

Scientific Visualization: Grids

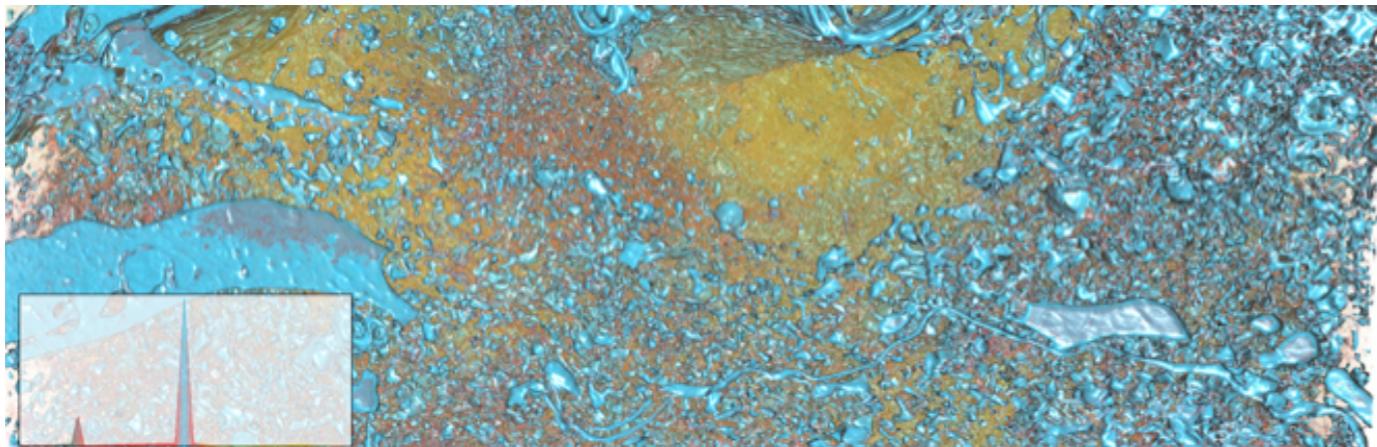
CS 6630, Fall 2015
Aaron Knoll

Recap from Alex's first lecture: the traditional “branches” of visualization

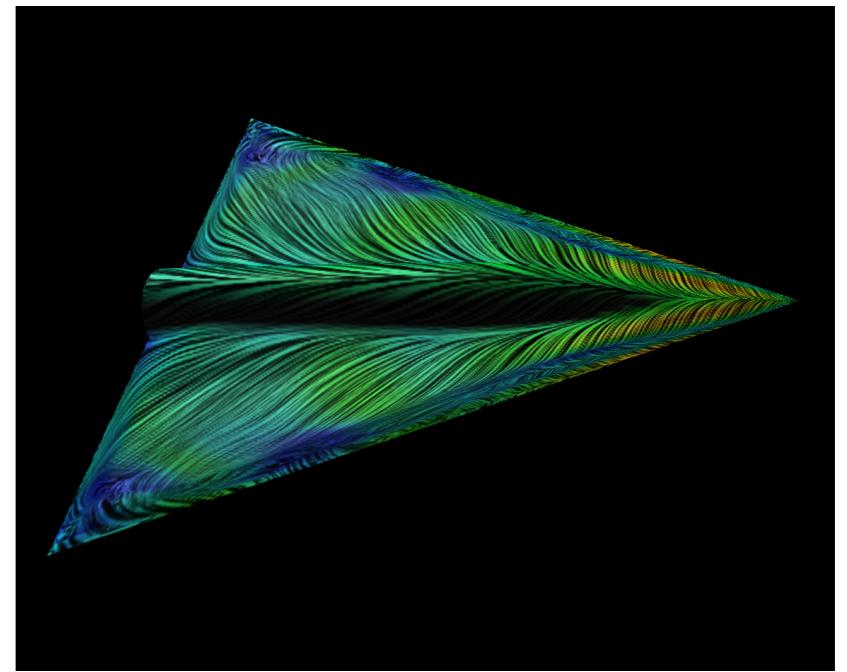
- Scientific Visualization
- Information Visualization
- Visual Analytics

Scientific visualization

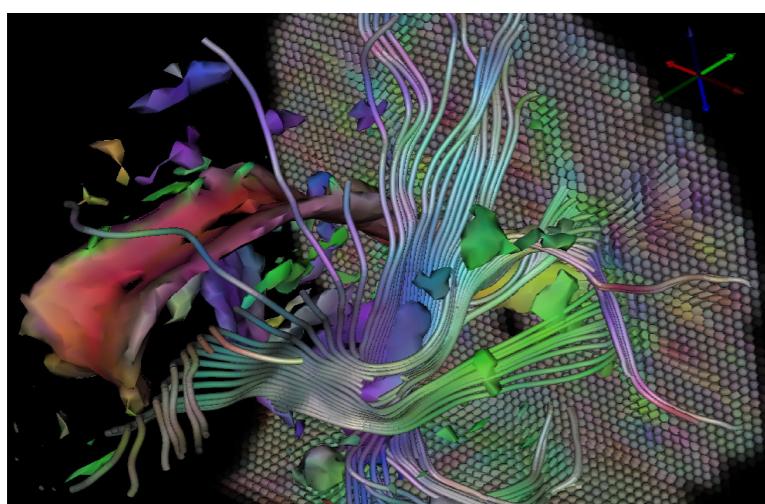
- Data have spatial context (usually from simulation or scan)
- Map spatial quantities to colors or geometry,
 $f(\text{space, time}) \rightarrow \text{rgba}$
- **2D or 3D graphics for visualization.**



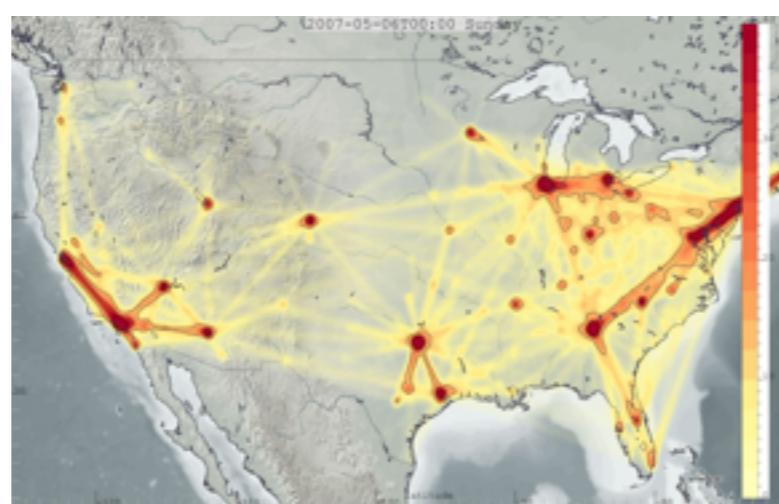
Volume rendering



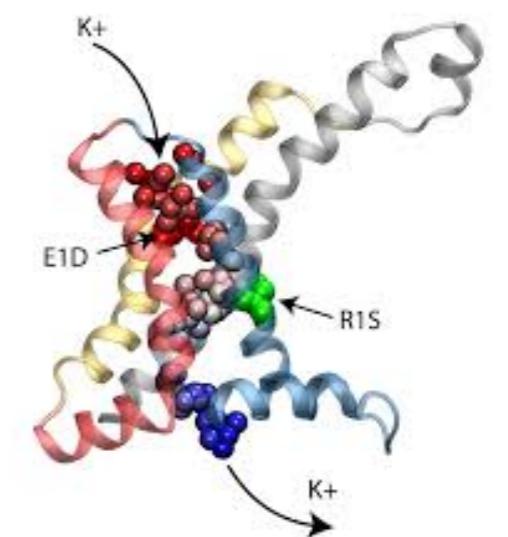
Flow visualization



Tensor field visualization



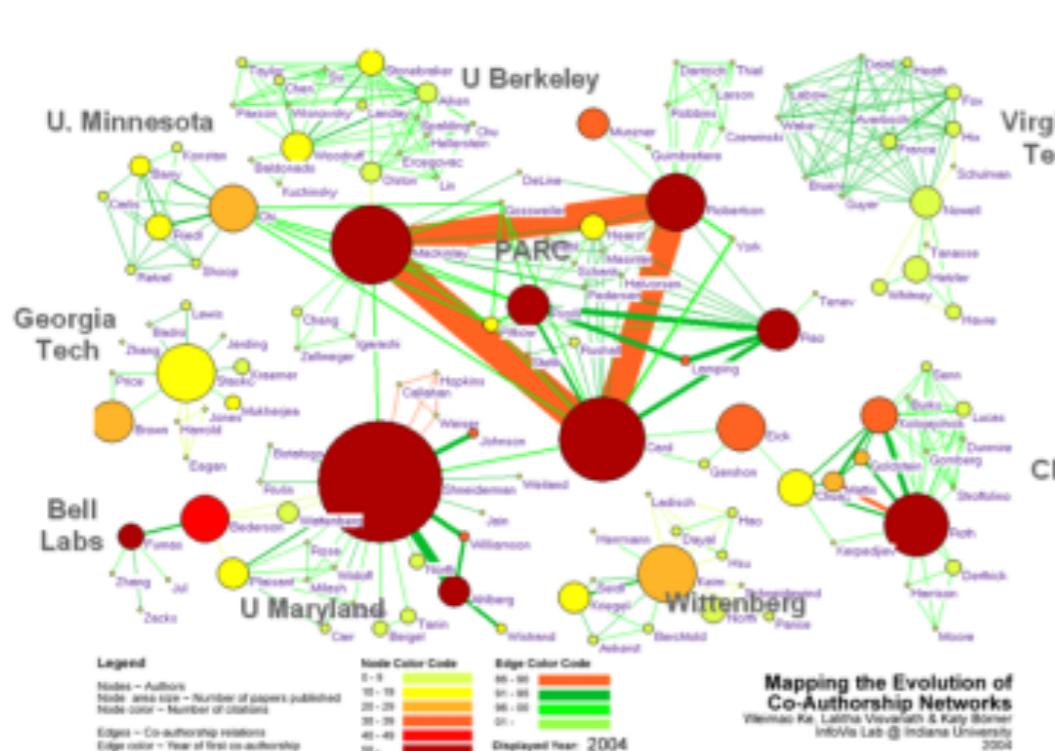
Map and GIS visualization



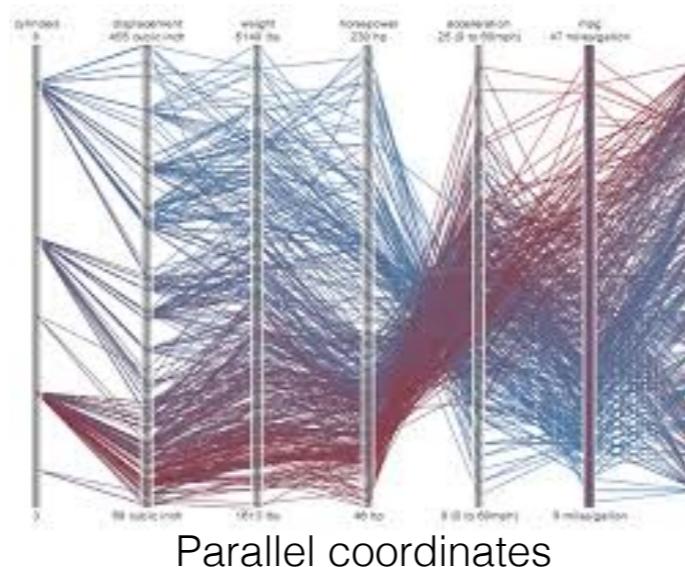
Molecular visualization

Information visualization

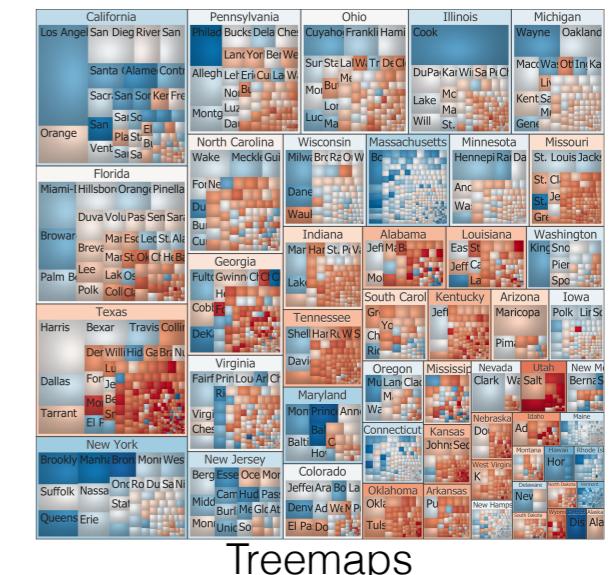
- Spatial position is secondary or non-existent.
- Illustrate relationships between abstract attributes.
- **Plots, charts, graphs, diagrams.**



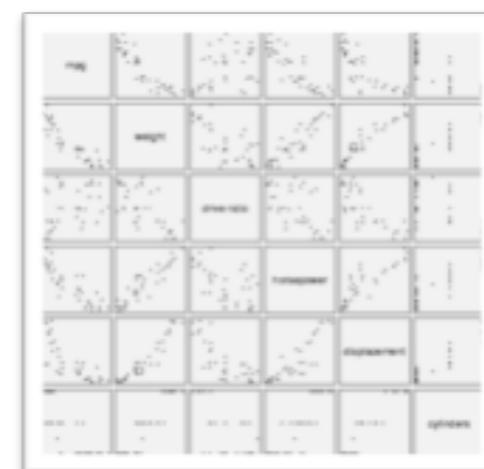
Graph and network visualization



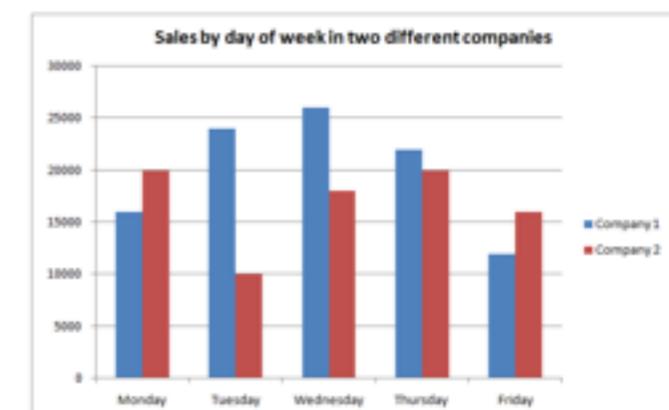
Parallel coordinates



Treemaps



Scatterplots



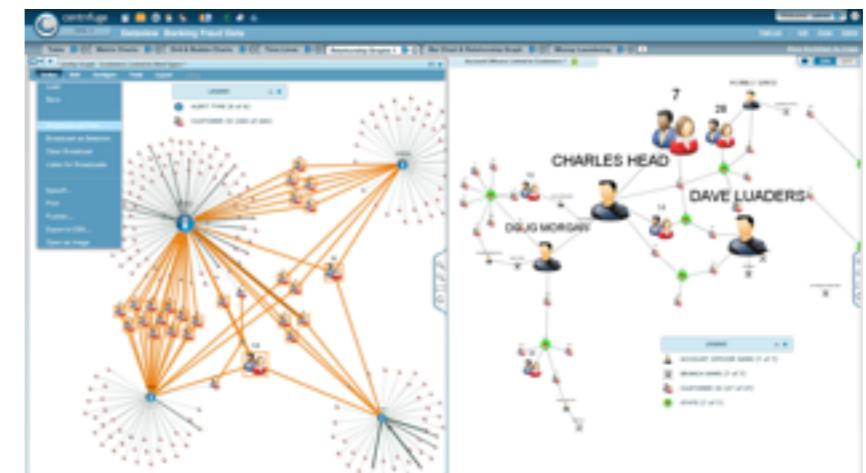
Charts

Visual Analytics

- More about **interactive user interfaces** for data analysis.
- Uses techniques from both scientific visualization and information visualization, as well as statistics, perception, cognition.
- D3+Javascript, R, Matlab
- “Putting it all together”



Management Information Systems (SAS)



Security visualization (Centrifuge)



Genomics (Meyer et al. "Mizbee")



Scientific Visualization

- **Sci-vis is about interpreting and rendering spatial data.**
- Today:
 - where do spatial data come from?
 - what do they look like?
 - what can we do with them?
 - *HW6 - your very own volume renderer, in a web browser!*
- Thurs, Oct 22: Volumes
Tues, Nov 10: Isosurfaces
Thurs, Nov 12: Vector and Tensor Fields

Scientific Visualization

- Data sources
- Data representation
 - fields
 - grids
- Data interpretation
 - The scientific visualization pipeline
 - Interpolation

Data sources

Computational Data

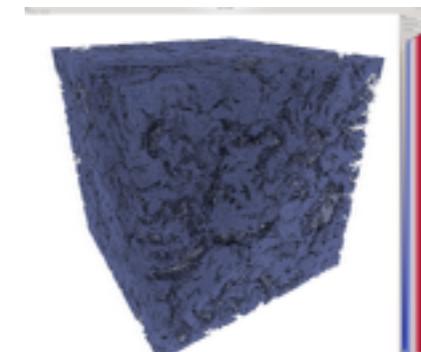
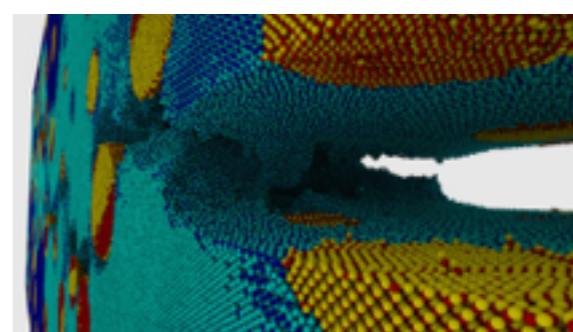
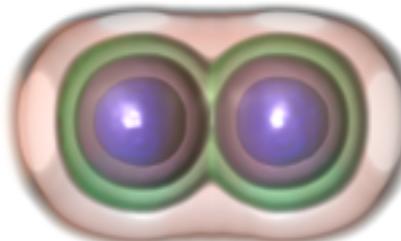
The output of scientific computing:

physics, chemistry, blood flow,
neurophysiology, meteorology,
climatology, astronomy...

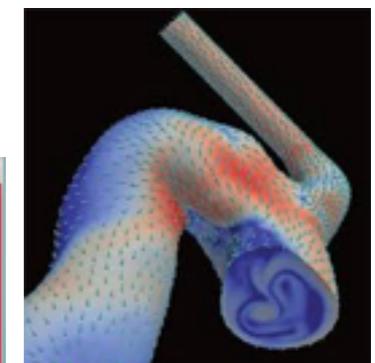


- Nuclear physics
- Quantum chemistry
- Molecular dynamics
- Computational fluid dynamics
- Rigid-body and structural mechanics
- Coarse-grained dynamics, agents simulations
- Meteorology
- Astrophysics
- Cosmology

kilobytes



petabytes



Scanned data

The output of instruments in medical imaging, microscopy, telescropy, GIS

- X-ray crystallography
- Synchrotron / radiation light sources
- Transmission electron microscopy
- Confocal microscopy
- Camera imagery
- Ultrasound
- Magnetic resonance imaging
- X-ray tomography
- Satellite
- Telescope

Angstroms



Megaparsecs



Data representation

Fields

- Mathematically, a *field* is a set of elements with addition, multiplication operators that satisfy the field axioms

name	addition	multiplication
associativity	$(a + b) + c = a + (b + c)$	$(a b) c = a (b c)$
commutativity	$a + b = b + a$	$a b = b a$
distributivity	$a (b + c) = a b + a c$	$(a + b) c = a c + b c$
identity	$a + 0 = a = 0 + a$	$a \cdot 1 = a = 1 \cdot a$
inverses	$a + (-a) = 0 = (-a) + a$	$a a^{-1} = 1 = a^{-1} a \text{ if } a \neq 0$

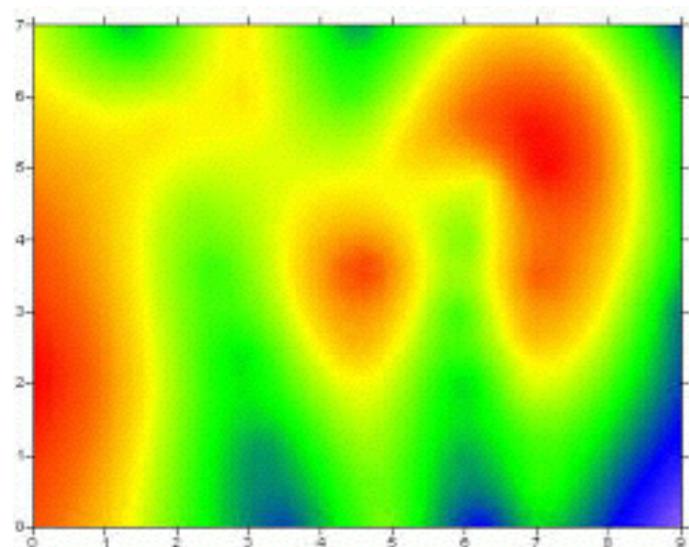
wolfram.com

- Intuitively, a field is a varying quantity defined continuously over space.

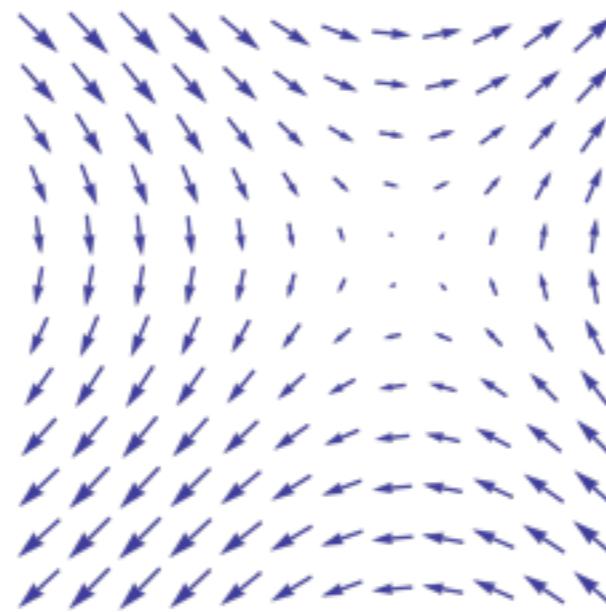
Fields

with a 2D domain

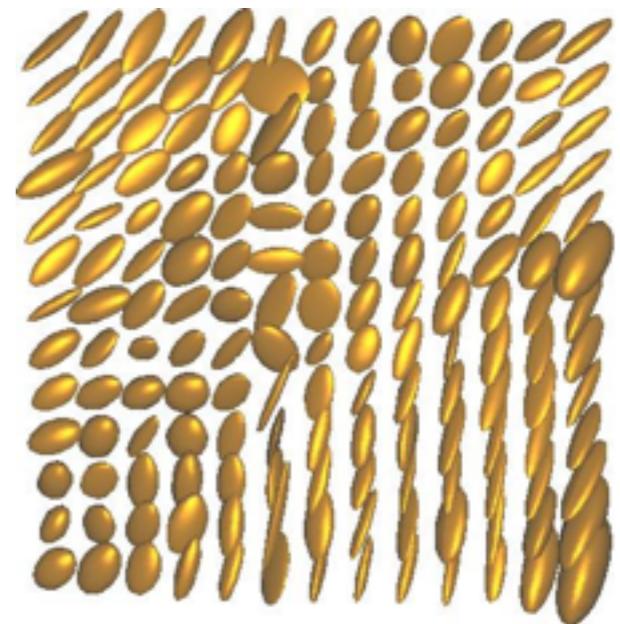
scalar field



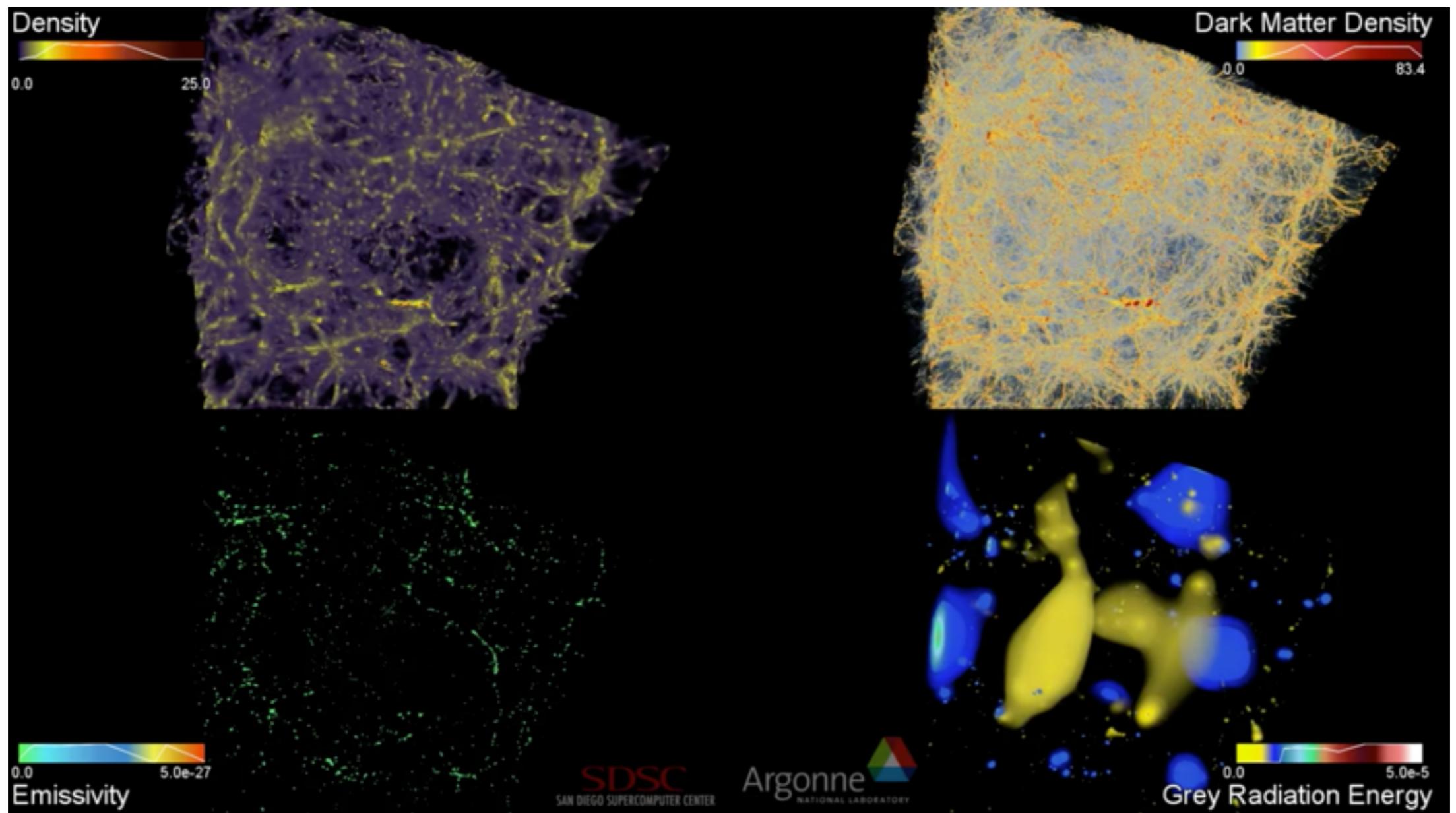
vector field



tensor field

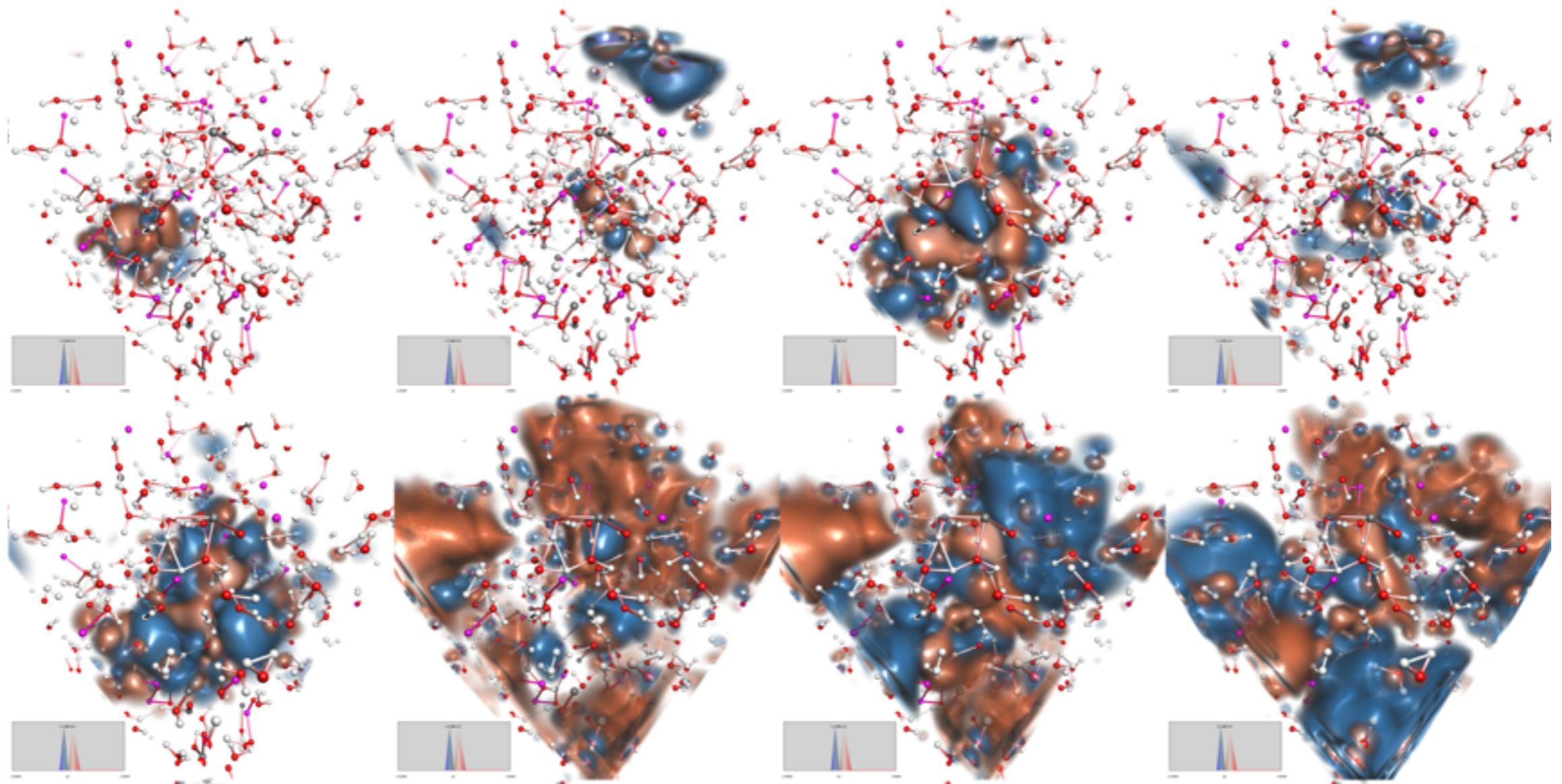


Multifields



Radiation hydrodynamics in Enzo: Joe Insley (ANL), Rick Wagner (SDSC)
<https://vimeo.com/17771397>

