

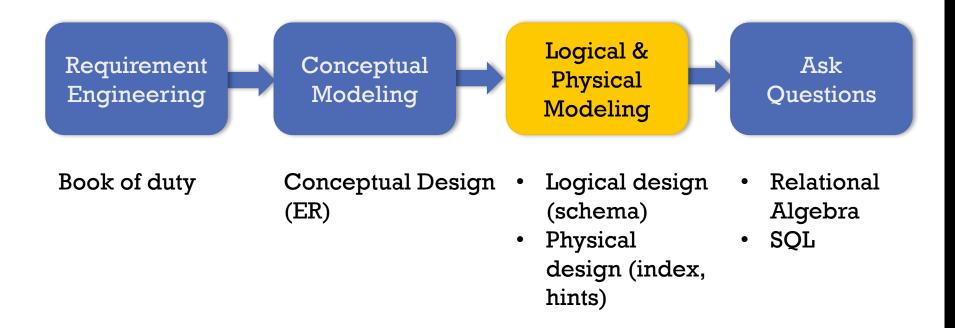
RELATIONAL-IMODEL

INTRODUCTION TO DATA SCIENCE



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DATABASES FOR DATA SCIENTIST



RELATIONAL MODEL

MODELING ELENTS

RELATIONAL MODEL - TERMS

	bname	acct_no	balance
Account =	Downtown	A-101	500
/toodant =	Brighton	A-201	900
	Brighton	A-217	500
Table name			

Attribute names

Terms

- Tables → Relations
- Columns → Attributes
- Rows → Tuples
- Schema (e.g.: Acct_Schema = (bname:string, acct_no:string, balance:double))

WHY ARE TABLES CALLED RELATIONS?

Relation:

- $R \subseteq D_1 \times ... \times D_n$
- O_1 , D_2 , ..., D_n are domains

Example: AddressBook ⊆ string x string x integer

WHY ARE TABLES CALLED RELATIONS?

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Example: AddressBook ⊆ string x string x integer

Tuple: $t \in R$

Example: t = (,/Mickey Mouse), "Main Street), 4711)

WHY ARE TABLES CALLED RELATIONS?

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- O_1 , D_2 , ..., D_n are domains

Example: AddressBook ⊆ string x string x integer

Tuple: $t \in R$

Example: t = (,/Mickey Mouse), "Main Street), 4711)

Schema: associates labels to domains

Example:

AddrBook: {[Name: string, Address: string, <u>Tel#:integer</u>]}

EXAMPLE: RELATIONS

bname	acct_no	balance
Downtown	A-101	500
Brighton	A-201	900
Brighton	A-217	500

Considered equivalent to...

```
{ (Downtown, A-101, 500),
(Brighton, A-201, 900),
(Brighton, A-217, 500) }
```

Relational database semantics are defined in terms of mathematical relations (i.e., sets)

KEYS AND RELATIONS

Kinds of keys

- Superkeys:
 set of attributes of table for which every row has distinct set of values
- Candidate keys: "minimal" superkeys
- Primary keys:
 DBA-chosen candidate key (marked in schema by underlining)

ISBN	Title	Author	Edition	Publisher	Price
0439708184	Harry Potter	J.K. Rowling	1	Scholastic	\$6.70
0545663261	Mockingjay	Suzanne Collins	1	Scholastic	\$7.39

KEYS AND RELATIONS

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- Primary keys:
 DBA-chosen candidate key (marked in schema by underlining)

Act as Integrity Constraints

i.e., guard against illegal/invalid instance of given schema e.g., Branch = (bname, bcity, assets)

