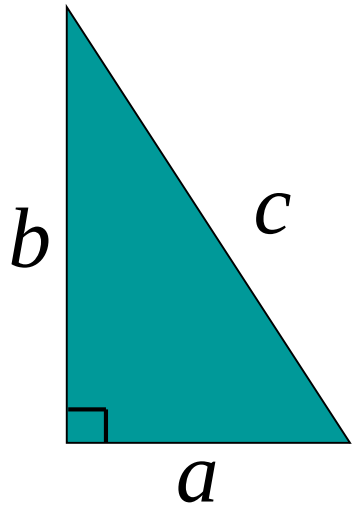
What is logic?

Mathematical proof (what and why)

Pythagorean theorem (毕达哥拉斯定理)

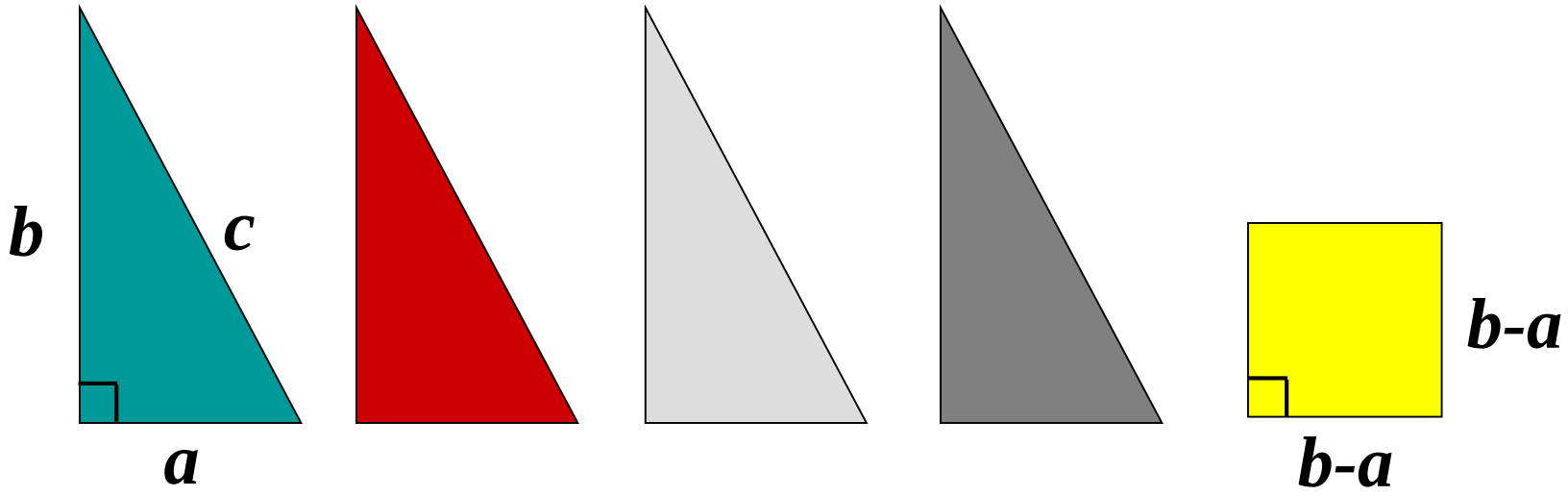


$$a^2 + b^2 = c^2$$

Familiar?

Obvious?

Good proof



We will show that these five pieces can be rearranged into:

