastic, glass, and chemically pure water.

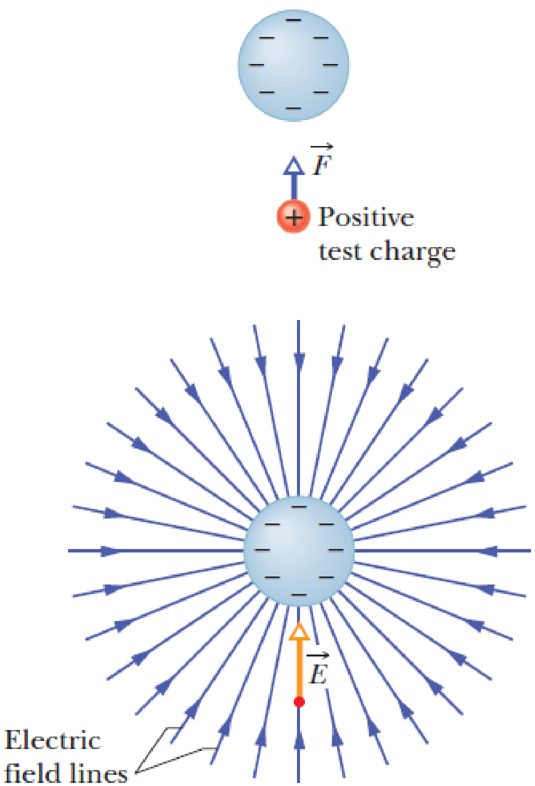
- **Semiconductors** are materials that are intermediate between conductors and insulators; examples include silicon and germanium in computer chips.

- **Superconductors** are materials that are perfect conductors, allowing charge to move without any hindrance.

Note: we will discuss only conductors and insulators.

Electric Field

A charged particle sets up an **electric field** (a vector quantity) in the surrounding space. If a second charged particle is located in that space, an electrostatic force acts on it due to the magnitude and direction of the field at its location.



The electric field at any point is defined in terms of the electrostatic force that would be exerted on a positive test charge q_0 placed there:

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

SI units: (N/C)

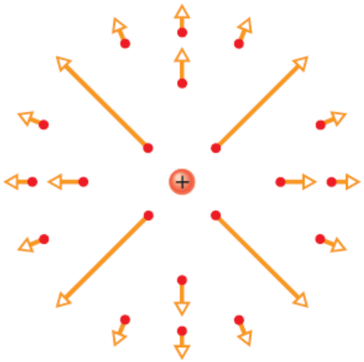
sometimes (V/m)
(see it below)

Electric field **lines** help us visualize the direction and magnitude of electric fields. The electric field vector at any point is tangent to the field line through that point. The density of field lines in that region is proportional to the magnitude of the electric field there.

Electric Field due to a Charged Particle

Rule: Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).

Force acting on the test charge: $$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{\mathrm{r}}$$



Electric field vector:

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathrm{r}}}$$

magnitude of electric field: $$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$$

In case of several charged particles: $$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}$$

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n.$$

superposition principle

Electric Field

QUIZ Check your understanding:

Suppose the electric field lines in a region of space are straight lines. If a charged particle is released from rest in that region, will the trajectory of the particle be along a field line?

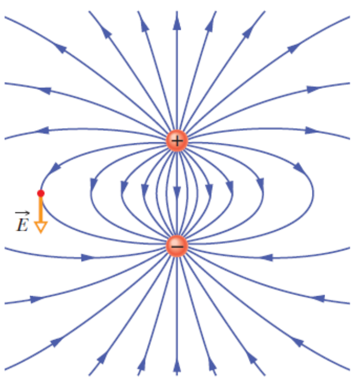
QUIZ Check your understanding:

(a) A negative point charge moves along a straight-line path directly toward a stationary positive point charge. Which aspect(s) of the electric force on the negative point charge will remain constant as it moves?
(i) Magnitude; (ii) direction; (iii) both magnitude and direction; (iv) neither magnitude nor direction.

(b) A negative point charge moves along a circular orbit around a positive point charge. Which aspect(s) of the electric force on the negative point charge will remain constant as it moves? (i) Magnitude; (ii) direction; (iii) both magnitude and direction; (iv) neither magnitude nor direction.

Electric Field due to an Electric Dipole

Electric dipole is a system of two particles with charges of equal magnitude *q* but opposite signs, separated by a small distance *d*.

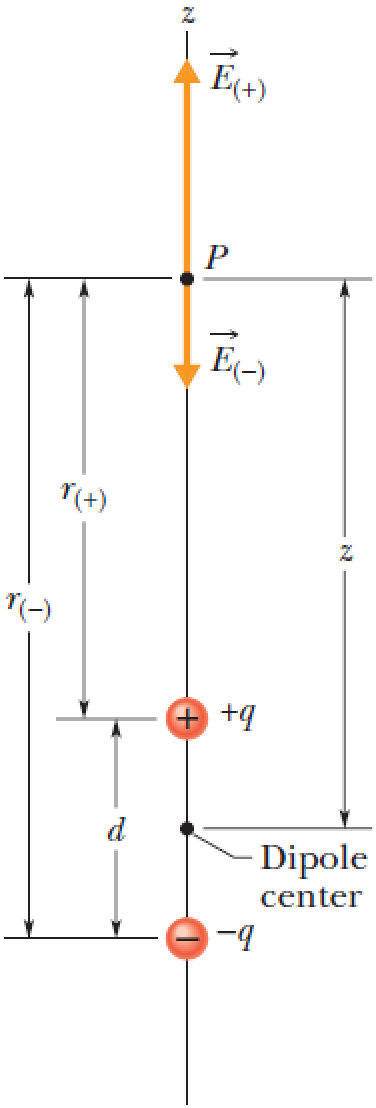


EXERCISE

$$E = E_{(+)} - E_{(-)}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2}$$
$$= \frac{q}{4\pi\varepsilon_0(z - \frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z + \frac{1}{2}d)^2}$$

After some algebra:

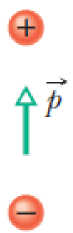
$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1 - \frac{d}{2z}\right)^2} - \frac{1}{\left(1 + \frac{d}{2z}\right)^2}\right)$$



Electric Field due to an Electric Dipole

Electric dipole is a system of two particles with charges of equal magnitude *q* but opposite signs, separated by a small distance *d*.

Up here the $+q$ field dominates.



Down here the $-q$ field dominates.

EXERCISE

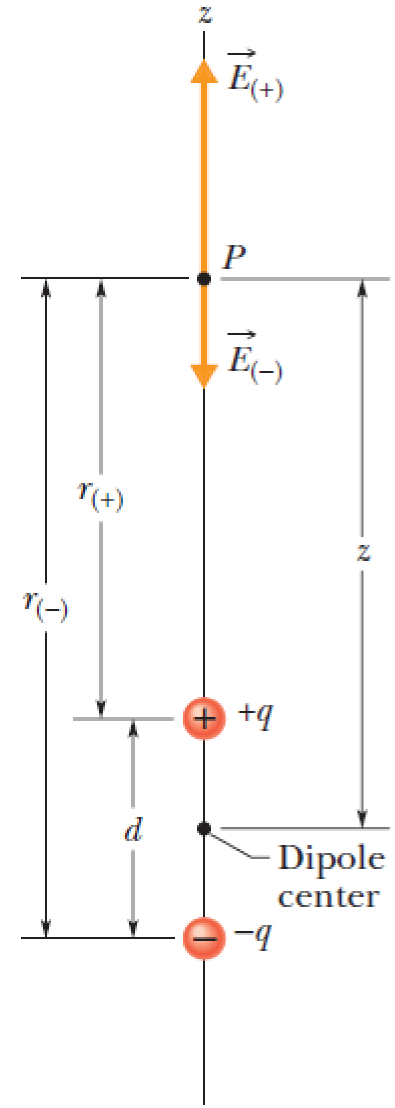
$$
\begin{aligned}
E &= E_{(+)} - E_{(-)} \\
&= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2} \\
&= \frac{q}{4\pi\varepsilon_0(z-\frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z+\frac{1}{2}d)^2}
\end{aligned}
$$

Usually it is of interest to consider: $z \gg d$

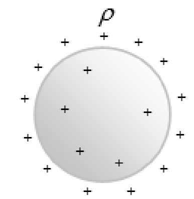
$$\boxed{E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}}$$

$$p = qd$$

electric dipole moment



Electric Field due to Extended Objects



The equation for the electric field set up by a particle **does not** apply to an **extended** object with charge (said to have a **continuous** charge distribution).

To find the electric field of an **extended** object at a point, we first consider the electric field set up by a charge element *dq* in the object, where the element is **small** enough for us to apply the equation for a particle. Then we sum, via **integration**, components of the electric fields ***dE*** from all the charge elements.

In order to do that, we introduce the **charge densities**:

$$\rho = \frac{\mathrm{d}q}{\mathrm{d}V}, \qquad \sigma = \frac{\mathrm{d}q}{dA}, \qquad \lambda = \frac{\mathrm{d}q}{ds}$$

volume c.d. surface c.d. linear c.d.

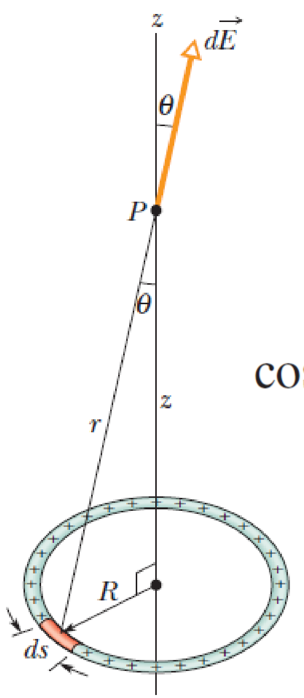
Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

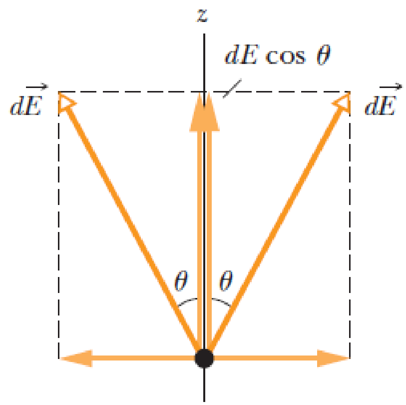
Because the individual electric fields ***dE*** have different magnitudes and point in different directions, we first see if symmetry allows us to cancel out any of the components of the fields, to simplify the integration.

Electric Field due to a Line of Charge

Consider the electric field of a uniformly charged **ring** at some arbitrary *P* point on its central axis at distance *z* from the center of the ring.

EXERCISE





$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2 + R^2)}$$

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}}$$

$$dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{z\lambda}{(z^2 + R^2)^{3/2}}\, ds$$

$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\int_0^{2\pi R} ds$$

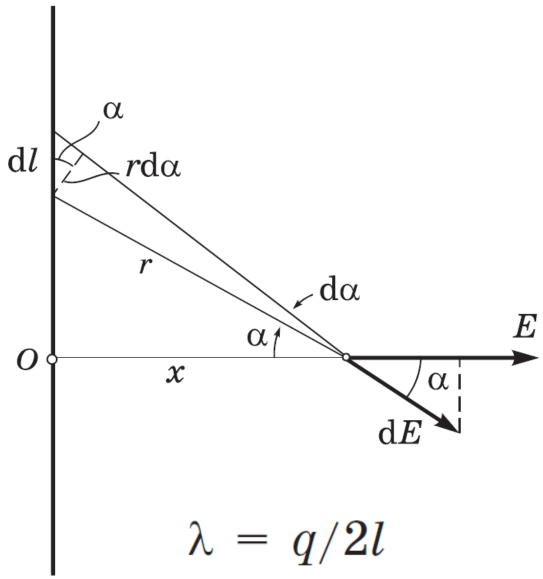
$$dq = \lambda\, ds$$

$$= \boxed{\frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}}$$

Electric Field due to a Line of Charge

Consider the electric field of a uniformly charged straight thin **filament** of length 2*l* at some arbitrary *P* point separated by a distance x from the midpoint of a filament and located symmetrically in respect to its ends.

EXERCISE



$$\lambda = q/2l$$

$$dE_x = dE\cos\alpha = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{r^2}\cos\alpha$$

$$dl\cos\alpha = r\,d\alpha \qquad r = x/\cos\alpha$$

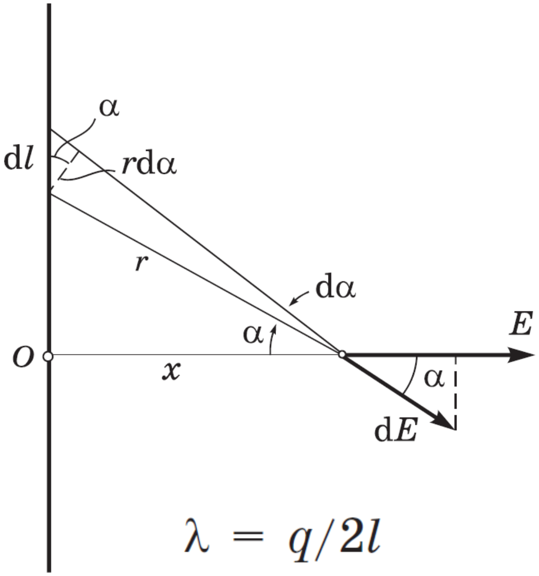
$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda r\,d\alpha}{r^2} = \frac{\lambda}{4\pi\varepsilon_0 x}\cos\alpha\,d\alpha$$

$$E = \frac{\lambda}{4\pi\varepsilon_0 x}2\int_0^{\alpha_0}\cos\alpha\,d\alpha = \boxed{\frac{\lambda}{4\pi\varepsilon_0 x}2\sin\alpha_0} \qquad \sin\alpha_0 = l/\sqrt{l^2+x^2}$$

Electric Field due to a Line of Charge

Consider the electric field of a uniformly charged straight thin **filament** of length 2*l* at some arbitrary *P* point separated by a distance x from the midpoint of a filament and located symmetrically in respect to its ends.