bname	bcity	assets
Brighton	Brooklyn	5M
Brighton	Boston	3M



NULLS

NULL are special values that can be used for any data type

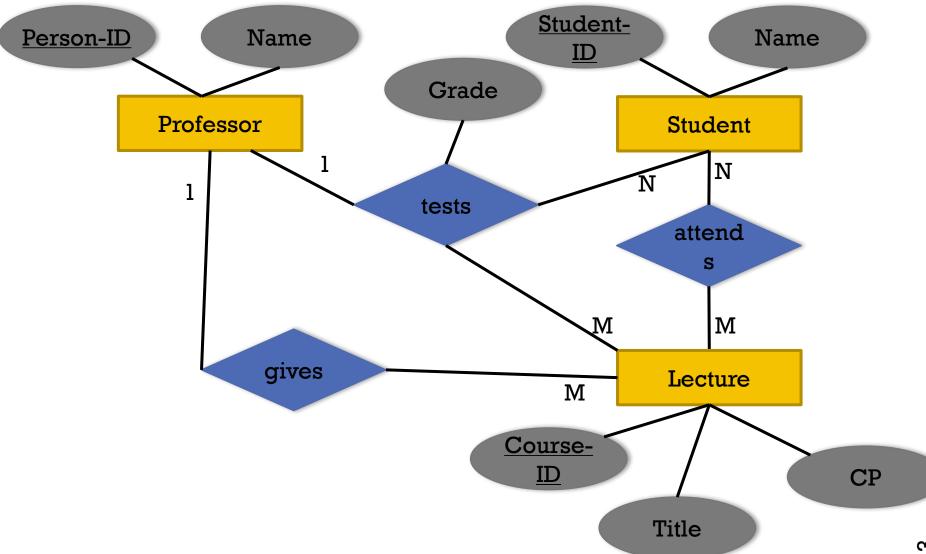
NULL typically means that value is not known

Keys can not be NULL

<u>ISBN</u>	Title	Author	Genre	Publisher	Price
0439708184	Harry Potter	J.K. Rowling	FANTASY	Scholastic	\$6.70
0545663261	Mockingjay	Suzanne Collins	NULL	Scholastic	\$7.39

TRANSLATION OF ER TO RELATIONAL MODELS

HOW TO TRANSLATE ER TO RELATIONS?

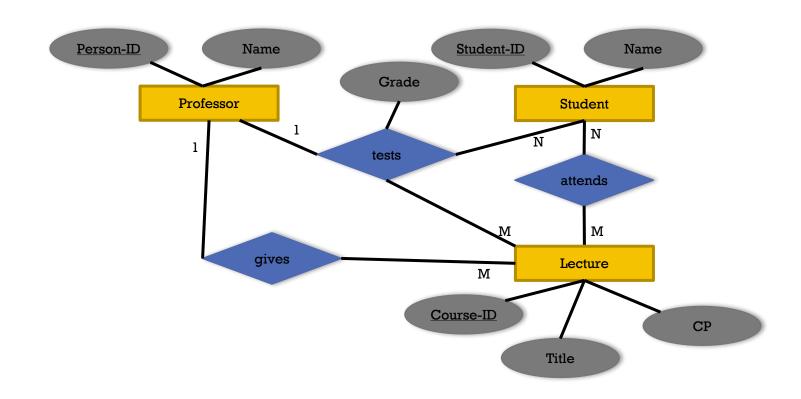


RULE #1: ENTITIES

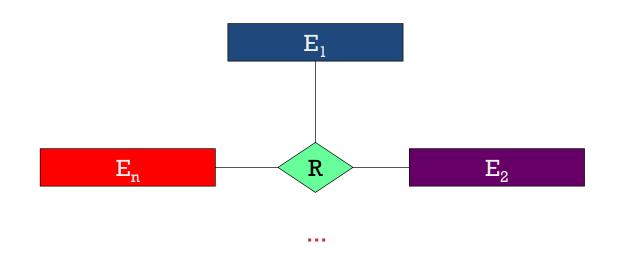
Professor(Person-ID:integer, Name:string)

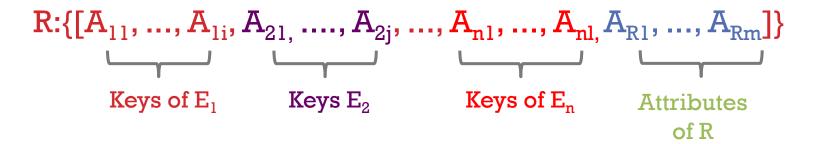
Student(Student-ID:integer, Name:string)

Lecture(Course-ID:string, Title:string, CP:float)



RULE #2: RELATIONSHIPS





RULE #2: RELATIONSHIPS

Professor(Person-ID:integer, Name:string)

Student(Student-ID:integer, Name:string)

Lecture(Course-ID:string, Title:string, CP:float)

