1. Which of these sentences are propositions? What are the truth values of those that are propositions?

a) Boston is the capital of Massachusetts.

b) Miami is the capital of Florida.

c) 2 + 3 = 5.

d) 5 + 7 = 10.

e) x+ 2 = 11.

f ) Answer this question.

2. What is the negation of each of these propositions?

a) Mei has an MP3 player.

b) There is no pollution in New Jersey.

c) 2 + 1 = 3.

d) The summer in Maine is hot and sunny.

3. Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of these propositions for the most recent fiscal year.

a) Quixote Media had the largest annual revenue.

b) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.

c) Acme Computer had the largest net profit or Quixote. Media had the largest net profit.

d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.

e) Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual revenue.

4. Let p and q be the propositions “Swimming at the New Jersey shore is allowed” and “Sharks have been spotted near the shore,” respectively. Express each of these compound propositions as an English sentence.

a) ¬q b) p ∧ q

c) ¬p ∨ q d) p →¬q

e) ¬q → p f ) ¬p →¬q

g) p ↔¬q h) ¬p ∧ (p∨ ¬q)

5. For each of these sentences, state what the sentence means if the logical connective or is an inclusive or (that is, a disjunction) versus an exclusive or. Which of these meanings of or do you think is intended?

a) To take discrete mathematics, you must have taken calculus or a course in computer science.

b) When you buy a new car from Acme Motor Company, you get $2000 back in cash or a 2% car loan.

c) Dinner for two includes two items from column A or three items from column B.

d) School is closed if more than 2 feet of snow falls or if the wind chill is below −100.

6. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows today, I will ski tomorrow.

b) I come to class whenever there is going to be a quiz.

c) A positive integer is a prime only if it has no divisors other than 1 and itself.

7 Construct a truth table for each of these compound propositions.

a) p ∧¬p b) p ∨¬p

c) (p ∨¬q) → q d) (p ∨ q) → (p ∧ q)

e) (p → q) ↔ (¬q →¬p)

f ) (p → q) → (q → p)

8. Construct a truth table for each of these compound propositions.

a) p →¬q b) ¬p ↔ q

c) (p → q) ∨ (¬p → q) d) (p → q) ∧ (¬p → q)

e) (p ↔ q) ∨ (¬p ↔ q)

f ) (¬p ↔¬q) ↔ (p ↔ q)

9. Explain, without using a truth table, why (p ∨ q ∨ r) ∧ (¬p ∨¬q ∨¬r) is true when at least one of p, q, and r is true and at least one is false, but is false when all three variables have the same truth value.

10. Use truth tables to verify these equivalences.

a) p ∧ T ≡ p b) p ∨ F ≡ p

c) p ∧ F ≡F d) p ∨ T ≡ T

e) p ∨ p ≡ p f ) p ∧ p ≡ p

11. Use truth tables to verify the commutative laws

a) p ∨ q ≡ q ∨ p. b) p ∧ q ≡ q ∧ p.

Answer:

12. Use De Morgan’s laws to find the negation of each of the following statements.

a) Jan is rich and happy.

b) Carlos will bicycle or run tomorrow.

c) Mei walks or takes the bus to class.

d) Ibrahim is smart and hard working.

Answer:

13. Show that each of these conditional statements is a tautology by using truth tables. Then prove them without truth table.

a) (p ∧ q) → p b) p → (p ∨ q)

c) ¬p → (p → q) d) (p ∧ q) → (p → q)

e) ¬(p → q) → p f ) ¬(p → q)→¬q

Answer:

14. Use truth tables to verify the absorption laws.

a) p ∨ (p ∧ q) ≡ p b) p ∧ (p ∨ q) ≡ p

Answer:

15. Determine whether (¬q ∧ (p → q))→¬p is a tautology.

Answer:

16. Show that ¬(p ↔ q) and p ↔¬q are logically equivalent.

Answer

17. Show that (p → r) ∧ (q → r) and (p ∨ q) → r are logically equivalent.

Answer:

18. Show that (p → q) → r and p → (q → r) are not logically equivalent.

Answer:

19. Show that (p → q) → (r → s) and (p → r) → (q → s) are not logically equivalent.

20.求下列各式的主合取范式和合取范式。

1. 
2. 
3. 
4. 

21.求命题公式(¬P ∧ Q) ↔ (P → Q)的合取范式。

22.求下列各式的析取范式。

1. (¬P ∨ Q) ↔ (P ∧ Q)
2. P ∧（Q → R）

23.试采用将公式化为主范式的方法，证明下列各式等价。

1. 
2. 
3. 

4）

24.使用常用恒等式证明下列各式，并给出下列各式的对偶式。

1. 
2. 

25.试证明下列合式公式是永真式。

1. 
2. 