EXERCISE



$$\lambda = q/2l$$

$$dE_x = dE\cos\alpha = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{r^2}\cos\alpha$$

$$dl\cos\alpha = r\,d\alpha \qquad r = x/\cos\alpha$$

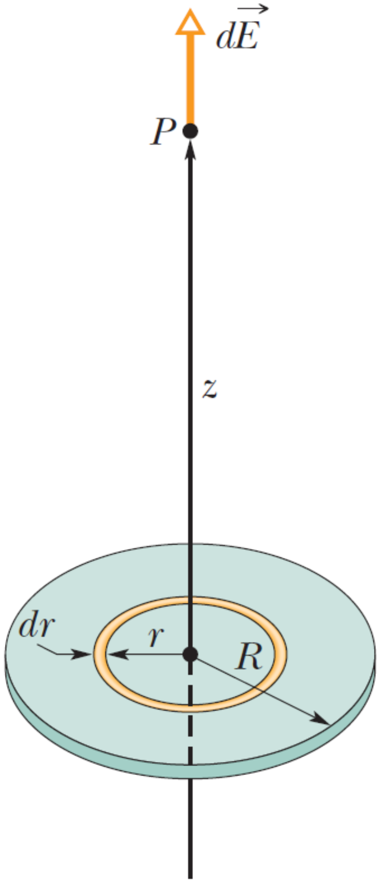
$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda r\,d\alpha}{r^2} = \frac{\lambda}{4\pi\varepsilon_0 x}\cos\alpha\,d\alpha$$

$$E = \frac{q/2l}{4\pi\varepsilon_0 x}2\frac{l}{\sqrt{l^2+x^2}} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{q}{x\sqrt{l^2+x^2}}}$$

Electric Field due to a Charged Disc

Consider the electric field of a uniformly charged **disc** at some arbitrary *P* point on its central axis at distance *z* from the center of the disc.

EXERCISE



We superimpose a ring on a disk for which we already know:

$$dE = \frac{dq\, z}{4\pi\varepsilon_0(z^2 + r^2)^{3/2}} \qquad dq = \sigma\, dA = \sigma\,(2\pi r\, dr)$$

$$E = \int dE = \frac{\sigma z}{4\varepsilon_0}\int_0^R (z^2 + r^2)^{-3/2}(2r)\, dr$$

$$= \frac{\sigma z}{4\varepsilon_0}\left[\frac{(z^2 + r^2)^{-1/2}}{-\frac{1}{2}}\right]_0^R$$

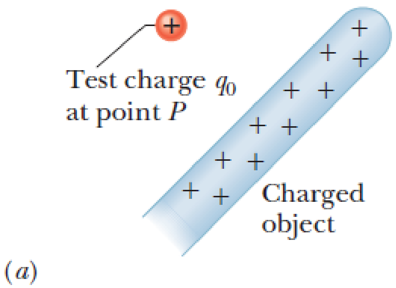
$$\boxed{E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)}$$

$$R \rightarrow \infty \qquad E = \frac{\sigma}{2\varepsilon_0}$$

infinite sheet (plane)

Electric Potential

The electric potential (or *potential* for short) can be defined in terms of the electric potential energy. We do it in the same way, as for the gravitational potential energy.



$$U = -W \quad \text{(potential energy)}$$

Reference point (*U* = *0*): infinitely far from the rod

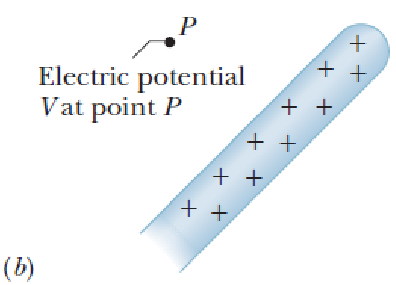
The rod sets up an electric potential, which determines the potential energy.

The electric potential:

$$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0}$$

SI units: (V) = (J/C)

Electric potential energy: $U = qV$



Note: (i) one should distinguish the terms potential and potential energy – although sounding similar, these terms are very different and not interchangeable; (ii) electric potential is scalar (not a vector).

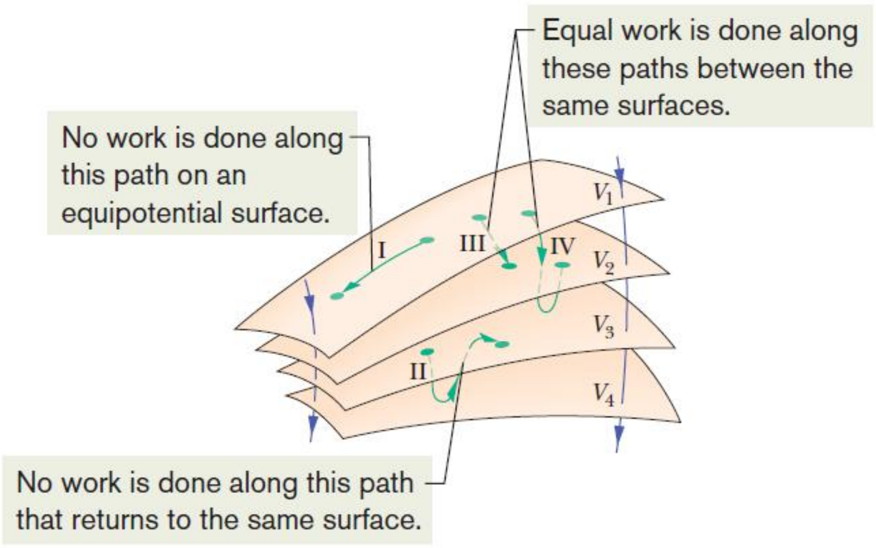
electric field → $1\ \text{N/C} = \left(1\frac{\text{N}}{\text{C}}\right)\left(\frac{1\ \text{V}}{1\ \text{J/C}}\right)\left(\frac{1\ \text{J}}{1\ \text{N}\cdot\text{m}}\right) = 1\ \text{V/m}$

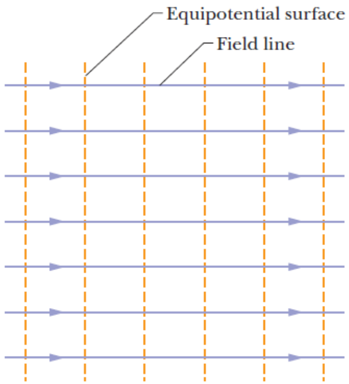
Equipotential Surfaces

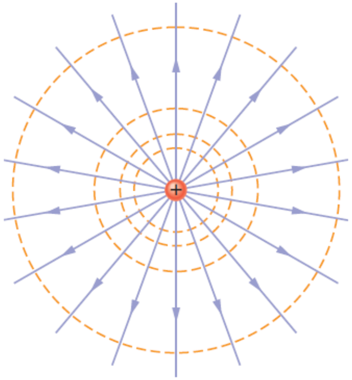
Adjacent points that have the **same** electric potential form an **equipotential** surface, which can be either an imaginary surface or a real, physical surface.

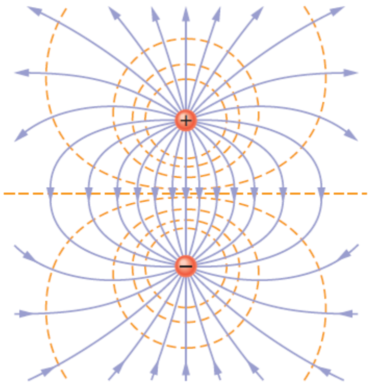
No net work W is done on a charged particle by an electric field when the particle moves between two points i and f on the same equipotential surface.

$$V_f = V_i$$



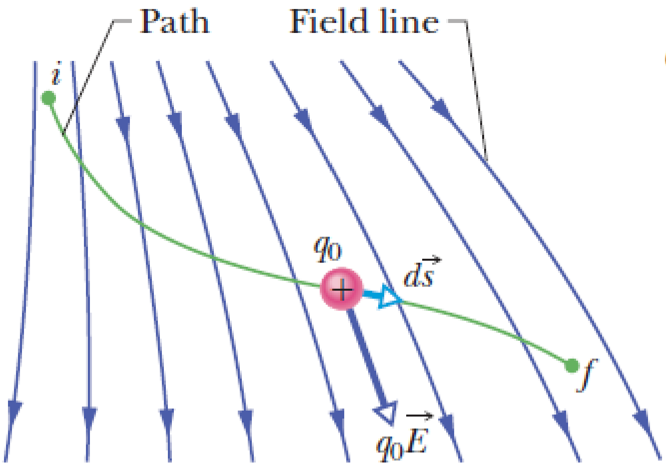






Calculating the Potential from the Field

We can calculate the potential difference between any two points *i* and *f* in an electric field if we know the electric field vector all along any path connecting those points.



$$dW = \vec{F}\cdot d\vec{s} = q_0\vec{E}\cdot d\vec{s}$$

Total work done on the particle:

$$W = q_0\int_i^f \vec{E}\cdot d\vec{s}$$

$$\Rightarrow \boxed{V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}}$$

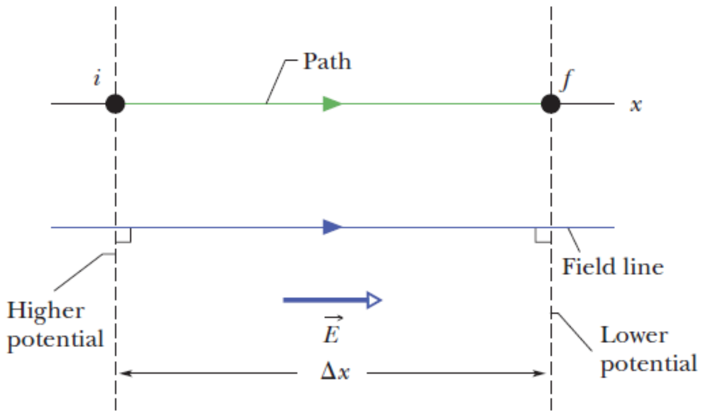
Reference point ($V_i = 0$): infinitely far from the rod:

$$\boxed{V = -\int_i^f \vec{E}\cdot d\vec{s}}$$

Note: because the electric force is conservative, all paths (whether easy or difficult to use) yield the same result.

Calculating the Potential from the Field

In case of a **uniform** field:

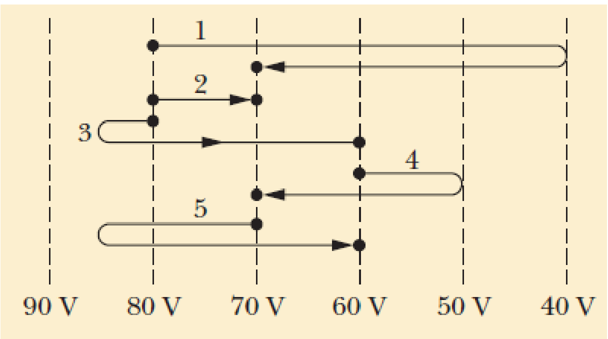


$$\vec{E} \cdot d\vec{s} = E\, ds \cos 0 = E\, ds$$

$$V_f - V_i = -E \int_i^f ds$$

$$\Rightarrow \quad \boxed{\Delta V = -E\,\Delta x}$$

Note: The electric field vector points from higher potential toward lower potential.



QUIZ

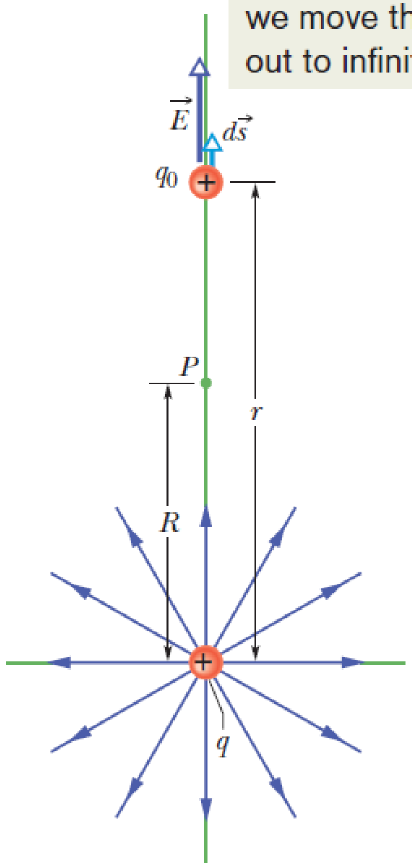
Check your understanding:

Assume the test charge is positive. (a) What is the direction of the electric field associated with the surfaces? (b) For each path, is the work we do positive, negative, or zero? (c) Rank the paths according to the work we do, greatest first.
(d) What happens if we consider an electron?

Potential due to a Charged Particle

Consider a point *P* at distance *R* from a fixed particle of positive charge *q*:

EXERCISE



To find the potential of the charged particle, we move this test charge out to infinity.

$$\vec{E}\cdot d\vec{s} = E\cos\theta\, ds \qquad \theta = 0, \quad \cos\theta = 1$$

$$V_f - V_i = -\int_R^{\infty} E\, dr \qquad V_f = 0 \ (\text{at } \infty), \quad V_i = V \ (\text{at } R)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \Rightarrow \quad \boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$

Note: A positively charged particle produces a positive electric potential.
A negatively charged particle produces a negative electric potential.

Group of *n* charged particles:

$$\boxed{V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}}$$

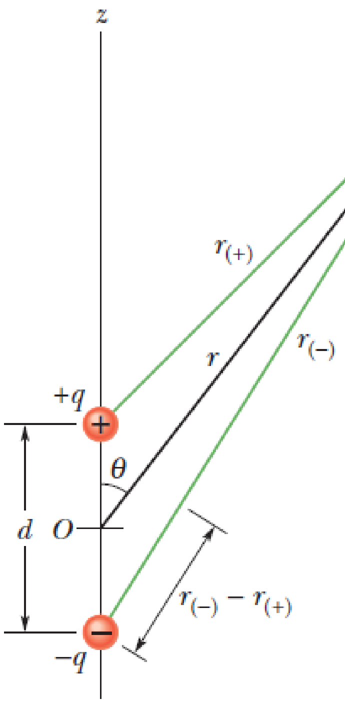
Potential due to an Electric Dipole

Consider the potential *V* at any given point due to an electric dipole, in terms of the magnitude *p* of the dipole moment or the product of the charge separation *d* and the magnitude *q* of either charge.

