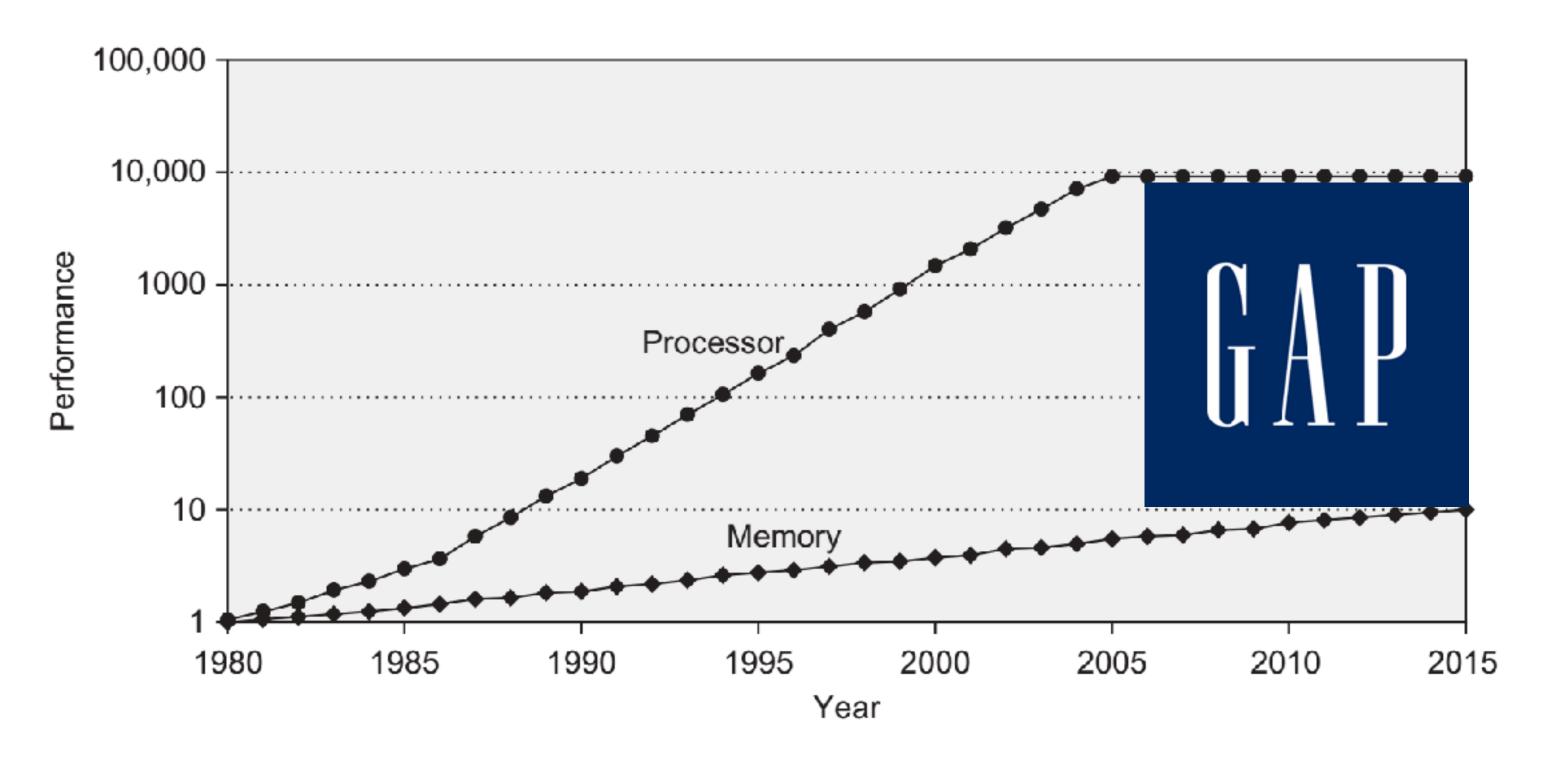
Memory Hierarchy Inside Out: (5) Cache misses and their remedies (III)