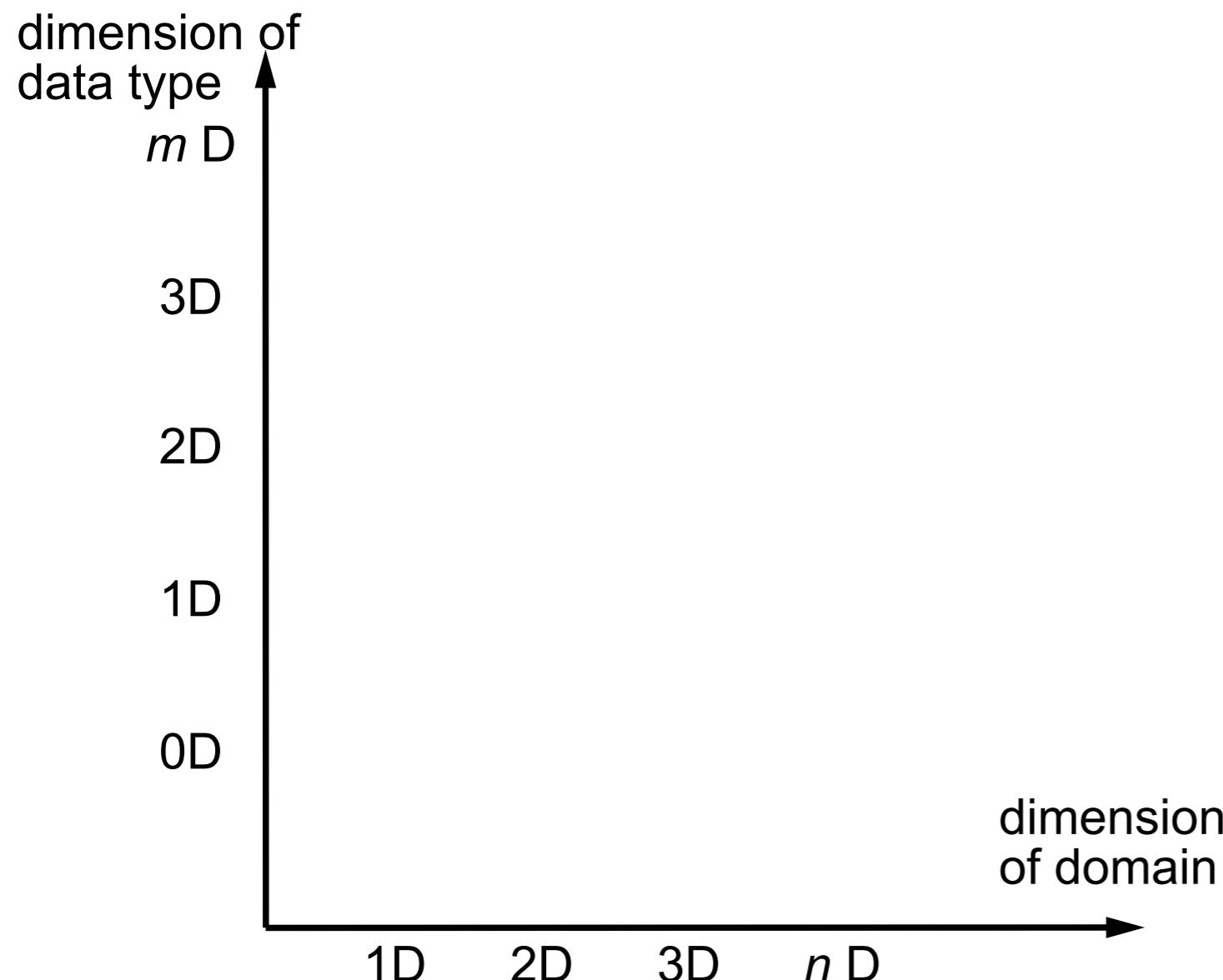
Multifields



8 molecular orbitals of a LiAlH₂O DFT simulation, courtesy Aiichiro Nakano, University of Southern California

Types and Classification of Field Data

- dimension of domain (the field)
- dimension of the data to visualize (the geometry)

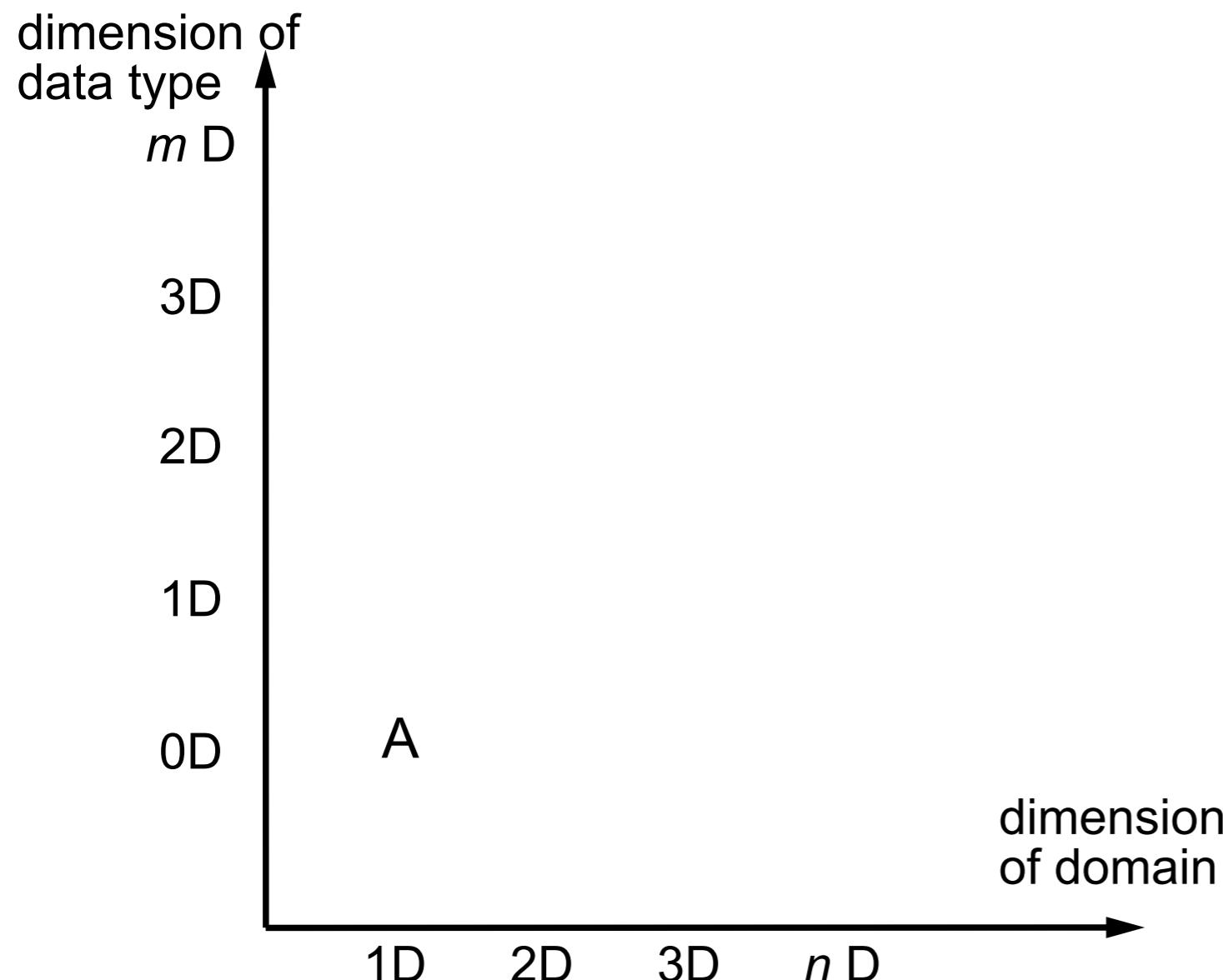


Examples:

- A: gas station along a road
- B: map of cholera in London
- C: temperature along a rod
- D: height field of a continent
- E: 2D air flow
- F: 3D air flow in the atmosphere
- G: stress tensor in a mechanical part
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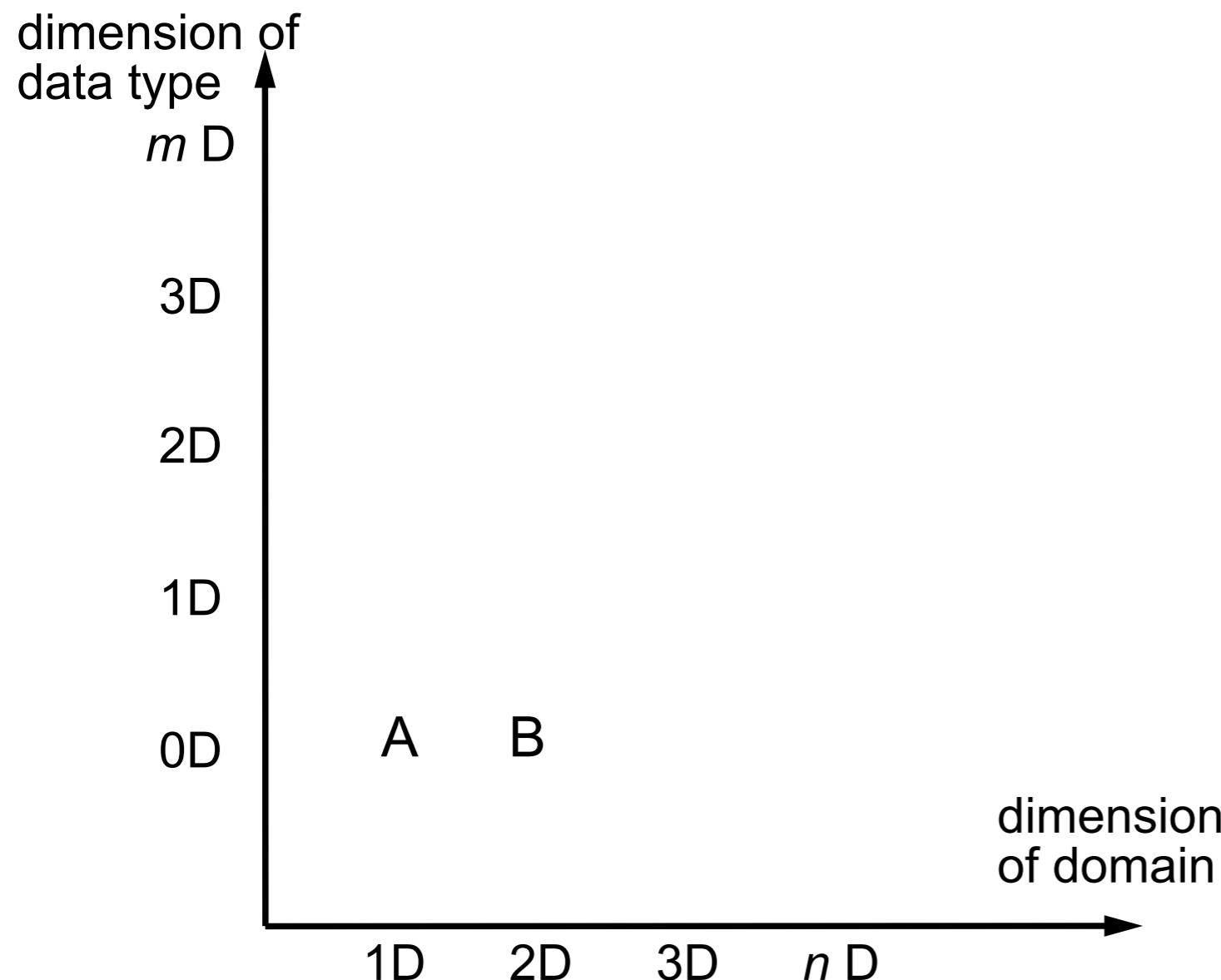


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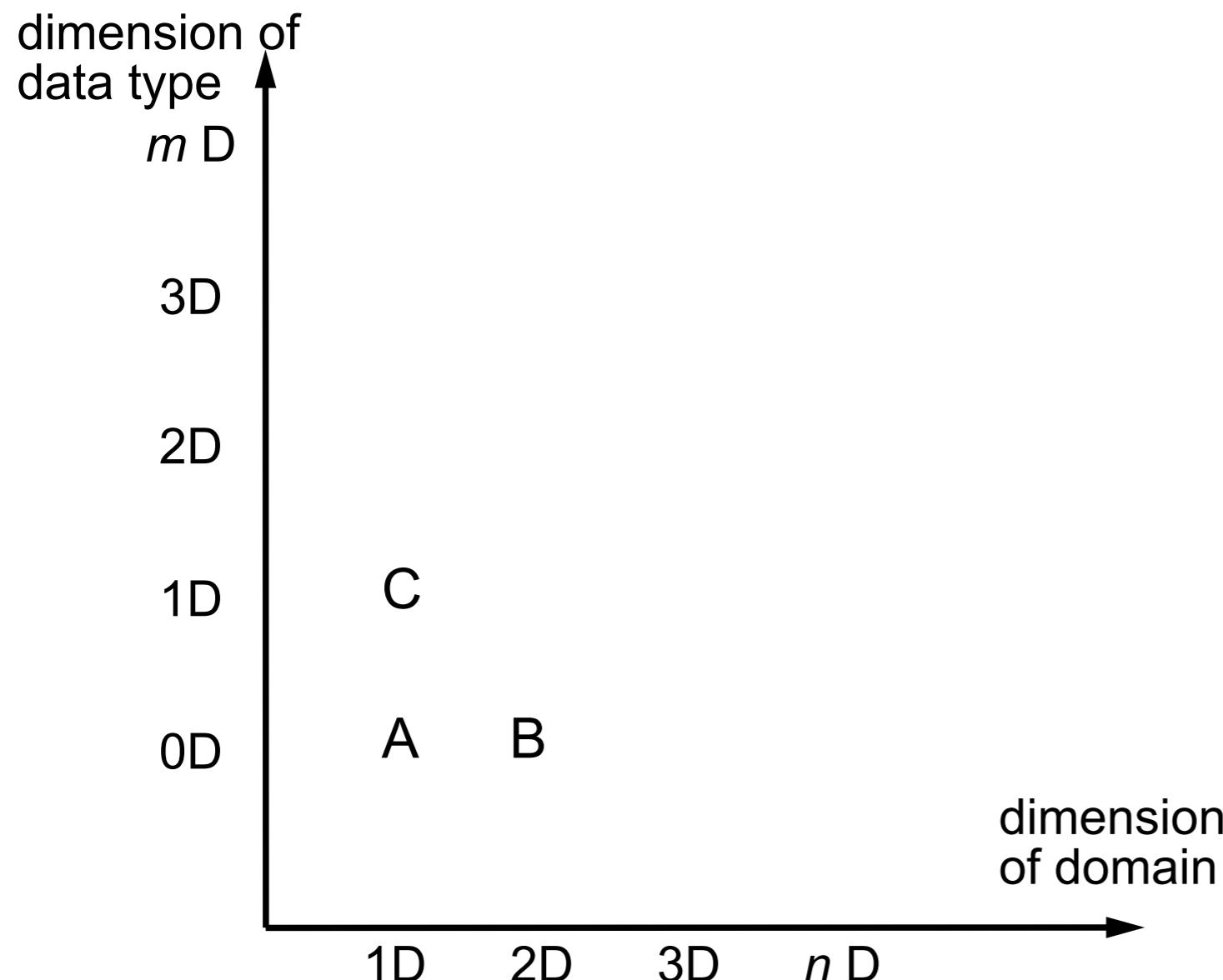


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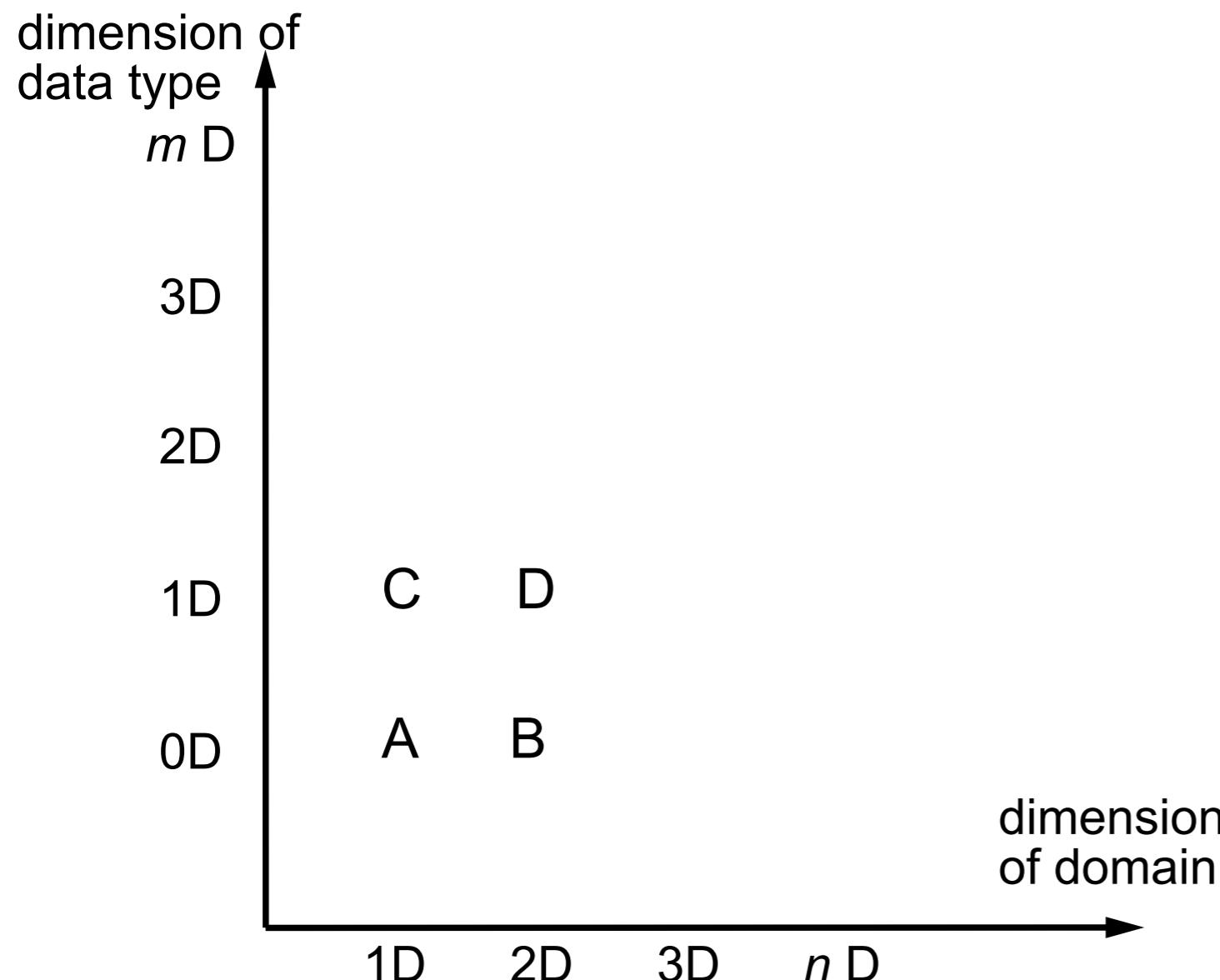


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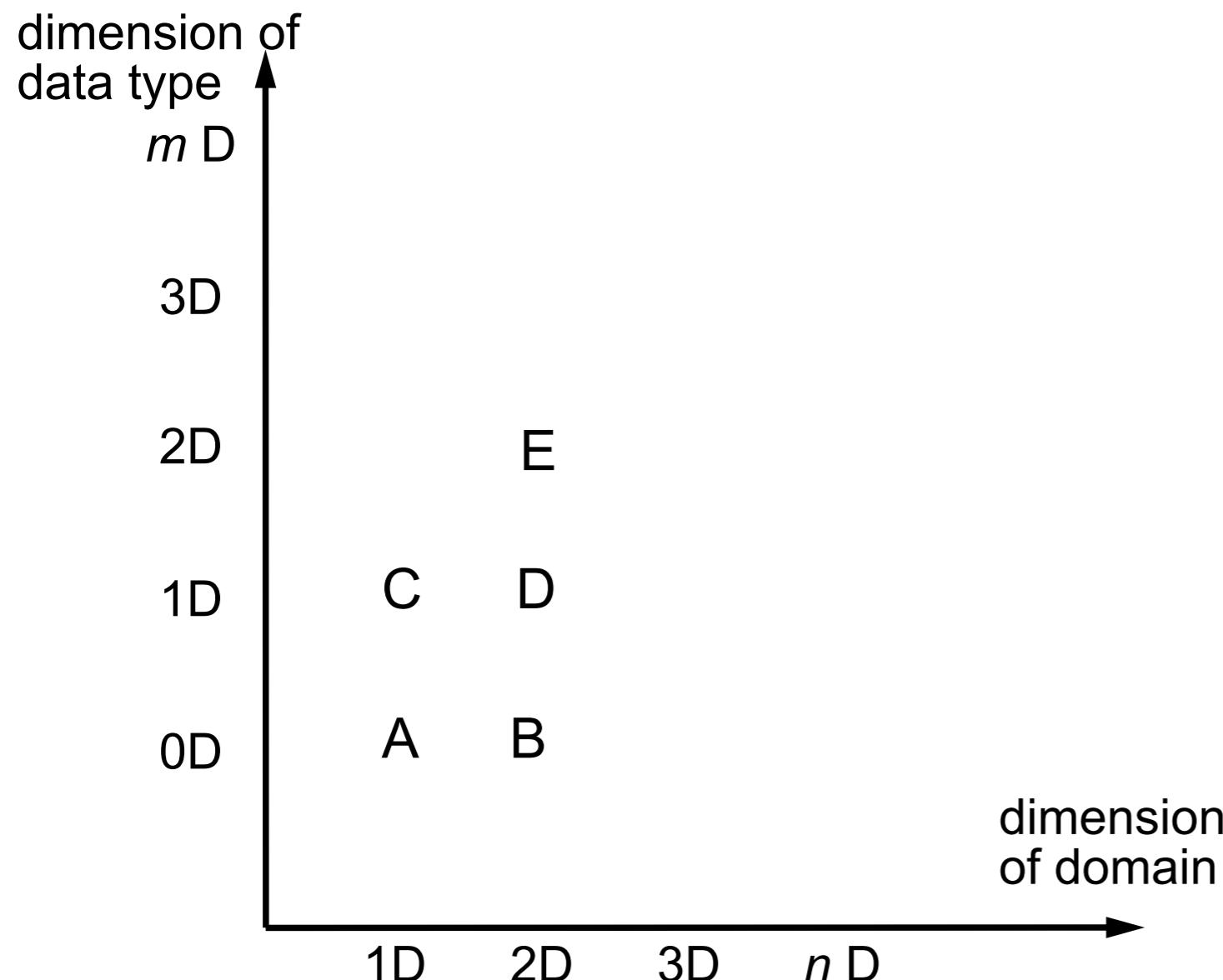


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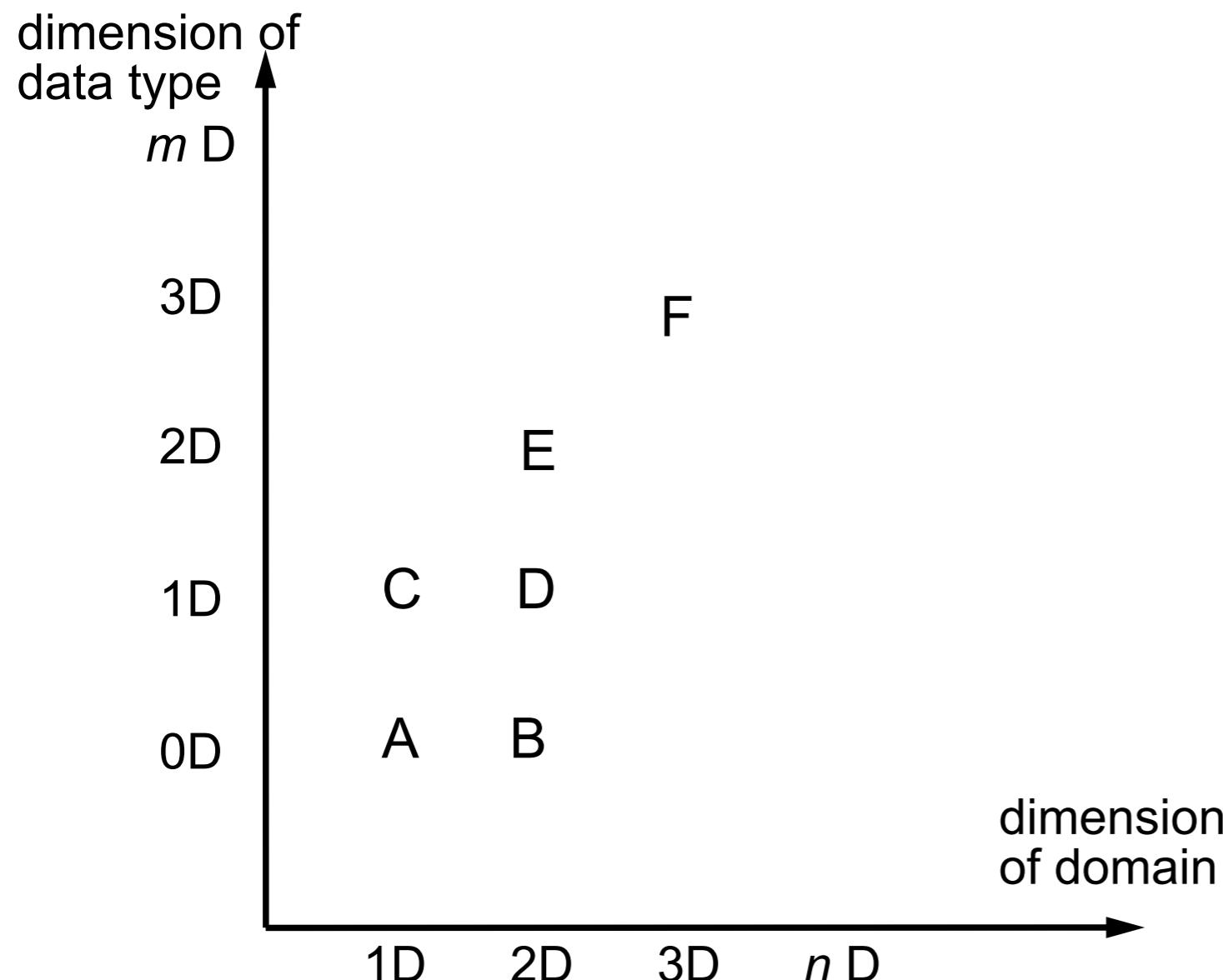