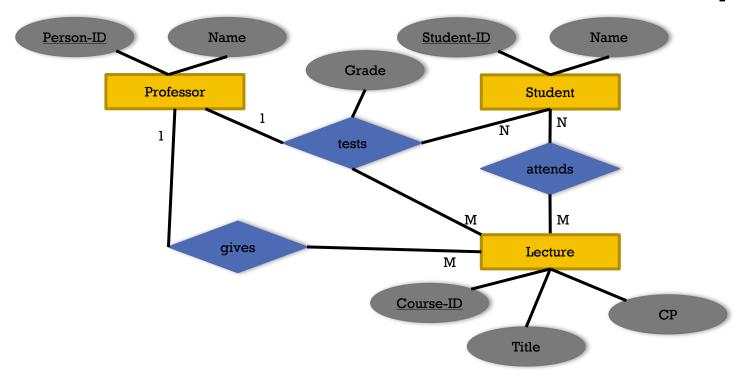
Gives(Person-ID:integer, Course-ID:string)

Attends(Student-ID:integer, Course-ID:string)

Tests(Student-ID:integer, Course-ID:string, Person-ID:integer,

Grade:String)

What about keys?

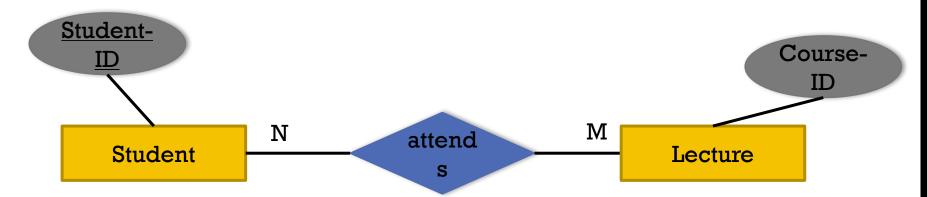


EXAMPLE: KEYS OF ATTENDS?

Student	
Student- ID	
1	
2	
4	
5	
6	
10	

Attends		
Student- ID	Course- ID	
1	CS1951a	
1	CS167	
2	CS1951a	
2	CS167	
3	CS18	
•••		

Lecture		
Course- ID		
CS1951a	•••	
CS195w	•••	
CS18	•••	
CS17	•••	
CS142		
CS167		



RULE #2: RELATIONSHIPS

Keys depends on type of relationship

1:1 - Take key of one of the connected entities

1:N - Take key from the N-side

N:M – Combine keys of connected entities

RULE #2: RELATIONSHIPS

Professor(Person-ID:integer, Name:string)

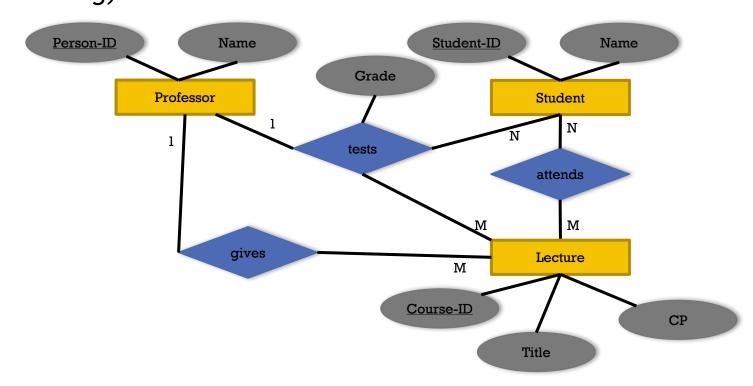
Student(Student-ID:integer, Name:string)

Lecture (Course-ID:string, Title:string, CP:float)

Gives(Person-ID:integer, Course-ID:string)

Attends(Student-ID:integer, Course-ID:string)

Tests(Student-ID:integer, Course-ID:string, Person-ID:integer,, Grade:string)



RULE #3: MERGE RELATIONS

Merge relations of relationship into relations of entities

Professor(Person-ID:integer, Name:string)

Lecture(Course-ID:string, Title:string, CP:float)

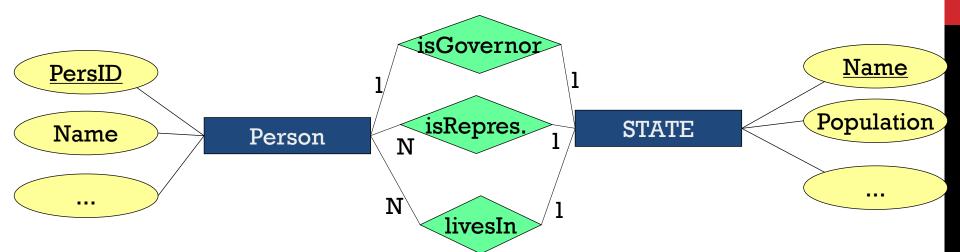
Gives(Person-ID:integer, Course-ID:string)