(i) a c×c square, and then

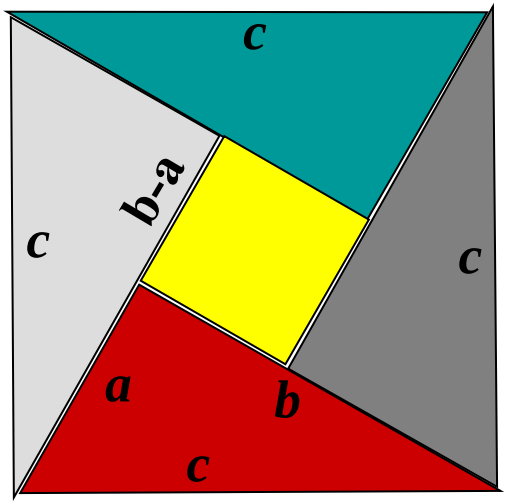
(ii) an a×a & a b×b square

And then we can conclude that $c^2 = a^2 + b^2$

Good proof

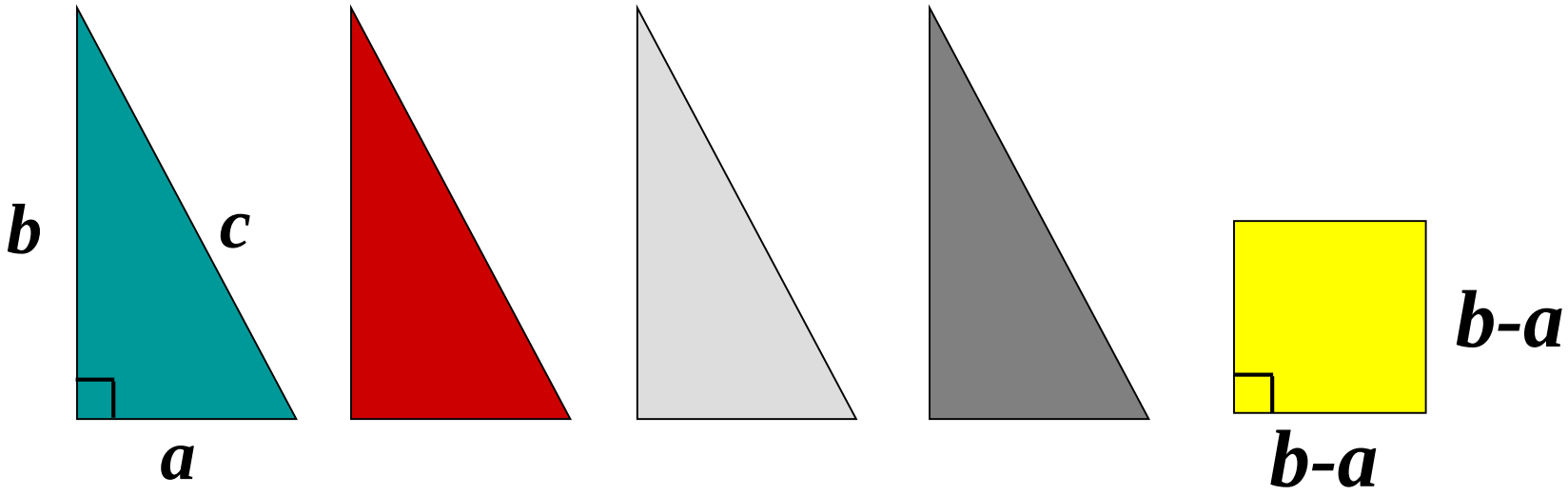
The five pieces can be rearranged into:

(i) a c×c square

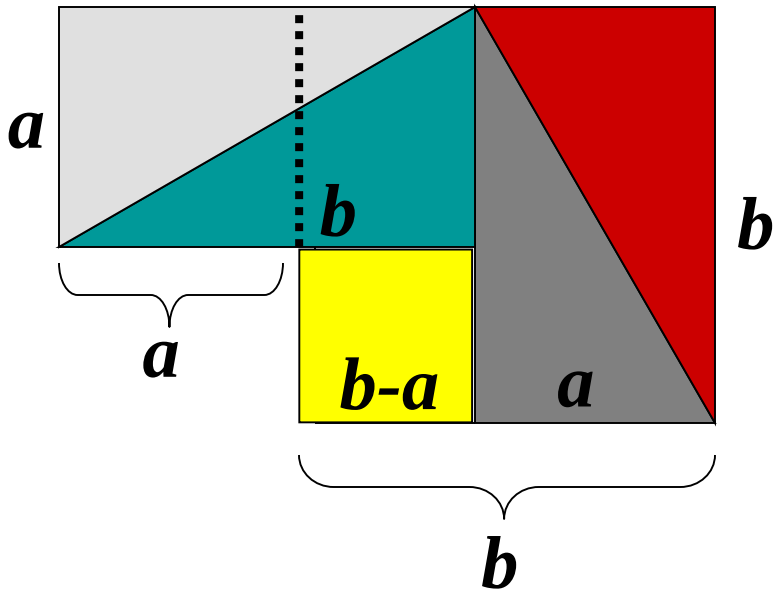


Good proof

How to rearrange them into an axa square and a bxb square?