26.不构造真值表证明下列蕴含式。

（1）

（2）

27.足坛四支劲旅举行友谊比赛。已知情况如下，请问结论是否成立？

1. 若大连阿尔滨队获得冠军，则北京国安队或上海申花队获得亚军。
2. 若上海申花队获得亚军，则大连阿尔滨队不能获得冠军。
3. 若广州恒大队获得亚军，则北京国安队不能获得亚军。
4. 最后大连阿尔滨队获得冠军。

结论：广州恒大队未能获得的亚军。

28. 将下列命题用谓词逻辑符号化。

1. 小王聪明而且好学。
2. 没有最大素数
3. 并非所有大学生都能成为科学家。
4. 每个自然数不是奇数就是偶数
5. 诗人李白游览所有名山大川。

29. Let P(x) be the statement “x spends more than five hours every weekday in class,” where the domain for x consists of all students. Express each of these quantifications in English.

a) ∃xP(x) b) ∀xP(x)

c) ∃x P(x) d) ∀x￢P(x)

30. Translate these statements into English, where C(x) is “x is a comedian” and F(x) is “x is funny” and the domain consists of all people.

a) ∀x(C(x) → F(x)) b) ∀x(C(x) ∧ F(x))

c) ∃x(C(x) → F(x)) d) ∃x(C(x) ∧ F(x))

31. Let P(x) be the statement “x can speak Russian” and let Q(x) be the statement “x knows the computer language C++.” Express each of these sentences in terms of P(x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

a) There is a student at your school who can speak Russian and who knows C++.

b) There is a student at your school who can speak Russian but who doesn’t know C++.

c) Every student at your school either can speak Russian or knows C++.

d) No student at your school can speak Russian or knows C++.

32. Suppose that the domain of the propositional function P(x) consists of the integers 1, 2, 3, 4, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

a) ∃xP(x) b) ∀xP(x)

c) ￢∃xP(x) d) ￢∀xP(x)

e) ∀x((x≠3) → P(x)) ∨ ∃x￢P(x)

33. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

a) No one is perfect.

b) Not everyone is perfect.

c) All your friends are perfect.

d) At least one of your friends is perfect.

e) Everyone is your friend and is perfect.

f ) Not everybody is your friend or someone is not perfect.

34. Translate these specifications into English where F(p) is “Printer p is out of service,” B(p) is “Printer p is busy,” L(j) is “Print job j is lost,” and Q(j) is “Print job j is queued.”

a) ∃p(F(p) ∧ B(p)) → ∃jL(j)

b) ∀pB(p) → ∃jQ(j)

c) ∃j (Q(j) ∧ L(j)) → ∃pF(p)

d) (∀pB(p) ∧ ∀jQ(j)) → ∃jL(j)

35. Determine whether ∀x(P(x) → Q(x)) and ∀xP(x) →∀xQ(x) are logically equivalent. Justify your answer.

36. Show that ∃xP(x) ∧ ∃xQ(x) and ∃x(P(x) ∧ Q(x)) are not logically equivalent.

37. Let P(x), Q(x), and R(x) be the statements “x is a professor,” “x is ignorant,” and “x is vain,” respectively. Express each of these statements using quantifiers; logical connectives; and P(x), Q(x), and R(x), where the domain consists of all people.

a) No professors are ignorant.

b) All ignorant people are vain.

c) No professors are vain.

d) Does (c) follow from (a) and (b)?

38. Let P(x), Q(x), R(x), and S(x) be the statements “x is a baby,” “x is logical,” “x is able to manage a crocodile,” and “x is despised,” respectively. Suppose that the domain consists of all people. Express each of these statements using quantifiers; logical connectives; and P(x), Q(x), R(x), and S(x).

a) Babies are illogical.

b) Nobody is despised who can manage a crocodile.

c) Illogical persons are despised.

d) Babies cannot manage crocodiles.

∗e) Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?

39. Use a direct proof to show that the sum of two odd integers is even.

40. Show that the square of an even number is an even number using a direct proof.

41. Prove or disprove that the product of two irrational numbers is irrational.

42. Use a proof by contraposition to show that if x + y ≥ 2, where x and y are real numbers, then x ≥ 1 or y ≥ 1.

43. Prove that at least one of the real numbers a1, a2, . . . , an is greater than or equal to the average of these numbers. What kind of proof did you use?

44. Determine whether each of these arguments is valid. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs?

a) If n is a real number such that n > 1, then n2 > 1. Suppose that n2 > 1. Then n > 1.

b) If n is a real number with n > 3, then n2 > 9. Suppose that n2 ≤ 9. Then n ≤ 3.

c) If n is a real number with n > 2, then n2 > 4. Suppose that n ≤ 2. Then n2 ≤ 4.

45. Justify the rule of universal modus tollens by showing that the premises ∀x(P(x) → Q(x)) and ¬Q(a) for a particular element a in the domain, imply ¬P(a).

46. Use resolution to show that the hypotheses “It is not raining or Yvette has her umbrella,” “Yvette does not have her umbrella or she does not get wet,” and “It is raining or Yvette does not get wet” imply that “Yvette does not get wet.”

47. Use resolution to show that the compound proposition (p ∨ q) ∧ (¬p ∨ q) ∧ (p ∨¬q) ∧ (¬p ∨¬q) is not satisfiable.