Professor(Person-ID:integer, Name:string)

Lecture(Course-ID:string, Title:string, CP:float, Person-ID:integer)

RULE #3: EXCEPTION



	Person				
PersID	Name	:	lives In	is Repr.	is Governor
4711	Binnig		RI	NULL	NULL
4813	Pyne		RI	NULL	NULL
	•••				
9011	Geller		RI	RI	NULL
9012	Raymondo		RI	RI	RI
		•••	•••	•••	

Problem: Many NULLs

Solution: Do no merge

relations!

FINAL RELATIONAL MODEL

Professor(Person-ID:integer, Name:string)

Student(Student-ID:integer, Name:string)

Lecture(Course-ID:string, Title:string, CP:float,

Person-ID:integer)

Attends(Student-ID:integer, Course-ID:string)

Tests(Student-ID:integer, Course-ID:string,

Person-ID:integer, Grade:string)

Why didn't we merge **Attends** and **Tests**?

RELATIONAL MODEL

WHAT IS A GOOD RELATIONAL MODEL?

WHAT IS A BAD RELATIONAL MODEL?

Redundancy can lead to anomalies?

PersonID	Name	•••	CourseID	Title
1	Upfal		1	Statistics
1	Upfal		2	Intro to Data Science
2	Felzen- schwalb		3	ML
3	Mr. X		NULL	NULL

- Update anomaly: not all values are updates
- Insert anomaly: NULL values must be inserted
- Delete anomaly: Information is lost

NORMALIZATION

Process of decomposing a relational schema to avoid redundancy

Decomposition must be lossless

Normal forms:

- First and Second NF: Contain some redundancy
- Third NF: Used in practice
- Boyce-Codd NF: Special form of Third NF
- Other NFs: Forth and Fifth NF

LOSSLESS DECOMPOSITION

PersonID	Name	•••	CourseID	Title
1	Upfal	•••	1	Statistics
1	Upfal		2	Intro to Data Science
2	Felzen- schwalb	•••	3	ML



Decomposition



<u>PersonID</u>	Name
1	Upfal
2	Felzen-
	schwalb

CourseID	Title	PersonID
1	Statistics	1
2	Intro to Data Science	1
3	ML	2



Reconstruction possible?

FIRST NORMAL FORM

"all columns represent atomic (non-complex) values"

<u>PersonID</u>	Name		Courses
1	Upfal	•••	{1, Stats}, {2, Intro to Data Science}
2	Felzen- schwalb		{3, ML}
3	Mr. X		NULL

Here: Courses is NOT atomic

SECOND NORMAL FORM

"all of a relation's nonkey attributes are dependent on all keys"

PersonID	Name	•••	CourseID	Title
1	Upfal		1	Statistics
1	Upfal		2	Intro to Data Science
2	Felzen- schwalb		3	ML
3	Mr. X		NULL	NULL
4	Binnig		2	Intro to Data Science

Here: Name only depends on PersonID and not on CourseID

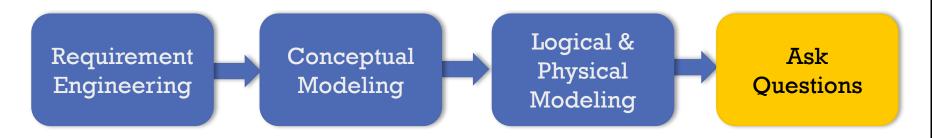
THIRD NORMAL FORM

"if it is in second normal form and has no transitive dependencies"

PersonID	Name	Zip	City
1	Upfal	02902	Providence
2	Felzen- schwalb	02905	Cranston
3	Mr. X	02902	Providence

Here: City depends on Zip and Zip on PersonID

DATABASES FOR DATA SCIENTIST



Book of duty

Conceptual Design (ER)

- Logical design (relational schema)
- Physical design (index, hints)
- Relational Algebra
- SQL