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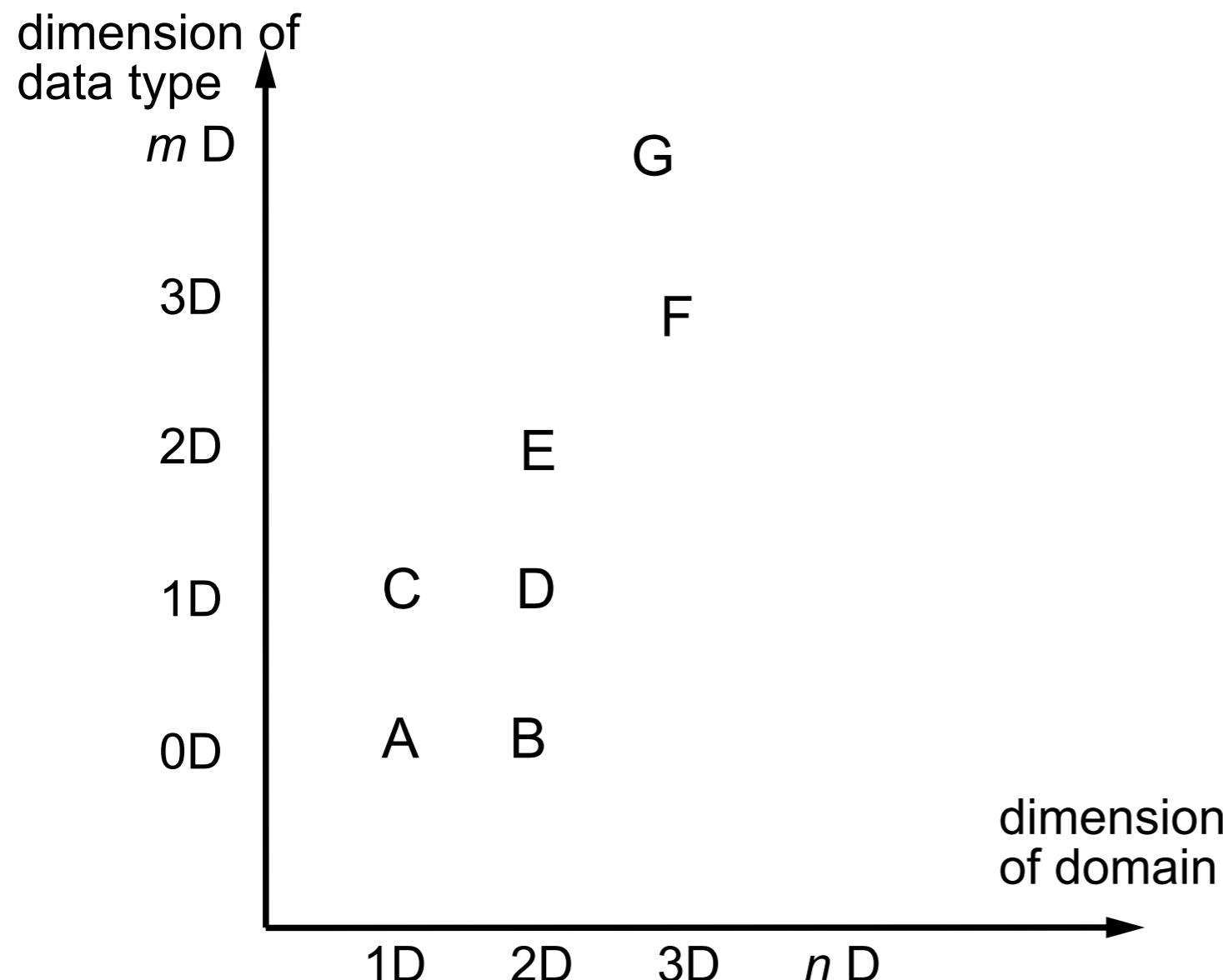


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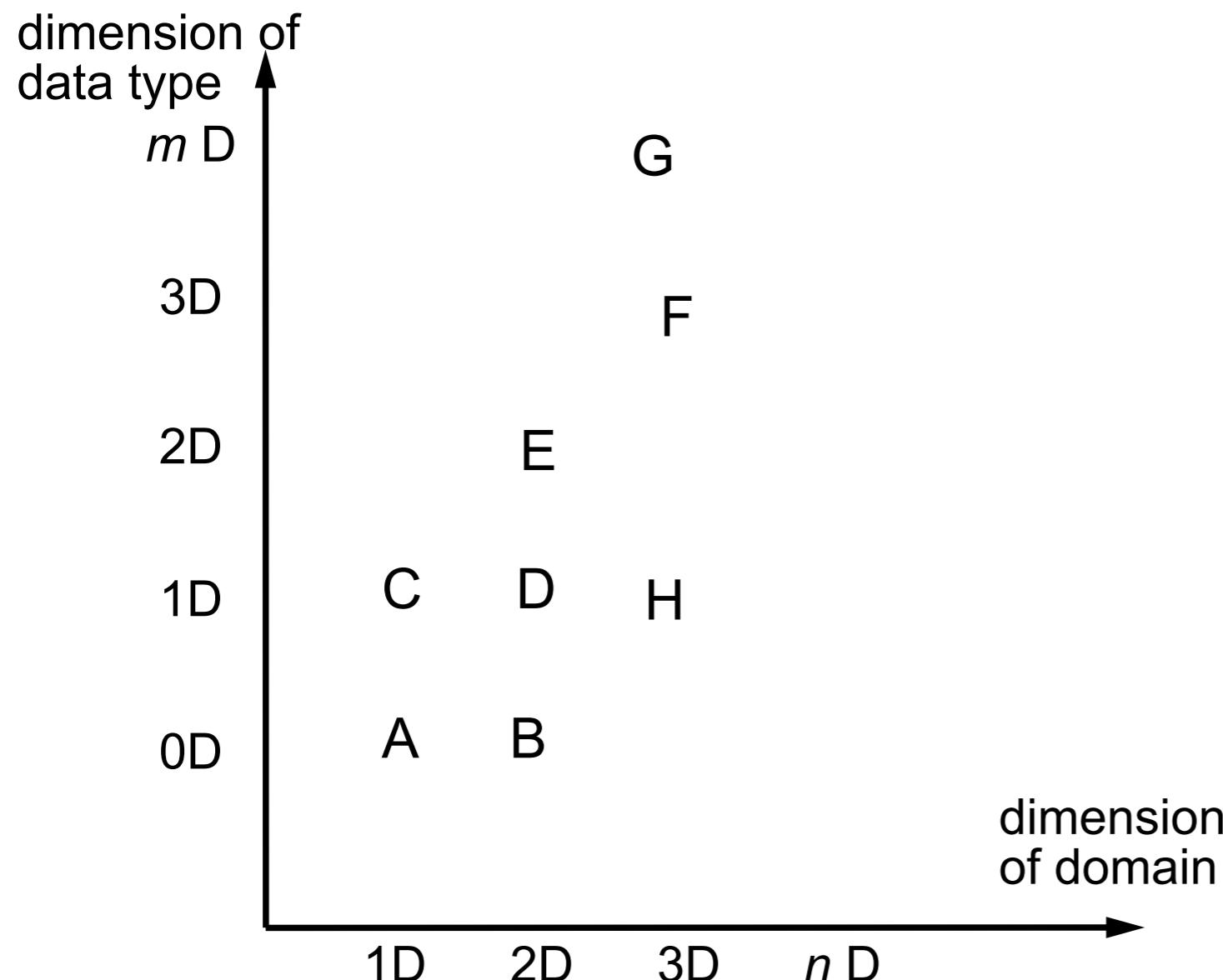


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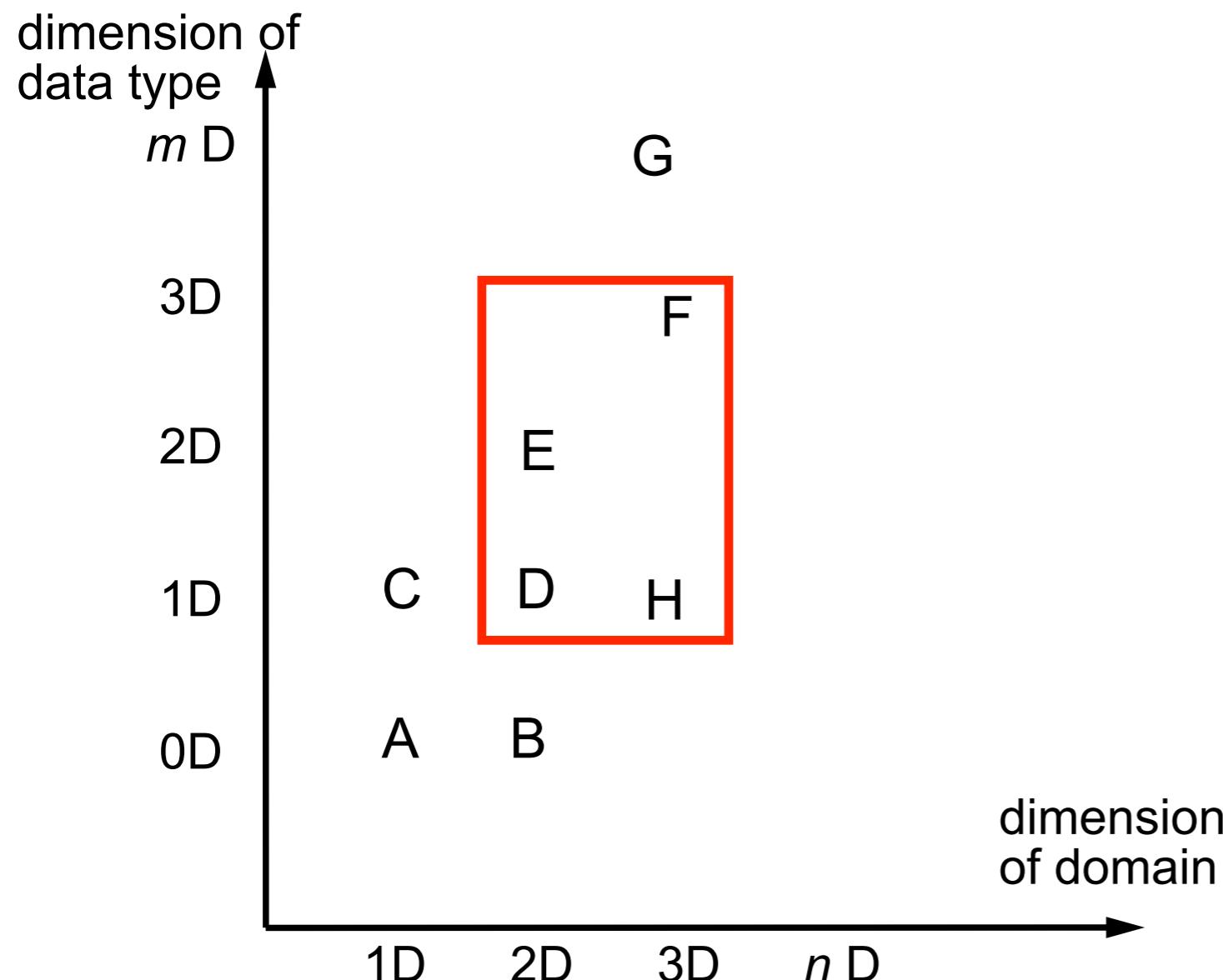


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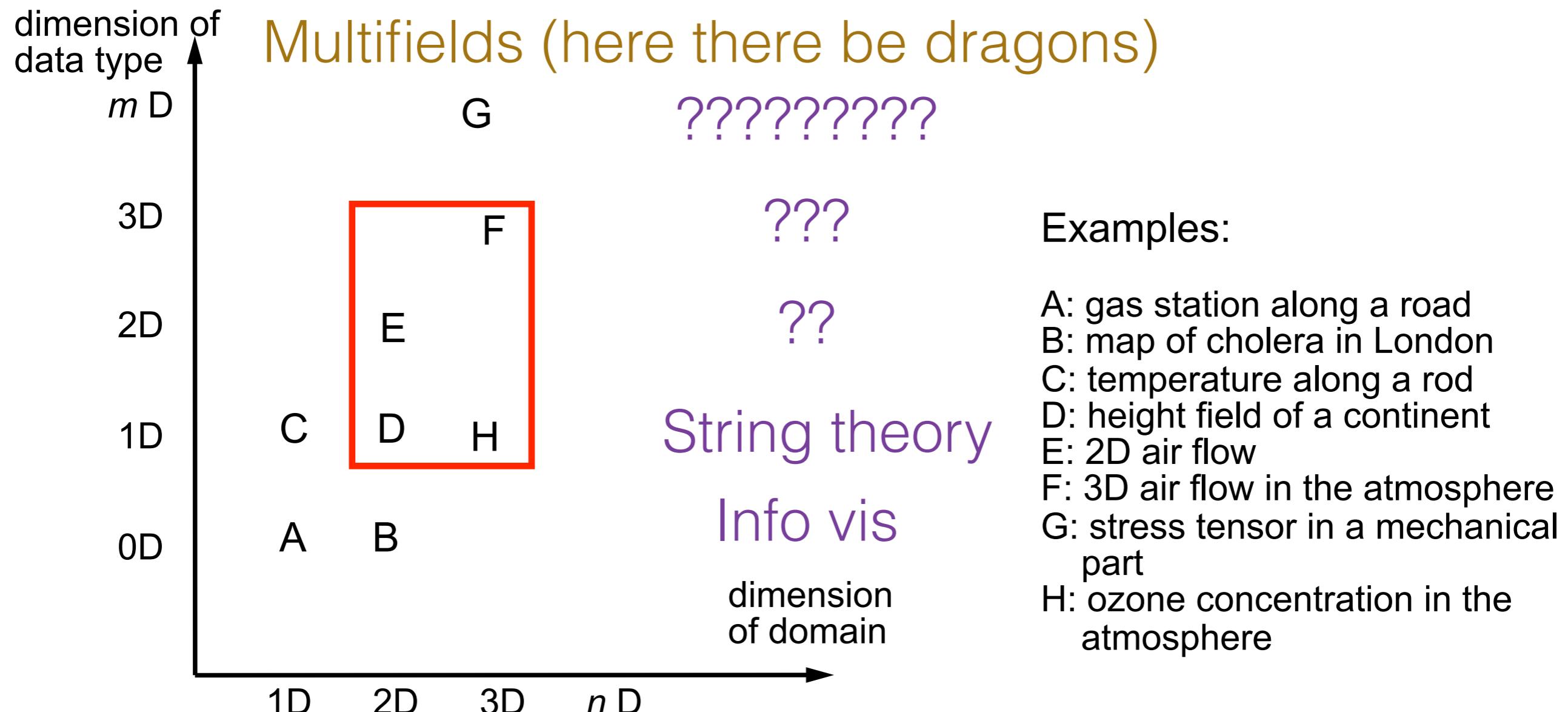


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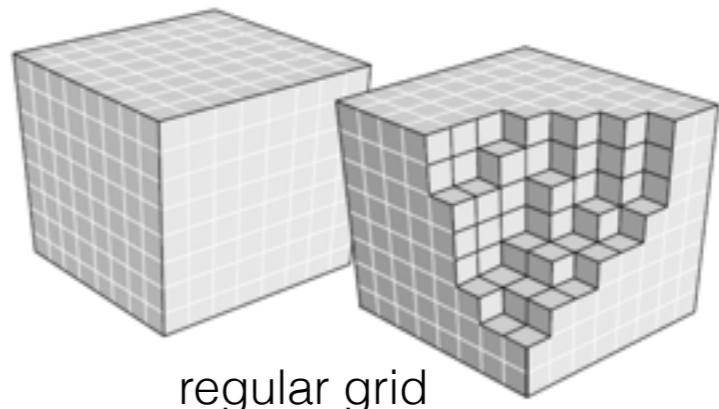
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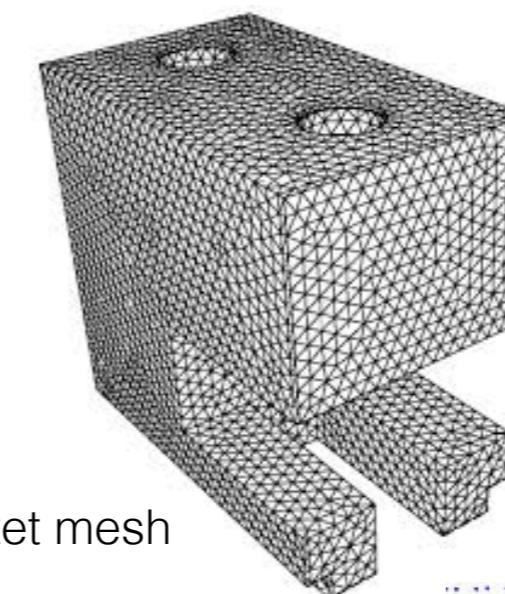


Grids

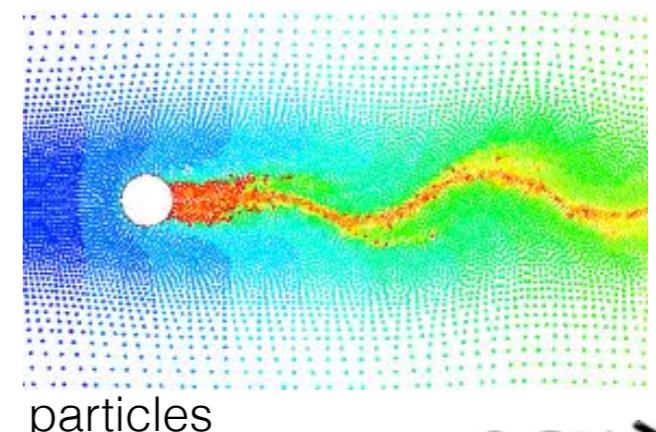
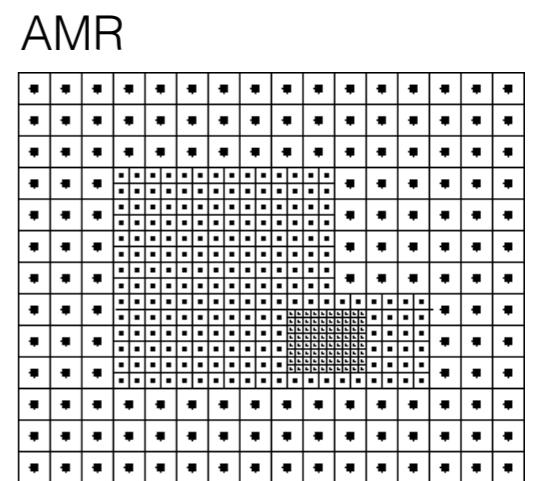
- Continuous fields are an illusion
- All data are discrete
- Meshes are chosen based on what is computationally efficient for the
- Visualization software must implement data models to handle a wide range of field and non-field data
 - (e.g., VTK)



regular grid



tet mesh



particles

Structured vs Unstructured

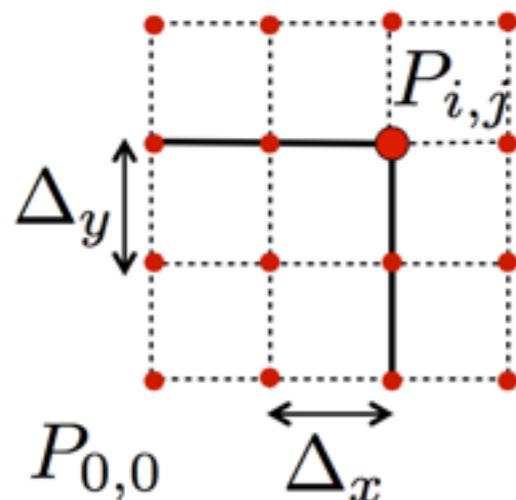
- In general, from the relational database world:
 - **Structured data** are data that are indexed, and can be accessed via a hash, array, or other query.
 - I.e., search time $O(1)$ or $O(\log N)$.
 - **Unstructured data** are not indexed — you have to brute-force search to find them.
 - I.e. search time $O(N)$
- In information/data visualization:
 - **structured** means data you've already indexed, organized (for example, in D3).
 - **unstructured** is everything else (i.e. text, imagery, video, foo) you have to search through.
- In scientific visualization, this can get a bit confusing...
- First we need to differentiate between geometry and topology.

Geometry vs Topology

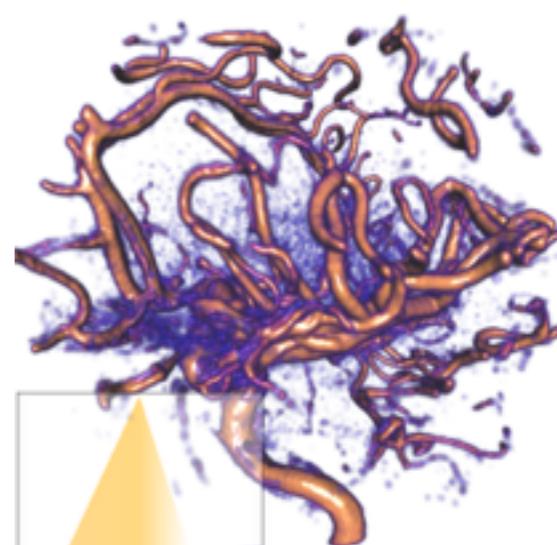
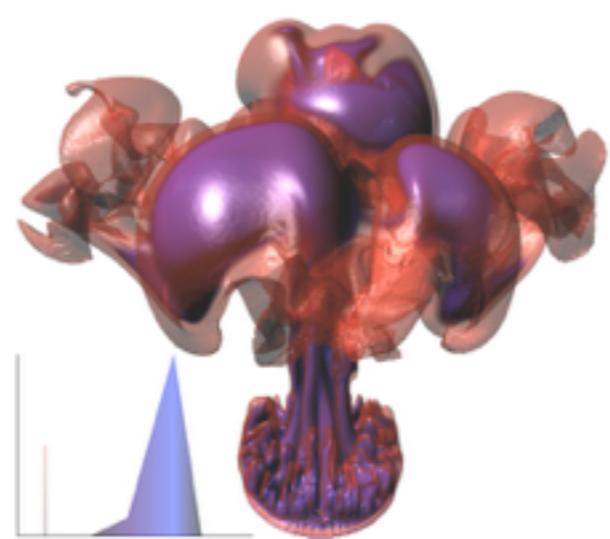
- Geometry
 - Position of vertices in Euclidean space
 - Can be uniform, structured or unstructured.
- Topology
 - Defines the “cells”, or connectivity of the vertices.
 - Can also be structured or unstructured.

Uniform grid geometry

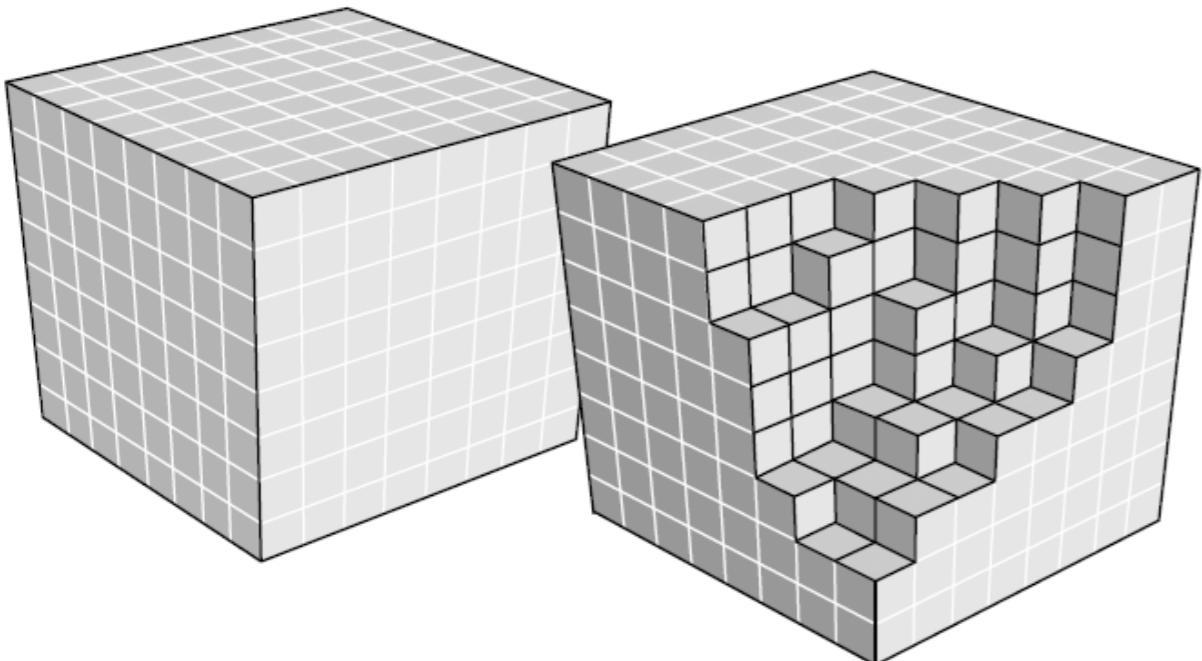
- Uniform spacing along the axes, also known as “raster data”.
- Most volume data look like this; structured data usually means this.
- You still need metadata to know the size of the axes!



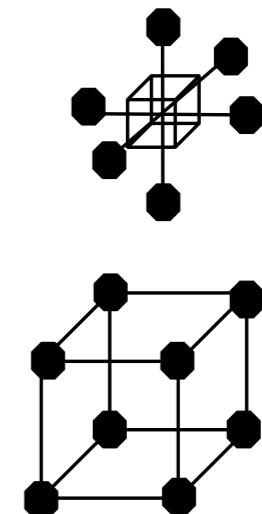
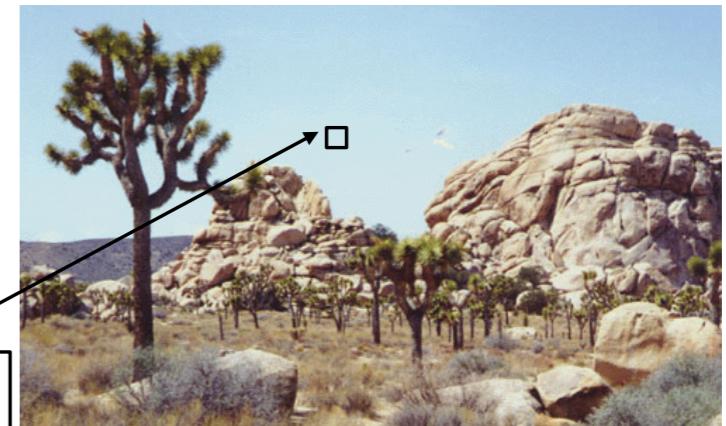
$$P_{i,j,k} = P_{0,0} + i\Delta_x \vec{e}_x + j\Delta_y \vec{e}_y$$



- Representation of scalar 3D data set $\Omega \in R^3 \rightarrow R$
- Analogy: pixel (picture element) 
- Voxel (volume element), with two interpretations:
 - Values between grid points are resampled by interpolation



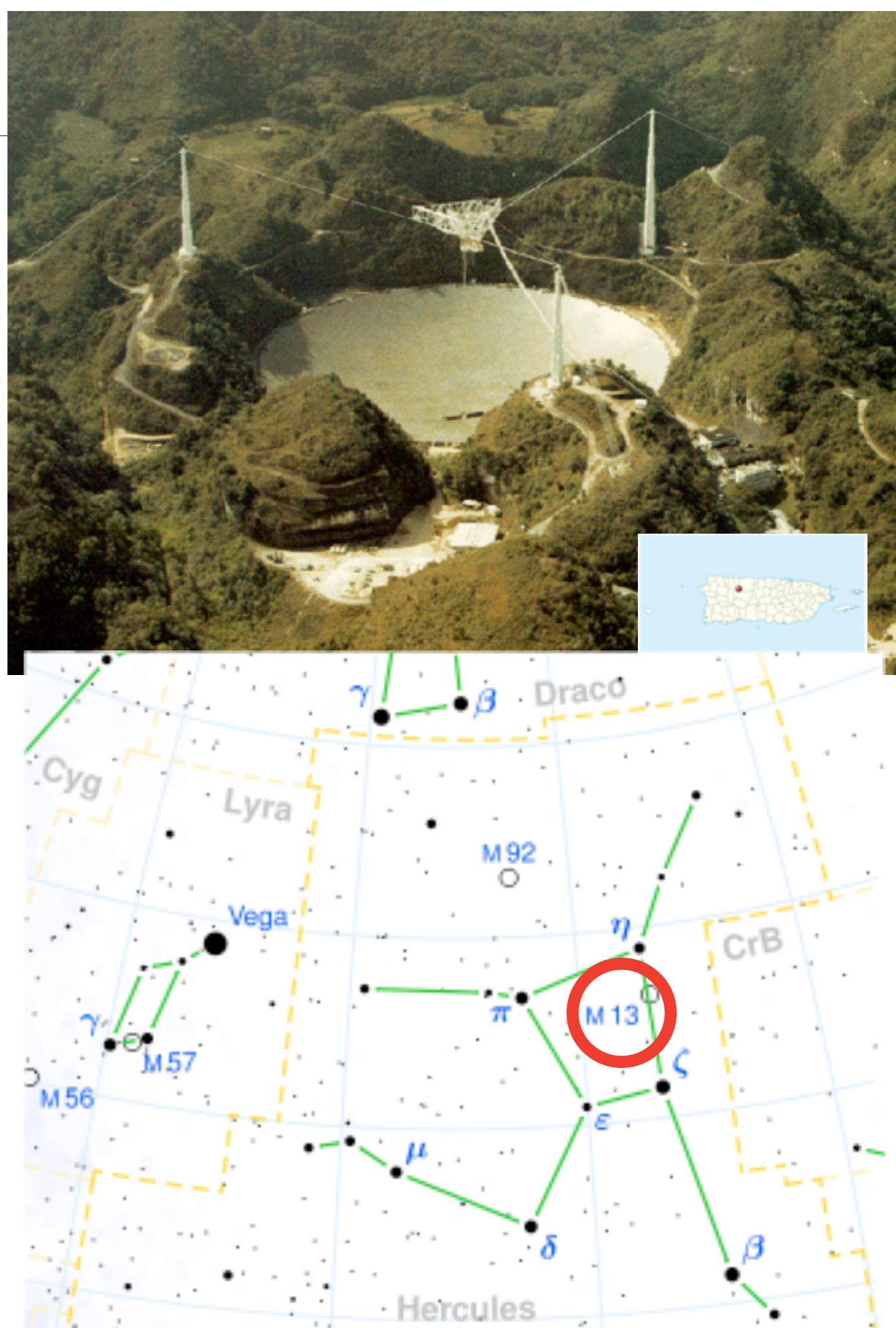
- Collection of voxels
- Uniform grid



Arecibo Message

http://en.wikipedia.org/wiki/Arecibo_message

- Way of understanding mechanics of raster image representation
- Radio telescope in Puerto Rico
- built in 1964, renovated in 1974
- To celebrate: Frank Drake and Carl Sagan (Cornell University) sent message to M13 in Hercules (25,000 light years away)
- 1679 bits, frequency modulate 2380 MHz



The Message

<http://www.physics.utah.edu/~cassiday/p1080/lec06.html>

1679 bits were encoded as 2380MHz plus and minus some frequency

```
000000101010100000000000101000010100000010010001000100010010010110010101  
0101010101010010010000000000000000000000000000000000000000000000000000000000  
000011010000000000000000000000000000000000000000000000000000000000000000000000  
111110000000000000000000000000000000000000000000000000000000000000000000000000  
001100100001101000110001100001101011110111101111000000000000000000000000000000  
00000000000100000000000000000000000000000000000000000000000000000000000000000000  
00001111100000000000000000000000000000000000000000000000000000000000000000000000  
00100000001000000000000000000000000000000000000000000000000000000000000000000000  
00000000000000000000000000000000000000000000000000000000000000000000000000000000  
10000011000000000000000000000000000000000000000000000000000000000000000000000000  
00001000000001000000000000000000000000000000000000000000000000000000000000000000  
01000011000000000000000000000000000000000000000000000000000000000000000000000000  
00000100000000100000000000000000000000000000000000000000000000000000000000000000  
00001000100000000000000000000000000000000000000000000000000000000000000000000000  
00000000110000000000000000000000000000000000000000000000000000000000000000000000  
00000000110000000000000000000000000000000000000000000000000000000000000000000000  
00010000011111000001100000000000000000000000000000000000000000000000000000000000  
11100001110000011011000000000000000000000000000000000000000000000000000000000000  
00001010000110000001000000000000000000000000000000000000000000000000000000000000  
00100000000000000000000000000000000000000000000000000000000000000000000000000000  
00010100000000000000000000000000000000000000000000000000000000000000000000000000  
11100000000000000000000000000000000000000000000000000000000000000000000000000000  
01100001000101000001010001000000000000000000000000000000000000000000000000000000  
00000000100001000010000000000000000000000000000000000000000000000000000000000000  
0111100111101001111000
```

This is a **1-D** sequence of bits in time
How will an alien understand this list of bits?
(will have different symbols than “0” “1”)
No meta-information!