Hung-Wei Tseng

Recap: Performance gap between Processor/Memory



Recap: Memory Hierarchy Processor fastest 1 **Processor** < 1ns Core fastest Registers L1\$ L2\$ SRAM\$ a few ns L3\$ **DRAM** tens of ns larger v TBs Storage us/ms larger

Review: C = ABS

- C: Capacity in data arrays
- A: Way-Associativity how many blocks within a set
 - N-way: N blocks in a set, A = N
 - 1 for direct-mapped cache
- **B**: **B**lock Size (Cacheline)
 - How many bytes in a block

memory address:

- **S**: Number of **S**ets:
 - A set contains blocks sharing the same index
 - 1 for fully associate cache
- number of bits in block offset lg(B)
- number of bits in set index: Ig(S)
- tag bits: address_length lg(S) lg(B)
 - address_length is 64 bits for 64-bit machine address
- $\frac{1}{block \ size}$ (mod S) = set index

set block tag index offset 0b000010000010000100001000

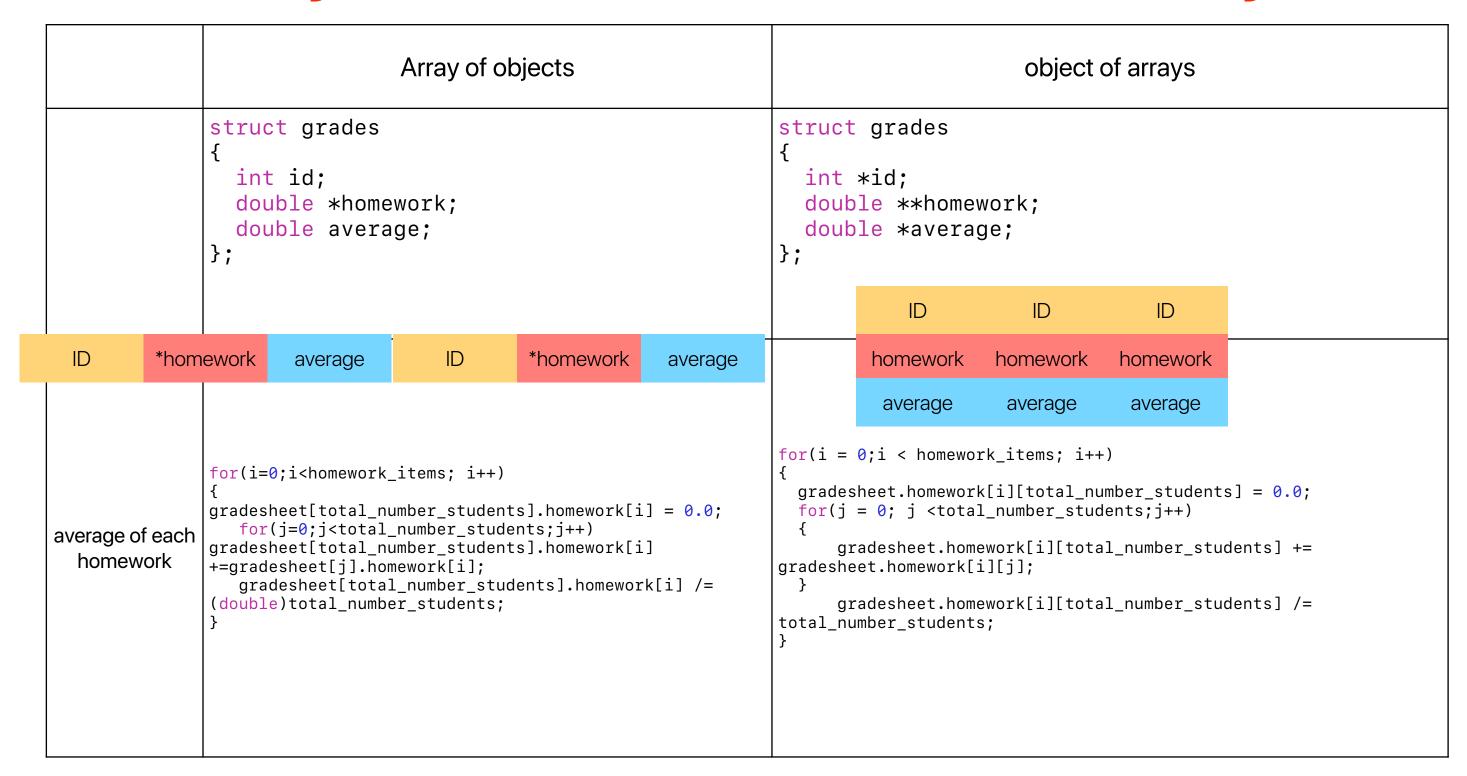
Review: 3Cs of misses

- Compulsory miss
 - Cold start miss. First-time access to a block
- Capacity miss
 - The working set size of an application is bigger than cache size
- Conflict miss
 - Required data replaced by block(s) mapping to the same set
 - Similar collision in hash

Recap: Summary of Optimizations

- Hardware
 - Prefetch compulsory miss
 - Write buffer miss penalty
 - Bank/pipeline miss penalty
 - Critical word first and early restart miss penalty
- Software
 - Data layout to improve locality compulsory/conflict/capacity misses

Array of structures or structure of arrays



Outline

- The remedies of cache misses the software version
- The basic of virtual memory

How can programmer improve memory performance?

Loop interchange/fission/fusion

Demo — programmer & performance

```