EXERCISE

$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

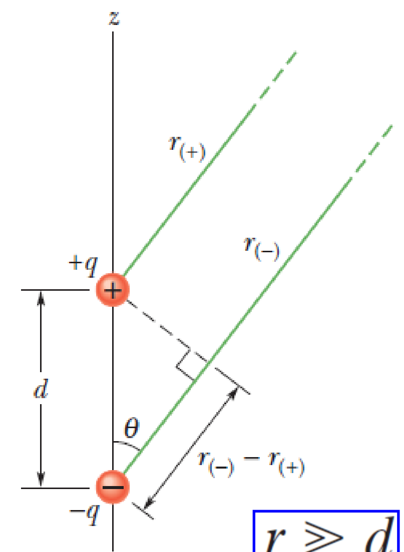
$$= \frac{q}{4\pi\varepsilon_0}\,\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}$$



$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\,\frac{p\cos\theta}{r^2}}$$

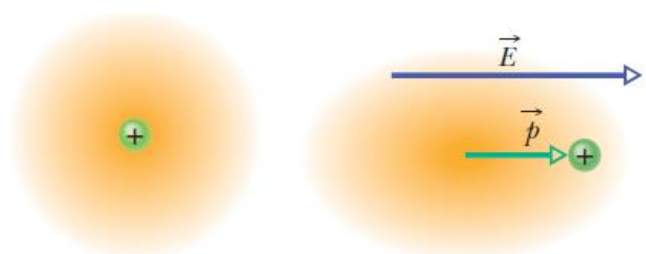
$$r_{(-)} - r_{(+)} \approx d\cos\theta$$

$$r_{(-)}r_{(+)} \approx r^2$$



$\boxed{r \gg d}$

Induced dipole moment:



Potential due to a Continuous Charge Distribution

Strategy is almost the same as in the case of calculating the electric fields:

- For a continuous distribution of charge (over an extended object), the potential is found by (i) dividing the distribution into charge elements *dq* that can be treated as particles and then (ii) summing the potential due to each element by integrating over the full distribution:

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$

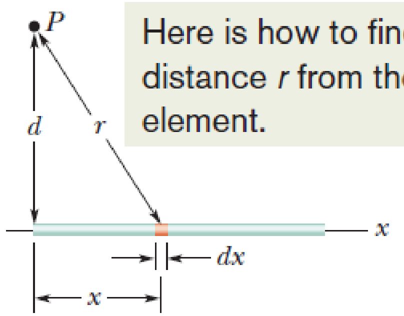
- In order to carry out the integration, *dq* is replaced with the product of either a linear charge density and a length element (such as *dx*), or a surface charge density and area element (such as *dx dy*), or by volume charge density and a volume element (such as *dx dy* dz).

Note: because the electric potential is a scalar, there are no vector components to consider.

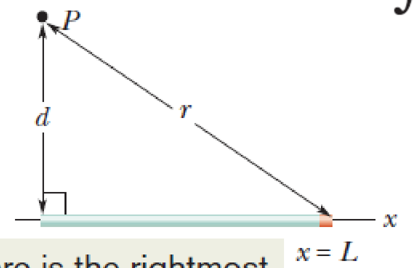
Potential due to a Line of Charge

Consider a thin nonconducting rod of length *L* having a positive charge of uniform linear density *λ*. Let us determine the electric potential *V* due to the rod at point *P*, a perpendicular distance *d* from the left end of the rod.

EXERCISE



Here is how to find distance *r* from the element.



Here is the rightmost element.

$$dq = \lambda\, dx$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(x^2 + d^2)^{1/2}}$$

$y = x + \sqrt{x^2 + d^2}$

$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{(x^2 + d^2)^{1/2}}\,dx = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(x^2 + d^2)^{1/2}}$$

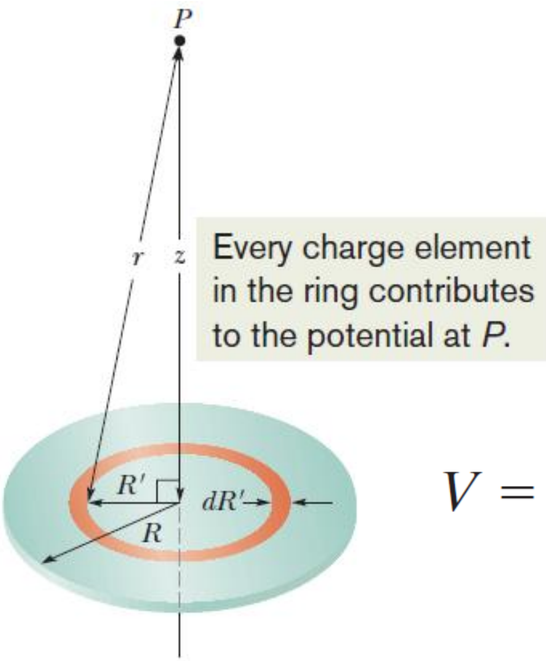
$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(x + (x^2 + d^2)^{1/2}\right)\right]_0^L$$

$$= \boxed{\frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{L + (L^2 + d^2)^{1/2}}{d}\right]}$$

Potential due to a Charged Disc

Consider the electric potential V due to the nonconducting uniformly charged disc at any point on the central axis.

EXERCISE



$$dq = \sigma(2\pi R')(dR')$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi R')(dR')}{\sqrt{z^2 + R'^2}}$$

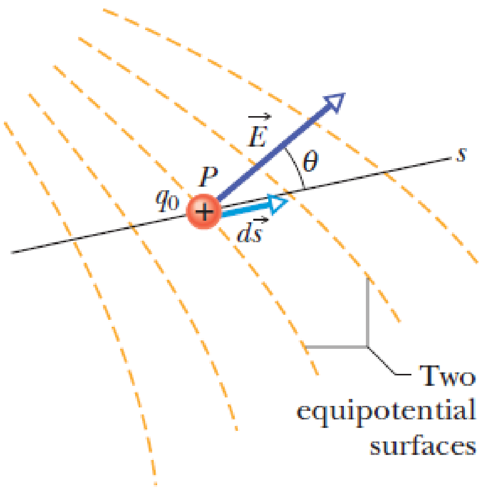
$$V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{R'\,dR'}{\sqrt{z^2 + R'^2}} = \boxed{\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right)}$$

Note: in evaluating the integral, we have assumed that $z \geq 0$.

Calculating the Field from the Potential

We already know how to find the potential at a point *f* if we know the electric field along a path from a reference point to point *f*.

Can we solve the inverse problem ?



$$-q_0\,dV = q_0 E(\cos\theta)\,ds$$

$$E\cos\theta = -\frac{dV}{ds} \Rightarrow E_s = -\frac{\partial V}{\partial s}$$

If we take the *s*-axis to be in turn *x*-, *y*- and *z*-axis:

$$E_x = -\frac{\partial V}{\partial x}; \qquad E_y = -\frac{\partial V}{\partial y}; \qquad E_z = -\frac{\partial V}{\partial z}$$

$$\Rightarrow \boxed{\vec{E} = -\nabla V}$$

Calculating the Field from the Potential

EXERCISE

Task #1: The electric potential at any point on the central axis of a uniformly charged disk is given by

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right)$$

Starting with this expression, derive an expression for the electric field at any point on the axis of the disk.

Solution:

For any value of *z*, the direction of ***E*** must be along that axis because the disk has circular symmetry. Thus, we want the component E_z in the direction of *z*.

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\frac{d}{dz}\left(\sqrt{z^2 + R^2} - z\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

Note: This is the same expression that we derived earlier by integration!

Electric Field and Electric Potential

QUIZ Check your understanding:

(a) If the electric potential at a certain point is zero, does the electric field at that point have to be zero?
(b) If the electric field at a certain point is zero, does the electric potential at that point have to be zero?

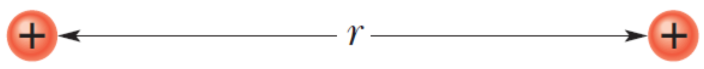
QUIZ Check your understanding:

In a certain region of space the potential is given by $V = A + Bx + Cy^3 + Dxy$, where A, B, C, and D are positive constants. Which of these statements about the electric field $\boldsymbol{E}$ in this region of space is correct? (There may be more than one correct answer.)
(i) Increasing the value of A will increase the value of $\boldsymbol{E}$ at all points; (ii) increasing the value of A will decrease the value of $\boldsymbol{E}$ at all points; (iii) $\boldsymbol{E}$ has no z-component;
(iv) the electric field is zero at the origin ($x = 0$, $y = 0$, $z = 0$).

Electric Potential Energy

In this part we will calculate the potential energy of a system of two charged particles and then discuss how to expand the result to a system of more than two particles.

Consider the work we must do to bring together two charged particles which are **initially infinitely far apart** and that end up near each other and stationary



q_1 — then we bring it to its final position

q_2 — we start with it fixed in place (q_1 is infinitely far away)

$$\Delta U = U_f - U_i$$

$$U_f - U_i = q_1(V_f - V_i) \qquad U_i = 0$$

U_i

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r} \qquad r = \infty, \quad V_i = 0$$

$$V_f = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r}$$

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$

(we drop the *f* index)

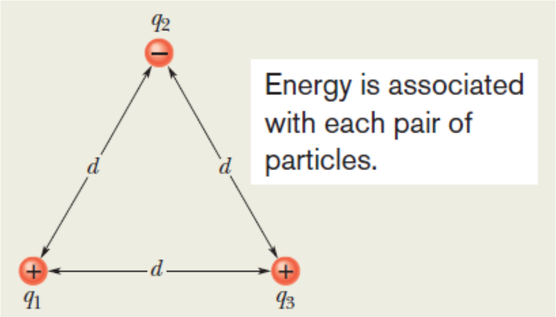
Note: This result applies also to situations where the charges are both negatively charged or have different signs

The total potential energy of a system of particles is the sum of the potential energies for every pair of particles in the system.

Electric Potential Energy

EXERCISE

Task #2: Calculate the potential energy of a system of three charged particles



$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

$$q = 150\ \text{nC} \qquad d = 12\ \text{cm}$$

Solution:

We start with fixing, say, the q_1. Then we bring q_2: $$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d}$$

Then we bring q_3: $$W_{13} + W_{23} = U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{d} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{d}$$

Finally:

$$U = U_{12} + U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\left(\frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d}\right)$$

$$= -\frac{10q^2}{4\pi\varepsilon_0 d} = -\frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10)(150 \times 10^{-9}\ \text{C})^2}{0.12\ \text{m}} = -1.7 \times 10^{-2}\ \text{J}$$

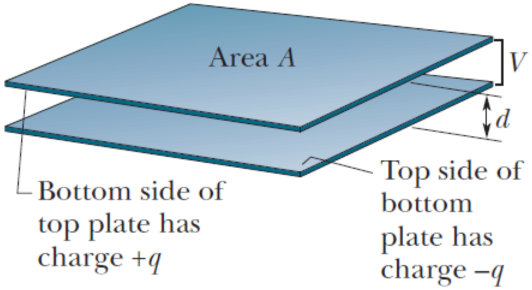
$$= -17\ \text{mJ}$$

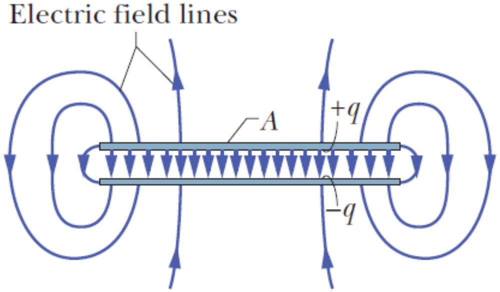
Note: The negative potential energy means that negative work should be done to assemble this structure, starting with the three charges infinitely separated and at rest

Capacitance

A **capacitor** is a device in which the electrical energy can be stored. Unlike battery, the capacitor can supply energy at a much greater rate.

- Consider how much charge can be stored in the capacitor. This "how much" is called **capacitance**.





electric scheme symbol:

$$\boxed{q = CV}$$

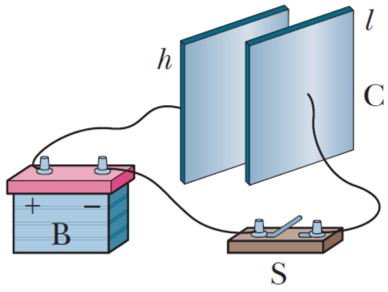
capacitance

SI unit: (F) = (C / V)

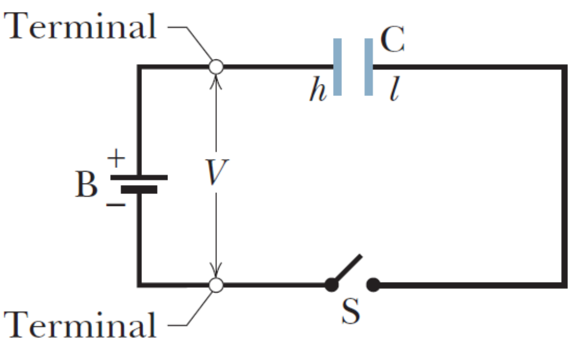
- When a capacitor is charged, its plates have charges of equal magnitudes and opposite signs.
- The capacitance is a measure of how much charge must be put on the plates to produce a certain potential difference between them.
- The greater the capacitance, the more charge is required.

Charging a Capacitor

One way to charge a capacitor is to place it in an **electric circuit** with a battery.