Understanding the message

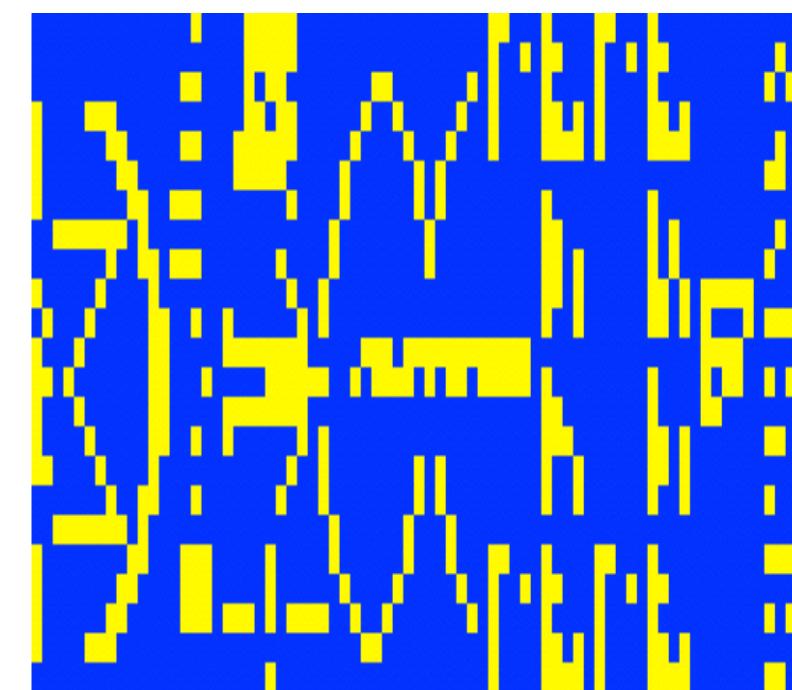
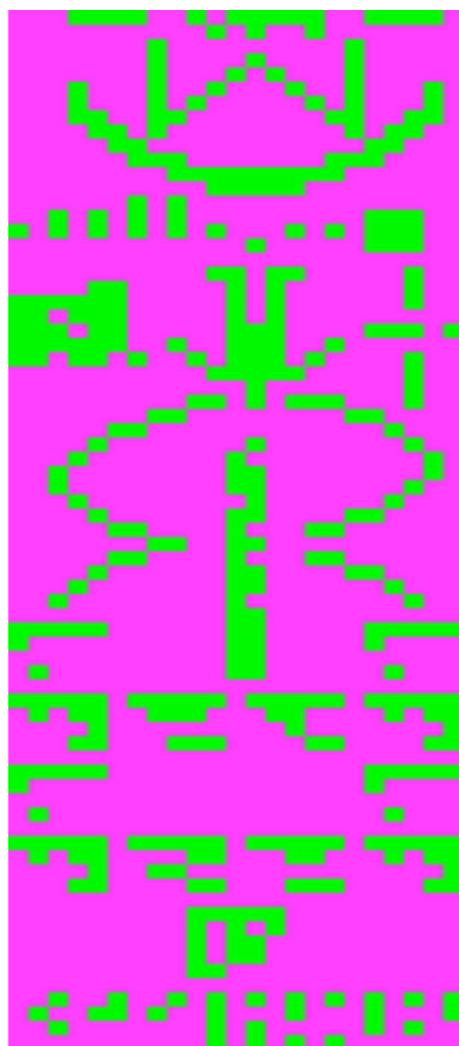
- Perhaps some “visual” representation of bits



- (what is black vs white?)
- Aliens notice $1679 = 23 \times 73$ (product of two primes)
- Perhaps its not a linear sequence: 2-D array
- Two ways of sequencing values in 2D array
- Various ways of laying them out in 2D space
- Then: have to decipher it!



73 x 23



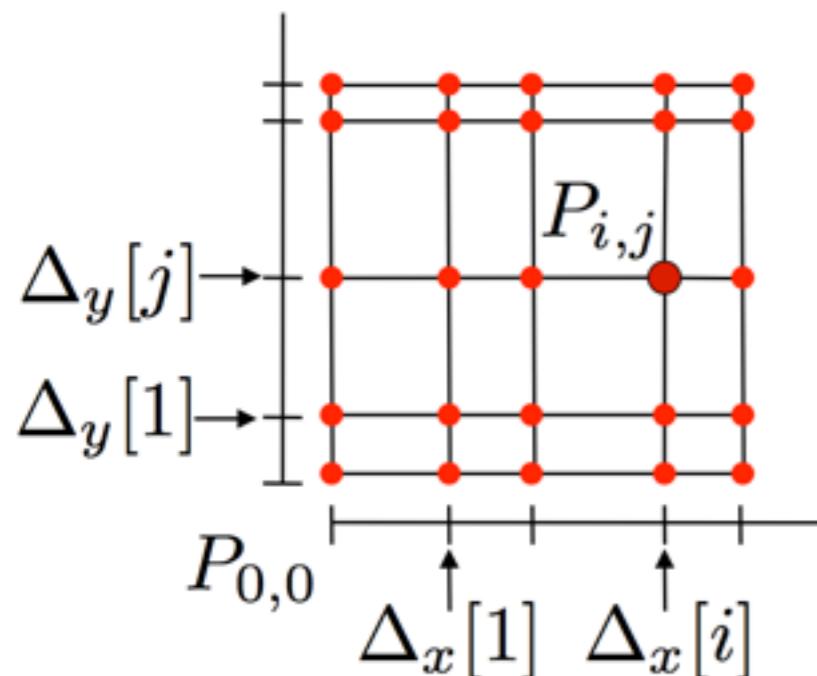
compare to:
http://en.wikipedia.org/wiki/Arecibo_message

23 x 73: what was different?

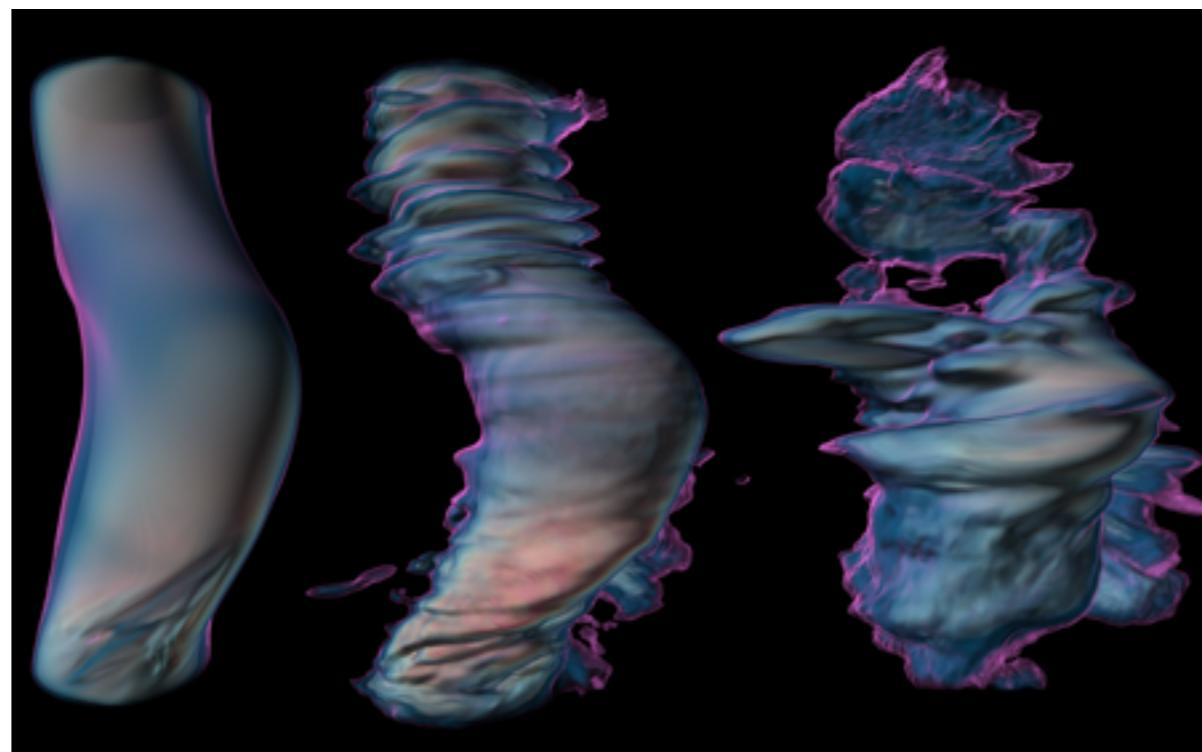


Structured (rectilinear) grid geometry

- Still structured, but with non-uniform spacing along the axes.
- Positions can still be computed procedurally
- Some meteorology, climate CFD data like this.



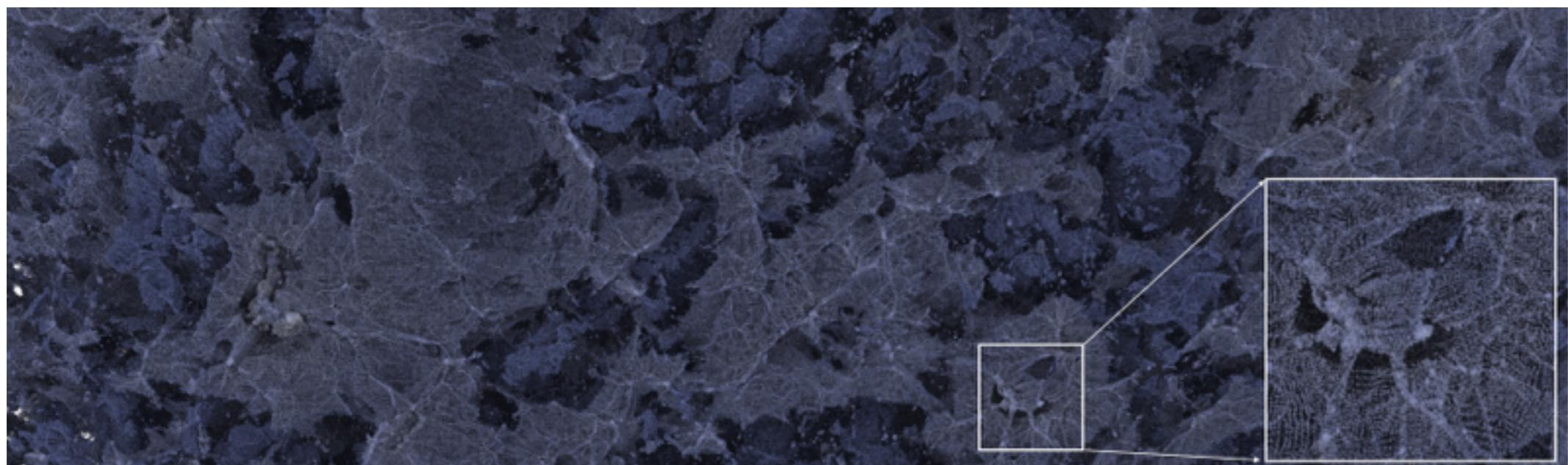
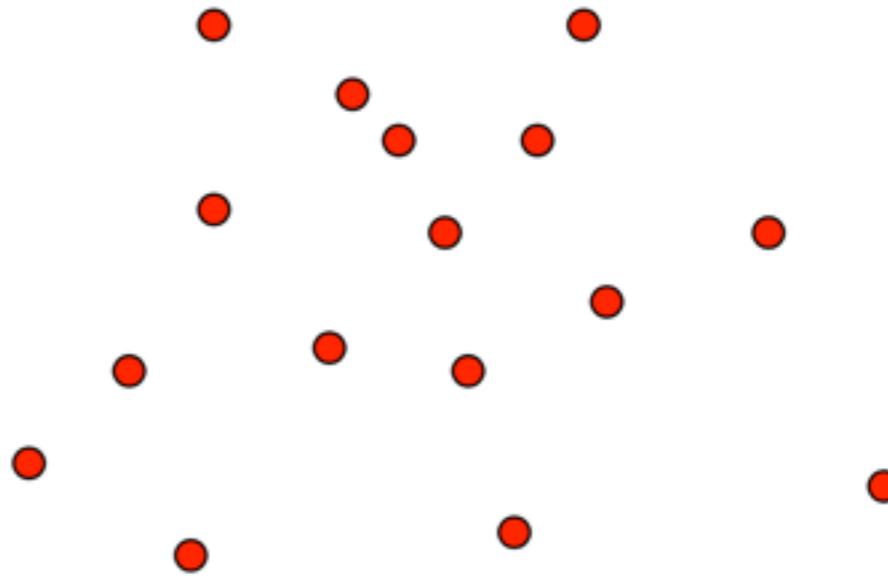
$$P_{i,j,k} = P_{0,0} + \Delta_x[i]\vec{e}_x + \Delta_y[j]\vec{e}_y$$



Turbulence in the Ionosphere - Greg Foss, TACC

Unstructured geometry

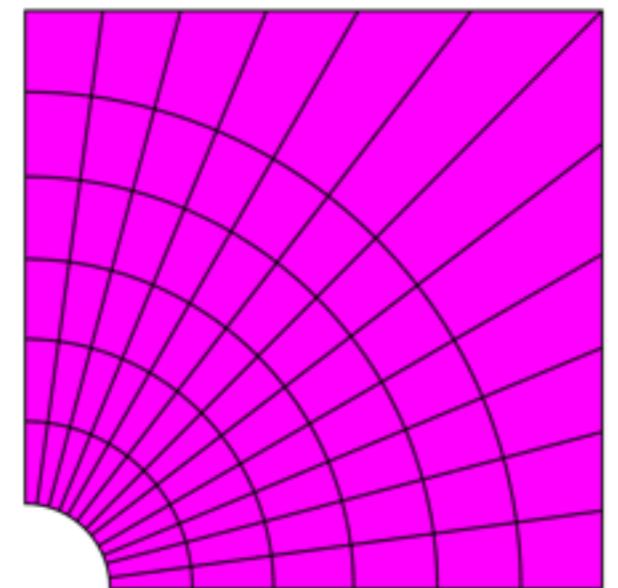
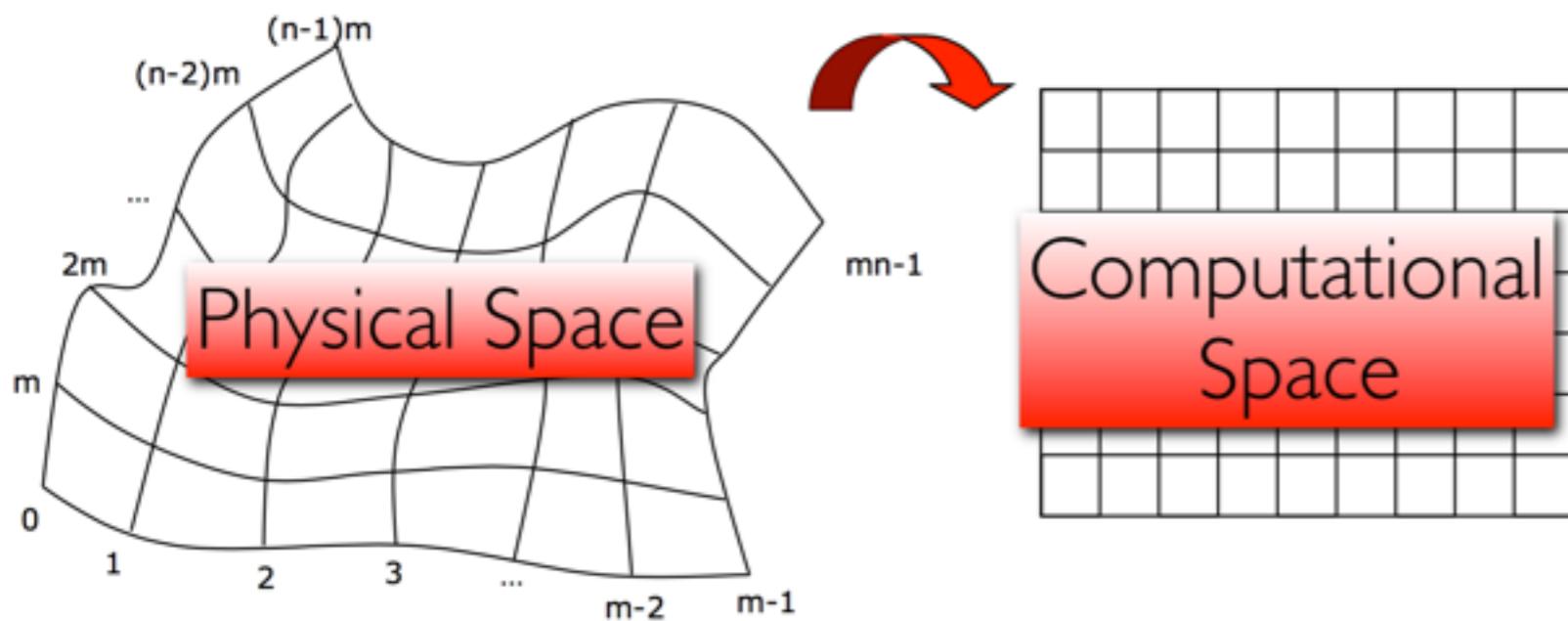
- Raw, unstructured point data.
- You actually need to store the x,y,z positions of vertices.
- Some of the largest computational and scanned data
 - LiDAR, RGB-D point clouds, range scans
 - n-body codes — molecular dynamics, cosmology
- Note: this is unstructured topology, too!



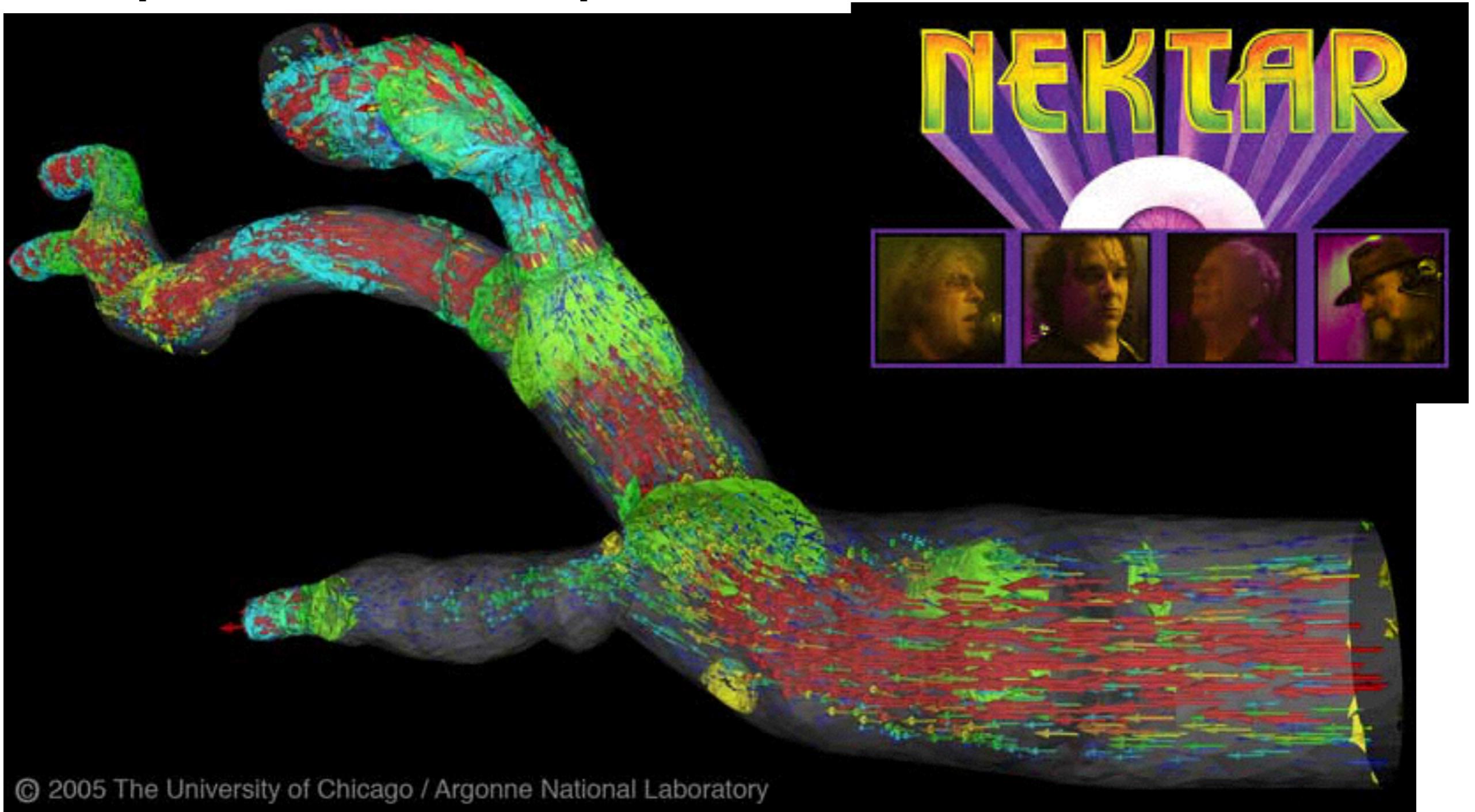
30-billion particle "Cosmic Web" data — Paul Shapiro, University of Texas at Austin
I Wald, A Knoll, G.P. Johnson, W. Usher, V. Pasccci & M.E. Papka: "CPU Ray Tracing Large Particle Data with P-k-d Trees" IEEE Visualization 2015 (to appear)

Structured grid topology

- You can have unstructured geometry but structured topology
 - Implicit definition of cells
 - Implicit connectivity between vertices
- More exotic options with structured grid topology:
 - Finite elements, finite differences on curvilinear grids
 - spectral F/E, some spline-based finite elements simulations
 - Good for precision-critical flow computations (blood flow, CFD)



Spectral/hp finite elements



© 2005 The University of Chicago / Argonne National Laboratory

Data: George Karniadakis, Brown University. Visualization: Joe Insley, ANL

Curvilinear grids

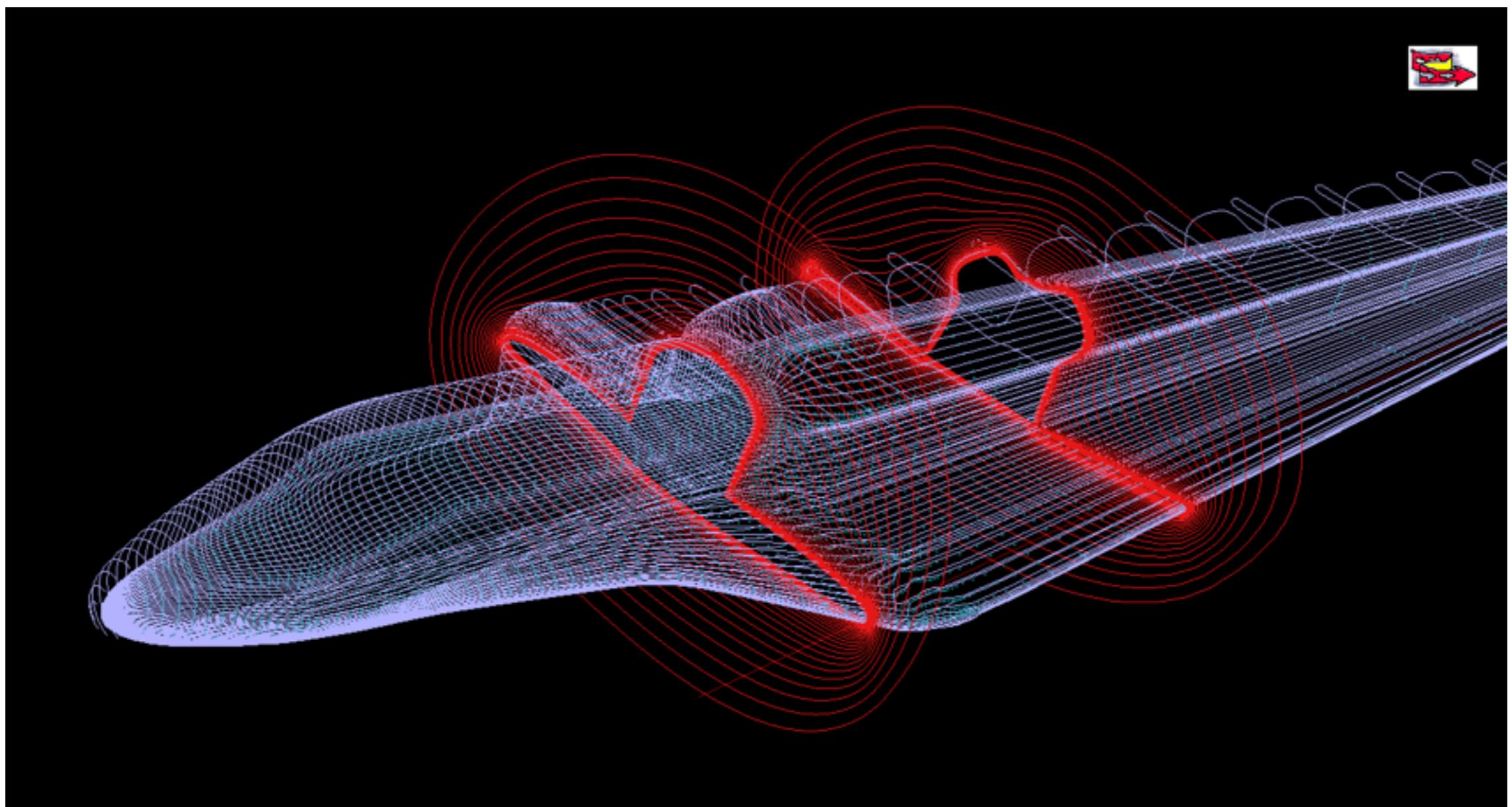
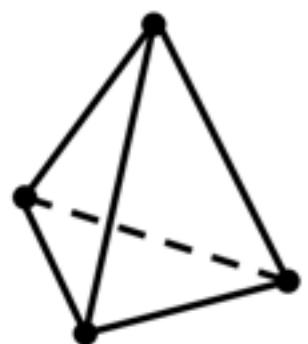


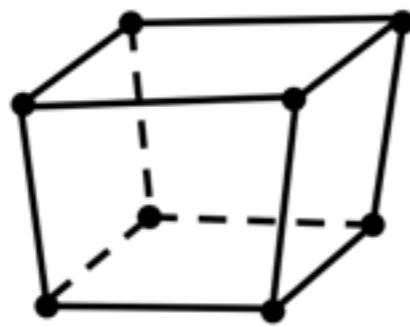
Image:T.U. Graz

Unstructured grid topology

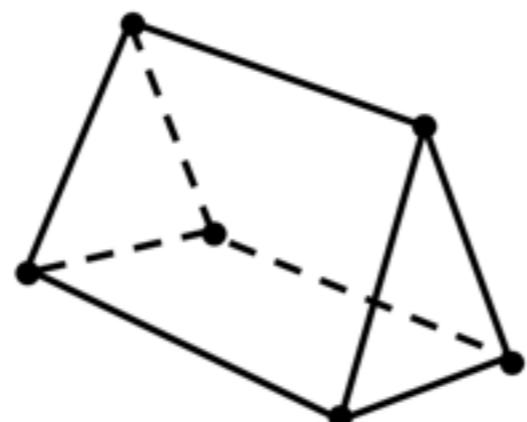
- Both uniform elements and “mixed elements” (allowing any cell type)
- Need to store vertices and indices separately
 - mixed elements: vertices, indices and count
- Many, many finite elements codes.
 - solid mechanics, CAD
 - bioelectric modeling