for(i = 0; i < ARRAY_SIZE; i++)
{
  for(j = 0; j < ARRAY_SIZE; j++)
    {
    c[i][j] = a[i][j]+b[i][j];
  }
}</pre>
```

```
for(j = 0; j < ARRAY_SIZE; j++)
{
   for(i = 0; i < ARRAY_SIZE; i++)
   {
      c[i][j] = a[i][j]+b[i][j];
   }
}</pre>
```

 $O(n^2)$

Complexity

 $O(n^2)$

Same

Instruction Count?

Same

Same

Clock Rate

Same

Better

CPI

Worse

Loop interchange

NVIDIA Tegra X1

- D-L1 Cache configuration of NVIDIA Tegra X1
 - Size 32KB, 4-way set associativity, 64B block, LRU policy, write-allocate, write-back, and assuming 64-bit address.

```
double a[8192], b[8192], c[8192], d[8192], e[8192]; 

/* a = 0x10000, b = 0x20000, c = 0x30000, d = 0x40000, e = 0x50000 */ 

for(i = 0; i < 512; i++) { 

    e[i] = (a[i] * b[i] + c[i])/d[i]; 

    //load a[i], b[i], c[i], d[i] and then store to e[i] 

}
```

What's the data cache miss rate for this code?

- A. 12.5%
- B. 56.25%
- C. 66.67%
- D. 68.75%
- E. 100%

What if the code look like this?

- D-L1 Cache configuration of NVIDIA Tegra X1
 - Size 32KB, 4-way set associativity, 64B block, LRU policy, write-allocate, write-back, and assuming 64-bit address.

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/* a = 0x10000, b = 0x20000, c = 0x30000, d = 0x40000, e = 0x50000 */ 

for(i = 0; i < 512; i++) 

    e[i] = a[i] * b[i] + c[i]; //load a, b, c and then store to e 

for(i = 0; i < 512; i++) 

    e[i] /= d[i]; //load e, load d, and then store to e
```

What's the data cache miss rate for this code?

- A. ~10%
- B. ~20%
- C. ~40%
- D. ~80%
- E. 100%



What if the code look like this?

- D-L1 Cache configuration of NVIDIA Tegra X1
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```

What's the data cache miss rate for this code?

