PASCAL’S IDENTITY & TRIANGLE

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OVERVIEW

Introduction

Pascal’s Identity

Pascal’s Triangle

Building Pascal’s Triangle

Applications

Binomial Expansion

Probability

Combinations

BLAISE PASCAL (1623-1662)

- Pascal's Identity and Triangle are named after a 17th century mathematician
- Along with Fermat, laid the foundations for the modern theory of probability
- Contributed to the advance of differential calculus

PASCAL'S IDENTITY

PASCAL'S IDENTITY Let n and k be positive integers with $n \geq k$. Then

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}.$$

- Is the basis for a geometric arrangement of the binomial coefficients in a triangle
- Together with the initial conditions $\binom{n}{0} = \binom{n}{n} = 1$ can be used to recursively define binomial coefficients
- Useful in the computation of binomial coefficients

PASCAL'S TRIANGLE

$$\binom{0}{0}$$
$$\binom{1}{0}\ \binom{1}{1}$$
$$\binom{2}{0}\ \binom{2}{1}\ \binom{2}{2}$$
$$\binom{3}{0}\ \binom{3}{1}\ \binom{3}{2}\ \binom{3}{3}$$
$$\binom{4}{0}\ \binom{4}{1}\ \binom{4}{2}\ \binom{4}{3}\ \binom{4}{4}$$
$$\binom{5}{0}\ \binom{5}{1}\ \binom{5}{2}\ \binom{5}{3}\ \binom{5}{4}\ \binom{5}{5}$$
$$\binom{6}{0}\ \binom{6}{1}\ \binom{6}{2}\ \binom{6}{3}\ \binom{6}{4}\ \binom{6}{5}\ \binom{6}{6}$$
$$\binom{7}{0}\ \binom{7}{1}\ \binom{7}{2}\ \binom{7}{3}\ \binom{7}{4}\ \binom{7}{5}\ \binom{7}{6}\ \binom{7}{7}$$
$$\binom{8}{0}\ \binom{8}{1}\ \binom{8}{2}\ \binom{8}{3}\ \binom{8}{4}\ \binom{8}{5}\ \binom{8}{6}\ \binom{8}{7}\ \binom{8}{8}$$

By Pascal's identity:

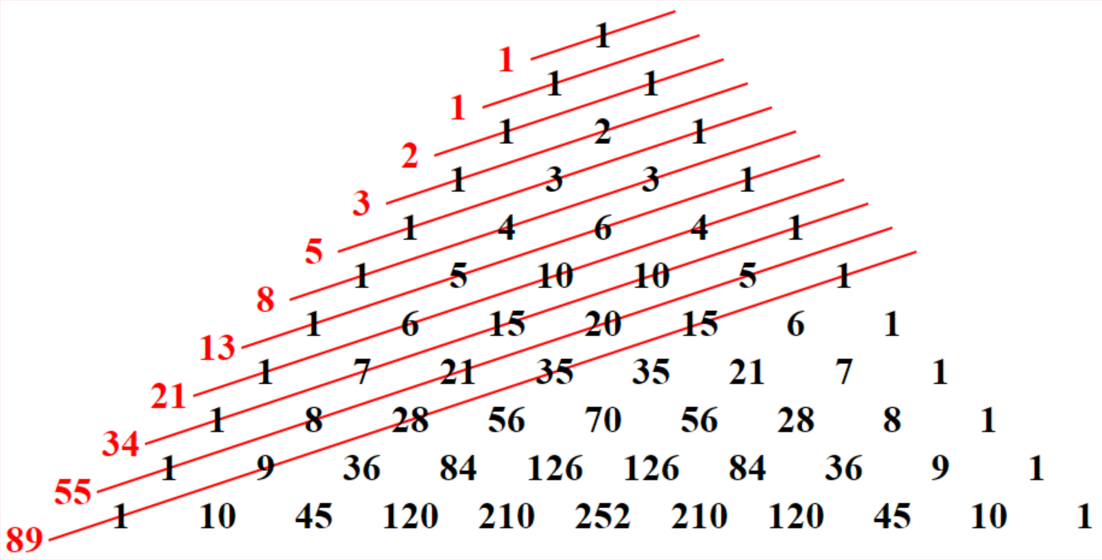
$$\binom{6}{4} + \binom{6}{5} = \binom{7}{5}$$