- An electric circuit is a path through which charge can flow



S – a switch; possible states: open or closed.

When the circuit is completed, electrons are driven through the wires by an electric field that the battery sets up in the wires.

- When the plates are uncharged, the potential difference between them is 0. As the plates become oppositely charged, that potential difference increases until it equals the potential difference V between the terminals of the battery.
- The capacitor is said to be **fully charged**, with a potential difference V and charge q, when the electric field in the wire is eliminated

Note: We assume that during the charging of a capacitor and afterwards, charge can not pass from one plate to the other across the gap separating them.

Capacitance of Various-Shaped Capacitors

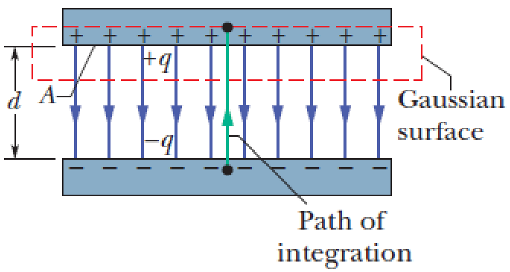
The capacitance of a capacitor can be calculated once we know its **geometry**.

This can be done by means of applying the **Gauss' law**:
(beyond the present consideration)

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

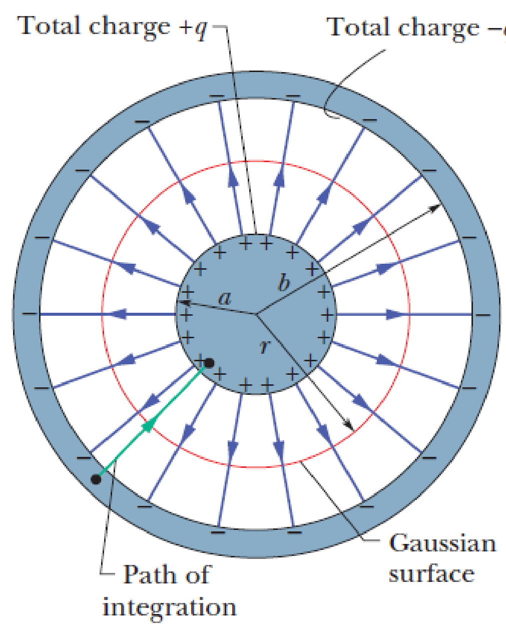
electric flux
through the surface

We use Gauss' law to relate *q* and *E*. Then we integrate the *E* to get the potential difference.



parallel-plate capacitor

$$C = \frac{\varepsilon_0 A}{d}$$



cylindrical capacitor

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$

Capacitance of Various-Shaped Capacitors

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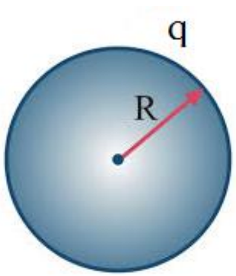
This can be done by means of applying the **Gauss' law**:
(beyond the present consideration)

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

electric flux
through the surface



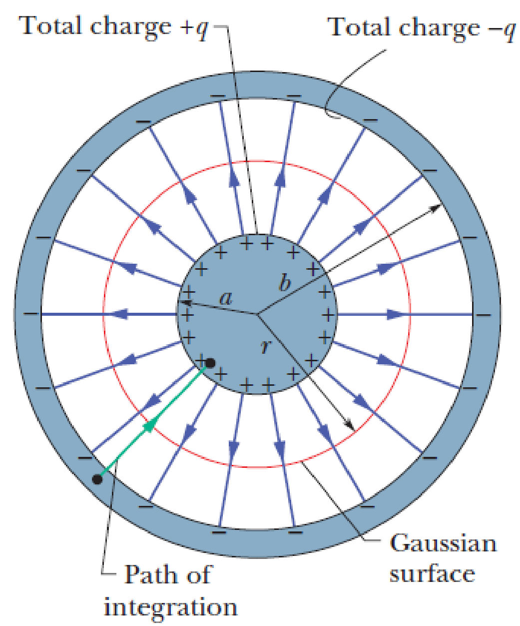
(we assume that the "missing plate" is a conducting sphere of infinite radius)

an isolated sphere

$$C = 4\pi\varepsilon_0 R$$

spherical capacitor

$$C = 4\pi\varepsilon_0 \frac{ab}{b - a}$$



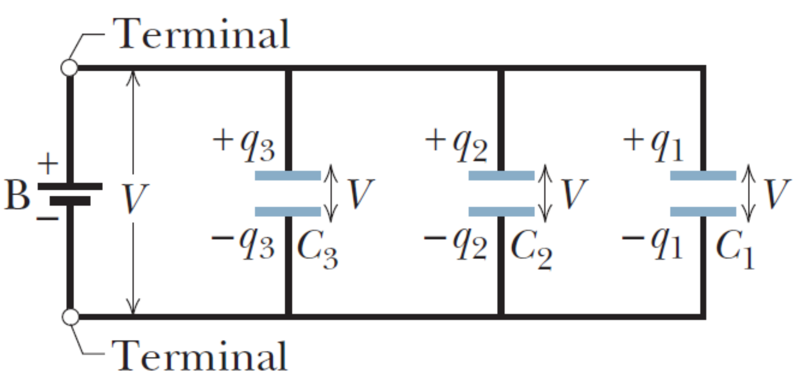
schematic figure is the same

Capacitors in Parallel and in Series

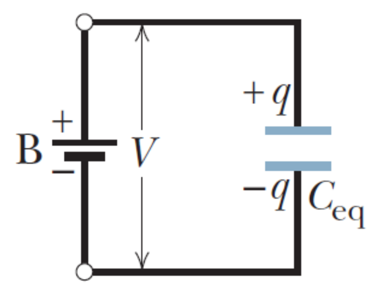
When there is a **combination** of capacitors in a circuit, we can sometimes replace that combination with an **equivalent capacitor** – that is, a single capacitor that has the same capacitance as the actual combination of capacitors.

such a replacement simplifies the circuit, affording easier solutions for unknown quantities of the circuit

Capacitors in Parallel



$$q_1 = C_1V, \quad q_2 = C_2V, \quad \text{and} \quad q_3 = C_3V.$$



$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$$

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3$$

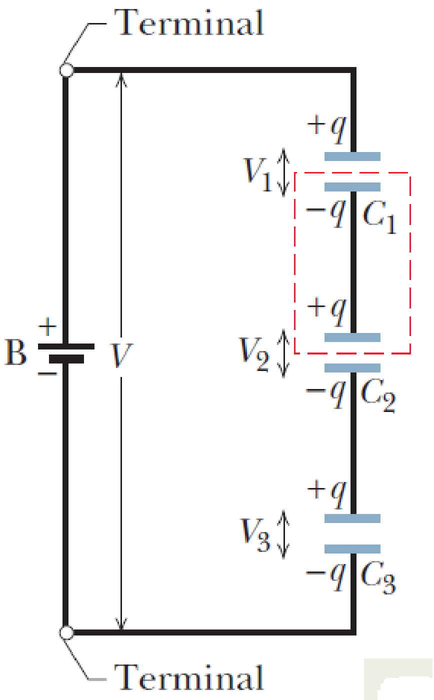
$$\boxed{C_{eq} = \sum_{j=1}^{n} C_j}$$

(n capacitors in parallel)

Capacitors in Parallel and in Series

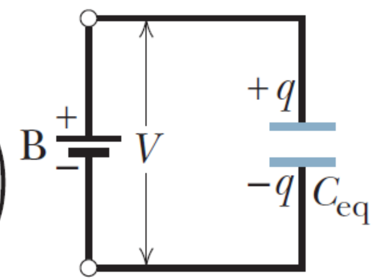
When there is a **combination** of capacitors in a circuit, we can sometimes replace that combination with an **equivalent capacitor** – that is, a single capacitor that has the same capacitance as the actual combination of capacitors.

➡ such a replacement simplifies the circuit, affording easier solutions for unknown quantities of the circuit



Capacitors in Series

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}$$



$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

➡

$$\boxed{\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}}$$

(n capacitors in series)

Capacitors in Parallel and in Series

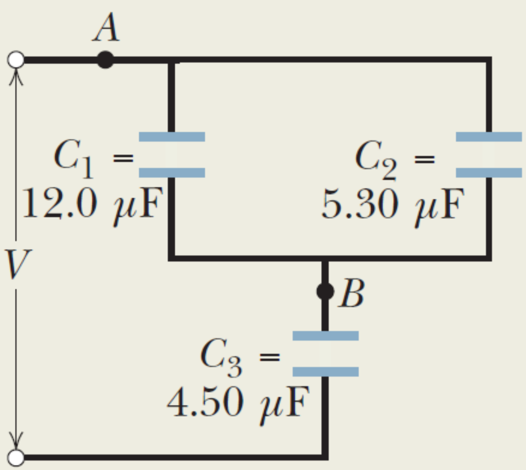
EXERCISE

Task #3: Find the equivalent capacitance of the following combinations

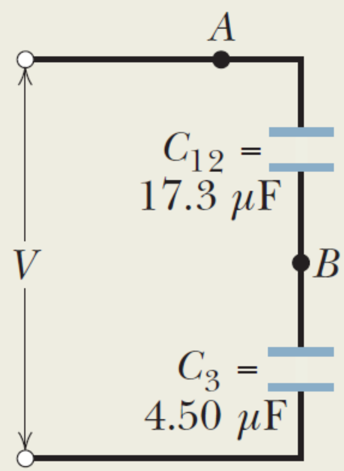
$$C_1 = 12.0\ \mu\text{F}, \quad C_2 = 5.30\ \mu\text{F}, \quad \text{and} \quad C_3 = 4.50\ \mu\text{F}.$$

Solution:

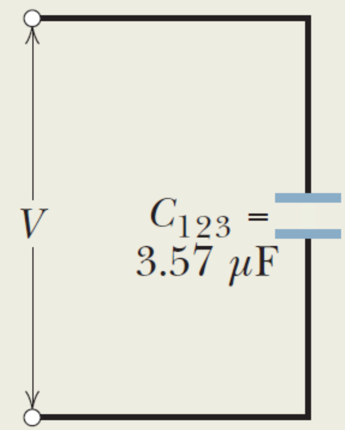
We first reduce the circuit to a single capacitor.



The equivalent of parallel capacitors is larger.



The equivalent of series capacitors is smaller.



Energy Stored in an Electric Field

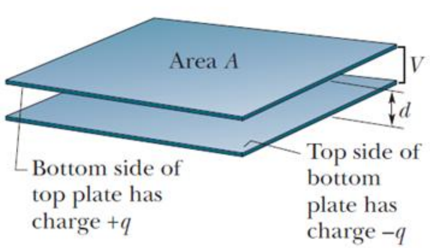
Work must be done by an **external** agent to charge a capacitor. We can imagine doing the work ourselves by transferring electrons from one plate to the other.

$$dW = V'\,dq' = \frac{q'}{C}\,dq' \qquad W = \int dW = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C}$$

$$\Rightarrow \boxed{U = \frac{q^2}{2C}} \quad \text{or} \quad \boxed{U = \tfrac{1}{2}CV^2}$$

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

In a **parallel-plate** capacitor (neglecting fringing), the electric field has the same value at all points between the plates. Thus, the **energy density *u*** – that is, the potential energy per unit volume between the plates – should also be uniform.



$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad} = \tfrac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 \qquad (C = \varepsilon_0 A/d)$$

$$\Rightarrow \boxed{u = \tfrac{1}{2}\varepsilon_0 E^2} \qquad (E = -\Delta V/\Delta s)$$

Note: this result is also valid for any electric field

Capacitors in Parallel and in Series

QUIZ Check your understanding:

(a) How should you connect a 4 μF capacitor and an 8 μF capacitor so that the 4 μF capacitor has a greater potential difference across it than the 8 μF capacitor?
(i) Series; (ii) parallel; (iii) either series or parallel; (iv) neither series nor parallel.

(b) How should you connect them so that the 4 μF capacitor has a greater charge than the 8 μF capacitor?
(i) Series; (ii) parallel; (iii) either series or parallel; (iv) neither series nor parallel.

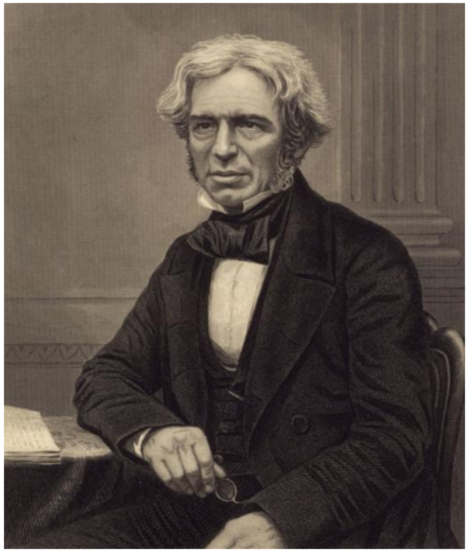
QUIZ Check your understanding:

You want to connect a 4 μF capacitor and an 8 μF capacitor. With which type of connection will the 4 μF capacitor have a greater amount of stored energy than the 8 μF capacitor?
(i) Series; (ii) parallel; (iii) either series or parallel; (iv) neither series nor parallel.

Capacitor with a Dielectric

What happens to the capacitance if we fill the space between the plates by an insulating material?

- In 1837 Michael Faraday experimentally discovered that the capacitance **increased** by a numerical factor κ, which he called the **dielectric constant** of the insulating material.
- The dielectric constant of **vacuum** is **unity** by definition.