```
A. ~10%
```

B. ~20%

C. ~40%

D. ~80%

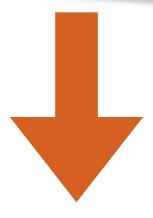
E. 100%

Loop fission

Loop fission

 \mathbf{m}

Loop fission



```
4
```

What if we change the processor?

- If we have an intel processor with a 32KB, 8-way, 64B-blocked L1 cache, which version of code performs better?
 - A. Version A, because the code incurs fewer cache misses
 - B. Version B, because the code incurs fewer cache misses
 - C. Version A, because the code incurs fewer memory references
 - D. Version B, because the code incurs fewer memory references
 - E. They are about the same

```
a
```

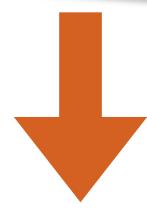
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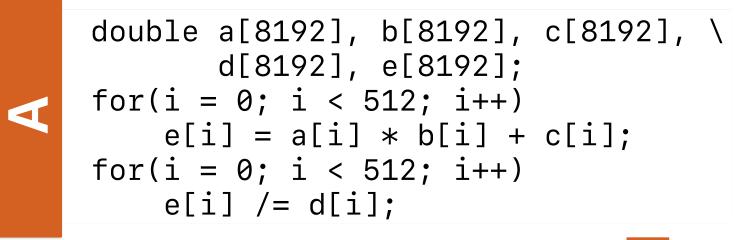
Loop optimizations

 \mathbf{m}

Loop fission







Loop fusion



m

Blocking/Tiling

Case study: Matrix Multiplication

Matrix Multiplication

```
Very likely a miss if
for(i = 0; i < ARRAY_SIZE; i++) {
   for(j = 0; j < ARRAY_SIZE; j++) {
     for(k = 0; k < ARRAY_SIZE; k++) {
        c[i][j] += a[i][k]*b[k][j];
     }
}
}</pre>
```

b

If each dimension of your matrix is 2048

C

- Each row takes 2048*8 bytes = 16KB
- The L1 \$ of intel Core i7 or AMD RyZen is 32KB, 8-way, 64-byte blocked
- You can only hold at most 2 rows/columns of each matrix!
- You need the same row when j increase!

a

It's show time!

Simply the addresses of B[][]

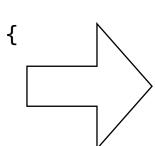
element	address	tag	index
b[0][0]	7F68297E1000	7F68297E1	0
b[1][0]	7F68297E1800	7F68297E1	20
b[2][0]	7F68297E2000	7F68297E2	0
b[3][0]	7F68297E2800	7F68297E2	20
b[4][0]	7F68297E3000	7F68297E3	0
b[5][0]	7F68297E3800	7F68297E3	20
b[6][0]	7F68297E4000	7F68297E4	0
b[7][0]	7F68297E4800	7F68297E4	20
b[8][0]	7F68297E5000	7F68297E5	0
b[9][0]	7F68297E5800	7F68297E5	20
b[10][0]	7F68297E6000	7F68297E6	0
b[11][0]	7F68297E6800	7F68297E6	20
b[12][0]	7F68297E7000	7F68297E7	0
b[13][0]	7F68297E7800	7F68297E7	20
b[14][0]	7F68297E8000	7F68297E8	0
b[15][0]	7F68297E8800	7F68297E8	20
b[16][0]	7F68297E9000	7F68297E9	0
b[17][0]	7F68297E9800	7F68297E9	20
b[18][0]	7F68297FA000	7F68297FA	0

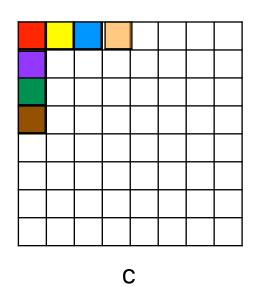
We only used $17 \times 64 = 1088$ bytes!

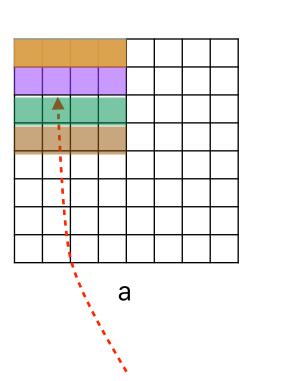
Conflict miss starts!

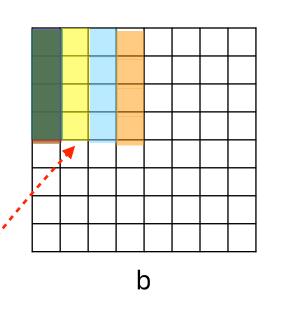
Block algorithm for matrix multiplication

