Heat Transfer - Cubic Interpolations

CS 131, RVCL, March 2018 - Machine Problem 1

User your favorite programming language. Be ready to upload a copy of all source files and a readme (.odt, .docx, .lyx, .tex or .pdf) to UVLe. The readme should contain your name, student number, section and date done in the header. Next is a 1-2 sentence description of the work. Include screenshots of your terminal to show matrices and answers. Remember to address all the discussion questions in your documentation. At the bottom, include a selfie with your program on your screen. As an appendix, include a brief but complete setup guide for running/ compiling in the language you chose. Then describe how to run the program.

Note, I might ask an SA (DCS 3rd year or 4th year) to run and your program, if he/ she can't get it running, you might not get any points. I will personally do the checking. This is normally done after the exam week at the end of the sem.

I estimate you will need around 2-3 weeks to do this given you have other subjects. This is scored over 100 points with bonuses that may increase it to 120. It shall represent 12.5% of your CS 131 grade.

Curve Fitting

1. Perform least squares regression to a cubic polynomial using the data in Table 1.

2. Determine a_0, a_1, a_2 and a_3 using Cholesky LDL^T

3. Solve for the inflection point, which is defined as the thermocline.

4. Plot the temperature versus water depth. You may use a spreadsheet program for plotting.

5. Solve for the gradient (first derivative). The thermocline is at the maximum slope. Use a function based on the Horner's method of solving polynomials.

6. Plot the gradient versus lake depth. You may use a spreadsheet program for plotting.

7. Use the gradient and determine the heat flux across the thermocline in terms of calories per square centimeter per day using Fourier's Law.

Monomial Basis Interpolation

1. Construct a Vandermonde Matrix for cubic single polynomial interpolation for the data in Table 1.

2. Determine a_0, a_1, a_2 , a_3 , a_4 , a_5 , a_6 and a_7 using LU decomposition.

Table 1: Summer temperature vs depth for Platte Lake

z (m)	0	2.3	4.9	9.1	13.7	18.3	22.9	27.2
T ($°C$)	22.8	22.8	22.8	20.6	13.9	11.7	11.1	11.1

3. Solve for the inflection point, using a function based on the Horner's method of solving polynomials.
4. Plot the temperature versus water depth curve. You may use a spreadsheet program for plotting.
5. Solve for the gradient (first derivative) using Horner's method. The thermocline is at the maximum slope.
6. Use the gradient and determine the heat flux across the thermocline in terms of calories per square centimeter per day using Fourier's Law.
7. Plot the temperature gradient versus water depth. You may use a spreadsheet program for plotting.

Piecewise Cubic Polynomial Interpolation

1. Use polynomial cubic splines to model the temperature to depth relationship in a lake in Michigan using the data in Table 1. Assume *clamped condition* for the endpoints (i.e. $dT/dz = 0$).
2. Determine the depth of the thermocline by calling a function based on Horner's method.
3. Solve for the gradient (first derivative) using Horner's method. The thermocline is at the maximum slope.
4. Use the gradient and determine the heat flux across the thermocline in terms of calories per square centimeter per day using Fourier's Law.
5. Plot the water temperature versus lake depth. You may use a spreadsheet program for plotting.
6. Plot the gradient versus lake depth. You may use a spreadsheet program for plotting.

Some Hints

Lakes in temperate regions, can become thermally stratified during the summer. A warm layer of water exists near the surface alled *epilimnion*. A cold band of water remains under called *hypolimnion*. Both laters are connected by a transition boundary called the *thermocline*.

This phenomena is important for environmental engineers and scientists. With temperature stratification, there is also less mixing. Organic decomposition products are more concentrated in the bottom part. Oxygen is significantly more rich in the upper part.