Tetrahedron



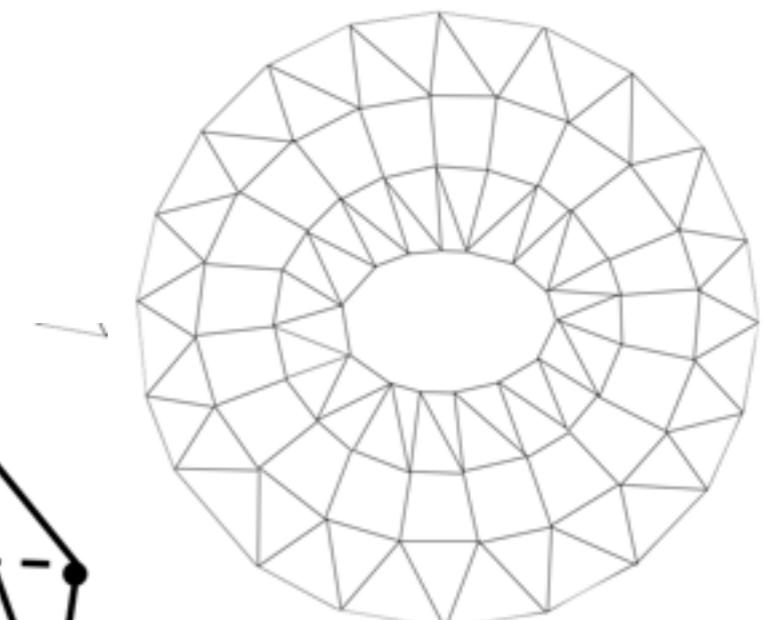
Hexahedron



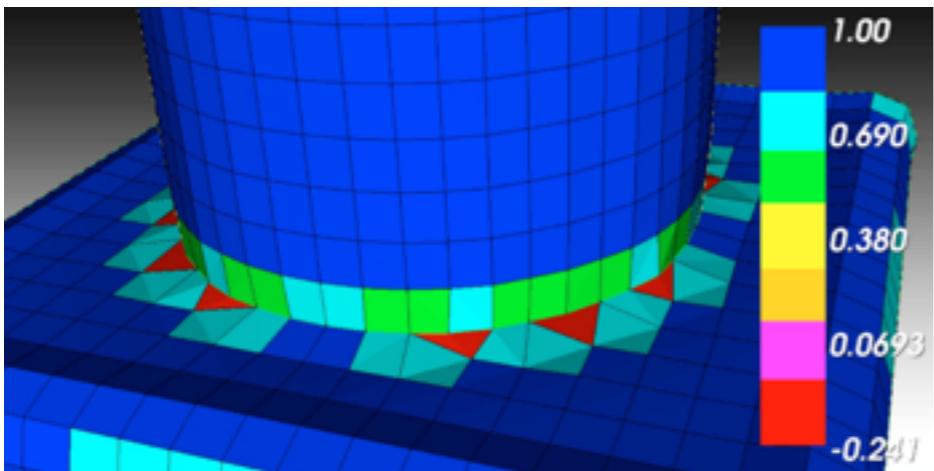
Wedge



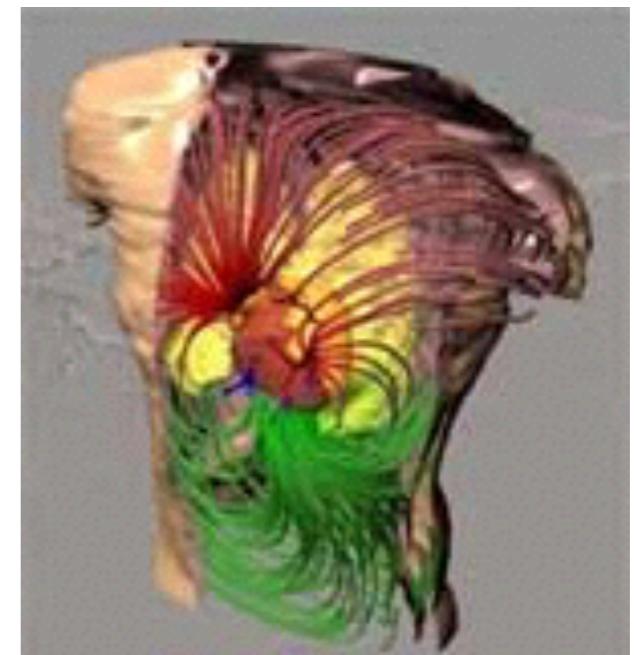
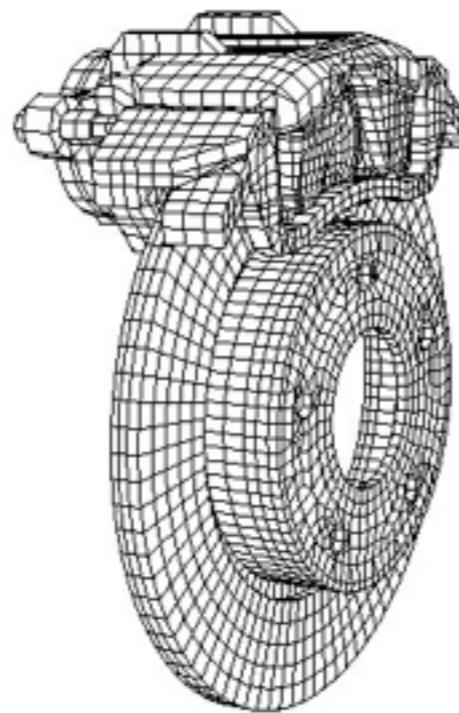
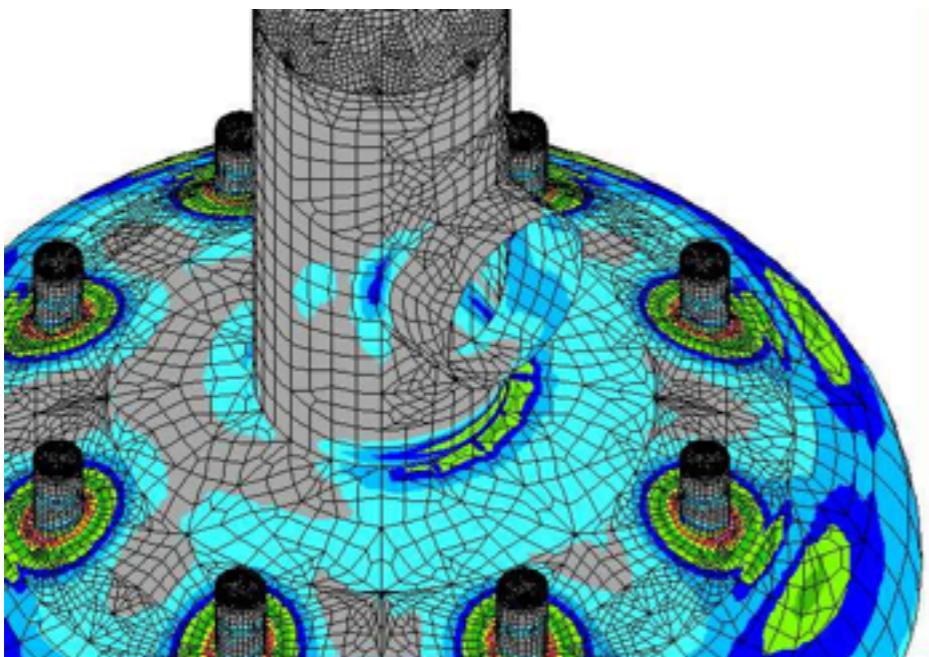
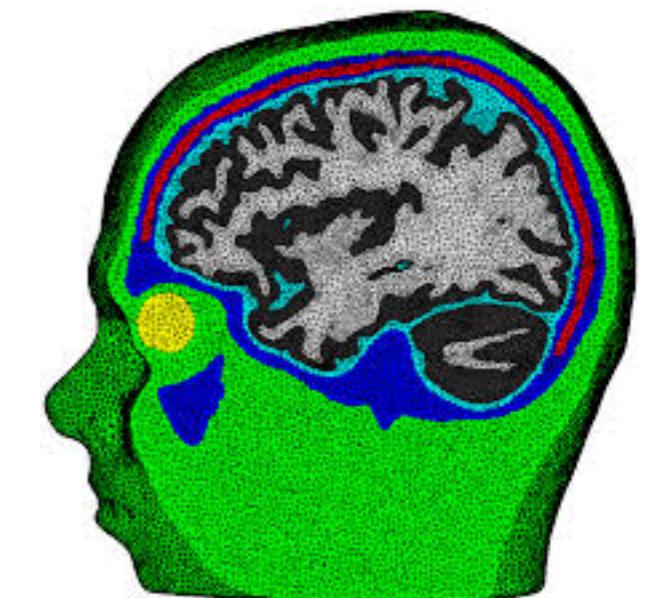
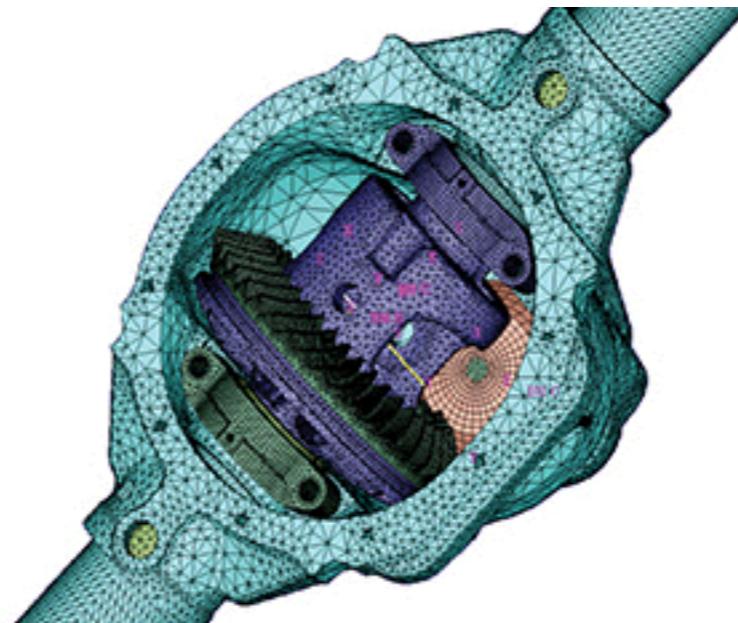
Pyramid



Finite elements



Visualization: Steve Owen, using Cubit



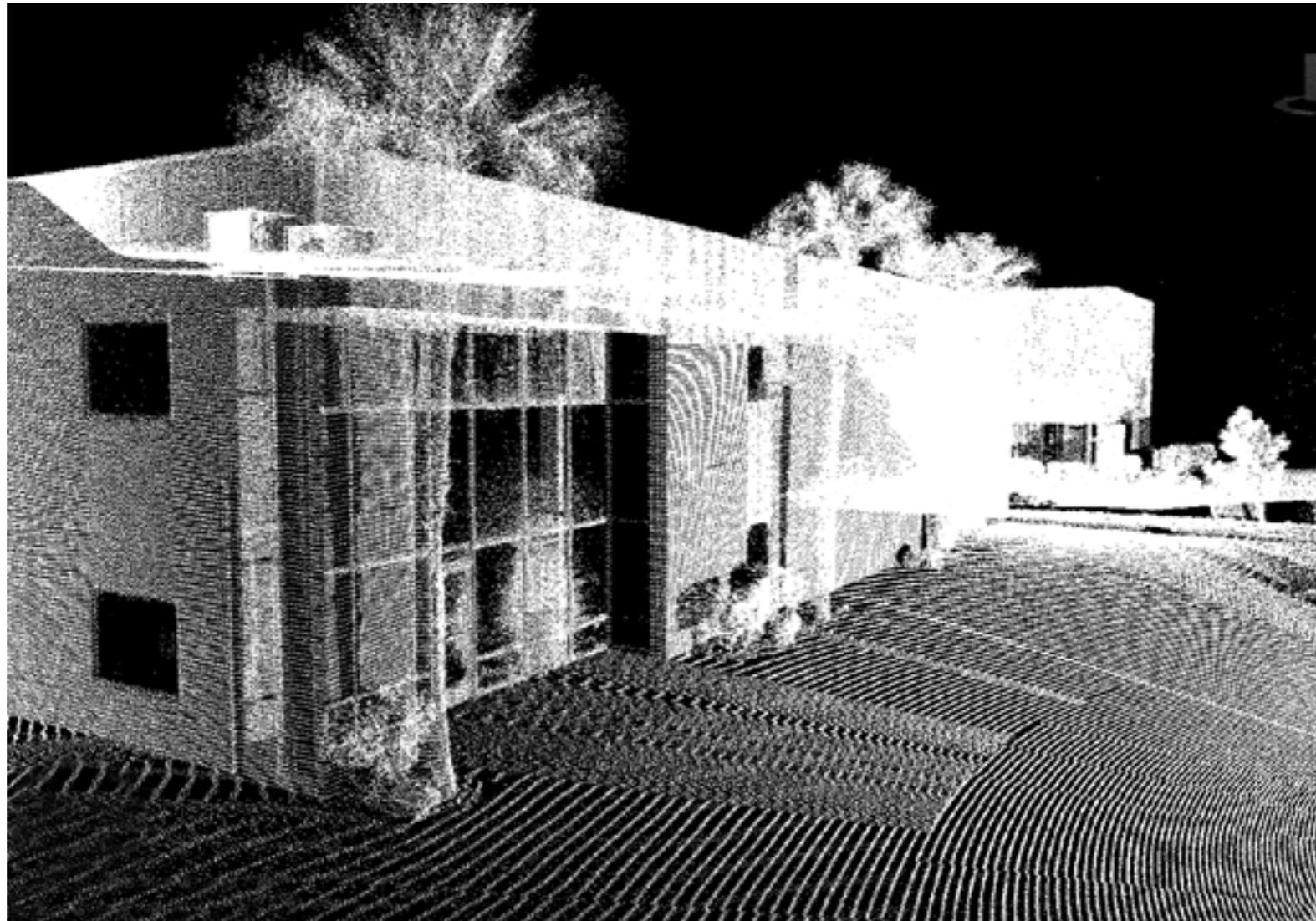
To summarize...

			Topology	
		Structured	Unstructured	
Geometry	Uniform	Image	Unstructured	
	Structured	Rectilinear	Unstructured	
	Unstructured	Curvilinear	Unstructured	

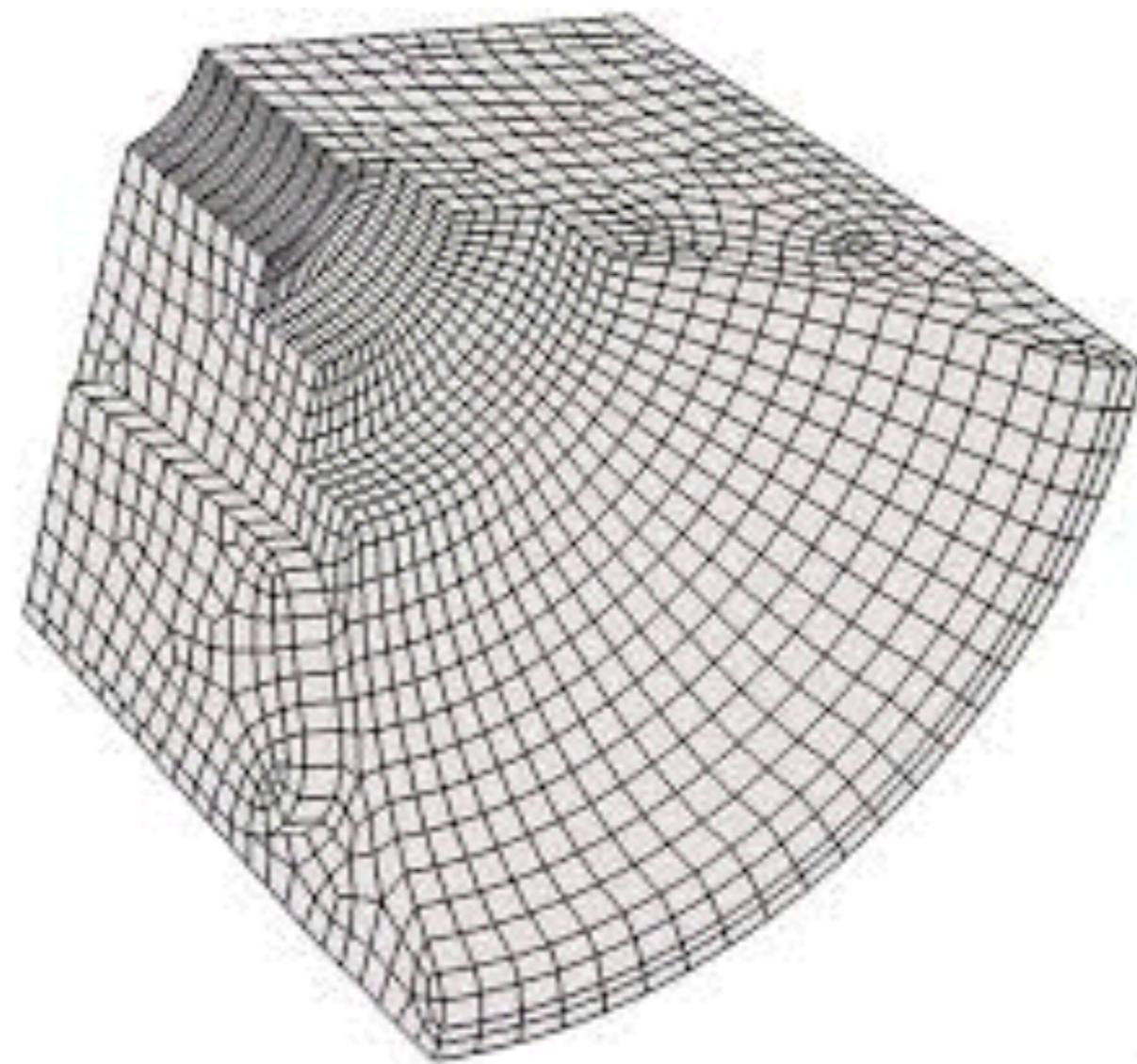
Colloquially

- In sci-vis we usually talk about geometry, thus:
 - **structured** means rectilinear grid (not *necessarily* uniform... but almost always).
 - **unstructured** means everything else (curvilinear grids, tetrahedra, hexahedra, points, etc.)

Is it (geometrically) structured or unstructured?

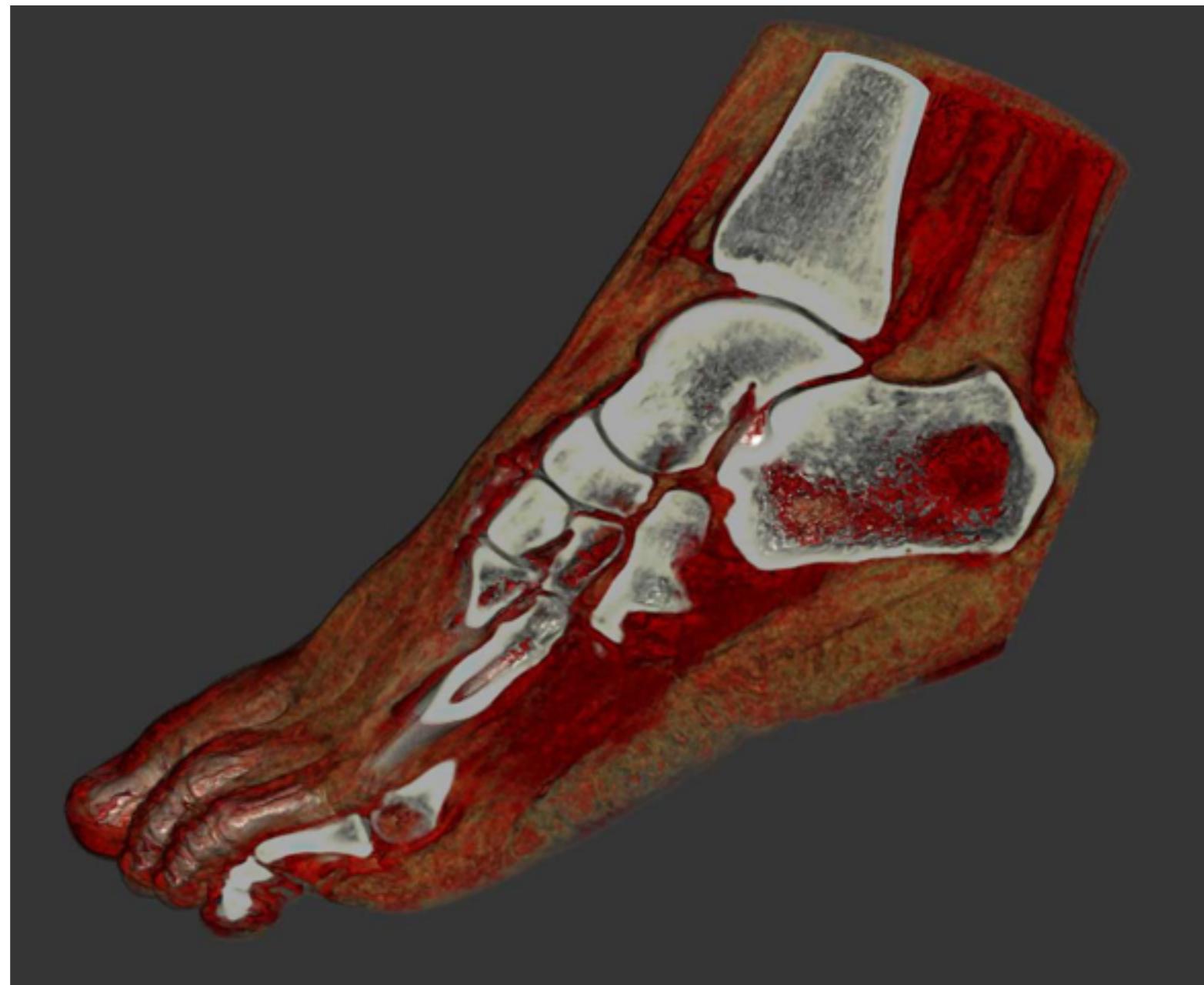


Is it (geometrically) structured or unstructured?

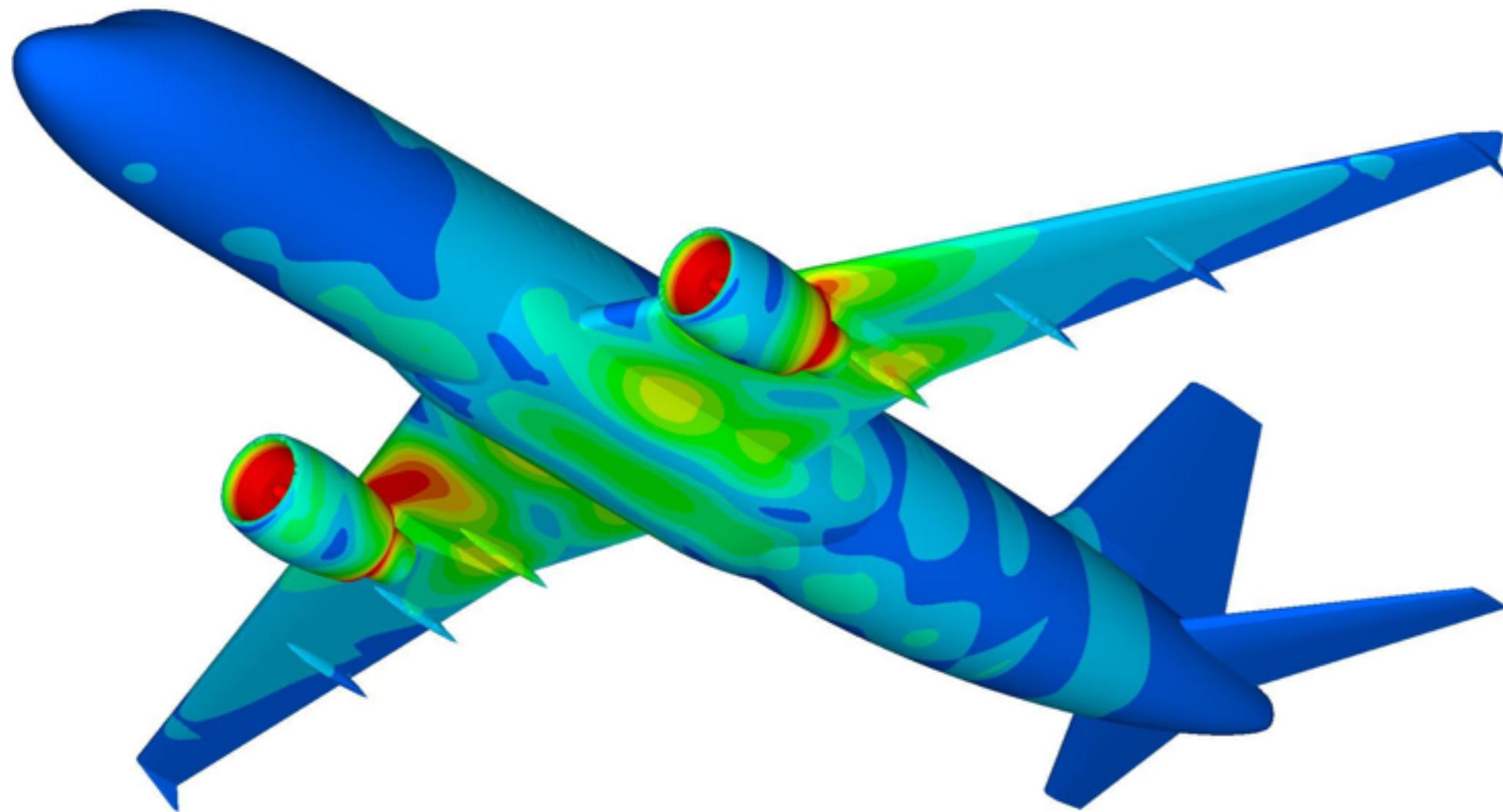


1

Is it (geometrically)
structured or unstructured?

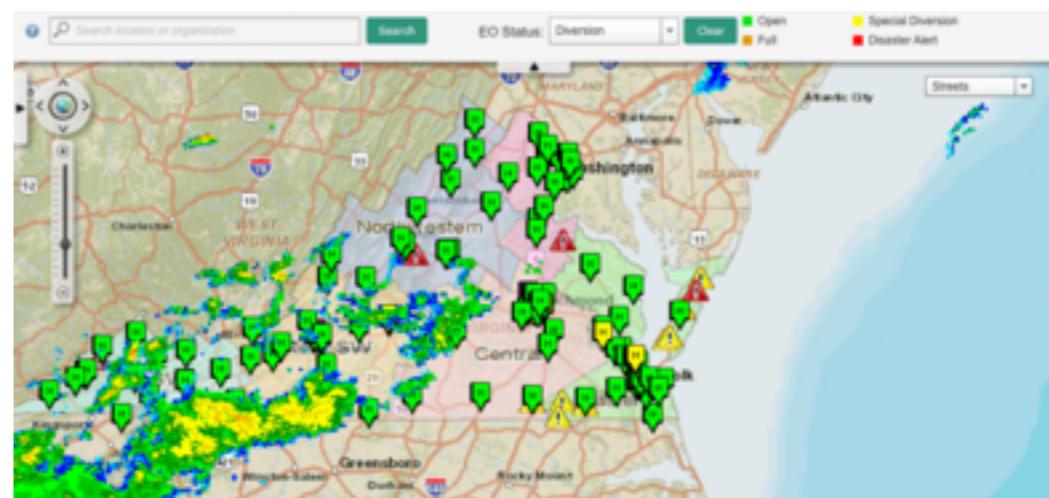
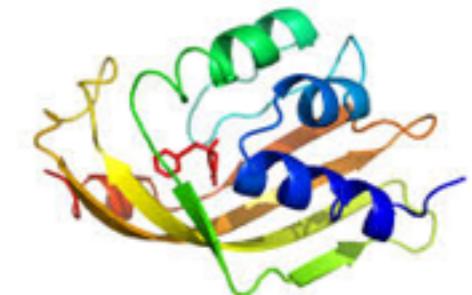
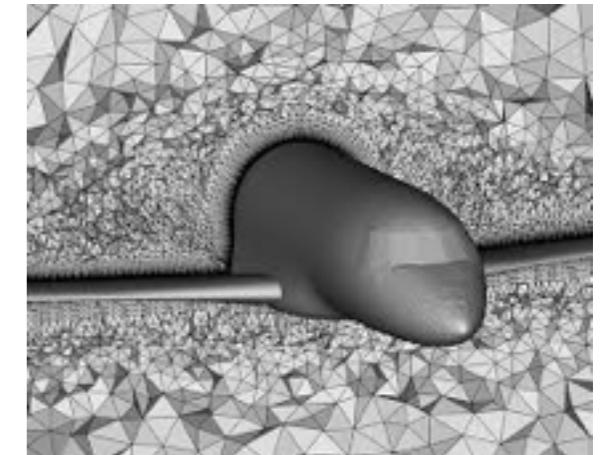


Is it (geometrically)
structured or unstructured?