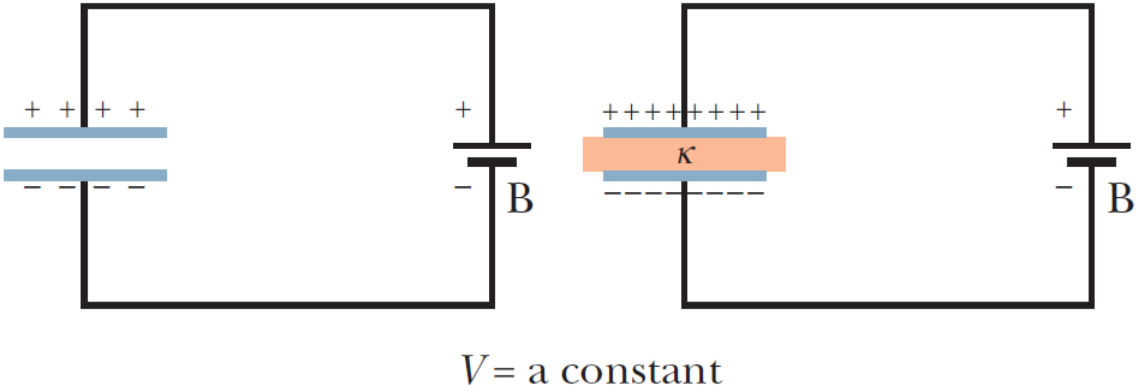


Michael Faraday
(1791-1867)

Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, κ = unity.

$$\boxed{C = \kappa C_{\text{air}}}$$



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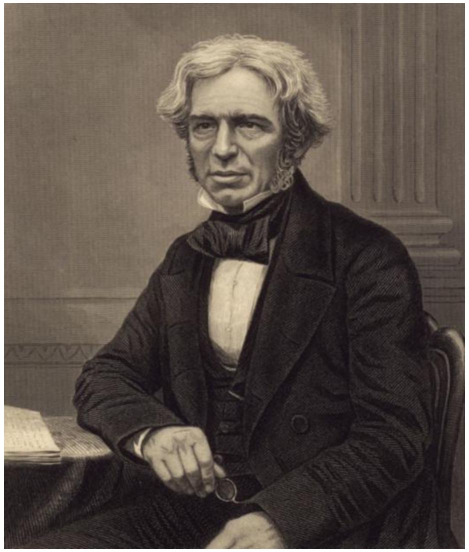

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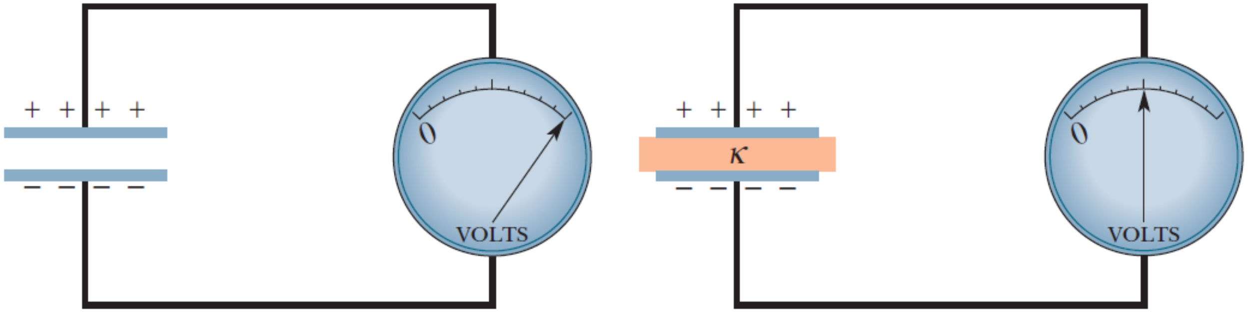


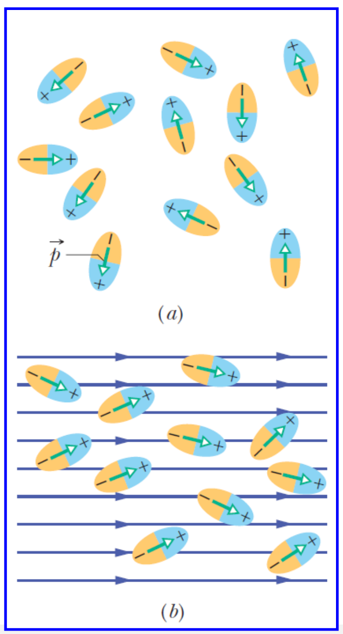
q = a constant

Capacitor with a Dielectric

In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$.

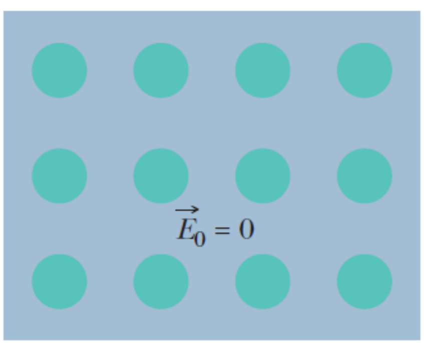
point charge: $$E = \frac{1}{4\pi\kappa\varepsilon_0}\frac{q}{r^2}$$

infinite sheet (plane): $$E = \frac{\sigma}{\kappa\varepsilon_0}$$

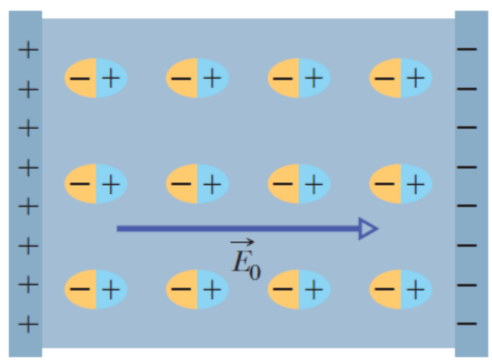


An Atomic View on Dielectrics

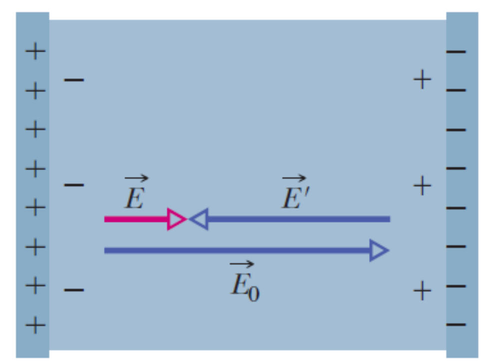
The initial electric field inside this nonpolar dielectric slab is zero.



The applied field aligns the atomic dipole moments.



The field of the aligned atoms is opposite the applied field.



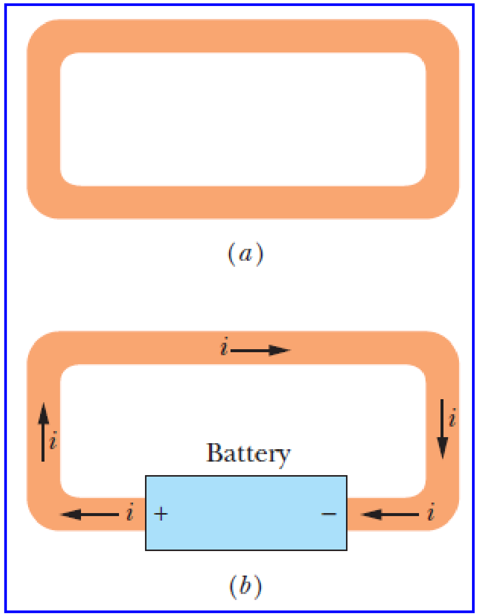
Capacitor with a Dielectric

QUIZ Check your understanding:

The space between the plates of an isolated parallel-plate capacitor is filled by a slab of dielectric with dielectric constant K. The two plates of the capacitor have charges Q and $-Q$. You pull out the dielectric slab. If the charges do not change, how does the energy in the capacitor change when you remove the slab?
(i) It increases; (ii) it decreases; (iii) it remains the same.

Electric Current

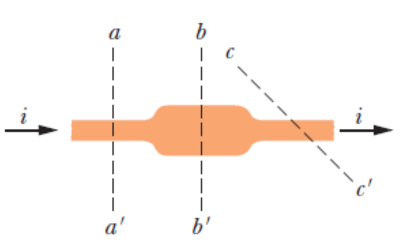
Up to now we considered electrostatics – the physics of stationary charges. Now let us discuss the physics of **electric currents** – that is, charges in **motion**. To be more precise, in this part we restrict ourselves to the study of **steady currents** of conduction electrons moving through metallic conductors such as copper wires.



Electric current: $$\boxed{i = \frac{dq}{dt}}$$

SI unit: (A) = (C / s)

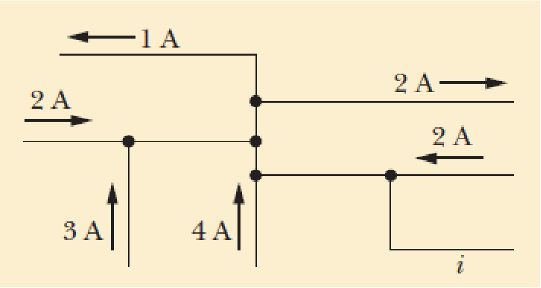
The current is the same in any cross section.



under steady-state conditions

The charge that passes through the plane: (in a time interval extending from 0 to t)

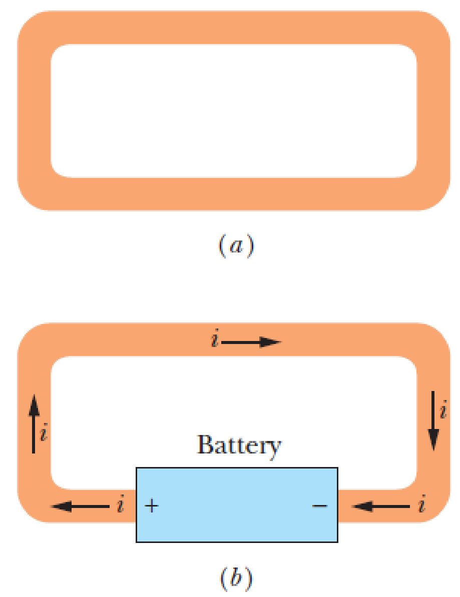
$$q = \int dq = \int_0^t i\,dt$$



Note: current is a scalar. Yet, we often represent a current with an arrow to indicate that charge is moving. Such arrows are not vectors and they do not require vector addition.

Electric Current

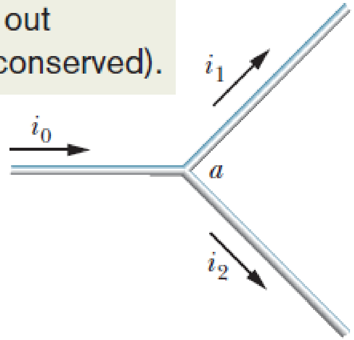
Up to now we considered electrostatics – the physics of stationary charges. Now let us discuss the physics of **electric currents** – that is, charges in **motion**. To be more precise, in this part we restrict ourselves to the study of **steady currents** of conduction electrons moving through metallic conductors such as copper wires.

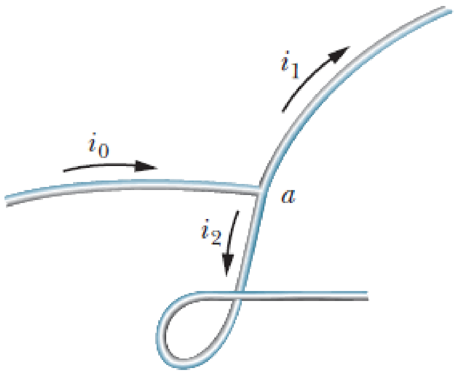


Electric current: $\boxed{i = \dfrac{dq}{dt}}$

SI unit: (A) = (C / s)

The current into the junction must equal the current out (charge is conserved).





$$\boxed{i_0 = i_1 + i_2}$$

this relation is true at junction ***a*** no matter what the orientation in space of the three wires is

Note: A current arrow is drawn in the direction in which **positive** charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.

Current Density

If we want to take a localized view and study the flow of charge through a cross section of the conductor at a particular point, we have to consider the **current density**.

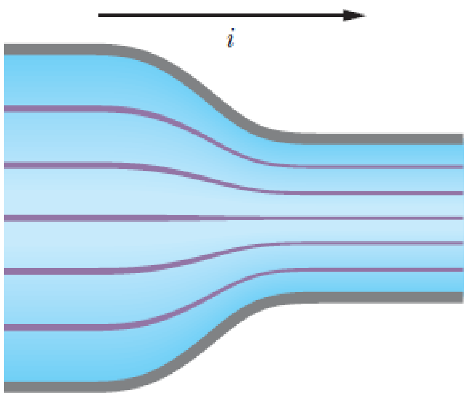
Current density = current per unit area

SI unit: (A / m^2)

$$i = \int \vec{J} \cdot d\vec{A}$$

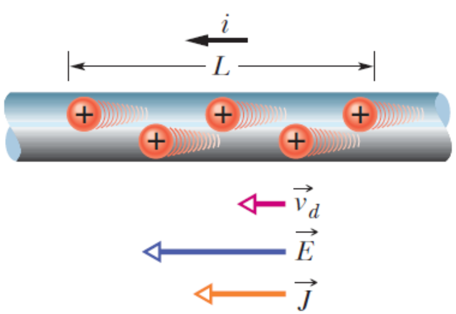
Uniform current across the surface:

$$i = \int J\,dA = J \int dA = JA$$



Streamlines represent current density in the flow of charge through a constricted conductor

Current is said to be due to positive charges that are propelled by the electric field.



Drift speed:

~ 10^{-5} – 10^{-4} m/s

no current → conduction electrons move randomly

current → conduction electrons still move randomly, but now they tend to drift in the direction opposite to the applied electric field

Current Density

The total charge of the carriers in the length *L*, each with charge *e*: $q = (nAL)e$

the number of carriers per unit volume

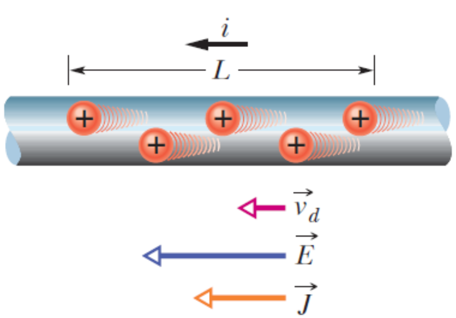
Assuming that all carriers move with the same drift speed:

$$t = \frac{L}{v_d} \Rightarrow i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d$$

$$v_d = \frac{i}{nAe} = \frac{J}{ne} \Rightarrow \boxed{\vec{J} = (ne)\vec{v}_d}$$

carrier charge density (C/m^3)

Current is said to be due to positive charges that are propelled by the electric field.