REMINDER

1) Clicker: Register in Banner

2) Assignment 1: Due by 2/9

3) Assignment 2: Out 2/9

4) Projects: Dan will come in class 2/9

RELATIONAL MODEL

RELATIONAL ALGEBRA

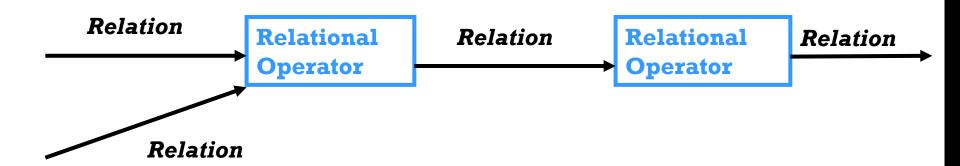
FORMAL DEFINITION OF REL. ALGEBRA

Operators (unary and binary):

- Selection: σ (E1)
- Projection: Π (E1)
- Cartesian Product: E1 x E2
- Rename: $\rho_{V}(E1)$, $\rho_{A\leftarrow B}(E1)$
- Union: El \cup E2
- **Minus**: E1 E2
- Other: Joins, Aggregation, ...

Input and Output: Relations

CLOSURE PROPERTY / COMPOSABILITY



Professor(<u>Person-ID:integer</u>, Name:varchar(30), Level:varchar(2))

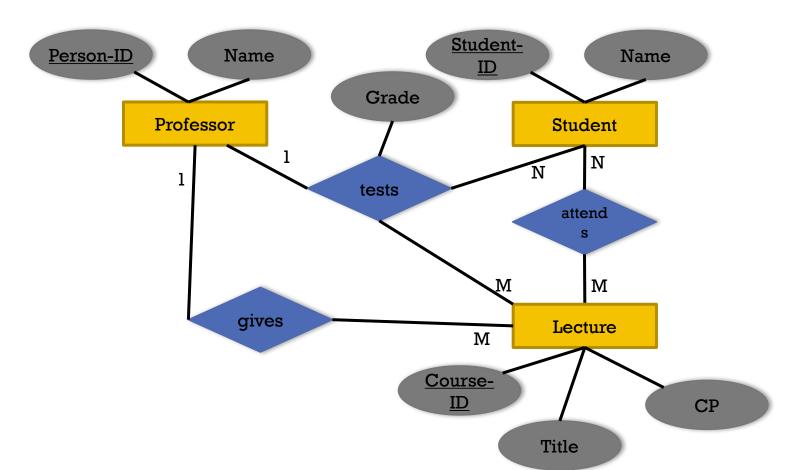
Student(Student-ID:integer, Name:varchar(30), Semester:integer)

Lecture(Course-ID:varchar(10), Title:varchar(50), CP:float)

Gives(Person-ID:integer, Course-ID:varchar(10))

Attends(Student-ID:integer, Course-ID:varchar(10))

Tests(Student-ID:integer, Course-ID:varchar(10), Person-ID:integer, Grade:char(2))



SELECTION AND PROJECTION

 $Professor(\underline{Person\text{-}ID\text{:}integer}, Name: varchar(30), Level: varchar(2))$

Student(Student-ID:integer, Name:varchar(30), Year:integer)

Selection

σ _{Year<5} (Student)		
Student-ID	Name	Year
24002	Sally	2
25403	Emily	3

Projection

$\Pi_{Name}(Professor)$
Name
Eli
Stephanie

CARTESIAN PRODUCT

X

L				
Α	В	С		
a ₁	b ₁	C ₁		
a ₂	b ₂	C ₂		

F	R			
D	Е			
d ₁	e ₁			
d ₂	e ₂			

Result						
A	В	С	D	Е		
a_1	b ₁	C ₁	d_1	e_1		
a_1	b ₁	C ₁	d ₂	e ₂		
a ₂	b ₂	C ₂	d_1	e_1		
a ₂	b ₂	C ₂	d ₂	e_2		

CARTESIAN PRODUCT (CTD.)

Student x attends?

Student					
Student- Name Year ID					
26120	Sally	2			
29555	Emily	3			

Attends				
Student-ID				
	ID			
26120	5001			
26120	5004			
29555	5004			
29555	5111			

CARTESIAN PRODUCT (CTD.)