The location of the thermocline is defined as the inflection point of the temperature depth curve.

$$d^2T/dz^2 = 0, \tag{1}$$

where T is the temperature and z is the water depth. This is where the absolute value of the first derivative (temperature gradient) is at the maximum. The temperature gradient or first derivative is important because it is related to the heat flux across the thermocline. Fourier's law give the heat flux, or the amount of calories of energy that goes from the epilimnion to the hypolimnion.

$$J = -\alpha \rho C \frac{dT}{dz} \tag{2}$$

where J is the heat flux $[cal/(cm^2 \cdot s)]$, α is the eddy diffusion coefficient (cm^2/s) , ρ is the density of water $(\approx 1g/cm^3)$, and C is the specific heat $[\approx 1cal/(g \cdot C)]$. For this case, a good assumption for eddy diffusion is $\alpha = 0.01cm^2/s$.

If I invert the axes, the output graphs are expected to look more or less like Figure 1

Documentation Template

```
MP 1 docu
//  <Name> <St #> <section> <date>
Given the data, _________________________,  I will do ___________________,
in order to get the ___________________.
Mathematically, what I'm doing can be expressed as
-------
<use the corresponding variable names in the math as in the code you'll submit>
<use the equation editors... or better yet, use LaTex code>
<make sure the flow of your code corresponds to your math,
which corresponds to your text description of what you did>
<make sure you code file has header information , usage example and comments>
The output of my code will look like
---------------------------
<have pictures here of your output graph with grid, axis labels, title and legend>
---------------------------
Things I learned while doing this MP are
--------------------
<enumerate  1. 2. 3.... >
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Things I thought of while doing the MP (ideas and comments)
```

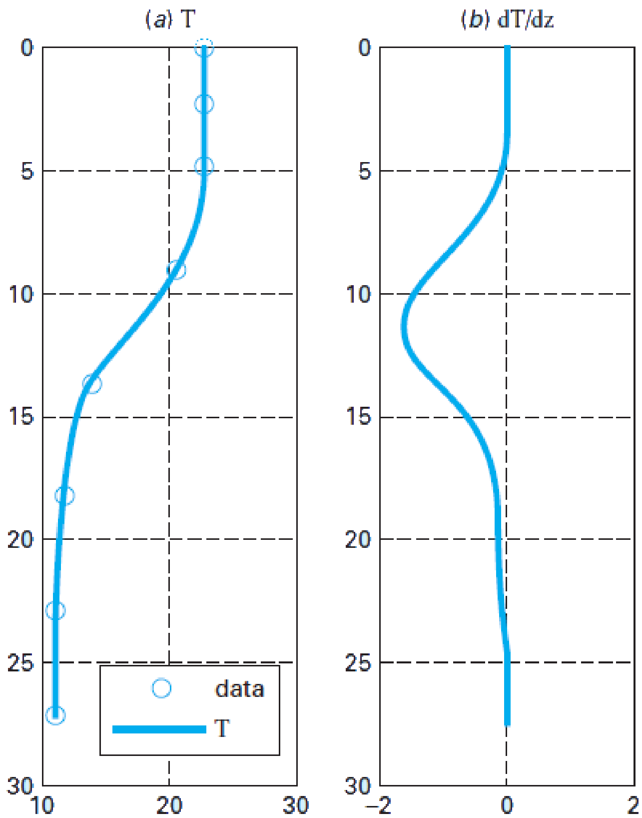

Figure 1: Plots of water depth versus (a) temperature and (b) gradient

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<list/ bullet>
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<lastly include a selfie with your computer screen showing your plot and desktop... smile>
<optionally you may add other things that may get you additional bonus points...>    :)
<the above are the minimum specs for the MP, missing anything means failing the MP...
please re-check everything prior to submission. thanks>
```

Bonus Opportunities

- Have your program do the plotting directly
- Have a graphical interface for your program
- Compare, contrast and discuss the analysis of the techniques used with respect to the problem being solved.
- Have something useful that I hadn't consider to ask for
- Make your submission pretty fun to read/ view.