Non-field and other data

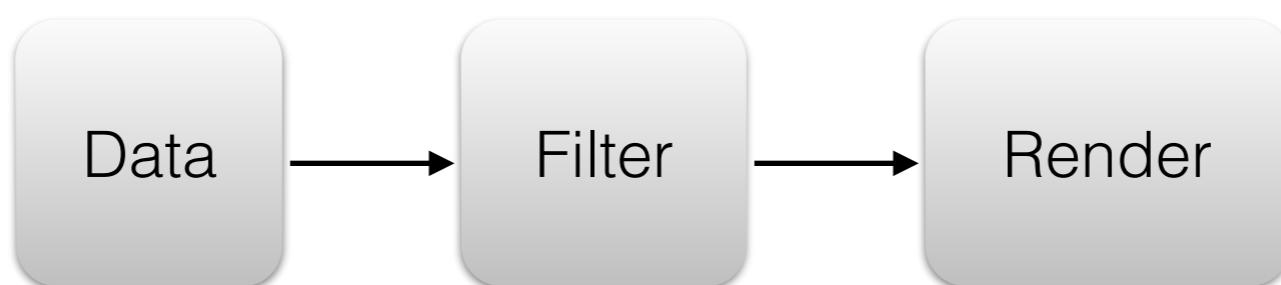
- In addition to structured/unstructured field data, you can have non-field geometry.
 - Boundary surface meshes
 - Atom positions, bonds, ribbons
- Non-geometric annotations
 - Especially in GIS.
 - Visualization data models are complex!



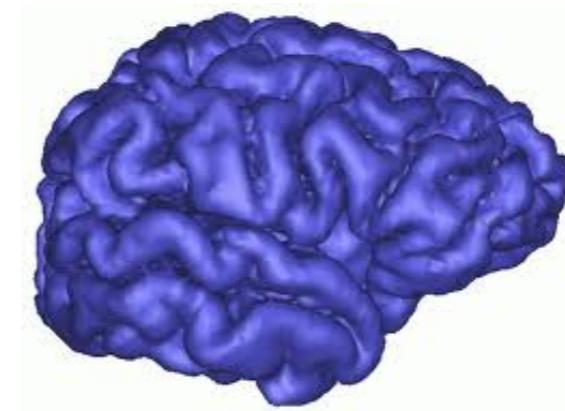
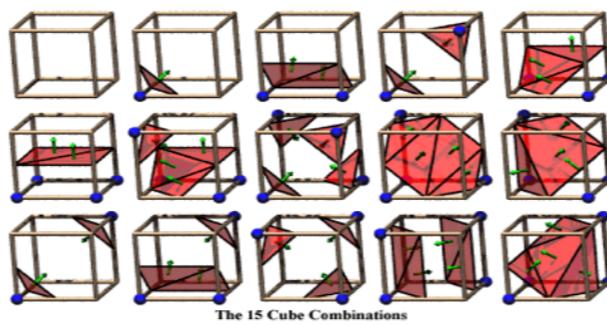
What do we do with these data?

- In computer graphics, life is “easy”
 - Have a triangle mesh, render it!
- Visualization is more than just rendering.
- Two approaches:
 - direct visualization:
i.e. render from a (usually 3D) field directly
 - indirect visualization:
i.e. convert the field to triangles and render those (usually with GPU rasterization)

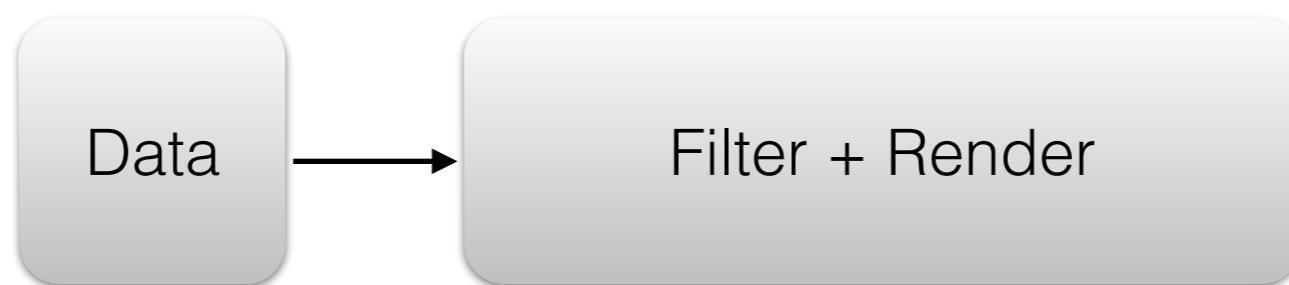
Indirect visualization



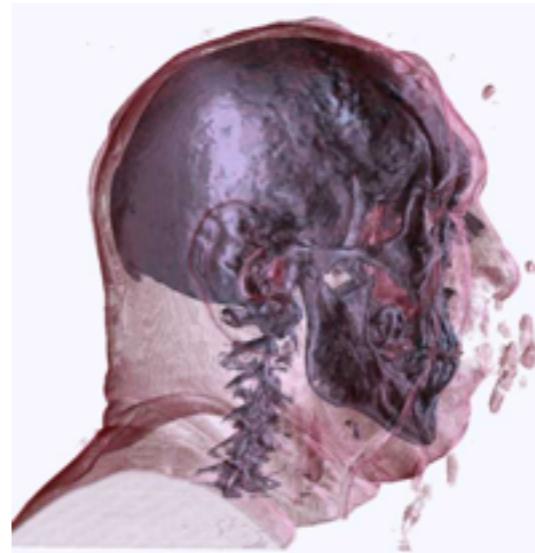
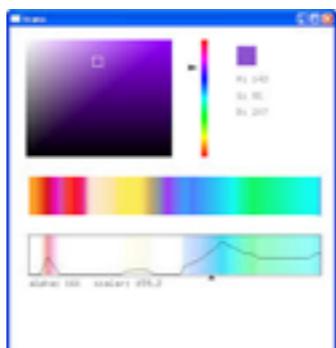
0	4	8	0
4	14	9	0
6	11	1	0
2	1	0	0



Direct visualization

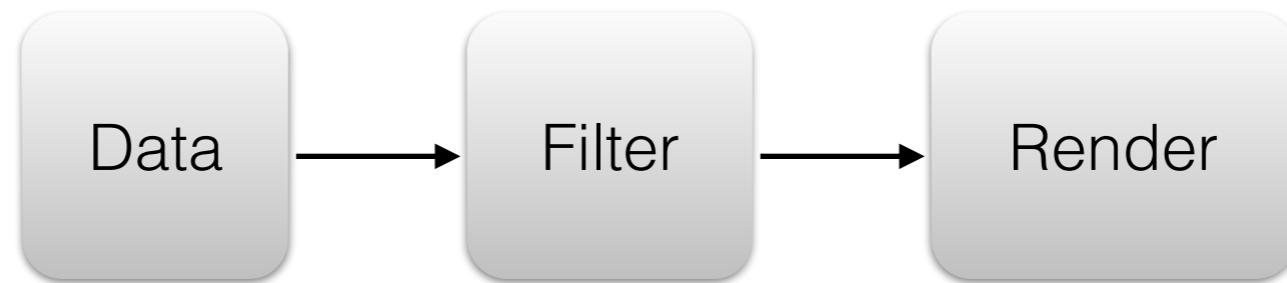


0	4	8	0
4	14	9	0
6	11	1	0
2	1	0	0



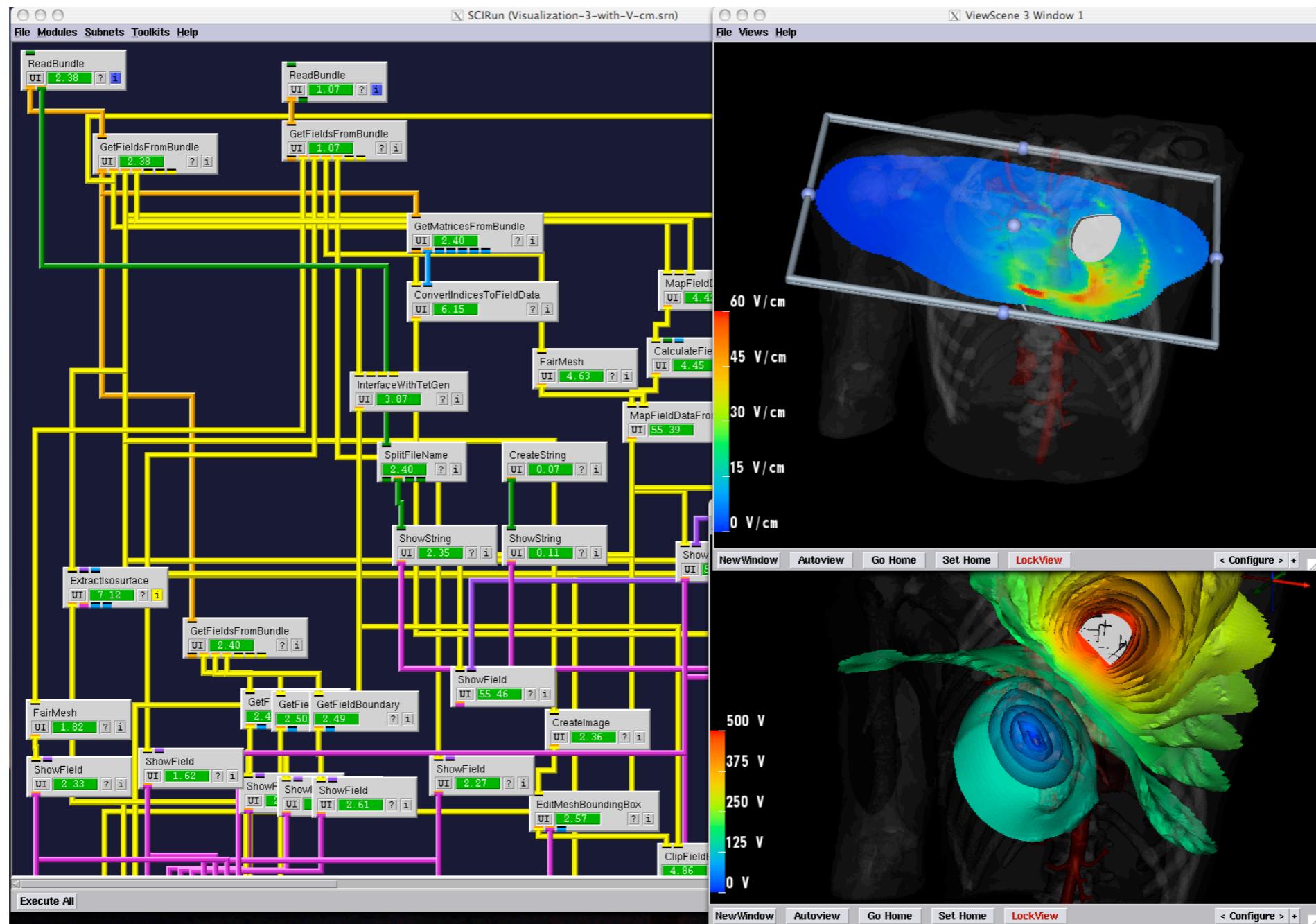
The visualization pipeline

- Even if we merge filtering and rendering, it is helpful to think of them as a chain of operations.

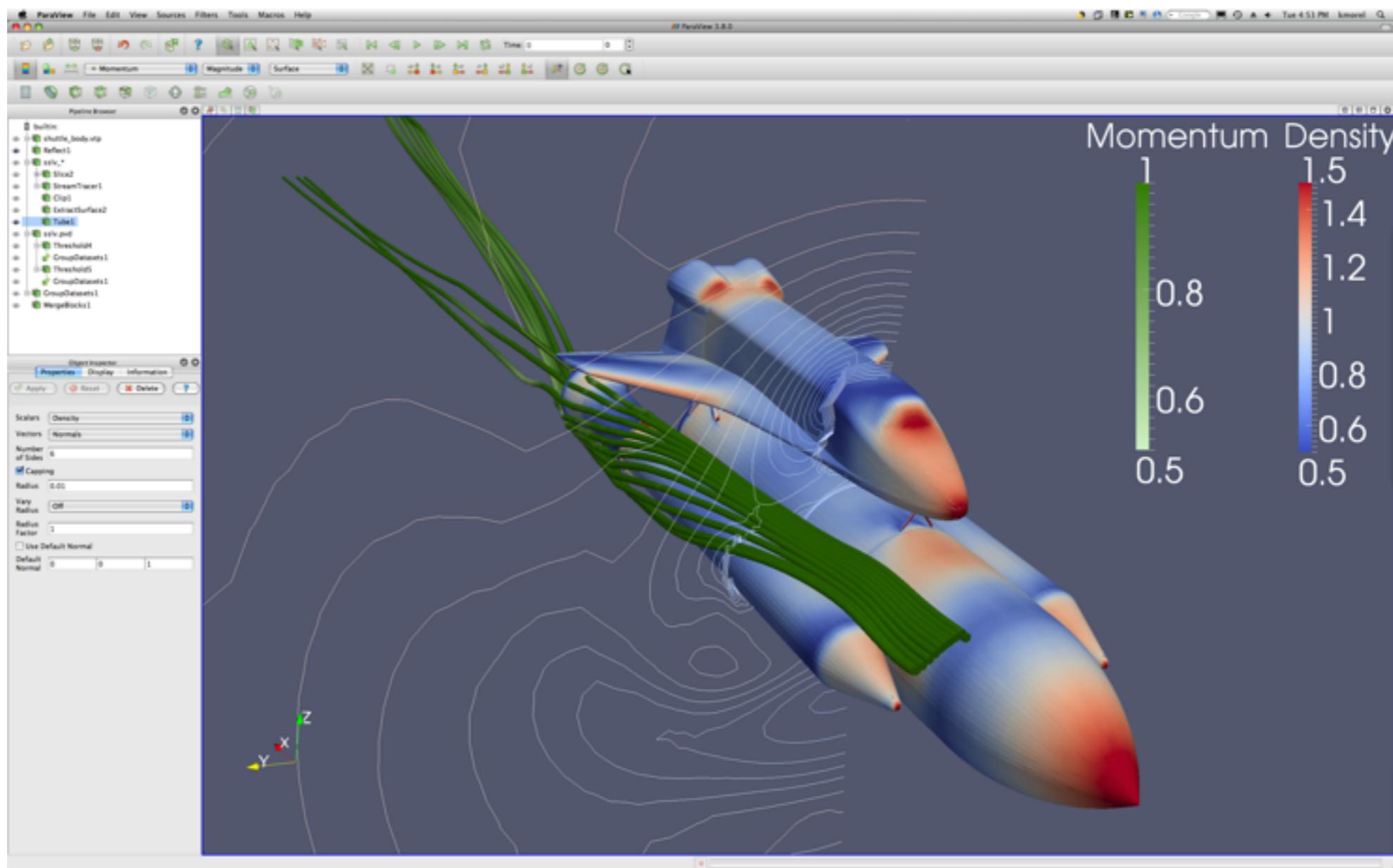


- Visualization workflows take the form of a flow chart, tree or network...

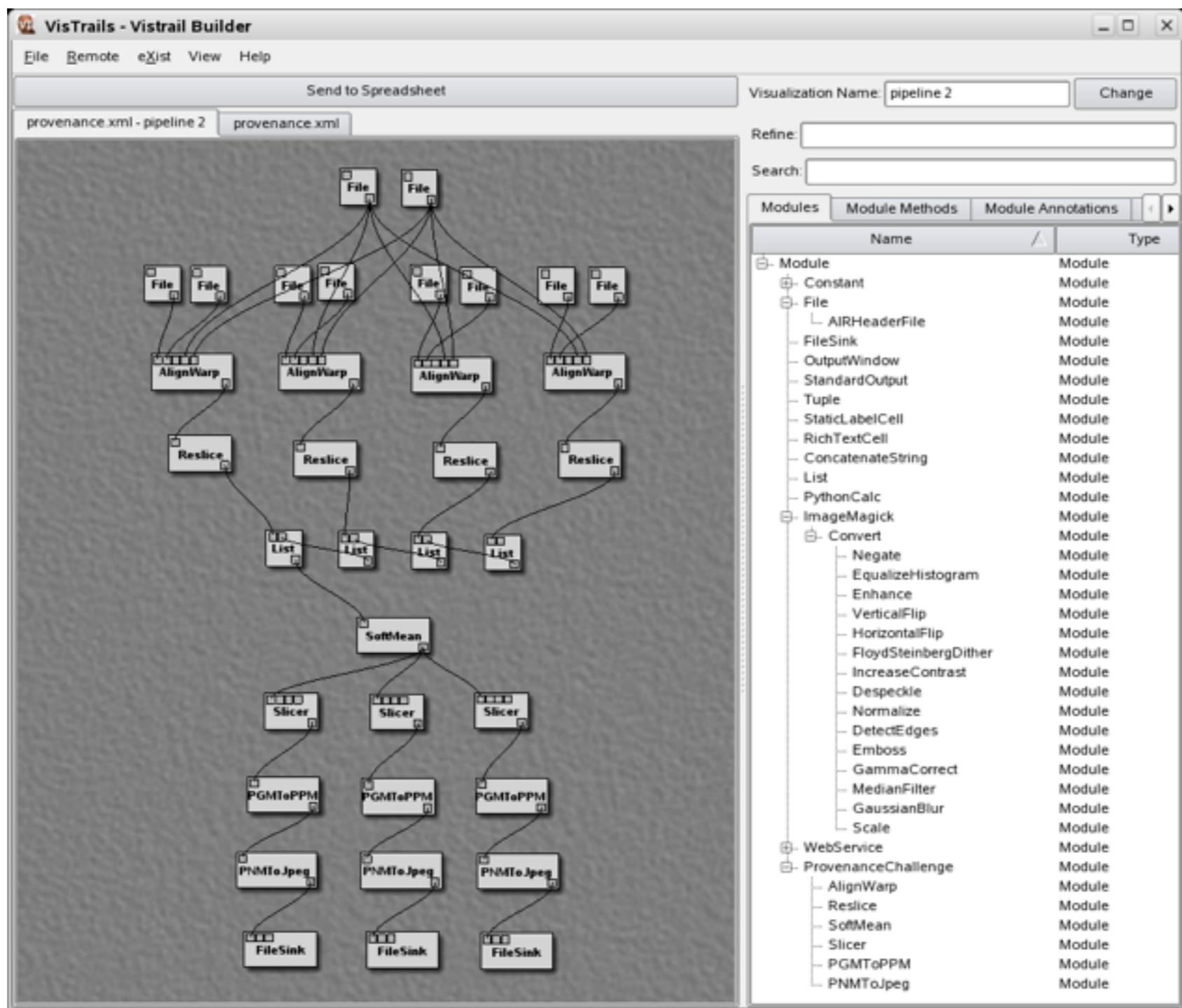
SCI Run



ParaView



VisTrails



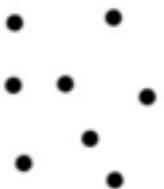
VTK

- <http://www.vtk.org>, open-source, developed and maintained by Kitware.
- The standard-bearer API for general-purpose scientific visualization
 - Full-fledged data model for structured, unstructured, particle data
 - Marching cubes, cut/clip planes, streamlines, etc.
 - Hundreds of other analysis filters
 - Numerous readers for common scientific formats
- Call as a library from C++, Java, Python, Tcl/Tk
- Limitations:
 - no UI — you need to code (or at least, script) your workflows.
 - No distributed rendering (see Paraview, VisIt)
 - does not exploit latest OpenGL features (OpenGL 2.x support currently in the works!)
 - filters and renderers do not support all “exotic” data structures (e.g., spectral finite elements)
 - data model can be “heavy”, memory-inefficient — but it nearly always works!

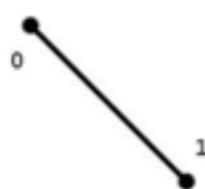
Cell types in VTK



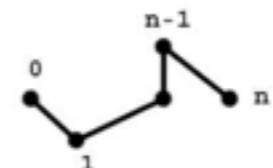
(1) Vertex



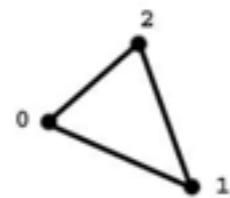
(2) Poly-vertex



(3) Line



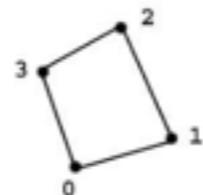
(4) Poly-line



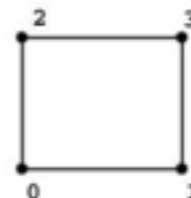
(5) Triangle



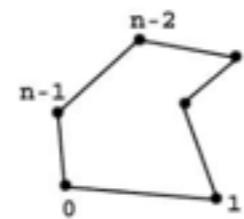
(6) Triangle strip



(7) Quadrilateral



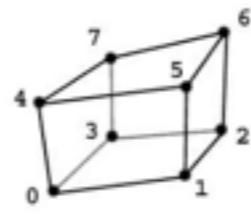
(8) Pixel



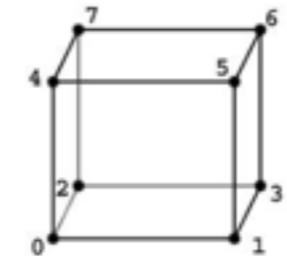
(9) Polygon



(10) Tetrahedron



(11) Hexahedron



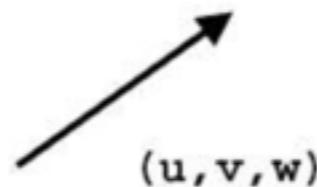
(12) Voxel

Attribute types in VTK

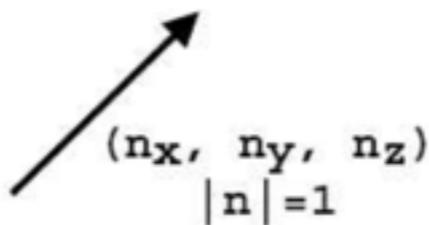


s

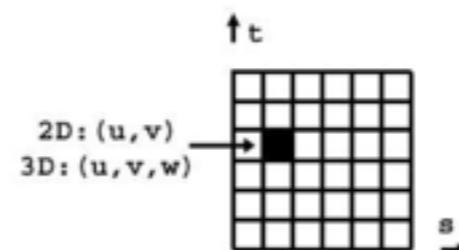
(1) Scalar



(2) Vector



(3) Normal



(4) Texture coordinate

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

(5) Tensor

S. Bruckner, "Data Structures in the Visualization Toolkit."

Simple data flow in VTK

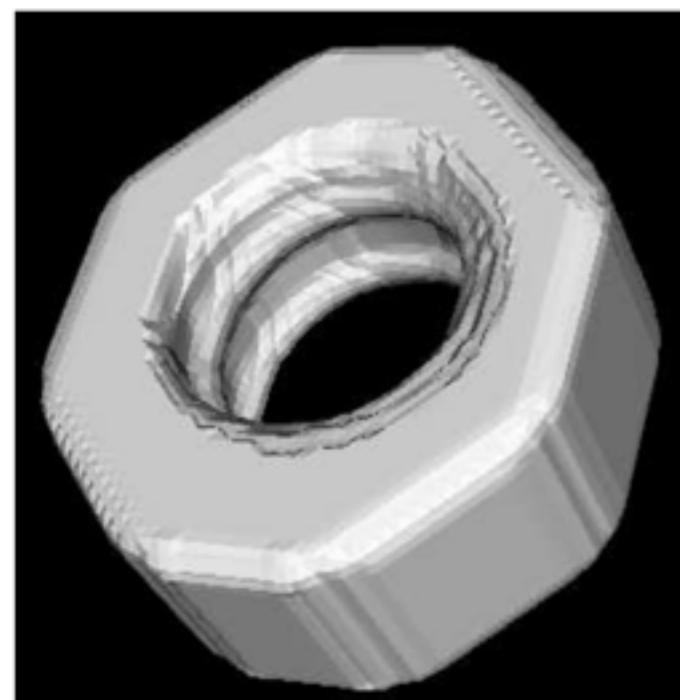


Figure 7: Resulting image for program 3.2

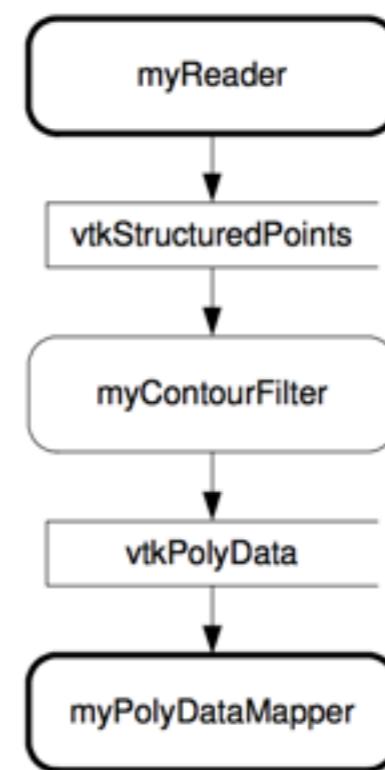


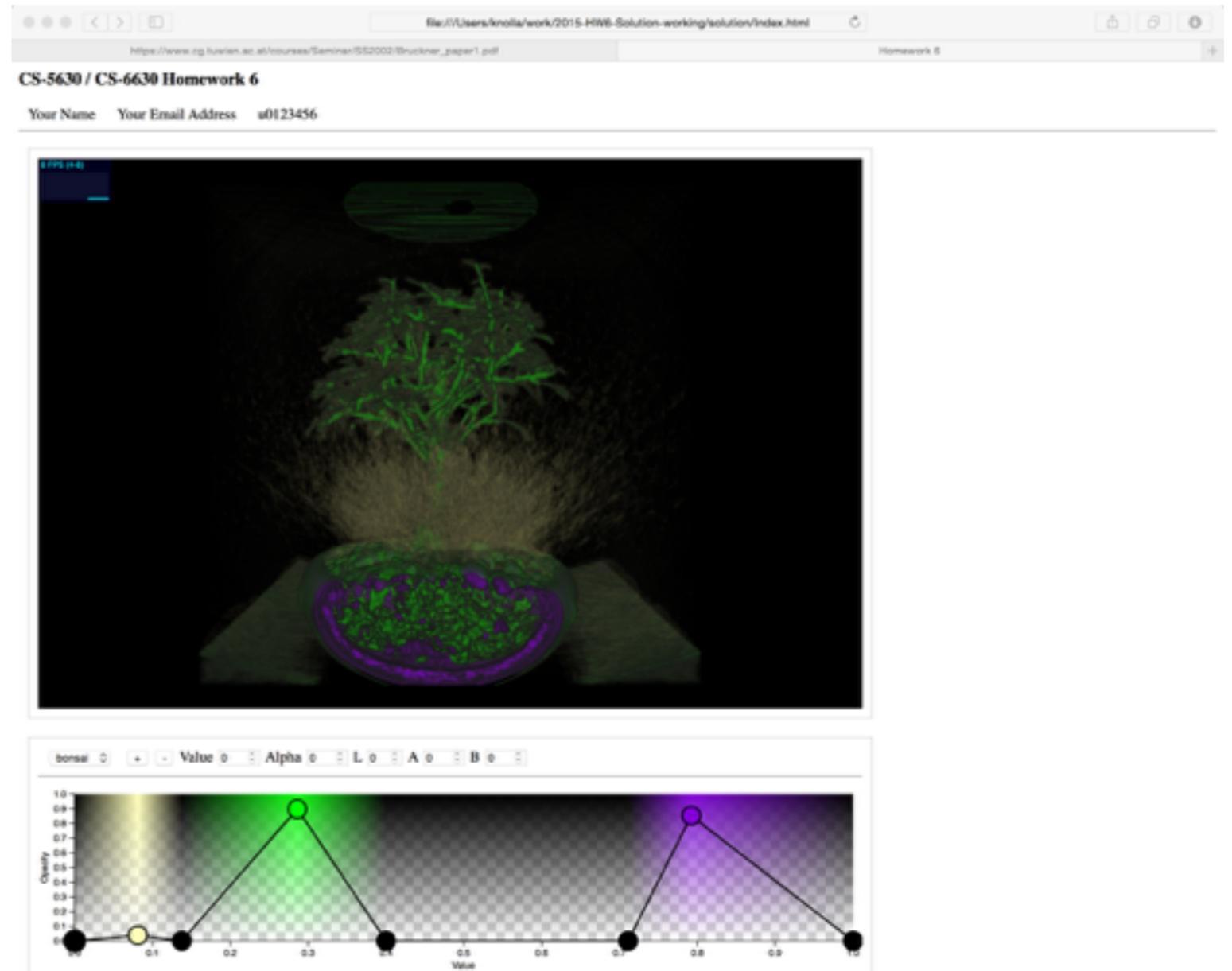
Figure 8: Data-flow chart for program 3.2

In practice...

- Most of the time, we want to visualize one or two scalar fields at a time, in 2D or 3D
- The type of visualization we do depends more on **context** than the MxN chart of fields, or even type of data. E.g.,
 - Volume rendering for 3D continuous phenomena
 - Heatmaps in 2D, GIS
 - Glyphs to represent explicit geometry (e.g. molecular vis, vectors or tensors)
 - Geometric abstractions and special geometry where appropriate (ribbons, streamlines, etc.)
- More on these later!