```
for(i = 0; i < ARRAY_SIZE; i++) {
  for(j = 0; j < ARRAY_SIZE; j++) {
    for(k = 0; k < ARRAY_SIZE; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
}</pre>
```









You only need to hold these sub-matrices in your cache

What kind(s) of misses can block algorithm remove?

• Comparing the naive algorithm and block algorithm on matrix multiplication, what kind of misses does block algorithm help to remove? (assuming an intel Core i7)

```
for(i = 0; i < ARRAY_SIZE; i++) {
  for(j = 0; j < ARRAY_SIZE; j++) {
    for(k = 0; k < ARRAY_SIZE; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
}</pre>
```

- A. Compulsory miss
- B. Capacity miss
- C. Conflict miss
- D. Capacity & conflict miss
- E. Compulsory & conflict miss

Block

```
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {
   for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {
     for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {
      for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)
          for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
          for(kk = k; kk < k+(ARRAY_SIZE/n); kk++)
          c[ii][jj] += a[ii][kk]*b[kk][jj];
   }
}</pre>
```

Matrix Transpose

```
// Transpose matrix b into b_t
                                                                for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {</pre>
                                                                  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {</pre>
                                                                      b_t[i][j] += b[j][i];
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {</pre>
  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {</pre>
    for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {</pre>
                                                                for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {</pre>
        for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)</pre>
                                                                  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {</pre>
          for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
                                                                     for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {</pre>
             for(kk = k; kk < k+(ARRAY_SIZE/n); kk++)
                                                                         for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)</pre>
               c[ii][jj] += a[ii][kk]*b[kk][jj];
                                                                           for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
                                                                             for(kk = k; kk < k+(ARRAY_SIZE/n); kk++)
                                                                                // Compute on b_t
                                                                                c[ii][jj] += a[ii][kk]*b_t[jj][kk];
```

Matrix Transpose

```
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                                                                      b_t[i][j] += b[j][i];
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {</pre>
  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {</pre>
    for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {</pre>
                                                                for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {</pre>
        for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)</pre>
                                                                  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {</pre>
          for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
                                                                     for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {</pre>
             for(kk = k; kk < k+(ARRAY_SIZE/n); kk++)
                                                                         for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)</pre>
               c[ii][jj] += a[ii][kk]*b[kk][jj];
                                                                           for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
                                                                             for(kk = k; kk < k+(ARRAY_SIZE/n); kk++)
                                                                                // Compute on b_t
                                                                                c[ii][jj] += a[ii][kk]*b_t[jj][kk];
```

What kind(s) of misses can matrix transpose remove?

• By transposing a matrix, the performance of matrix multiplication can be further improved. What kind(s) of cache misses does matrix transpose help to remove?

```
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {
   for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {
     for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {
      for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)
          for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
          for(kk = k; kk < k+(ARRAY_SIZE/n); kk++)
          c[ii][jj] += a[ii][kk]*b[kk][jj];
   }
}</pre>
```

- A. Compulsory miss
- B. Capacity miss
- C. Conflict miss
- D. Capacity & conflict miss
- E. Compulsory & conflict miss

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   for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {</pre>
      for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {</pre>
          b_t[i][j] += b[j][i];
    for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {</pre>
      for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {</pre>
        for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {</pre>
            for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)</pre>
               for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
Block
                 for(kk = k; kk < k+(ARRAY_SIZE/n); kk++</pre>
                   // Compute on b_t
                   c[ii][jj] += a[ii][kk]*b_t[jj][kk];
```

What kind(s) of misses can matrix transpose remove?