Student x attends

	Student	Atten	ds	
Student- ID	Name	Year	Student-ID	Course- ID
26120	Sally	2	26120	5001
26120	Sally	2	26120	5004
26120	Sally	2	29555	5004
26120	Sally	2	29555	5111
29555	Emily	3	26120	5001
	•••	•••	•••	

- Huge result set (n * m)
- Usually only useful in combination with a selection (-> Join)

NATURAL JOIN

Two relations:

```
•R(A_1,...,A_m,B_1,...,B_k)
```

•
$$S(B_1,...,B_k,C_1,...,C_n)$$

$$R \bowtie S = \prod_{A_{1,...,Am,R.B_{1,...,R.Bk},C_{1,...,C_{n}}} (\sigma_{R.B_{1}=S.B_{1}})$$

$$\wedge ... \wedge R.B_{k} = S.B_{k}(RxS)$$

$R \bowtie S$											
$R-S$ $R\cap S$ $S-R$											
Δ	Λ		Λ	$B_1 \mid B_2 \mid \dots \mid B_k$							
^ 1	\mathbf{A}_2		$ \mathbf{A}_{m} $	D_1	B ₂		B_k	C_1	C_2		C_n

NATURAL JOIN

(Student \bowtie attends)

	Attends		
Student- ID	Name	Year	Course- ID
26120	Sally	2	5001
26120	Sally	2	5004
29555	Emily	3	5004
29555	Emily	3	5011

THETA-JOIN

Two Relations:

- R(A1, ..., An)
- S(B1, ..., Bm)

$$R \bowtie_{\theta} S = \sigma_{\theta} (R \times S)$$

$\mathbf{R}\bowtie_{\theta}\mathbf{S}$							
R S							
A_1	$oldsymbol{A}_1 \qquad oldsymbol{A}_2 \qquad \dots \qquad oldsymbol{A}_n$				${\mathtt B}_2$		B _m
		:	:	:	:	:	:

• natural join

L				
Α	В	C		
a_1	b_1	C ₁		
a ₂	b_2	C_2		



	R	
С	D	Е
C_1	d_1	e_1
C ₃	d_2	e_2

	Result					
=	Α	В	C	D	Е	
	a_1	b_1	C ₁	d_1	e_1	

• left outer join

	L	
Α	В	U
a_1	b_1	C ₁
a ₂	b_2	C ₂



	R	
C	D	Е
C_1	d_1	e_1
C ₃	d_2	e_2

	Result				
	Α	В	O	О	Е
=	a_1	b_1	C ₁	d_1	e_1
	a ₂	b_2	C ₂	1	1

• right outer join

T		
Α	В	C
a_1	b_1	C ₁
a_2	b_2	C ₂



	R	
O	D	Ш
C_1	d_1	e_1
C ₃	d_2	e_2

	Result			
Α	В	C	D	Е
a_1	b_1	C ₁	d_1	e_1
-	1	C ₃	d_2	e_2

• (full) outer join

L		
Α	В	O
a_1	b_1	C_1
a ₂	b ₂	C_2



	R	
C	D	Е
C ₁	d_1	e_1
C ₃	d_2	e_2

	Result				
	A	В	U	D	Ш
=	a_1	b_1	C_1	d_1	e_1
	a_2	b_2	C ₂	1	1
	1	1	C ₃	d_2	e_2

• left semi join

L		
Α	В	C
a_1	b_1	C ₁
a_2	b_2	C ₂



	R	
С	D	Е
C_1	d_1	e_1
C ₃	d_2	e_2

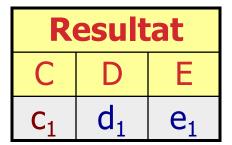
	Result		
=	Α	В	С
	a_1	b_1	C ₁

• right semi join

F			
A B C			
a_1	b_1	C_1	
a_2	b_2	C ₂	



R			
C D E			
C_1	d_1	e_1	
C ₃	d_2	e_2	



AGGREGATION OPERATOR

 $\chi_{group,aggreation}$

orderID	custID	custName	orderTotal
1	1	Sally	100
2	1	Sally	500
3	1	Sally	400
4	2	Emily	333

 χ {custID, custName}, {sum(oTotal), min(oTotal)}

custID	custID	Sum	Min
1	Sally	1000	100
2	Emily	333	333

RENAME OPERATOR

Renaming of relation names

- Needed to process self-joins and recursive relationships
- E.g., two-level dependencies of lectures (,,grandparents ")

Renaming of attribute names

 $\rho_{Requirement \;\leftarrow \; Prerequisite} \; (requires)$

Ρεθυιρεσ

course-id	prerequisite
CS1951A	CS160
CS160	CS320
CS2270	CS1270
CS1270	CS160

SET DIFFERENCE (–)