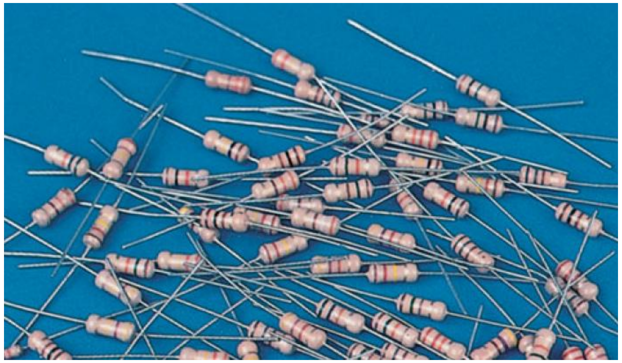
Drift speed:

~ 10^{-5} – 10^{-4} m/s

no current → conduction electrons move randomly

current → conduction electrons still move randomly, but now they tend to drift in the direction opposite to the applied electric field

Resistance and Resistivity



If we apply the same potential difference between the ends of geometrically similar rods of copper and of glass, very different currents result. The characteristic of the conductor that enters here is its **electrical resistance**:

$$\boxed{R = \frac{V}{i}}$$

electric scheme symbol: 

SI unit: (Ω) = (V / A)

Resistor is a conductor whose function in a circuit is to provide a specified resistance

If we want to focus on the electric field at a point of the resistive material instead of the potential difference across the resistor, we deal with the **resistivity**:

$$\rho = \frac{E}{J} \qquad \vec{E} = \rho\vec{J}$$

Note: these relations hold for **isotropic** materials only!

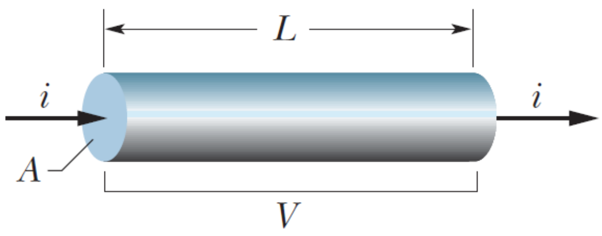
Conductivity: $\sigma = \frac{1}{\rho}$ ⇒ $\boxed{\vec{J} = \sigma\vec{E}}$

Material	Resistivity, ρ ($\Omega \cdot m$)
	Typical Metals
Silver	1.62×10^{-8}
Copper	1.69×10^{-8}
Gold	2.35×10^{-8}
Aluminum	2.75×10^{-8}
Manganin[a]	4.82×10^{-8}
Tungsten	5.25×10^{-8}
Iron	9.68×10^{-8}
Platinum	10.6×10^{-8}

Resistance and Resistivity

Resistance is a property of an object. Resistivity is a property of a material.

Current is driven by a potential difference.



(assuming the uniform streamlines for current)

$$E = V/L \qquad J = i/A \quad \Rightarrow \quad \rho = \frac{E}{J} = \frac{V/L}{i/A}$$

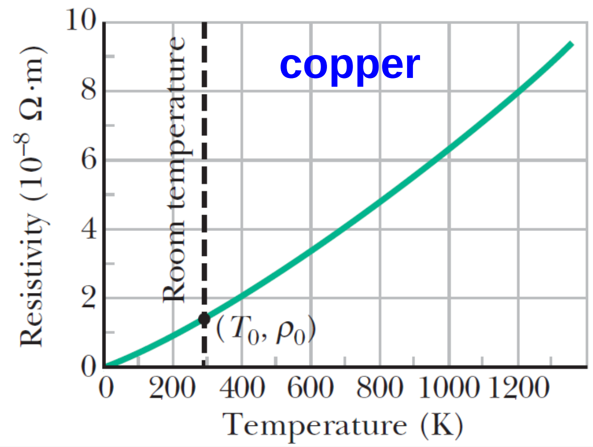
$$\boxed{R = \rho\frac{L}{A}}$$

Note: this expression can be applied only to a homogeneous isotropic conductor of uniform cross section with the potential difference applied as shown on the figure.

Variation with temperature:

$$\rho - \rho_0 = \rho_0\alpha(T - T_0)$$

	Temperature Coefficient of Resistivity, α (K^{-1})
Silver	4.1×10^{-3}
Copper	4.3×10^{-3}
Gold	4.0×10^{-3}
Aluminum	4.4×10^{-3}
Manganin[a]	0.002×10^{-3}
Tungsten	4.5×10^{-3}
Iron	6.5×10^{-3}
Platinum	3.9×10^{-3}



Resistance and Resistivity

EXERCISE

Task #4: A rectangular block of iron has dimensions 1.2 cm x 1.2 cm x 15 cm. A potential difference is to be applied to the block between parallel sides and in such a way that those sides are equipotential surfaces. What is the resistance of the block if the two parallel sides are (a) the square ends (with dimensions 1.2 cm x 1.2 cm) and (b) two rectangular sides (with dimensions 1.2 cm x 15 cm)?

Solution:

(a)

$$L = 15\text{ cm} = 0.15\text{ m} \qquad A = (1.2\text{ cm})^2 = 1.44 \times 10^{-4}\text{ m}^2$$

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega\cdot\text{m})(0.15\text{ m})}{1.44 \times 10^{-4}\text{ m}^2}$$

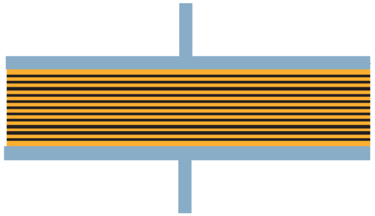
$$= 1.0 \times 10^{-4}\ \Omega = 100\ \mu\Omega$$

(b)

$$L = 1.2\text{ cm} \qquad A = (1.2\text{ cm})(15\text{ cm})$$

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega\cdot\text{m})(1.2 \times 10^{-2}\text{ m})}{1.80 \times 10^{-3}\text{ m}^2}$$

$$= 6.5 \times 10^{-7}\ \Omega = 0.65\ \mu\Omega$$



Resistance and Resistivity

QUIZ Check your understanding:

Suppose you increase the voltage across the copper wir. The increased voltage causes more current to flow, which makes the temperature of the wire increase. (The same thing happens to the coils of an electric oven or a toaster when a voltage is applied to them.) If you double the voltage across the wire, the current in the wire increases. By what factor does it increase? (i) 2; (ii) greater than 2; (iii) less than 2.

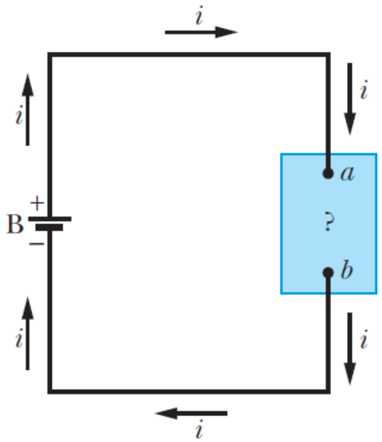
Power in Electric Circuits

Consider a circuit consisting of a battery *B* that is connected by wires, which we assume have negligible resistance, to an unspecified conducting device. The device might be a resistor, a storage battery (a rechargeable battery), a motor, or some other electrical device.

$$dq = i\,dt$$

– the amount of charge that moves between the terminals in *dt*

The battery at the left supplies energy to the conduction electrons that form the current.



This process goes through a decrease in the electric potential energy by the amount:

$$dU = dq\,V = i\,dt\,V$$

→ this is accompanied by a transfer of energy to some other form (*conservation of energy*)

$$\boxed{P = iV}$$

Note: this expression holds for all types of energy transfer

Resistive dissipation: $\boxed{P = \dfrac{V^2}{R}}$ or $\boxed{P = i^2 R}$

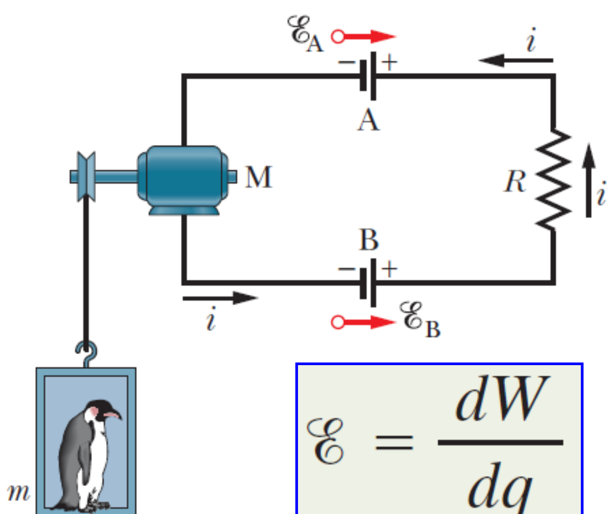
Single Loop Circuits

We are surrounded by **electric circuits**. In this part we will consider the circuits through which charge flows in **one** direction, which are called either **direct** current circuits or **DC** circuits.

To produce a steady flow of charge, you need a "charge pump," a device that – by doing work on the charge carriers – maintains a potential difference between a pair of terminals. We call such a device an **emf** device.

emf = electromotive force

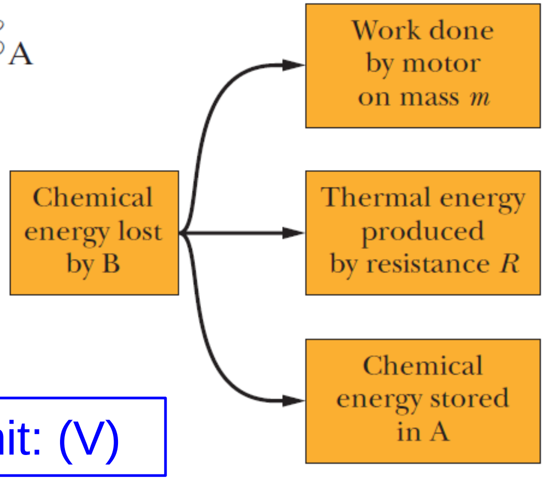
as a rule, this device is just a battery



$$\mathscr{E}_B > \mathscr{E}_A$$

$$\mathscr{E} = \frac{dW}{dq}$$

SI unit: (V)

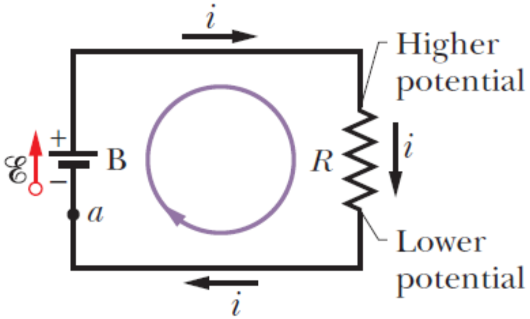


ideal emf device:
→no internal resistance

real emf device:
→has internal resistance

Calculating the Current in a Single Loop Circuit

The battery drives current through the resistor, from high potential to low potential.



We consider the two ways to calculate the current in the simple single loop circuit consisting of an **ideal** emf and a resistor.

Energy method: $dW = \mathscr{E}\, dq = \mathscr{E} i\, dt$

$i^2R\, dt$ dissipation $\Rightarrow$ $\mathscr{E} i\, dt = i^2 R\, dt$ $\quad \boxed{i = \dfrac{\mathscr{E}}{R}}$

Potential method:

LOOP RULE: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

$V_a + \mathscr{E} - iR = V_a$ $\Rightarrow$ $\mathscr{E} - iR = 0$ $\Rightarrow$ $\boxed{i = \dfrac{\mathscr{E}}{R}}$

Considering counterclockwise direction: $-\mathscr{E} + iR = 0$

Calculating the Current in a Single Loop Circuit

Before we proceed to more complex circuits, let us introduce two more rules for finding potential differences as we move around a loop:

RESISTANCE RULE: For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.

EMF RULE: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathscr{E}$; in the opposite direction it is $-\mathscr{E}$.