• By transposing a matrix, the performance of matrix multiplication can be further improved. What kind(s) of cache misses does matrix transpose help to remove?

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                 for(kk = k; kk < k+(ARRAY_SIZE/n); kk++</pre>
                   // Compute on b_t
                   c[ii][jj] += a[ii][kk]*b_t[jj][kk];
```

Tips of software optimizations

- Carefully layout your data structure can improve capacity misses!
- Make your data structures align with the access pattern can better exploit cache locality — improve conflict misses
- Implementing algorithms in a more cache friendly way!

Summary of Optimizations

Software

- Data layout capacity miss, conflict miss, compulsory miss
- Blocking capacity miss, conflict miss
- Loop fission conflict miss when \$ has limited way associativity
- Loop fusion capacity miss when \$ has enough way associativity
- Loop interchange conflict/capacity miss

Hardware

- Prefetch compulsory miss
- Write buffer miss penalty
- Bank/pipeline miss penalty
- Critical word first and early restart miss panelty

Let's dig into this code

```
int main(int argc, char *argv[])
    int i,j;
    double **a;
    double sum=0, average;
    int dim=32768;
    if(argc < 2)
        fprintf(stderr, "Usage: %s dimension\n", argv[0]);
        exit(1);
    dim = atoi(argv[1]);
    a = (double **)malloc(sizeof(double *)*dim);
    for(i = 0 ; i < dim; i++)
        a[i] = (double *)malloc(sizeof(double)*dim);
    for(i = 0 ; i < dim; i++)
        for(j = 0 ; j < dim; j++)
            a[i][j] = rand();
    for(i = 0 ; i < dim; i++)
        for(j = 0 ; j < dim; j++)
            sum+=a[i][i];
    average = sum/(dim*dim);
    fprintf(stderr, "average: %lf\n", average);
    for(i = 0 ; i < dim; i++)
        free(a[i]);
    free(a);
    return 0;
```

What will happen?

- If we execute the code on the right-hand side code on a machine with only 32 GB of physical memory installed and the dim is "70000" (requires 70000*70000*8 bytes ~ 37 GB memory at least), What will happen?
 - A. The program will crash in one of the malloc function call
 - B. The program will crash due to a "segmentation fault" that caused by accessing NULL pointer
 - C. The program will be killed automatically by the OS as it uses more than installed physical main memory
 - D. The program will finish without any issue

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    dim = atoi(argv[1]);
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    for(i = 0 ; i < dim; i++)
        a[i] = (double *)malloc(sizeof(double)*dim);
    for(i = 0 ; i < dim; i++)
        for(j = 0 ; j < dim; j++)
            a[i][j] = rand();
   for(i = 0 ; i < dim; i++)
        for(j = 0 ; j < dim; j++)
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    for(i = 0 ; i < dim; i++)
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        for(j = 0 ; j < dim; j++)
            a[i][j] = rand();
   for(i = 0 ; i < dim; i++)
        for(j = 0 ; j < dim; j++)
            sum+=a[i][i];
    average = sum/(dim*dim);
    fprintf(stderr, "average: %lf\n", average);
    for(i = 0 ; i < dim; i++)
        free(a[i]);
    free(a);
    return 0;
```

