# PASCAL'S IDENTITY & TRIANGLE

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# **OVERVIEW**

Introduction

Pascal's Identity

Pascal's Triangle

**Building Pascal's Triangle** 

**Applications** 

**Binomial Expansion** 

**Probability** 

Combinations

### **BLAISE PASCAL (1623-1662)**

- Pascal's Identity and Triangle are named after a 17<sup>th</sup> century mathematician
- Along with Fermat, laid the foundations for the modern theory of probability
- Contributed to the advance of differential calculus







**PASCAL'S IDENTITY** Let *n* and *k* be positive integers with  $n \ge k$ . Then

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}.$$

- Is the basis for a geometric arrangement of the binomial coefficients in a triangle
- Together with the initial conditions  $\binom{n}{0} = \binom{n}{n} = 1$  can be used to recursively define binomial coefficients
- Useful in the computation of binomial coefficients



### PASCAL'S TRIANGLE

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} \qquad \text{By Pascal's identity:} \qquad 1 \qquad 2 \qquad 1$$

$$\begin{pmatrix} 3 \\ 0 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \end{pmatrix} \begin{pmatrix} 6 \\ 4 \end{pmatrix} + \begin{pmatrix} 6 \\ 5 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} \qquad 1 \qquad 3 \qquad 3 \qquad 1$$

$$\begin{pmatrix} 4 \\ 0 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix} \qquad \qquad \qquad 1 \qquad 4 \qquad 6 \qquad 4 \qquad 1$$

$$\begin{pmatrix} 5 \\ 0 \end{pmatrix} \begin{pmatrix} 5 \\ 1 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \end{pmatrix} \begin{pmatrix} 5 \\ 4 \end{pmatrix} \begin{pmatrix} 5 \\ 5 \end{pmatrix} \begin{pmatrix} 5 \\ 6 \end{pmatrix} \qquad \qquad 1 \qquad 5 \qquad 10 \qquad 10 \qquad 5 \qquad 1$$

$$\begin{pmatrix} 6 \\ 0 \end{pmatrix} \begin{pmatrix} 6 \\ 1 \end{pmatrix} \begin{pmatrix} 6 \\ 2 \end{pmatrix} \begin{pmatrix} 6 \\ 3 \end{pmatrix} \begin{pmatrix} 6 \\ 4 \end{pmatrix} \begin{pmatrix} 6 \\ 5 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \end{pmatrix} \qquad \qquad 1 \qquad 6 \qquad 15 \qquad 20 \qquad 15 \qquad 6 \qquad 1$$

$$\begin{pmatrix} 7 \\ 0 \end{pmatrix} \begin{pmatrix} 7 \\ 1 \end{pmatrix} \begin{pmatrix} 7 \\ 2 \end{pmatrix} \begin{pmatrix} 7 \\ 3 \end{pmatrix} \begin{pmatrix} 7 \\ 4 \end{pmatrix} \begin{pmatrix} 7 \\ 5 \end{pmatrix} \begin{pmatrix} 7 \\ 6 \end{pmatrix} \begin{pmatrix} 7 \\ 7 \end{pmatrix} \qquad \qquad 1 \qquad 7 \qquad 21 \quad 35 \quad 35 \quad 21 \qquad 7 \qquad 1$$

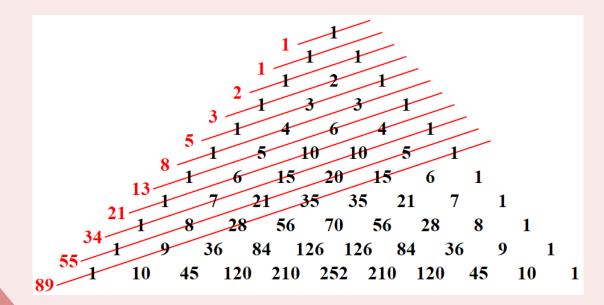
$$\begin{pmatrix} 8 \\ 0 \end{pmatrix} \begin{pmatrix} 8 \\ 1 \end{pmatrix} \begin{pmatrix} 8 \\ 2 \end{pmatrix} \begin{pmatrix} 8 \\ 3 \end{pmatrix} \begin{pmatrix} 8 \\ 4 \end{pmatrix} \begin{pmatrix} 8 \\ 5 \end{pmatrix} \begin{pmatrix} 8 \\ 6 \end{pmatrix} \begin{pmatrix} 8 \\ 7 \end{pmatrix} \begin{pmatrix} 8 \\ 8 \end{pmatrix} \qquad 1 \qquad 8 \qquad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \end{pmatrix} \begin{pmatrix} 1 \\ 5 \end{pmatrix} \begin{pmatrix} 1 \\ 5 \end{pmatrix} \begin{pmatrix} 1 \\ 5 \end{pmatrix} \begin{pmatrix} 1 \\ 6 \end{pmatrix} \begin{pmatrix} 1 \\ 5 \end{pmatrix} \begin{pmatrix} 1$$