LOOP RULE: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

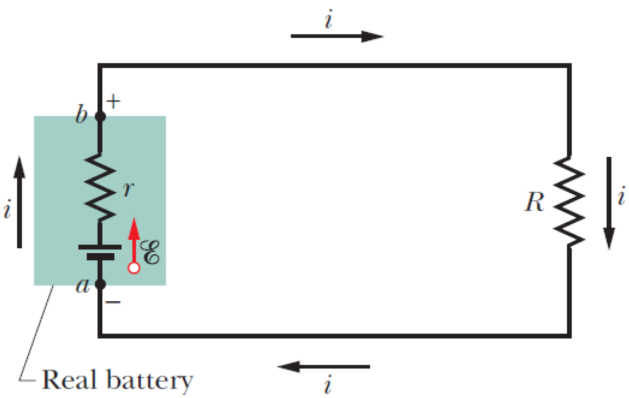
$$V_a + \mathscr{E} - iR = V_a \Rightarrow \mathscr{E} - iR = 0 \Rightarrow \boxed{i = \frac{\mathscr{E}}{R}}$$

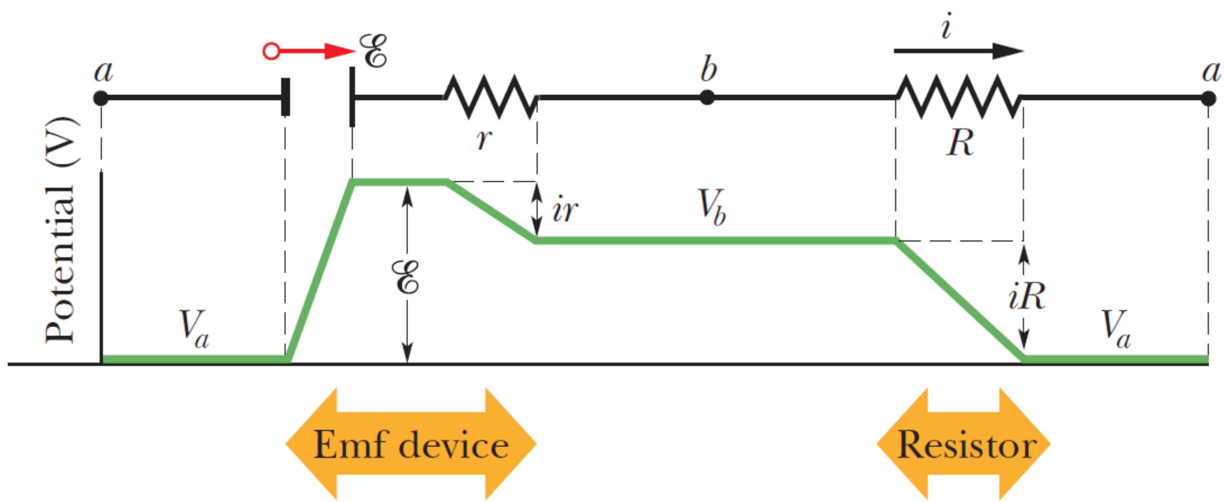
Considering counterclockwise direction: $-\mathscr{E} + iR = 0$

Internal Resistance

The real batteries always possess an **internal resistance** (resistance of the conducting materials of the battery) and thus is an unremovable feature of the battery.

we can draw the battery as if it could be separated into an ideal battery with emf ε and a resistor of resistance *r*. The order in which the symbols for these separated parts are drawn does not matter





$$\mathscr{E} - ir - iR = 0$$

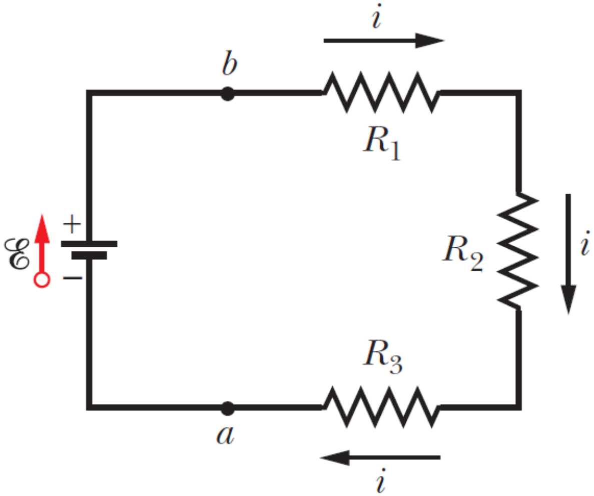
$$\boxed{i = \frac{\mathscr{E}}{R + r}}$$

$r = 0$ $\quad i = \mathscr{E}/R$

Note: traversing the circuit is like walking around a mountain back to your starting point – you return to the starting elevation.

Resistances in Series

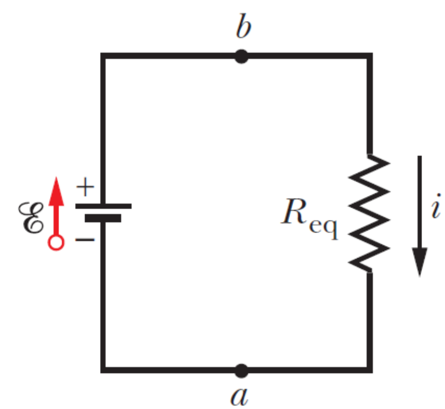
Consider the resistances which are wired one after another and that a potential difference *V* is applied across the two ends of the series. Our goal is to replace the resistances with an equivalent resistance that has the same current and the same total potential difference V as the actual resistances.



$$\mathscr{E} - iR_1 - iR_2 - iR_3 = 0$$

$$i = \frac{\mathscr{E}}{R_1 + R_2 + R_3}$$

$$\mathscr{E} - iR_{eq} = 0 \qquad i = \frac{\mathscr{E}}{R_{eq}}$$



When a potential difference *V* is applied across resistances connected in series, the resistances have identical currents *i*. The sum of the potential differences across the resistances is equal to the applied potential difference *V*.

$$R_{eq} = R_1 + R_2 + R_3$$

$$\boxed{R_{eq} = \sum_{j=1}^{n} R_j \quad (n \text{ resistances in series})}$$

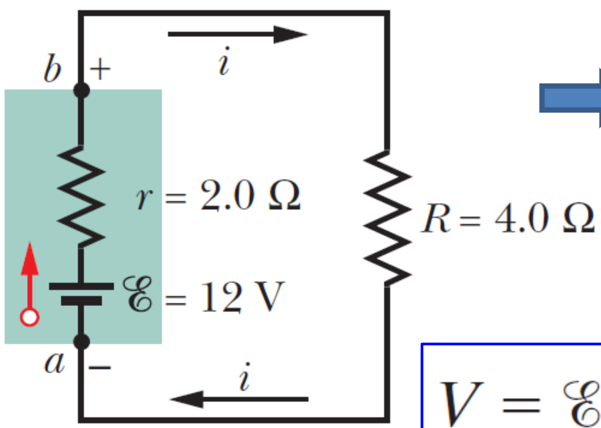
Potential Difference Between Two Points

Sometimes it is necessary to find the potential difference between two points in a circuit.

The internal resistance reduces the potential difference between the terminals.

$$V_a + \mathscr{E} - ir = V_b \qquad i = \frac{\mathscr{E}}{R + r}$$

$$V_b - V_a = \mathscr{E} - ir$$



$$\boxed{V_b - V_a =} \mathscr{E} - \frac{\mathscr{E}}{R + r} r \boxed{= \frac{\mathscr{E}}{R + r} R}$$

$$V_b - V_a = \frac{12\text{ V}}{4.0\ \Omega + 2.0\ \Omega} 4.0\ \Omega = 8.0\text{ V}$$

$$\boxed{V = \mathscr{E} - ir}$$ – potential difference across the real battery

To find the potential between any two points in a circuit, start at one point and traverse the circuit to the other point, following any path, and add algebraically the changes in potential you encounter.

Single Loop Circuits

EXERCISE

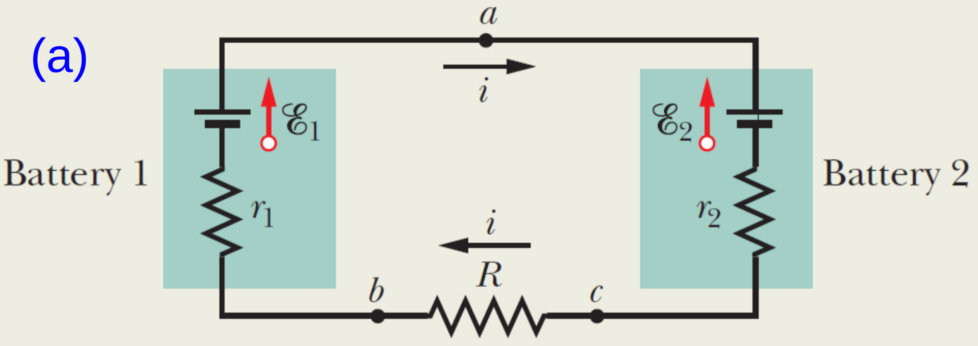
Task #5: The emfs and resistances in the circuit shown below have the following values:

$$\mathscr{E}_1 = 4.4\ \text{V}, \quad \mathscr{E}_2 = 2.1\ \text{V},$$
$$r_1 = 2.3\ \Omega, \quad r_2 = 1.8\ \Omega, \quad R = 5.5\ \Omega.$$

(a) What is the current i in the circuit? (b) What is the potential difference between the terminals of battery 1?

Solution:

(a)



$$-\mathscr{E}_1 + ir_1 + iR + ir_2 + \mathscr{E}_2 = 0$$

$$i = \frac{\mathscr{E}_1 - \mathscr{E}_2}{R + r_1 + r_2} =$$

$$= \frac{4.4\ \text{V} - 2.1\ \text{V}}{5.5\ \Omega + 2.3\ \Omega + 1.8\ \Omega} = 0.2396\ \text{A} \approx 240\ \text{mA}$$

Single Loop Circuits

EXERCISE

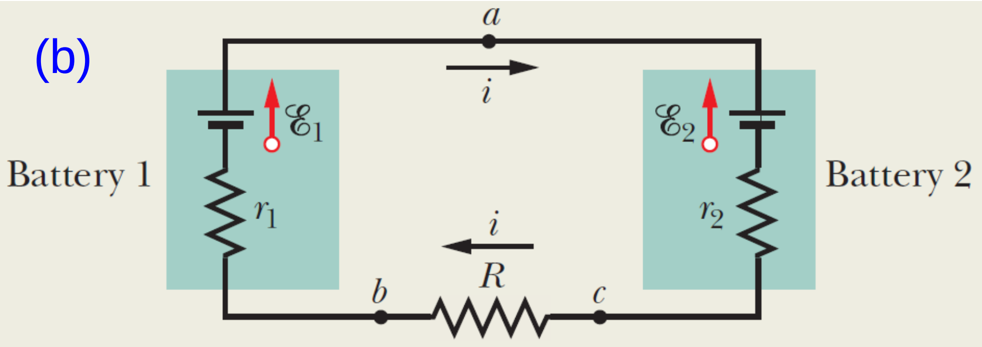
Task #5: The emfs and resistances in the circuit shown below have the following values:

$$\mathscr{E}_1 = 4.4\text{ V}, \quad \mathscr{E}_2 = 2.1\text{ V},$$
$$r_1 = 2.3\ \Omega, \quad r_2 = 1.8\ \Omega, \quad R = 5.5\ \Omega.$$

(a) What is the current i in the circuit? (b) What is the potential difference between the terminals of battery 1?

Solution:

(b)



$$V_b - ir_1 + \mathscr{E}_1 = V_a$$

$$V_a - V_b = -ir_1 + \mathscr{E}_1$$

$$= -(0.2396\text{ A})(2.3\ \Omega) + 4.4\text{ V} = +3.84\text{ V} \approx 3.8\text{ V}$$

Single Loop Circuits

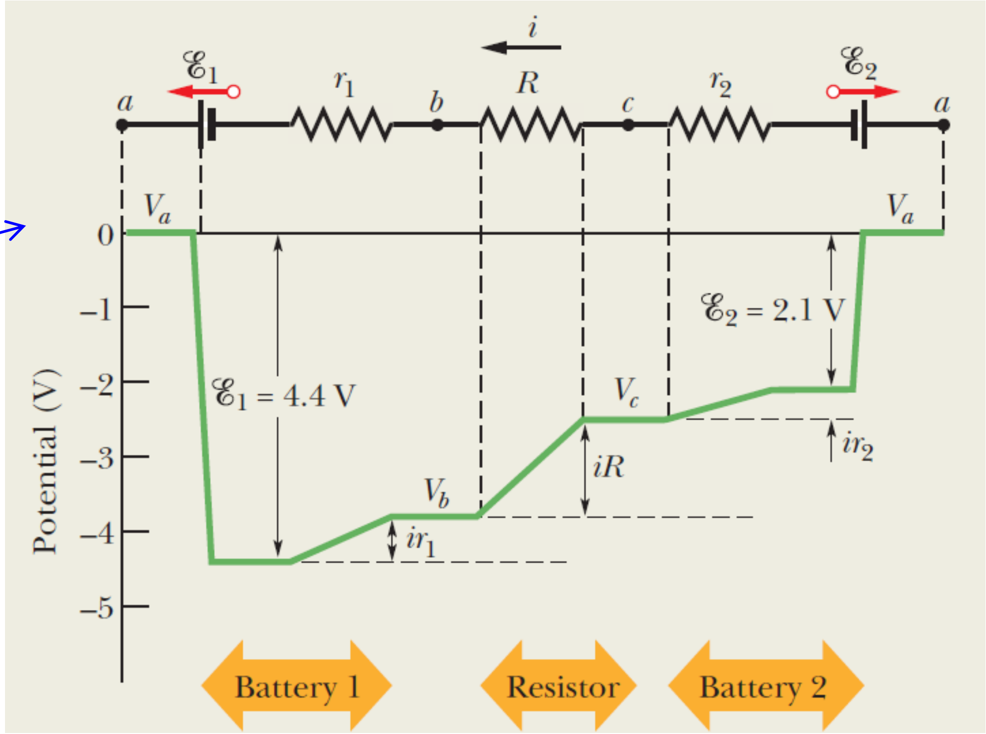
EXERCISE

Task #5: The emfs and resistances in the circuit shown below have the following values:

$$\mathscr{E}_1 = 4.4\text{ V}, \quad \mathscr{E}_2 = 2.1\text{ V},$$
$$r_1 = 2.3\ \Omega, \quad r_2 = 1.8\ \Omega, \quad R = 5.5\ \Omega.$$

(a) What is the current i in the circuit? (b) What is the potential difference between the terminals of battery 1?

Solution:



A graph of the potentials, counterclockwise from point *a*, with the potential at *a* arbitrarily taken to be zero.

