Matrices and Their Applications

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Matrices

- A Matrix is a rectangular array of numbers consisting of m rows and n columns
- The dimension of a matrix is defined by the number of rows and columns in which we can classify it as a m x n matrix
- In classes like Linear Algebra, matrices have more specific names

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}.$$

The matrix $\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix}$ is a 3×2 matrix.

Types of Matrices

1. Row Matrices and Column Matrices

$$\begin{array}{cc} \text{1 column} & \\ \downarrow & \\ A = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} & \longleftarrow \; n \text{ rows} \end{array}$$

$$\begin{array}{cc} n \text{ columns} & \\ \downarrow & \\ B = [b_1 \quad b_2 \quad \dots \quad b_n] & \longleftarrow \; 1 \text{ row} \end{array}$$

Types of Matrices

2. Square Matrix

$$Square\ Matrix\ M = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

Types of Matrices

3. Diagonal Matrix

$$\textit{Diagonal matrix } d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Types of Matrices

4. Identity Matrix

$$I_1 = [1],\ I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Matrix Operations

1. Addition

- Two matrices cannot be added if they are different dimensions
- $A + B = [a_{ij} + b_{ij}]$.

ADD

$$A + B = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} + \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 + b_1 & a_2 + b_2 \\ a_3 + b_3 & a_4 + b_4 \end{bmatrix}$$

$$A + C = \begin{bmatrix} -5 & 2 & 0 \\ 7 & -3 & 4 \\ -1 & 3 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 8 \\ 6 & -14 & 2 \\ 9 & 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (-5)+(0) & (2)+(-1) & (0)+(8) \\ (7)+(6) & (-3)+(-14) & (4)+(2) \\ (-1)+(9) & (3)+(5) & (2)+(1) \end{bmatrix}$$

$$A + C = \begin{bmatrix} -5 & 1 & 8 \\ 13 & -17 & 6 \\ 8 & 8 & 3 \end{bmatrix}$$

Matrix Operations

2. Subtraction

- Two matrices cannot be subtracted if they are different dimensions
- $A - B = [a_{ij} - b_{ij}]$.

SUBTRACT

$$A - B = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} - \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 - b_1 & a_2 - b_2 \\ a_3 - b_3 & a_4 - b_4 \end{bmatrix}$$

$$F - D = \begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix} - \begin{bmatrix} 6 & -2 \\ 3 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} (1)-(6) & (-2)-(-2) \\ (-3)-(3) & (4)-(7) \end{bmatrix}$$

$$F - D = \begin{bmatrix} -5 & 0 \\ -6 & -3 \end{bmatrix}$$

Matrix Operations

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Matrix Operations

3. Multiplication

- Two matrices can be multiplied even if they are different dimensions
- We are looking at whether the number of columns of A equals the number of rows of B.
- (m x n) x (n x p) = m x p

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1k} \\ a_{21} & a_{22} & \cdots & a_{2k} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ik} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mk} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2j} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{k1} & b_{k2} & \cdots & b_{kj} & \cdots & b_{kn} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & c_{ij} & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 10 & 11 \\ 20 & 21 \\ 30 & 31 \end{bmatrix}$$

$$= \begin{bmatrix} 1\text{x}10 + 2\text{x}20 + 3\text{x}30 & 1\text{x}11 + 2\text{x}21 + 3\text{x}31 \\ 4\text{x}10 + 5\text{x}20 + 6\text{x}30 & 4\text{x}11 + 5\text{x}21 + 6\text{x}31 \end{bmatrix}$$

$$= \begin{bmatrix} 10+40+90 & 11+42+93 \\ 40+100+180 & 44+105+186 \end{bmatrix} = \begin{bmatrix} 140 & 146 \\ 320 & 335 \end{bmatrix}$$

Matrices and Computer Science

- The applications of Linear Algebra are important in computer science
- Matrices represent many types of data and the linear transformations involved with it allow 3D objects to be rendered on 2D monitors.
- Such linear transformations include rotation, scaling, translation, reflection, shearing, etc.

Matrices and Computer Science

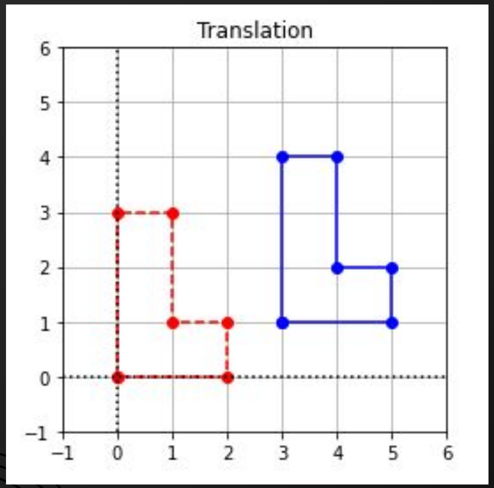
Simple Translation Transformation

```python
coords = np.array([[0,0],[0,3],[1,3],[1,1],[2,1],[2,0],[0,0]])
coords = coords.transpose()

x = coords[0,:]
y = coords[1,:]

## Compute translation by adding 3 to x coordinates and 1 to y coordinates
x_translated =  np.copy(x)
y_translated =  np.copy(y)

for i in range(x.shape[0]):
    x_translated[i] += 3
    y_translated[i] += 1
```

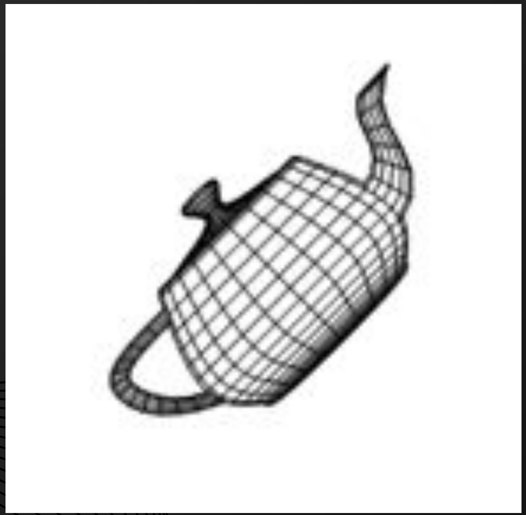


Matrices and Computer Science

Simple Rotation Transformation

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

```
R_z(t) = [ cosd(t)  -sind(t)        0        0
           sind(t)   cosd(t)        0        0
                 0         0        1        0
                 0         0        0        1]
```



Credits.

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Credits.

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