Notation: Relation₁ - Relation₂

R - S valid only if:

- 1. R, S have same number of columns (arity)
- 2. R, S corresponding columns have same domain (compatibility)

Example:

$$(\Pi_{\text{bname}} (\sigma_{\text{amount} \ge 1000} (\text{loan}))) - (\Pi_{\text{bname}} (\sigma_{\text{balance} < 800} (\text{account})))$$

loan

bname	lno	amount
Downtown	L-17	1000
Redwood	L-23	2000
Perry	L-15	1500
Downtown	L-14	500
Perry	L-16	300

account

bname	acct_no	balance
Mianus	A-215	700
Brighton	A-201	900
Redwood	A-222	700
Brighton	A-217	850

Result?

(3)

(2)

bname

bnamebnameMianusDowntownRedwoodRedwoodPerry

SET DIFFERENCE (–)

Notation: Relation₁ - Relation₂

R - S valid only if:

- 1. R, S have same number of columns (arity)
- 2. R, S corresponding columns have same domain (compatibility)

Example:

$$(\Pi_{\text{bname}} (\sigma_{\text{amount} \ge 1000} (\text{loan}))) - (\Pi_{\text{bname}} (\sigma_{\text{balance} < 800} (\text{account})))$$

loanbnamelnoamountDowntownL-171000RedwoodL-232000PerryL-151500DowntownL-14500PerryL-16300

account			
bname	acct_no	balance	
Mianus	A-215	700	
Brighton			
Redwood	A-222	700	
Brighton		850	

account

= (1)

bname

Mianus
Redwood

bname

Downtown
Redwood
Perry

Result?

bname

Downtown

Perry

(2)

INTERSECTION

$$\Pi_{Person-ID}(Lecture) \cap \Pi_{Person-ID}(\sigma_{Area=DB}(Professor))$$

Only works if both relations have the same schema

Same attribute names and attribute domains

Intersection can be simulated with minus:

$$\mathbf{R} \cap \mathbf{S} = \mathbf{R} - (\mathbf{R} - \mathbf{S})$$

Union should be trivial ...

RELATIONAL DIVISION

Relational Division $R \div S$ can be used for universal quantfication

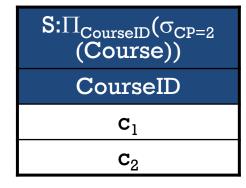
Example: Find StudentIDs (Student), that attended all 2CP lectures

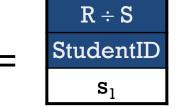
Algebra: attend \div S while S = $\Pi_{\text{CourseID}}(\sigma_{\text{CP=2}}(\text{Course}))$

Universal Quantfier: $\{s \mid s \in Student \land \forall l \in Course(c.CP=2\Rightarrow \exists a \in attend(a.CourseID=c.CourseID \land a.StudentID=s.StudentID))\}$

EXAMPLE: RELATIONAL DIVISION

R: attend		
Student ID	Course ID	
\mathbf{s}_1	$\mathtt{c}_{\scriptscriptstyle 1}$	
\mathbf{s}_1	$c_{\scriptscriptstyle 2}$	
\mathbf{s}_1	c_3	
\mathbf{s}_2	$c_{\scriptscriptstyle 2}$	
\mathbf{s}_2	c_3	





CODD'S THEOREM

3 Languages:

- Relational Algebra
- Tuple Relational Calculus (safe expressions only)
- Domain Relational Calculus (safe expressions only)

are equivalent.

Impact of Codd's theorem:

- SQL is based on the relational calculus (but with duplicates!)
- SQL implementation is based on relational algebra
- Codd's theorem shows that SQL is correct and complete.

FURTHER MATERIAL

Not covered here

- Aggregate Functions
- Codd's Proof
- •

But why not?