Homework 6 preview

- volume rendering in WebGL and Javascript
- D3 transfer function editor
- (extra credit)
phong lighting



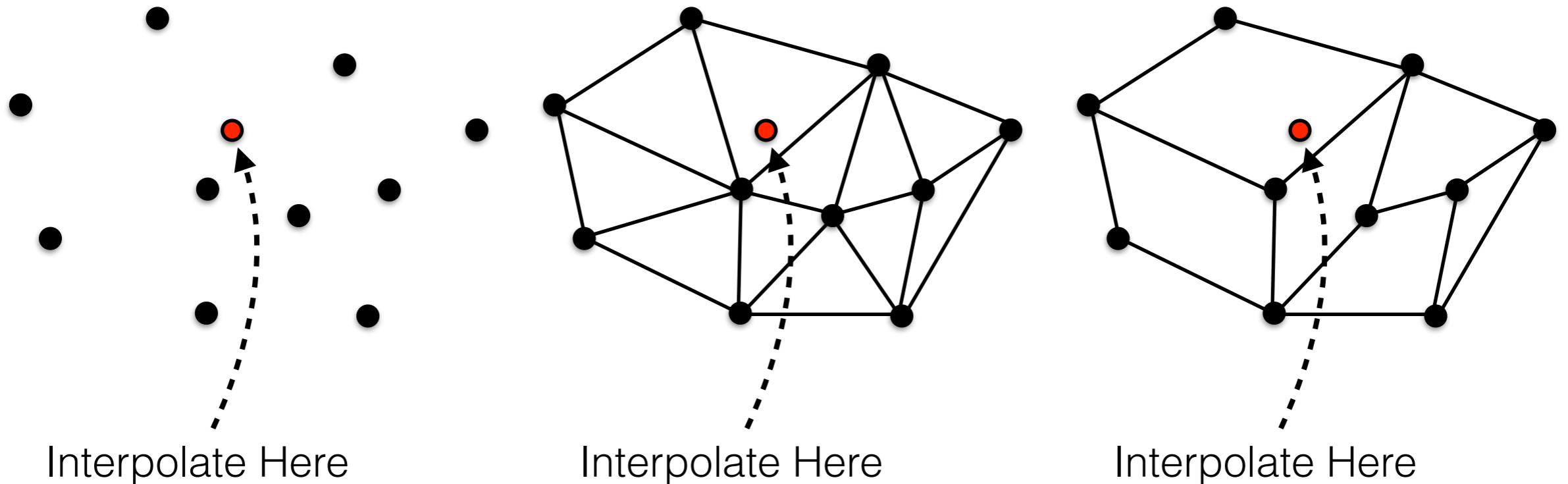
Next sci-vis lecture: Oct 22

- Interpolation
- Isosurfaces and Marching cubes
- Volume rendering
- Introduction to GLSL / ELSL (for HW6).

Interpolation

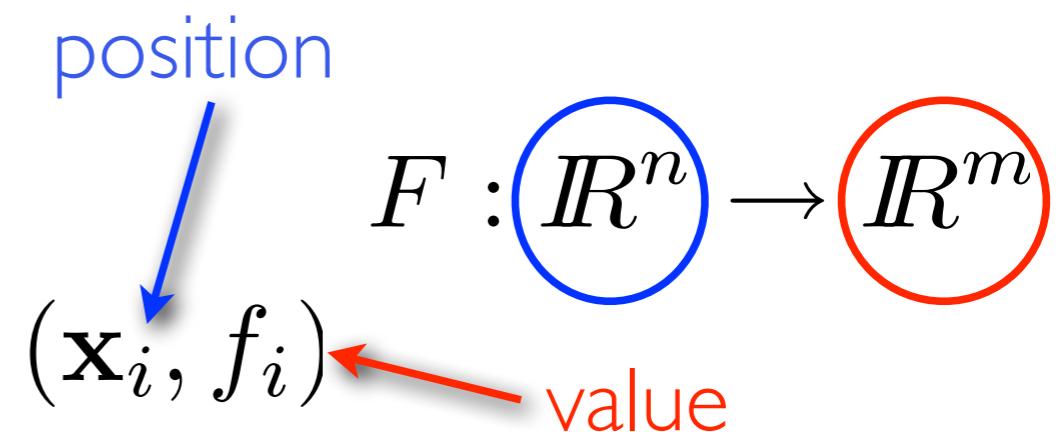
Mesh Choice Impacts How the Continuous Data is Interpreted

- Two key questions:
 - Sampling, or the choice of where attributes are measured
 - Interpolation, or how to model the attributes in the rest of space



Interpolation

- **Continuous** reconstruction of **discrete** input data

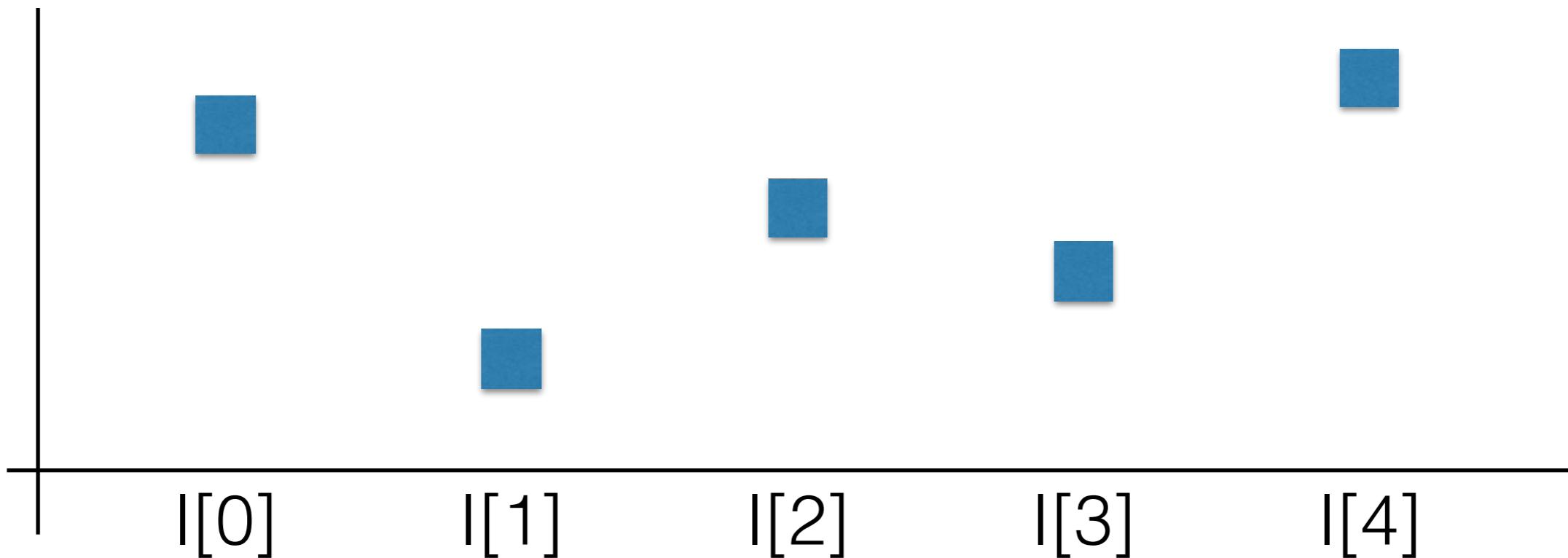


$$\forall i \in \{1, \dots, n\}, F(\mathbf{x}_i) = f_i$$

- Depends on grid structure (when available)
- Interpolation vs. approximation

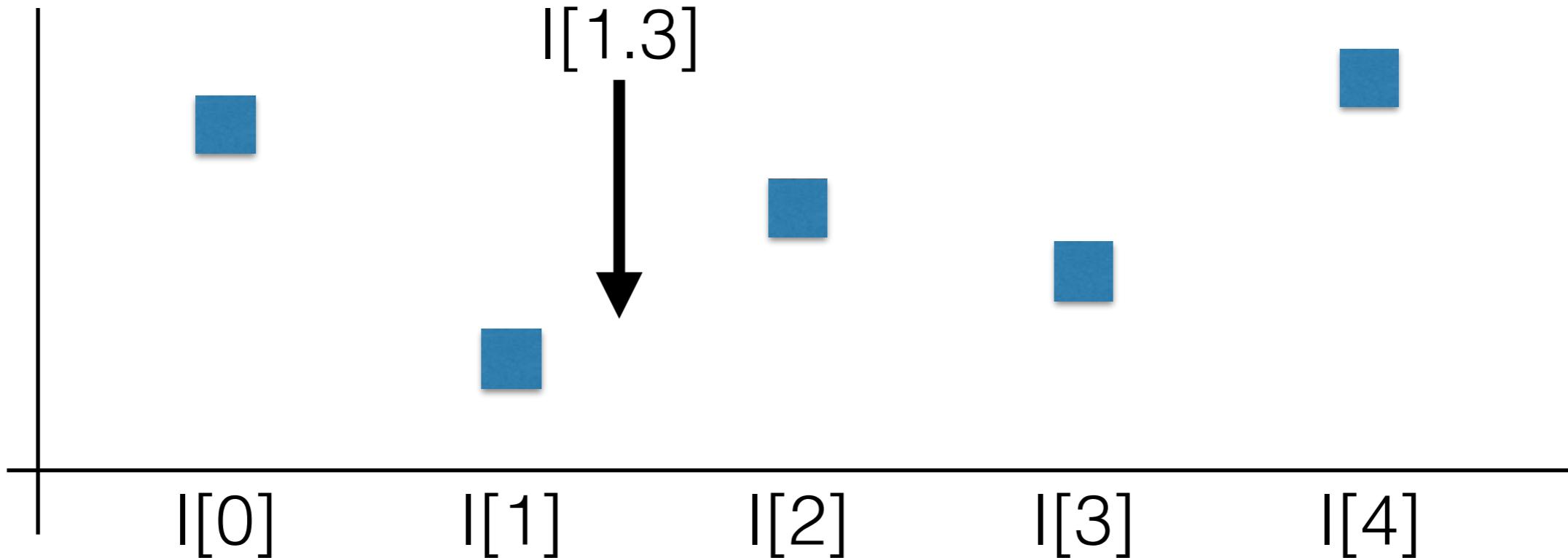
Nearest Neighbor Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is $I[1.3]$?



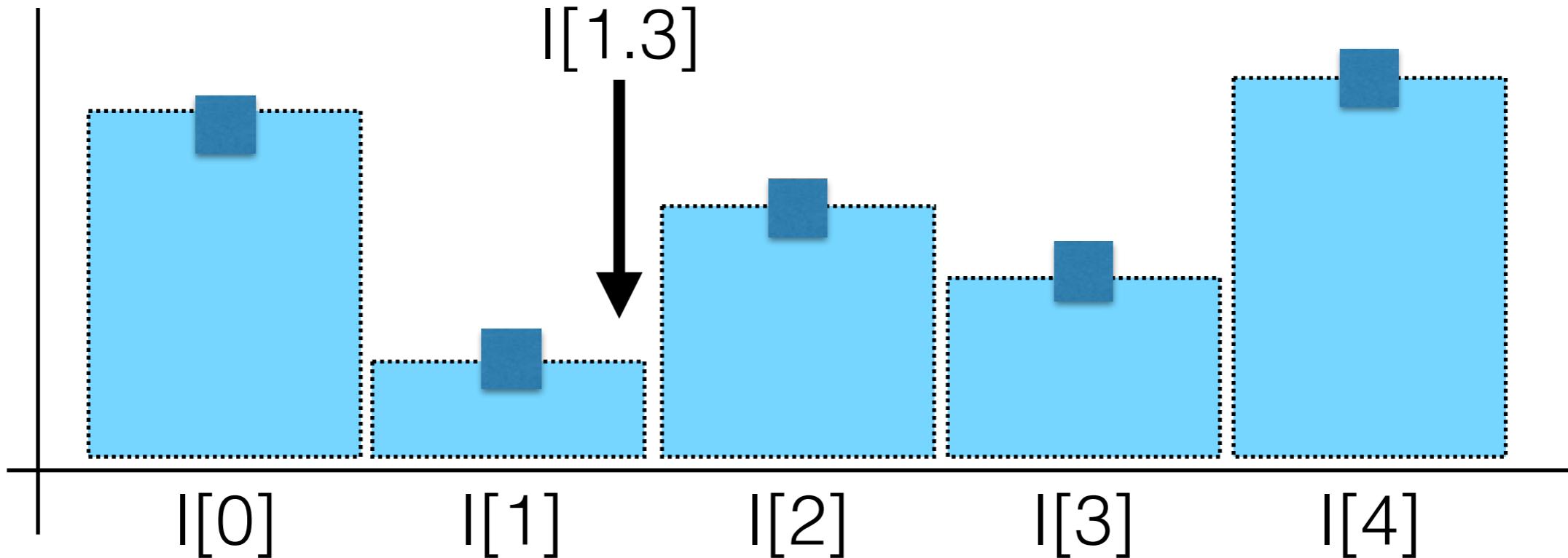
Nearest Neighbor Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is $I[1.3]$?



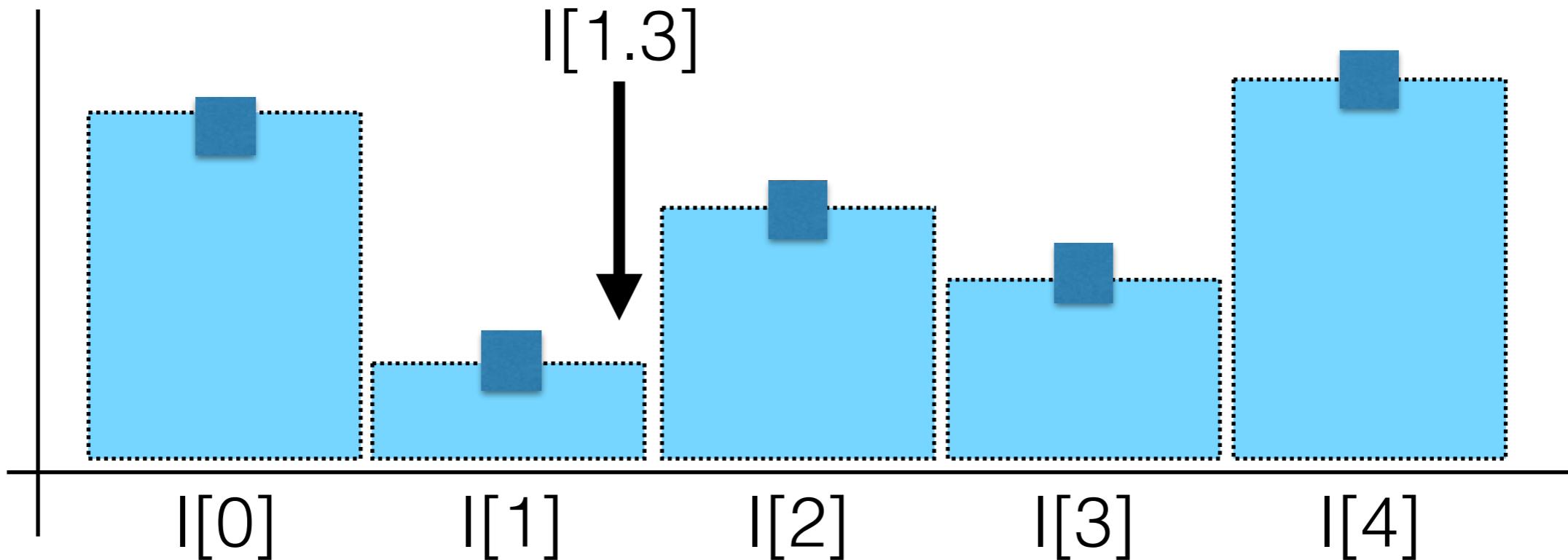
Nearest Neighbor Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is $I[1.3]$?



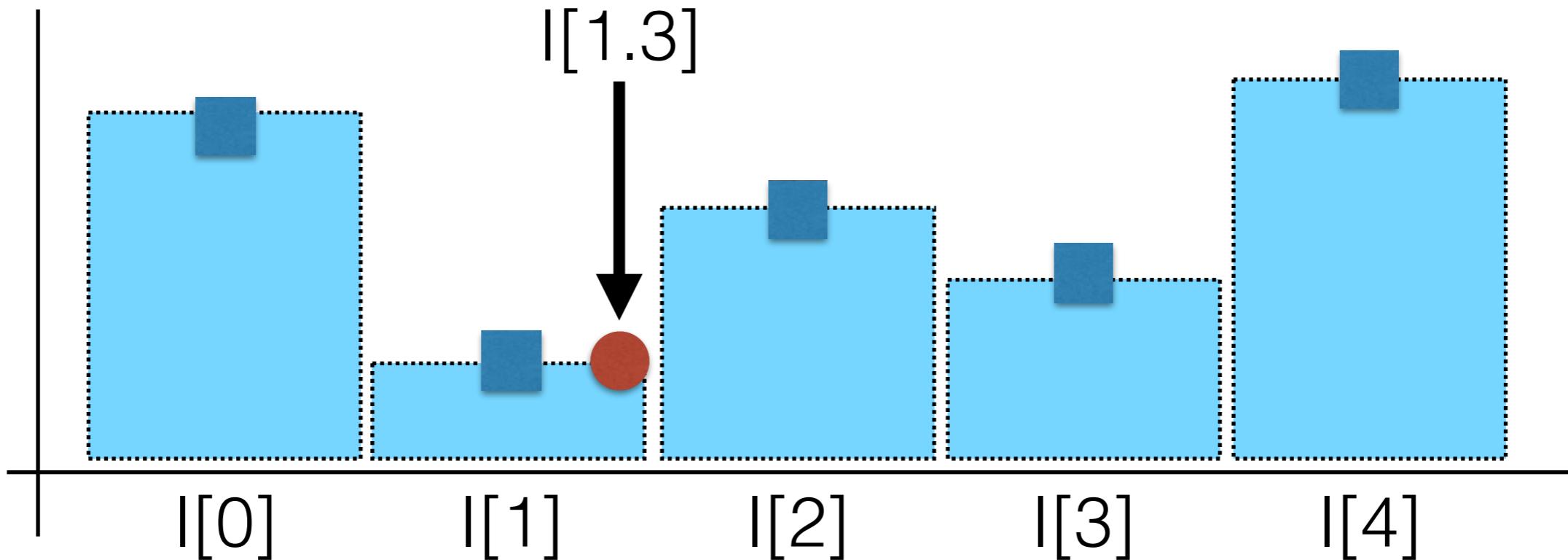
Nearest Neighbor Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is $I[1.3]$?
 - $I[1.3] = I[\text{round}(1.3)] = I[1]$



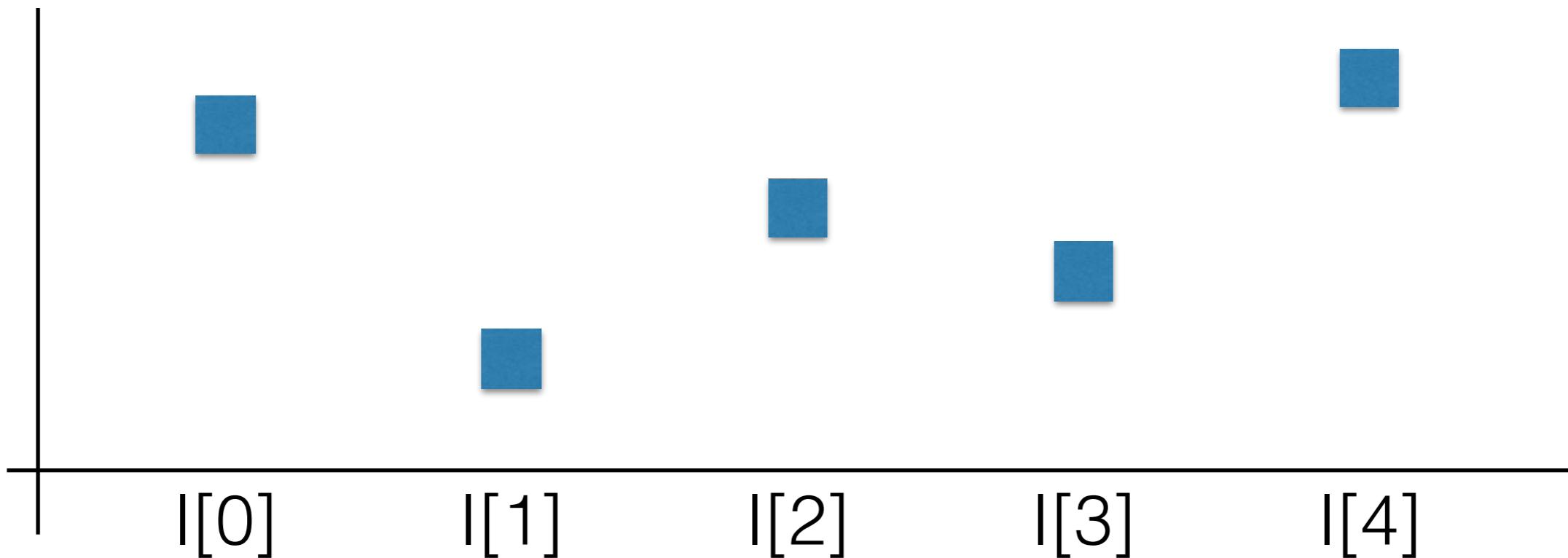
Nearest Neighbor Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is $I[1.3]$?
 - $I[1.3] = I[\text{round}(1.3)] = I[1]$



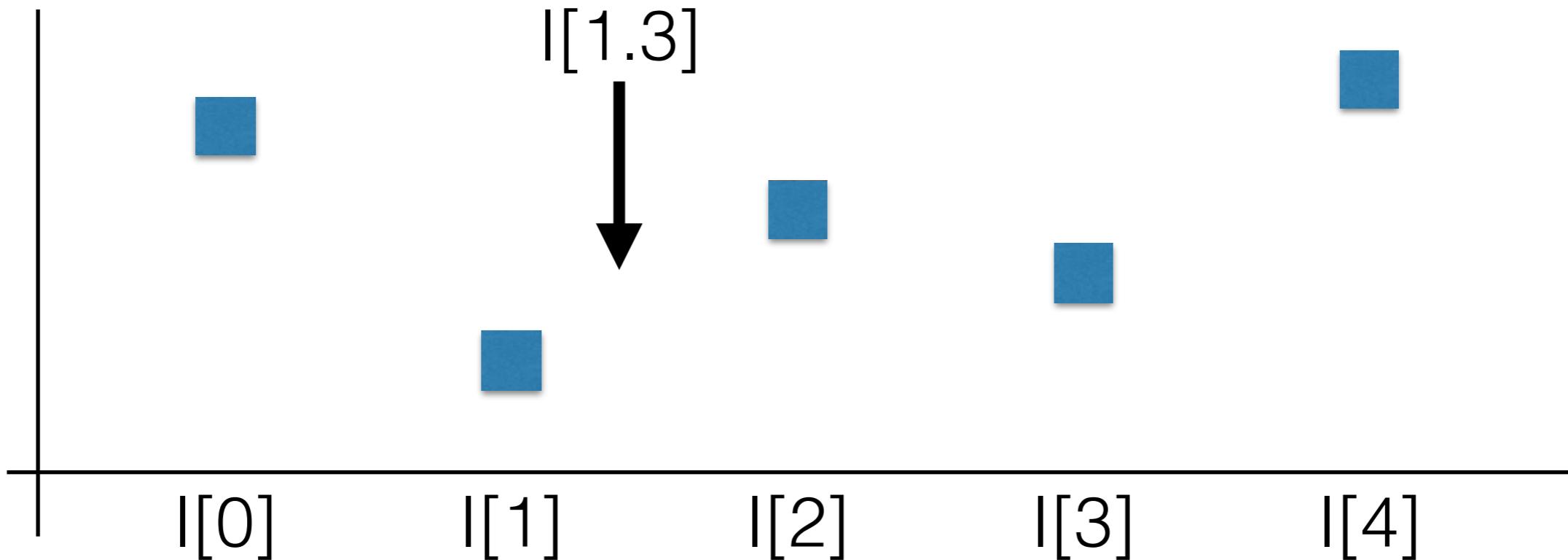
Linear Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is $I[1.3]$?



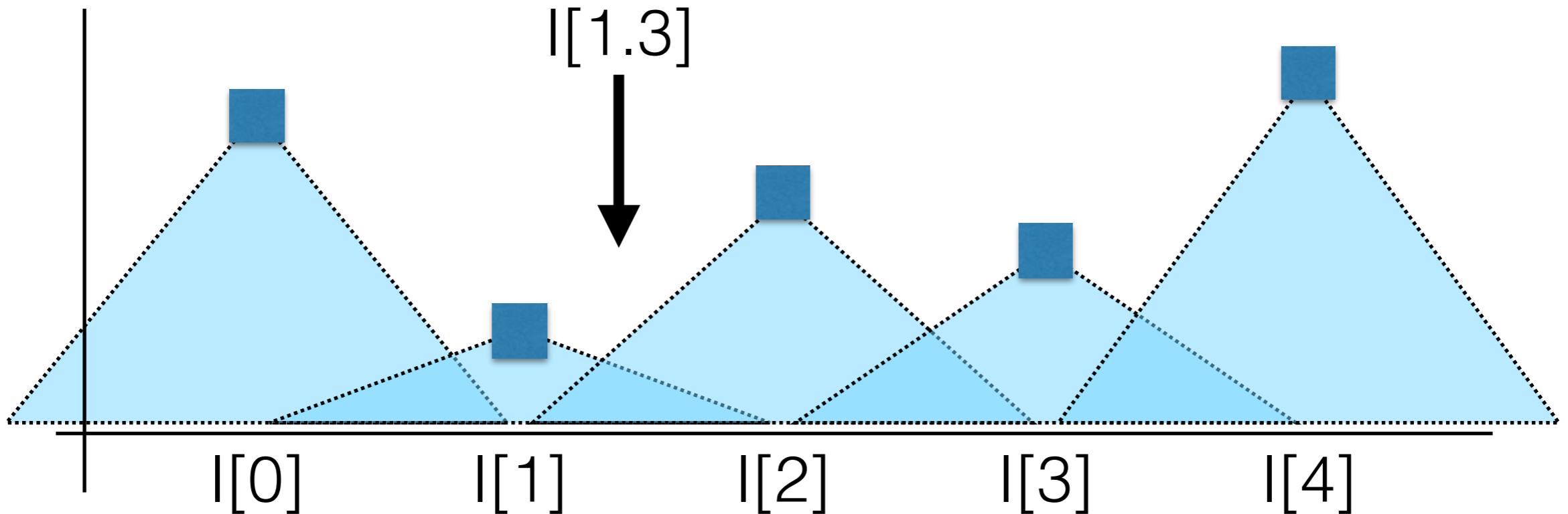
Linear Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is $I[1.3]$?



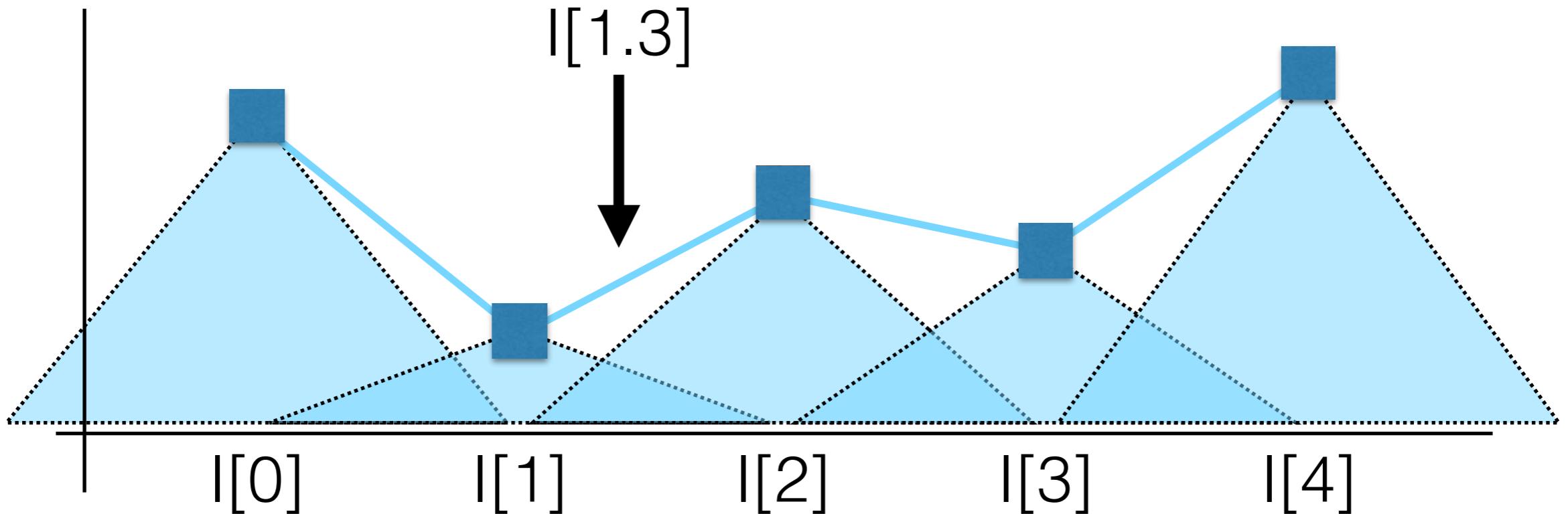
Linear Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is $I[1.3]$?