Good proof

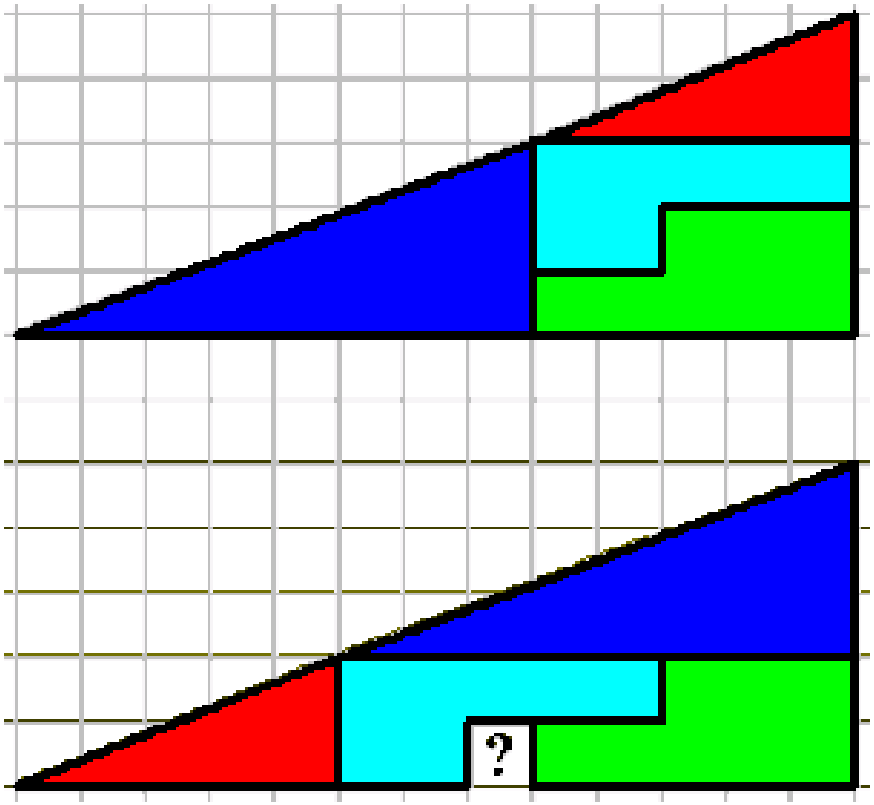


74 proofs in http://www.cut-the-knot.org/pythagoras/index.shtml

Missing square puzzle

A similar rearrangement technique shows that 65=64…

What's wrong with the proof?



Dissection Paradox (裁剪悖论)！！

Detective puzzle

A detective (侦探) has interviewed four witnesses(证人) to a crime. From the stories of the witnesses the detective has concluded that

- If the butler (管家) is telling the truth then so is the cook;
- The cook (厨师) and the gardener (园丁) cannot both be telling the truth;
- The gardener and the handyman (杂役) are not both lying;
- If the handyman is telling the truth then the cook is lying.

For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Explain your reasoning.

Solution: The detective can determine that the butler and cook are lying but cannot determine whether the gardener is telling the truth or whether the handyman is telling the truth.

What is logic

- Logic = the study of correct reasoning (推理)
- On an elementary level, logic provides rules and techniques for determining whether a given argument (论证) is valid.
- Use of logic
  - In mathematics:

    to prove theorems
  - In computer science:

    to prove that programs do what they are supposed to do
  - In natural and physical sciences

    to draw conclusions from experiments
  - In social sciences and in our everyday lives

    to solve a multitude of problems

    …
- Indeed, we are constantly using logical reasoning. In this part, we discuss a few of the basic ideas

What is logic

- Logic focuses on the relationship among statements. For example:
  - "My watch is digital."
  - "All digital devices run on batteries."
  - "Therefore, my watch runs on batteries."
- Note that logic is not concerned with the truth of the first two statements.
- But if they were true, then the inference is true