$$1$$
$$1\quad 1$$
$$1\quad 2\quad 1$$
$$1\quad 3\quad 3\quad 1$$
$$1\quad 4\quad 6\quad 4\quad 1$$
$$1\quad 5\quad 10\quad 10\quad 5\quad 1$$
$$1\quad 6\quad 15\quad 20\quad 15\quad 6\quad 1$$
$$1\quad 7\quad 21\quad 35\quad 35\quad 21\quad 7\quad 1$$
$$1\quad 8\quad 28\quad 56\quad 70\quad 56\quad 28\quad 8\quad 1$$

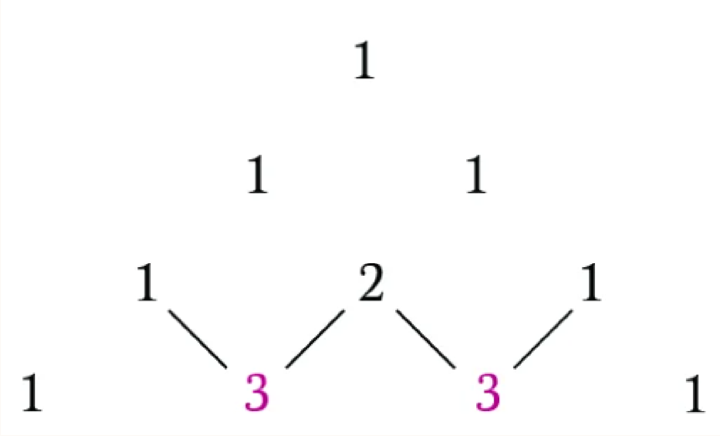
$\binom{n}{k}$, $k = 0, 1, \ldots, n$

PASCAL'S TRIANGLE

- The relationship between Fibonacci sequence and Pascal's triangle



BUILDING PASCAL'S TRIANGLE



1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

APPLICATIONS

BINOMIAL EXPANSION

$(x + y)^0$	1
$(x + y)^1$	$1x + 1y$
$(x + y)^2$	$1x^2 + 2xy + 1y^2$
$(x + y)^3$	$1x^3 + 3x^2y + 3xy^2 + 1y^3$
$(x + y)^4$	$1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$
$(x + y)^5$	$1x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1y^5$

PROBABILITY

1. HHHH
2. HHHT, HHTH, HTHH, THHH
3. TTHH, HHTT, HTHT, HTTH,THHT, THTH
4. HTTT, THTT, TTHT, TTTH
5. TTTT

COMBINATIONS

$$C(n,k) = C_k^n = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

BINOMIAL EXPANSION

- Expanding a binomial expression using Pascal's triangle

$(x+y)^0$	1
$(x+y)^1$	$1x + 1y$
$(x+y)^2$	$1x^2 + 2xy + 1y^2$
$(x+y)^3$	$1x^3 + 3x^2y + 3xy^2 + 1y^3$
$(x+y)^4$	$1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$
$(x+y)^5$	$1x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1y^5$

PROBABILITY

- Pascal's triangle can show how heads and tails can combine

1. HHHH
2. HHHT, HHTH, HTHH, THHH
3. TTHH, HHTT, HTHT, HTTH,THHT, THTH
4. HTTT, THTT, TTHT, TTTH
5. TTTT

COMBINATIONS

- The number of combinations of n things taken k at a time can be found with this equation

$$C(n,k) = C_k^n = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

CONCLUSION

PASCAL'S IDENTITY Let n and k be positive integers with $n \geq k$. Then

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}.$$

PASCAL'S IDENTITY

```
            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
  1   5  10  10   5   1
1   6  15  20  15   6   1
```

PASCAL'S TRIANGLE

$(x+y)^0$	1
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APPLICATIONS

WORKS CITIED

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