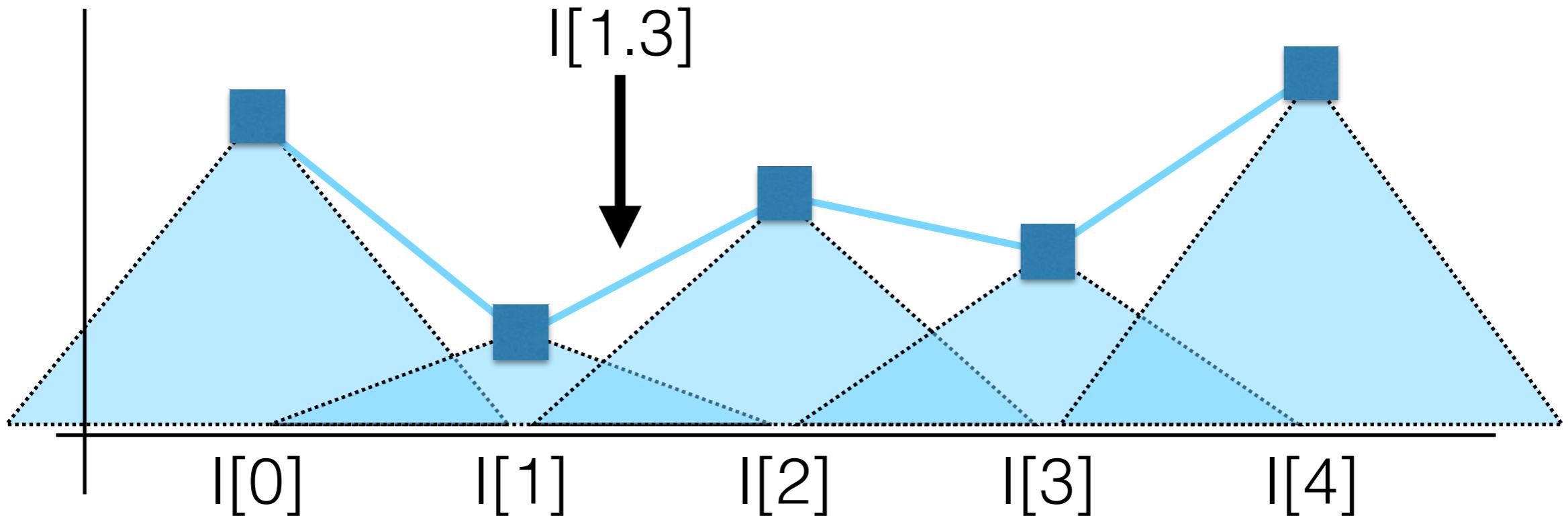
Linear Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is $I[1.3]$?



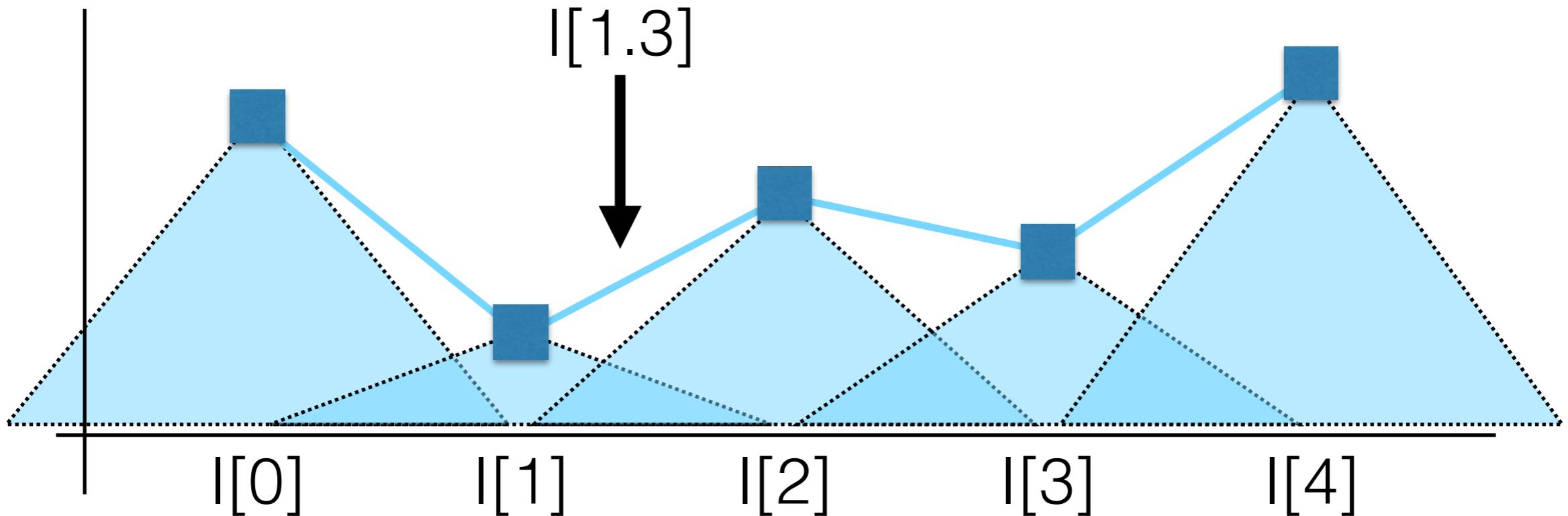
Linear Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is $I[1.3]$?
 - Let $s = 1.3 - \text{round}(1.3)$



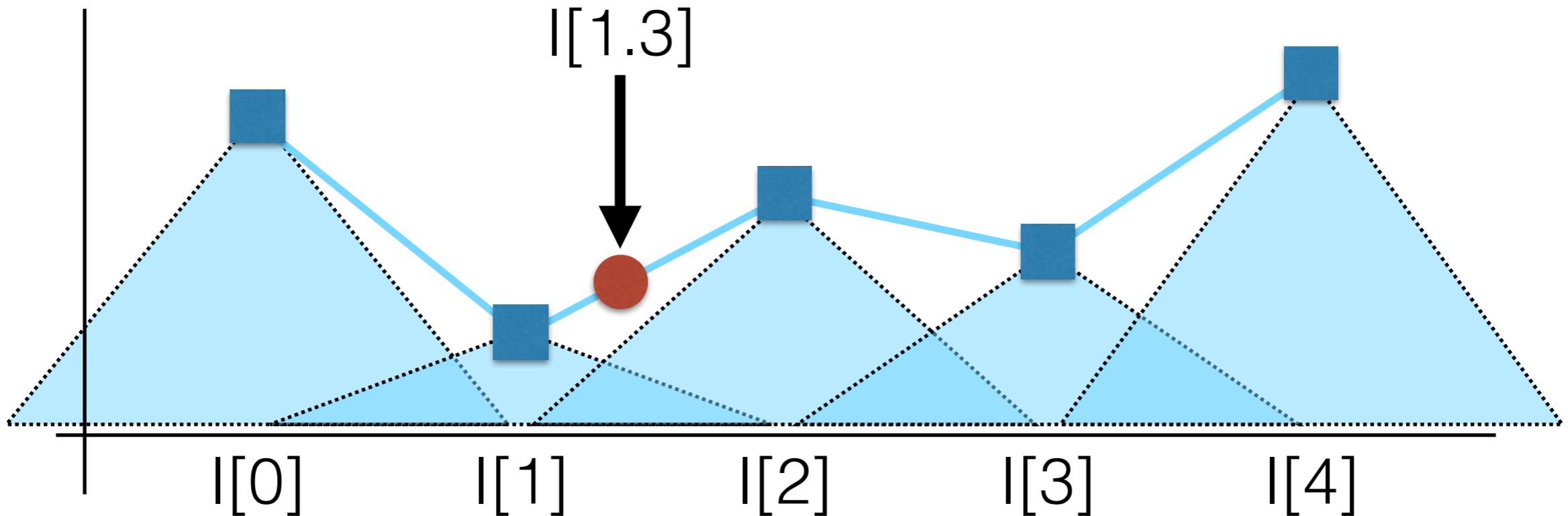
Linear Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is $I[1.3]$?
 - Let $s = 1.3 - \text{round}(1.3)$
 - $I[1.3] = 0.7*I[1] + 0.3*I[2] = (1-s)*I[1] + s*I[2]$



Linear Interpolation

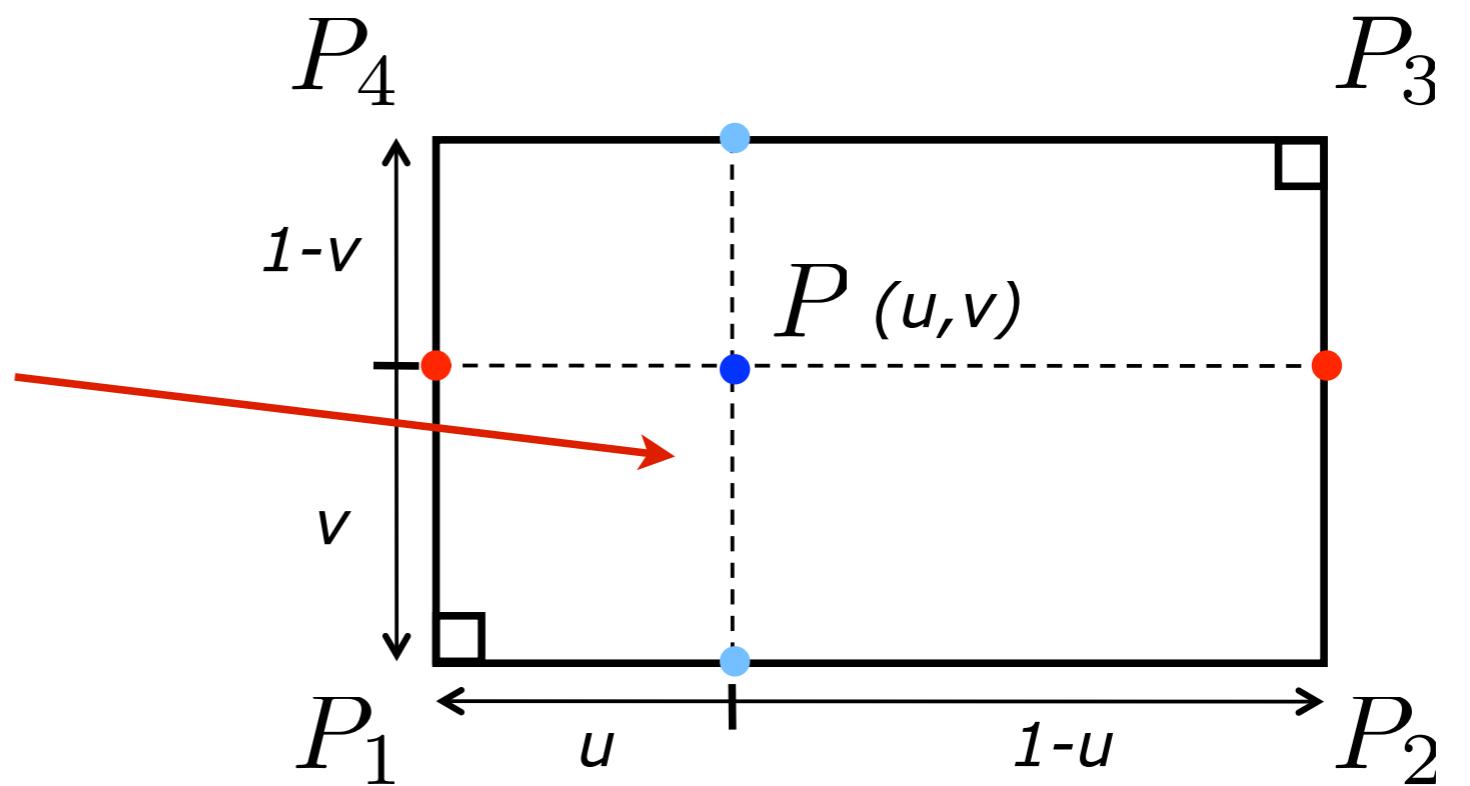
- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is $I[1.3]$?
 - Let $s = 1.3 - \text{round}(1.3)$
 - $I[1.3] = 0.7*I[1] + 0.3*I[2] = (1-s)*I[1] + s*I[2]$



Bilinear Interpolation

- In rectangle

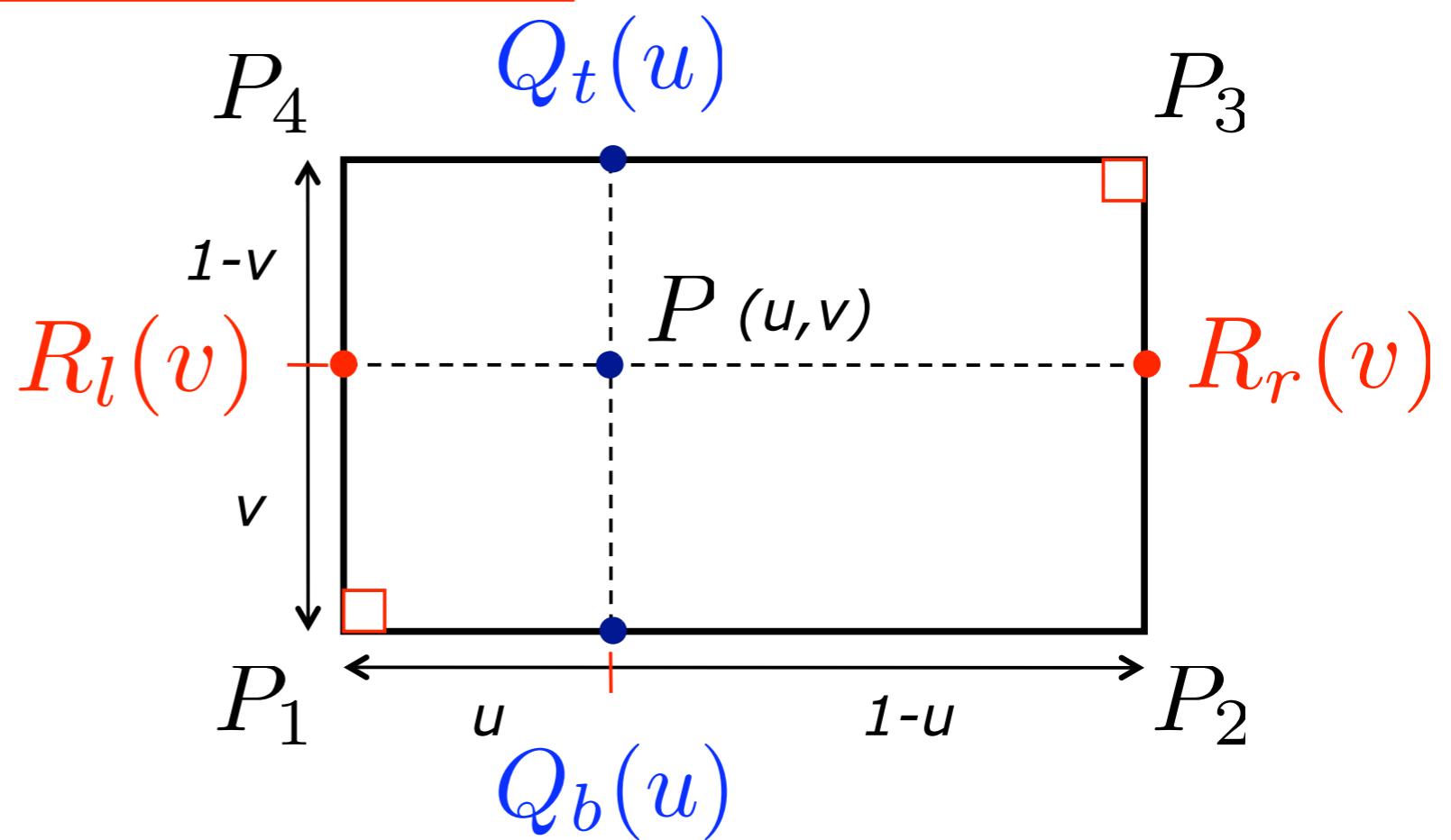
Combination of two consecutive linear interpolation



Bilinear Interpolation

- In rectangle

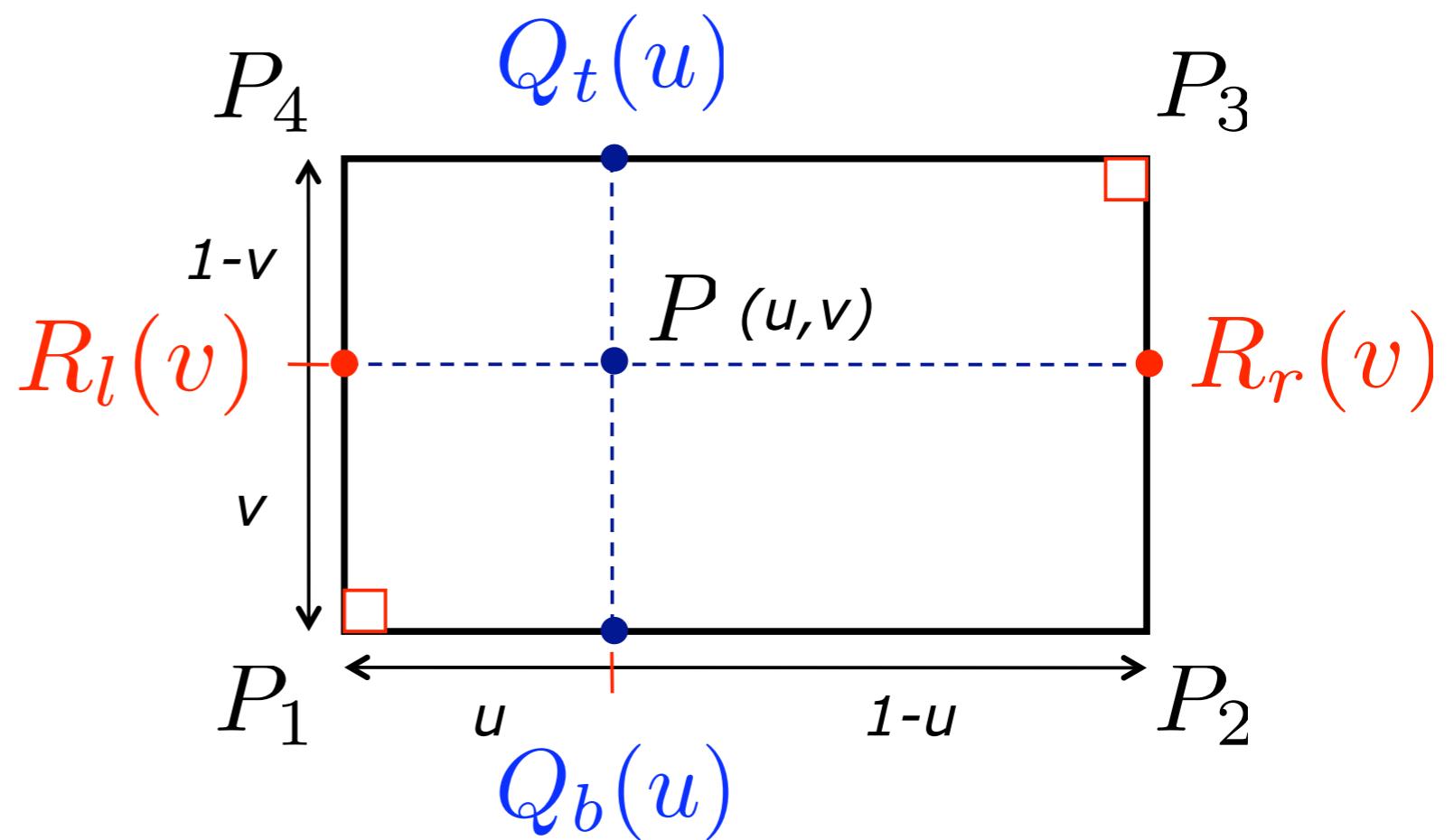
$$\begin{aligned} P &= (1 - v)Q_b(u) + vQ_t(u) \\ &= (1 - u)R_l(v) + uR_r(v) \end{aligned}$$



Bilinear Interpolation

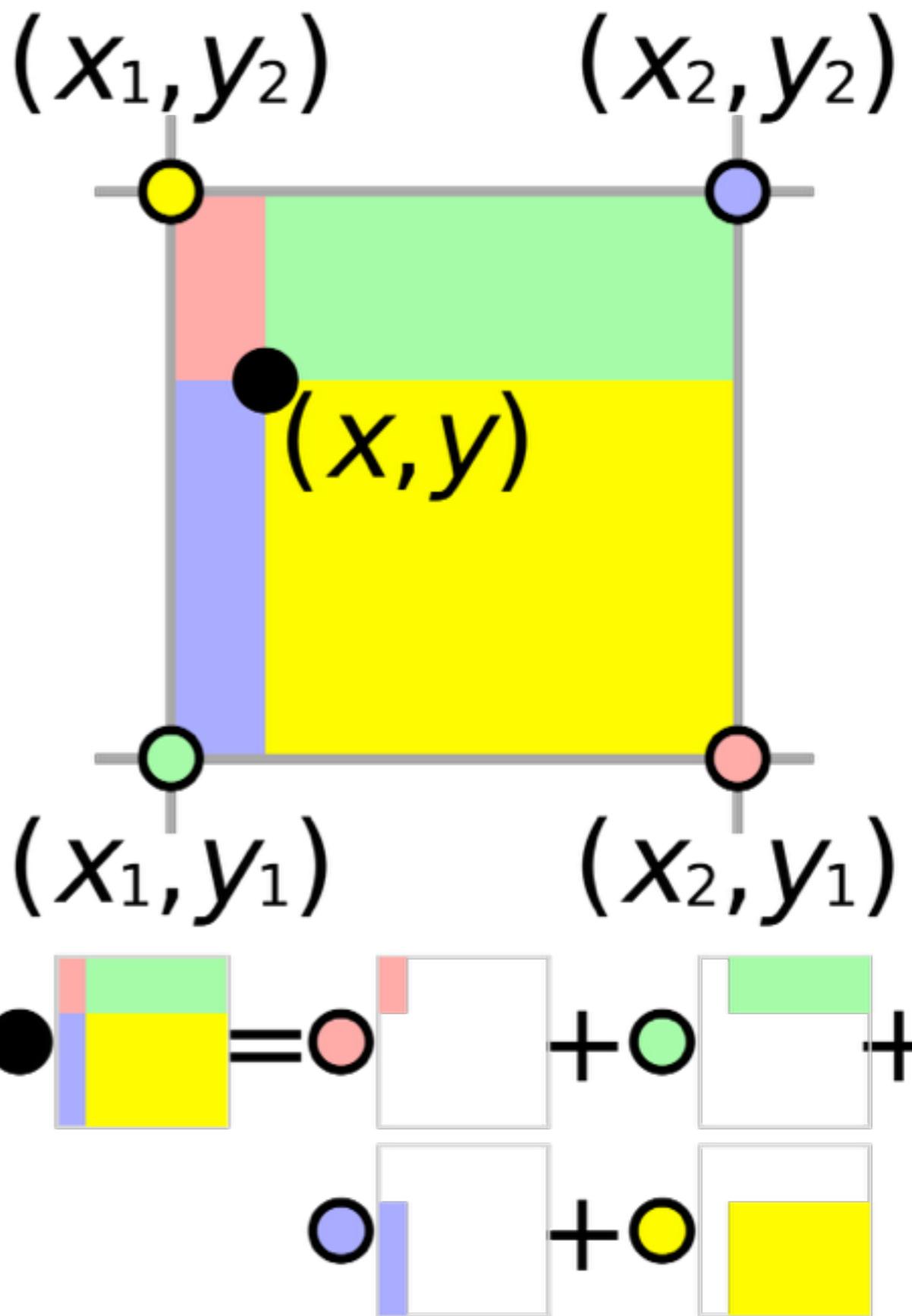
- In rectangle

$$P = P_1 + u(P_2 - P_1) + v(P_4 - P_1) \\ + uv(P_1 - P_2 + P_3 - P_4)$$



Bilinear Interpolation

- Alternate interpretation is a weighted sum of the four pixel values
- Weights defined by the area opposite each corner



Trilinear Interpolation

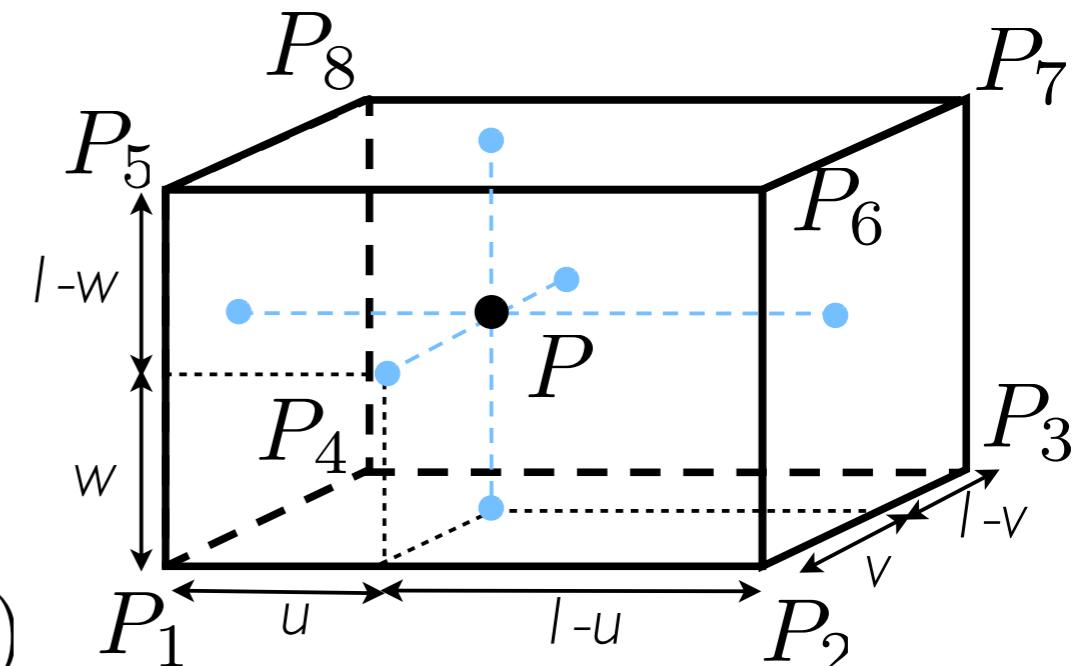
- In a cuboid (axis parallel)

- general formula

$$\phi(x, y, z) = axyz + bxy + cxz + dyz + ex + fy + gz + h$$

- with local coordinates

$$\begin{aligned} P &= P_1 \\ &\quad + u(P_2 - P_1) \\ &\quad + v(P_4 - P_1) \\ &\quad + w(P_5 - P_1) \\ &\quad + uv(P_1 - P_2 + P_3 - P_4) \\ &\quad + uw(P_1 - P_2 + P_6 - P_5) \\ &\quad + vw(P_1 - P_4 + P_8 - P_5) \\ &\quad + uvw(P_1 - P_2 + P_3 - P_4 + P_5 - P_6 + P_7 - P_8) \end{aligned}$$



Easier formula for trilinear interpolation

$$f(x, y, z) = \sum_{i,j,k=\{0,1\}} x_i y_j z_k v_{ijk}$$

Where $x_0 = i + 1 - x, x_1 = x - i$, ditto for y and z

And v_{ijk} is the value of the voxel at that vertex.

Even easier: pseudocode for trilinear interpolation

Just 7 linear interpolations!

```
#define lerp(a,b,t) (1-t) * a + t*b
```

```
//Given voxel vertices vXXX and the (x,y,z) position within the voxel [0,1]^3
```

```
//lerp along z direction.
```

```
float v000_z = lerp(v000, v001, z);  
float v010_z = lerp(v010, v011, z);  
float v100_z = lerp(v100, v101, z);  
float v110_z = lerp(v110, v111, z);
```

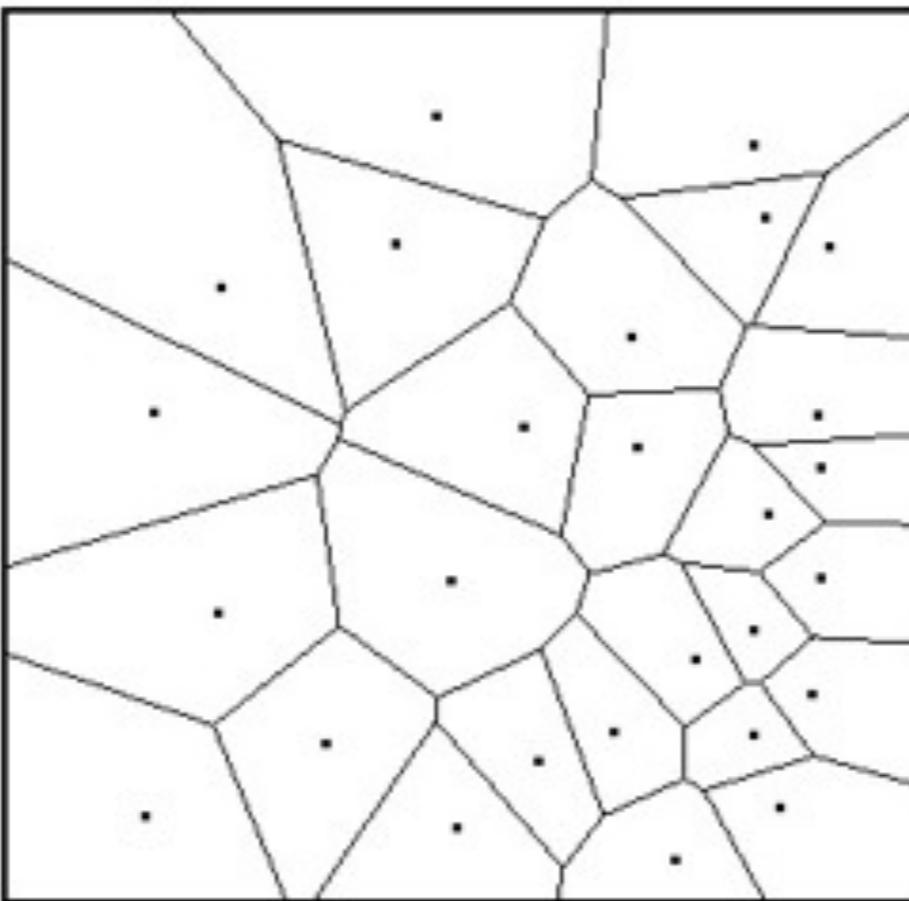
```
//lerp along y direction
```

```
float v000_yz = lerp(v000_z, v010_z, y);  
float v100_yz = lerp(v100_z, v110_z, y);
```

```
//lerp along x direction
```

```
return lerp(v000_yz, v100_yz, x);
```

Neighbor interpolation



Voronoi diagram

But Also...

- Higher-order interpolation schemes
 - splines, local polynomial fit (interpolation, least sq., ...)
 - smooth reconstruction kernels (on uniform grids)