- Sorting
- Duplicate Elicitation

IN CLASS TASKS

Player

PlayerID	Name	Age	Team
1	Russel	27	Seahawks

Team

Team	State
Seahawks	Washington

Played

PlayerID	Date	Place	Score
1	2/1/15	Phoenix	3

In relational algebra:

- 1) Return all teams, who played at least once in Phoenix
- 2) Return all Seahawks player names, who did not play in the entire season

CLICKER QUESTION I

Player

PlayerID	Name	Age	Team
1	Russel	27	Seahawks

Team

Team	State
Seahawks	Washington

Played

PlayerID	Date	Place	Score
1	2/1/15	Phoenix	3

Return all teams, who played at least once in Phoenix

- A) Π_{Team} ($\sigma_{\text{Place}=\text{`Phoenix'}}$ (Player X Played))
- B) Π_{Team} (Player \bowtie ($\sigma_{\text{Place}=\text{`Phoenix}}$ (Played)))
- C) Π_{Team} ($\sigma_{\text{Place}=\text{`Phoenix'}}$ (Player \bowtie Played))

CLICKER QUESTION I

Player

PlayerID	Name	Age	Team
1	Russel	27	Seahawks
	•••	•••	

Team

Team	State
Seahawks	Washington

Played

PlayerID	Date	Place	Score
1	2/1/15	Phoenix	3

Return all teams, who played at least once in Phoenix

- A) Π_{Team} ($\sigma_{\text{Place}=\text{`Phoenix'}}$ (Player X Played))
- B) Π_{Team} (Player \bowtie ($\sigma_{\text{Place}=\text{`Phoenix}}$ (Played)))
- C) Π_{Team} ($\sigma_{\text{Place='Phoenix'}}$ (Player \bowtie Played))

CLICKER QUESTION II

- 1. Π_{Team} ($\sigma_{\text{Place}=\text{`Phoenix'}}$ (Player \bowtie Played))
- 2. Π_{Team} (Player \bowtie ($\sigma_{\text{Place}=\text{`Phoenix}}$ Played))
- 3. Π_{Team} ($\sigma_{\text{Place}=\text{`Phoenix'}}$ (Player \bowtie Played \bowtie Team))

Which of these expressions are equivalent?

- A) All
- B) l and 2
- C) 1 and 3
- D) 2 and 3
- E) None

CLICKER QUESTION II

- 1. Π_{Team} ($\sigma_{\text{Place}=\text{`Phoenix'}}$ (Player \bowtie Played))
- 2. Π_{Team} (Player \bowtie ($\sigma_{\text{Place}=\text{`Phoenix}}$ Played))
- 3. Π_{Team} ($\sigma_{\text{Place}=\text{`Phoenix'}}$ (Player \bowtie Played \bowtie Team))

Which of these expressions are equivalent?

- A) All
- B) l and 2
- C) 1 and 3
- D) 2 and 3
- E) None

CLICKER QUESTION III

Player

PlayerID	Name	Age	Team
1	Russel	27	Seahawks

Team

Team	State
Seahawks	Washington

Played

PlayerID	Date	Place	Score
1	2/1/15	Phoenix	3

Return all Seahawks player names, who did not play so far

- A) Π_{Name} ($\sigma_{Team=`Seahawks`}$ (Player \longrightarrow Played))
- B) Π_{Name} ($\sigma_{\text{Team=`Seahawks`}}$ (Player)) Π_{Name} ($\sigma_{\text{Team=`Seahawks`}}$ (Player \bowtie Played))
- C) Π_{Name} ($\sigma_{\text{Team=`Seahawks`}}$ (Player \times Played))
- D) Π_{Name} ($\sigma_{\text{Team=`Seahawks`} \land \text{Date is null)}}$ (Player \blacktriangleright Played))

CLICKER QUESTION III

Player

PlayerID	Name	Age	Team
1	Russel	27	Seahawks

Team

Team	State
Seahawks	Washington

Played

PlayerID	Date	Place	Score
1	2/1/15	Phoenix	3

Return all Seahawks player names, who did not play so far

- A) Π_{Name} ($\sigma_{Team=`Seahawks`}$ (Player \longrightarrow Played))
- B) Π_{Name} ($\sigma_{\text{Team=`Seahawks`}}$ (Player)) Π_{Name} ($\sigma_{\text{Team=`Seahawks`}}$ (Player \bowtie Played))
- C) Π_{Name} ($\sigma_{\text{Team=`Seahawks`}}$ (Player \times Played))
- D) Π_{Name} ($\sigma_{\text{Team=`Seahawks`} \land \text{Date is null)}}$ (Player \blacktriangleright Played))

SUMMARY

Relational Model

- Tables / columns / rows
- set-oriented
- Keys
- NULL values

Translation of ER-Models into Relational Model

Relational Algebra

- Query Relations
- Composable
- Duplicate-free