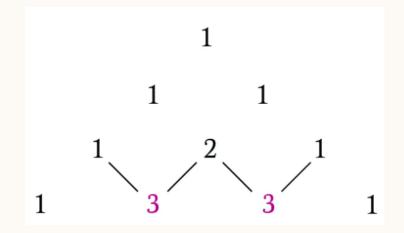
# PASCAL'S TRIANGLE

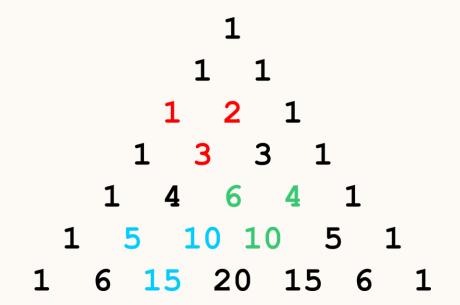
• The relationship between Fibonacci sequence and Pascal's triangle





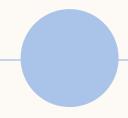
### **BUILDING PASCAL'S TRIANGLE**



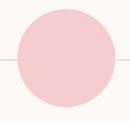




# **APPLICATIONS**



#### **BINOMIAL EXPANSION**



#### **PROBABILITY**

- 1. HHHH
- 2. HHHT, HHTH, HTHH, THHH
- 3. TTHH, HHTT, HTHT, HTTH, THTH
- 4. HTTT, THTT, TTHT, TTTH
- 5. TTTT



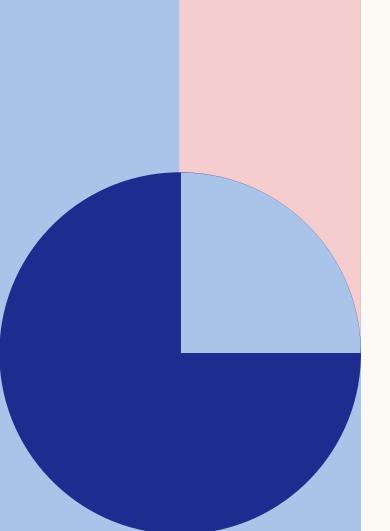
#### **COMBINATIONS**

$$C(n,k) = C_k^n = inom{n}{k} = rac{n!}{k!(n-k)!}$$



# **BINOMIAL EXPANSION**

• Expanding a binomial expression using Pascal's triangle



### **PROBABILITY**

• Pascal's triangle can show how heads and tails can combine

- 1. HHHH
- 2. HHHT, HHTH, HTHH, THHH
- 3. TTHH, HHTT, HTHT, HTTH, THTH
- 4. HTTT, THTT, TTHT, TTTH
- 5. TTTT



### COMBINATIONS

• The number of combinations of n things taken k at a time can be found with this equation

$$C(n,k) = C_k^n = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$



### CONCLUSION

**PASCAL'S IDENTITY** Let n and k be positive integers with  $n \ge k$ . Then  $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}.$ 

**PASCAL'S IDENTITY** 

1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1 1 6 15 20 15 6 1

**PASCAL'S TRIANGLE** 

```
 \begin{array}{lll} (x+y)^0 & 1 \\ (x+y)^1 & 1x+1y \\ (x+y)^2 & 1x^2+2xy+1y^2 \\ (x+y)^3 & 1x^3+3x^2y+3xy^2+1y^3 \\ (x+y)^4 & 1x^4+4x^3y+6x^2y^2+4xy^3+1y^4 \\ (x+y)^5 & 1x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+1y^5 \end{array}
```

**APPLICATIONS** 



### **WORKS CITIED**

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