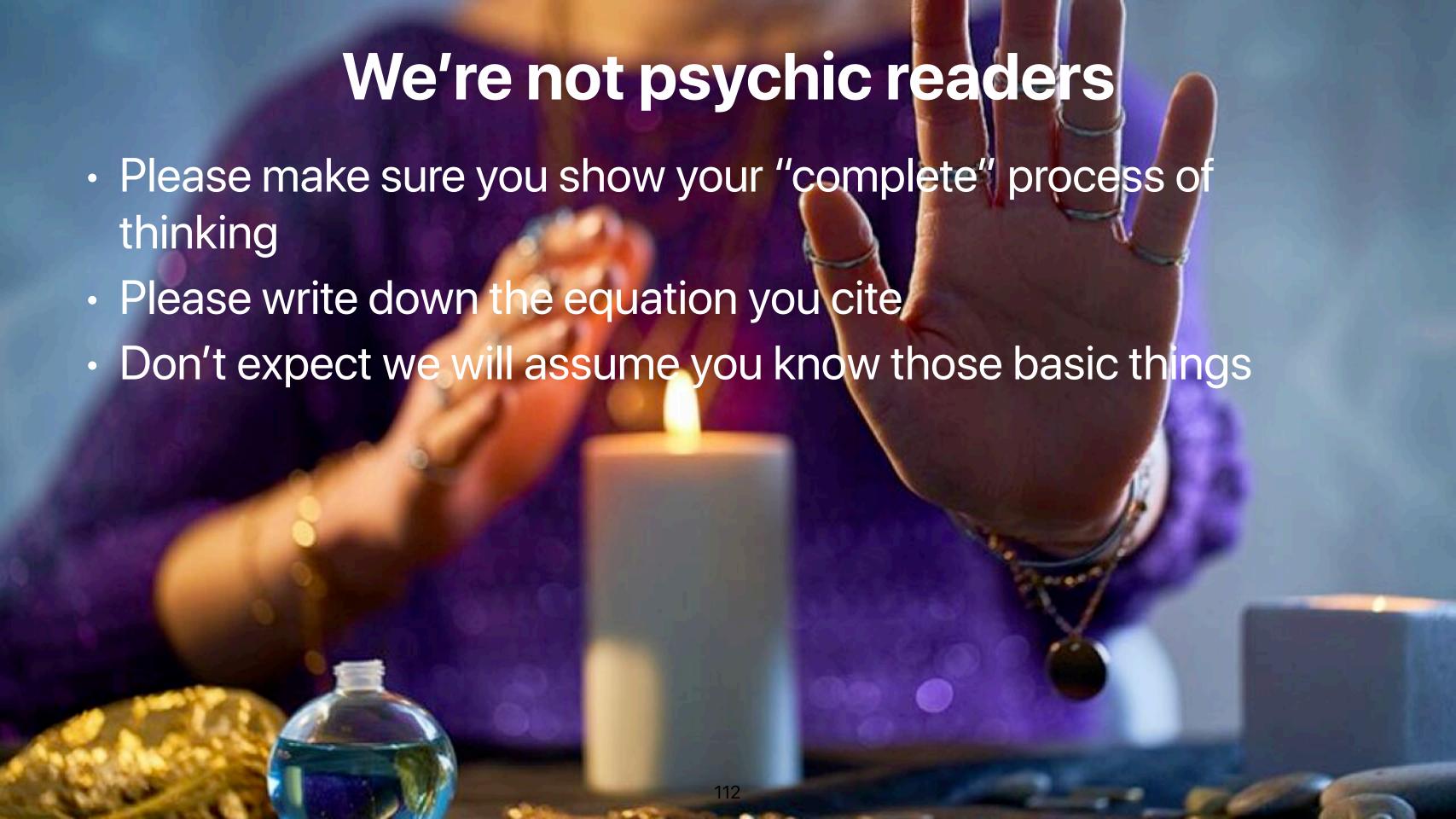
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   for(i = 0 ; i < dim; i++)
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            sum+=a[i][i];
    average = sum/(dim*dim);
    fprintf(stderr, "average: %lf\n", average);
    for(i = 0 ; i < dim; i++)
        free(a[i]);
    free(a);
    return 0;
```

Announcement

- Assignment #2 due this Thursday
 - Please start early some programs take very long time to finish
 - As mentioned in the beginning of each assignment server busy cannot be a reason for late submission or extension
 - If you have questions that cannot be address online, please come to office hours
 - Today: Hung-Wei's 2:30p—3:30p and Kuan-Chieh's 3:30p-4:30p
- Midterm next Tuesday
 - You can only take the exam in-person, closed-book, closed-note
 - Please bring your student ID and we will check
 - You may review/focus on the materials/topics covered in lectures
 - You SHOULD review your assignments
 - Cover topics including this Thursday



Computer Science & Engineering

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