①

1. This lazy student only needs to learn 2 topics, i.e. the two topics assigned by the teacher.

2. The lazy student needs to study a minimum of A topics assigned by the professor to ensure that the content studied covers the exam topics.

②

1. There are n read steps in the “horizontal” sub-round and there are n read steps in the “vertical” sub-round. The total number of read/write steps of the round of a processor is 2n.

2. Values at the nodes will be all 1.

$$\begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ 1 & 2 & 3 & \cdots & n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 3 & \cdots & n \end{pmatrix}.$$

③

1. Since the nodes in G are naturally sorted from left to right

Algorithm 1 Coloring a linear unit disk graph G sequentially

Require: $G := (V, E)$
  for $v \in V$ **do** ▷ In the order from left to right
    Color v using a minimum color which does not conflict with already colored nodes
  end for

Algorithm 2 Compute the clique number $\omega(G)$ for a linear unit disk graph G

Require: $G := (V, E)$
  $w \leftarrow 0$
  for $v \in V$ **do** ▷ In the order from left to right
    $w_v \leftarrow 1$ ▷ One vertex is a 1-clique
    for $u \in \{x | x \in V$ and x is at right of $v\}$ **do** ▷ In the order from left to right
      if $(u, v) \in E$ **then**
        $w_v \leftarrow w_v + 1$
      else
        break ▷ Not this clique
      end if
    end for
    $w = \max(w_v, w)$ ▷ Update clique number
  end for
  return w

2.
3. Since u and v in any edge (u, v) ∈ E either belong to the same unit interval or are adjacent to each other. Therefore their coloring must be different.
   Algorithm 1 and Algorithm 2 indicate that for a linear unit disk graph, its minimum number of colorings and its clique number are equal, i.e. χ(G) = ω(G). Thus Algorithm 1 is the optimal coloring solution. Therefore the competitive ratio of the given coloring method is $\frac{2\omega(G)}{\omega(G)} = 2$

If Anthony is right, then Henry and George are wrong. At this point only one statement is correct. Fred has more than 1000 dollars.

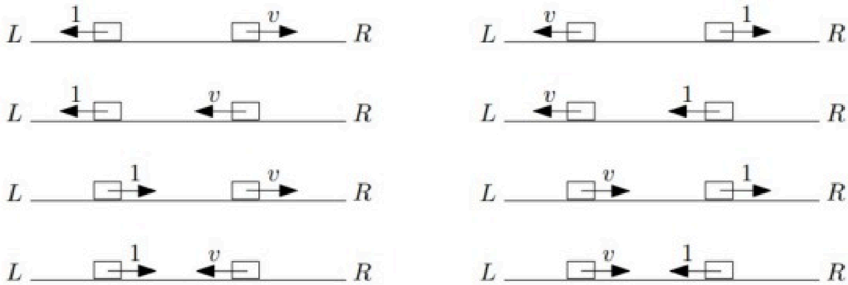
If George is right, then Henry and Anthony are wrong. At this point only one statement is correct. Fred has 1000 dollars.

If Henry is right, then George is right and Anthony is wrong. At this point two statements are correct. It's impossible.

Fred should have 1000 or more dollars.

⑤

1. The two robots are indistinguishable except for their speed, so the possible configurations are shown below.



2. The optimal strategy should be that first each robot tries to get the package to the left as soon as possible, and then delivers it to the right. After a robot gets the package, when the two robots meet, the package should be passed to the robot with the higher speed for further delivery

```
Each robot executes the following
Move to left
if arrive L then
    if Package is still here then
        Pick up the package
    end if
end if
Move to right
if meet another robot then
    Compare speed with the other robot
    Give the package to the robot with higher speed
end if
Move to right
if arrive R then
    if Package is still here then
        Put down the package
    end if
end if
```

⑥

$$1-\left(1-\frac{1}{k}\right)^{n}=1-\left(1-\frac{1}{13}\right)^{130}\approx 1-3.026\times 10^{-5}=0.99996974$$

Some students with the same score on both exams.

Find minimum value on a ring

Each node executes the following
for each clock cycle **do**
    if this node is the initiator **then**
        Send v to its two neighbors ▷ where v is the value this node having
        Terminate
    end if
    if Receive a value v_1 from a neighbor **then**
        Send $\min(v, v_1)$ to another neighbor ▷ where v is the value this node having
        Terminate
    else if Receive two value v_1, v_2 from a neighbor **then**
        $\min(v, v_1, v_2)$ is the minimum of the ring
        Terminate
    end if
end for

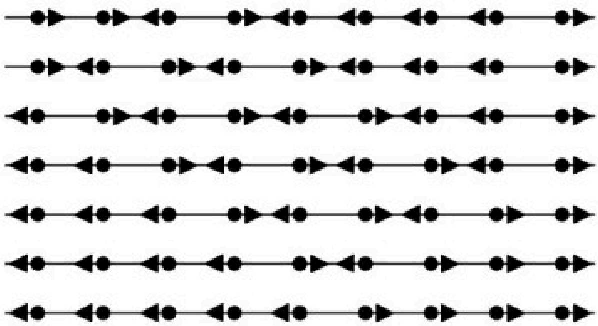
This algorithm requires $\lceil \frac{n}{2} \rceil$ steps to execute the whole algorithm.

Consider that for a given chick the probability of not being pecked is ¼. The expectation of the number of times it is pecked is ¼.

Note that the fact that the ith chick is not pecked implies that the i − 2th and i + 2th chicks are necessarily pecked. This shows that they are not independent of each other.
$E[X_1 + X_2] = E[X_1] + E[X_2]$, where X_1 and X_2 need not be independent of each other.

$$E\Big[\sum_{i=1}^{n} X_i\Big] = \sum_{i=1}^{n} \Big(E[X_i]\Big) = \frac{n}{4}$$

⑧



When n = 1, it is obvious that the flipping rule results in a terminating arrangement. Assume that this proposition is correct on the line graph with a size of n − 1. Then let us face the question of whether the n line graph will terminate. If the n-th node is not taken into account, then the subgraph consisting of the first n − 1 nodes can reach a terminating arrangement.

Consider several cases of adding the n-th node:

The arrow of the n-th node is to the right: then this line graph enters a terminating arrangement.

The arrow of the n-th node is to the left: If during the flipping operation of the first n − 1 nodes, the arrow of the n − 1-th node faces right, then this will cause the arrow of the n-th node to face right and eventually all nodes will enter a terminating arrangement. Or, if the arrow of the n − 1-th node always faces left, then the nodes will enter a terminating arrangement.

In total, the n − 1 size line graph results in a terminating arrangement that propagates the conclusions to n size graphs. Thus we prove inductively that for any line graph and any initial arrangement of arrows the application of the flipping rule results in a terminating arrangement. Adding a new node will increase the worst runtime by a constant.
The runtime should be linear as a function of n.

⑨

1. $\frac{d}{1+v}$
2. $\frac{d}{v}$
3. $\frac{v+1}{v}$