Single Loop Circuits

QUIZ Check your understanding:

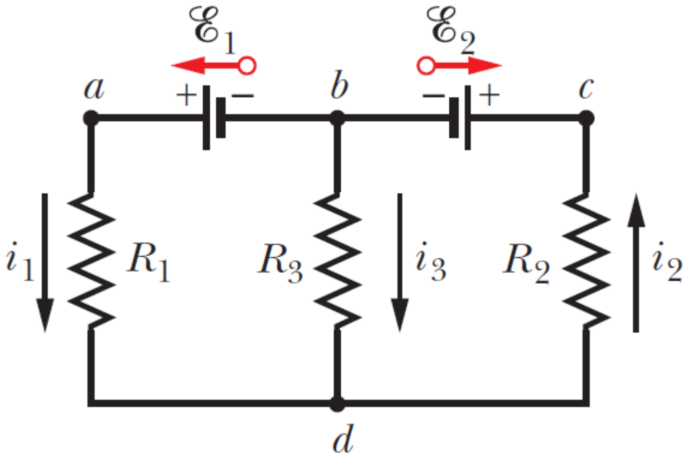
Rank the following circuits in order from highest to lowest current: (i) A 1.4 Ω resistor connected to a 1.5 V battery that has an internal resistance of 0.10 Ω; (ii) a 1.8 Ω resistor connected to a 4.0 V battery that has a terminal voltage of 3.6 V but an unknown internal resistance; (iii) an unknown resistor connected to a 12.0 V battery that has an internal resistance of 0.20 Ω and a terminal voltage of 11.0 V.

QUIZ Check your understanding:

Rank the following circuits in order from highest to lowest values of the net power output of the battery. (i) A 1.4 Ω resistor connected to a 1.5 V battery that has an internal resistance of 0.10 Ω; (ii) a 1.8 Ω resistor connected to a 4.0 V battery that has a terminal voltage of 3.6 V but an unknown internal resistance; (iii) an unknown resistor connected to a 12.0 V battery that has an internal resistance of 0.20 Ω and a terminal voltage of 11.0 V.

Multiloop Circuits

Multiloop circuits contain more than one loop, thus consisting of several branches.



→ left branch (bad)

→ right branch (bcd)

→ central branch (bd)

for junction *d*:

$$i_1 + i_3 = i_2$$

JUNCTION RULE: The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

$$\mathscr{E}_1 - i_1R_1 + i_3R_3 = 0$$

→ traverse of the left-hand loop in a counterclockwise direction from point *b*

$$-i_3R_3 - i_2R_2 - \mathscr{E}_2 = 0$$

→ traverse of the right-hand loop in a counterclockwise direction from point *b*

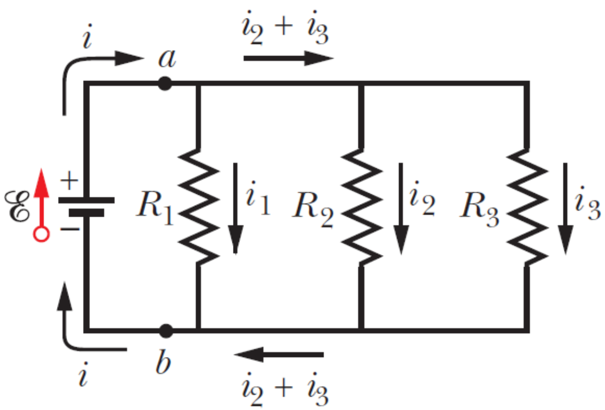
$$\mathscr{E}_1 - i_1R_1 - i_2R_2 - \mathscr{E}_2 = 0$$

→ traverse of the big loop (however, this is merely the sum of previous two)

Resistances in Parallel

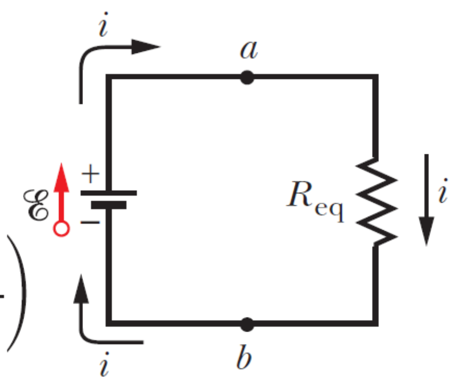
The term “in parallel” means that the resistances are directly wired together on one side and directly wired together on the other side, and that a potential difference *V* is applied across the pair of connected sides.

When a potential difference V is applied across resistances connected in parallel, the resistances all have that same potential difference V.



$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad i_3 = \frac{V}{R_3}$$

$$i = i_1 + i_2 + i_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$



$$i = \frac{V}{R_{eq}} \qquad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$$

(n resistances in parallel)

Series and Parallel Resistors and Capacitors

Let us summarize the equivalence relations

Series	Parallel
Resistors	
$R_{eq} = \sum_{j=1}^{n} R_j$	$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$
Same current through all resistors	Same potential difference across all resistors
Capacitors	
$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$	$C_{eq} = \sum_{j=1}^{n} C_j$
Same charge on all capacitors	Same potential difference across all capacitors

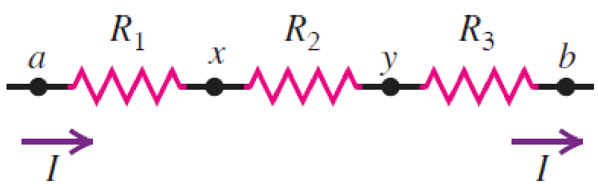
Resistors in Parallel and in Series

QUIZ

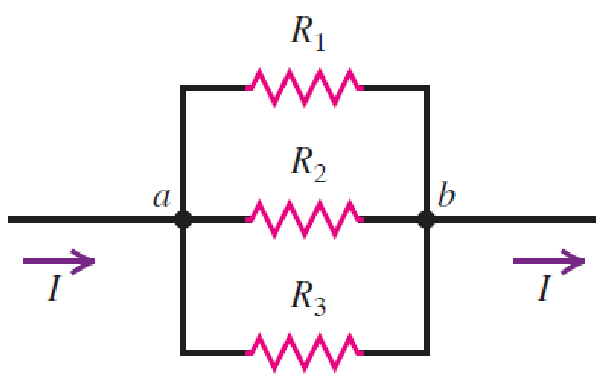
Check your understanding:

Suppose all three of the resistors shown in figures have the same resistance, so $R_1 = R_2 = R_3 = R$. Rank the four arrangements shown in parts (a)–(d) in order of their equivalent resistance, from highest to lowest.

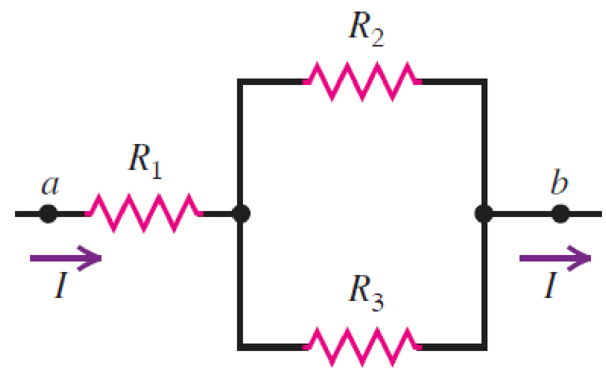
(a) R_1, R_2, and R_3 in series



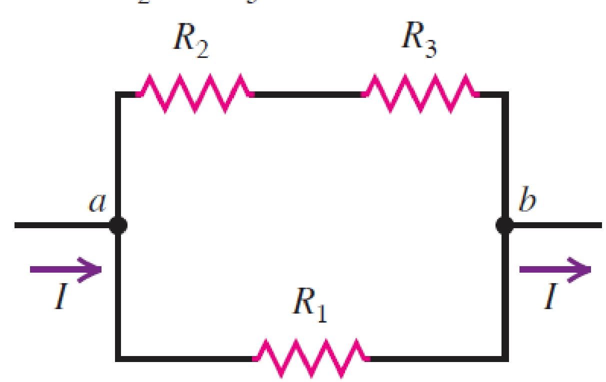
(b) R_1, R_2, and R_3 in parallel



(c) R_1 in series with parallel combination of R_2 and R_3

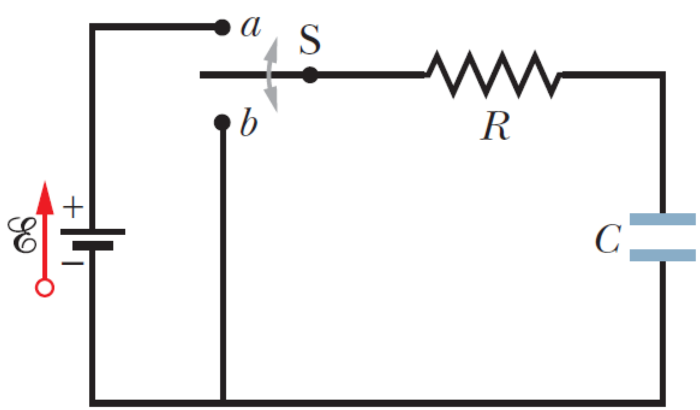


(d) R_1 in parallel with series combination of R_2 and R_3



RC Circuits

In preceding slides we dealt only with circuits in which the currents did not vary with time. Here we begin a discussion of **time-varying** currents.



The capacitor is supposed to be initially uncharged !

$$\mathscr{E} - iR - \frac{q}{C} = 0 \qquad i = \frac{dq}{dt}$$

$$\Rightarrow \boxed{R\frac{dq}{dt} + \frac{q}{C} = \mathscr{E}}$$

charging equation (ODE)

$$\boxed{q = C\mathscr{E}\left(1 - e^{-t/RC}\right)}$$

$$\boxed{V_C = \frac{q}{C} = \mathscr{E}\left(1 - e^{-t/RC}\right)}$$

EXERCISE

$$\boxed{i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-t/RC}}$$

$$\boxed{\tau = RC}$$ capacitive time constant

A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.

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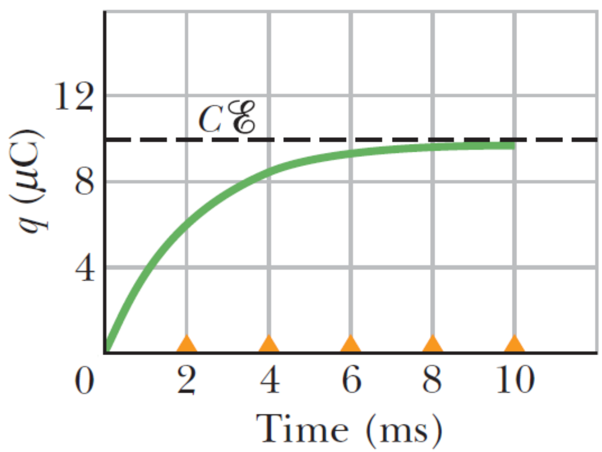
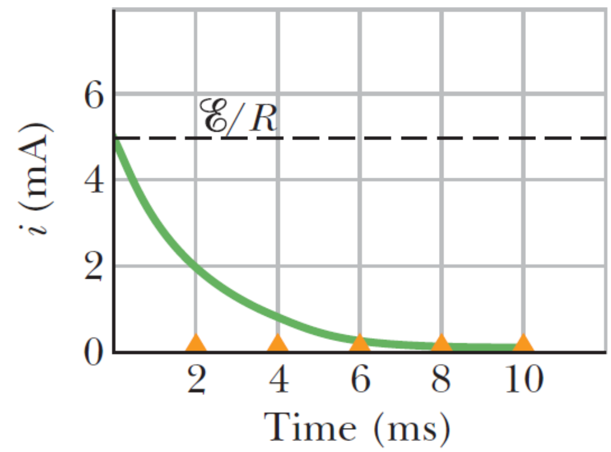
The capacitor's charge grows as the resistor's current dies out.

$R = 2000\ \Omega$
$C = 1\ \mu\text{F}$
$\mathscr{E} = 10\ \text{V}$





$$\boxed{q = C\mathscr{E}(1 - e^{-t/RC})}$$

$$\boxed{V_C = \frac{q}{C} = \mathscr{E}(1 - e^{-t/RC})}$$

EXERCISE

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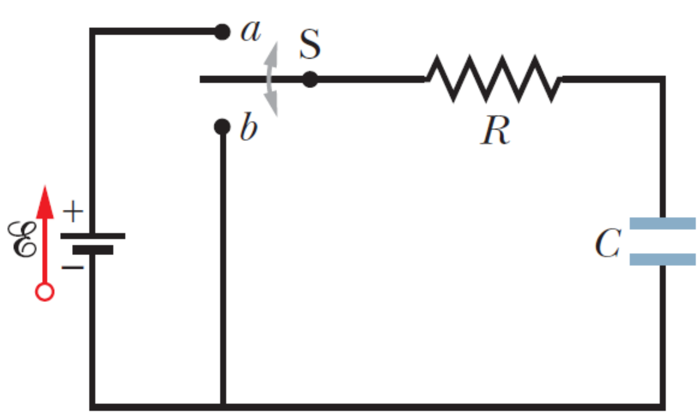

Now the capacitor is supposed to be fully charged, and we put the switch from *a* to *b.*

$$R\frac{dq}{dt} + \frac{q}{C} = 0$$

$$\boxed{q = q_0 e^{-t/RC}}$$

$$\boxed{i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC}}$$

Note: the charge decreases exponentially with time, at a rate that is set by the capacitive time constant. Same does the current.

At $t = 0$: $i_0 = V_0/R = (q_0/C)/R = q_0/RC$ $\qquad q_0 = CV_0$

Conclusions

• there exist two types of **charges**: **positive** and **negative.** Their magnitude is always equal to the **integer** number of elementary charges. The net electric charge of any isolated system is always **conserved**

• electric charges interact with each other (either attract or repel) by means of the **Coulomb** force which is a **conservative** force

• a charged object sets up an **electric field** in the surrounding space. In order to visualize it, we introduce **electric field lines**. Electric field obeys the **superposition** principle

• **electric potential** is a scalar quantity that equals the work needed to bring the point charge from the infinity to the considered position in space divided by its magnitude and taken with the minus sign

• in order to calculate electric fields and potentials of the **extended** (continuum) **bodies**, we split them into the tiny pieces, so that the laws for point charges could be applied. The net result is just a “sum” (integral) over these small pieces

• we introduced other terms describing electrical phenomena, like **current**, **resistance**, **capacitance**, and **emf**. You should understand their physical meaning

• there exists a set of very important **rules** and ways of treating both single loop and multiloop circuits, as well as junctions of elements (like capacitors and resistors)