Memory Hierarchy (4): Cache Misses and How to Address Them — the Software Version

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Data structures

Array of structures or structure of arrays

	Array of objects	object of arrays
	`struct grades { int id; double assignment_1, assignment_2, assignment 3, …; };`	`struct grades { int *id; double *assignment_1, *assignment_2, *assignment_3, …; };`
average of each homework	see code below	see code below

Array of objects:

```
struct grades {
  int id;
  double assignment_1, assignment_2,
assignment 3, …;
};
```

object of arrays:

```
struct grades {
  int *id;
  double *assignment_1, *assignment_2,
*assignment_3, …;
};
```

ID | assignment_1 | assignment_2 | assignment_3 | ... | ID | assignment_1 | assignment_2 | assignment_3 | ...

average of each homework

Array of objects:

```
for(i=0;i<homework_items; i++)
{
gradesheet[total_number_students].homework[i] = 0.0;
   for(j=0;j<total_number_students;j++)
gradesheet[total_number_students].homework[i]
+=gradesheet[j].homework[i];
   gradesheet[total_number_students].homework[i] /=
(double)total_number_students;
}
```

object of arrays:

```
for(i = 0;i < homework_items; i++)
{
  gradesheet.homework[i][total_number_students] = 0.0;
  for(j = 0; j <total_number_students;j++)
  {
      gradesheet.homework[i][total_number_students] +=
gradesheet.homework[i][j];
  }
      gradesheet.homework[i][total_number_students] /=
total_number_students;
}
```

ID | ID | ID | assignment_1 | assignment_1 | assignment_1 | assignment_2 | assignment_2 | assignment_2 | assignment_3 | assignme

Column-store or row-store

- If you're designing an in-memory database system for the following table, will you be using

RowId	EmpId	Lastname	Firstname	Salary
1	10	Smith	Joe	40000
2	12	Jones	Mary	50000
3	11	Johnson	Cathy	44000
4	22	Jones	Bob	55000

  - Column-store — stores data tables column by column

```
10:001,12:002,11:003,22:004;
Smith:001,Jones:002,Johnson:003,Jones:004;
Joe:001,Mary:002,Cathy:003,Bob:004;
40000:001,50000:002,44000:003,55000:004;
```

if the most frequently used query looks like —

```
select Lastname, Firstname from table
```

  - Row-store — stores data tables row by row

```
001:10,Smith,Joe,40000;
002:12,Jones,Mary,50000;
003:11,Johnson,Cathy,44000;
004:22,Jones,Bob,55000;
```

Takeaways: Software Optimizations

- Data layout — capacity miss, conflict miss, compulsory miss

Loop interchange/fission/fusion

Demo — programmer & performance

A

```
for(i = 0; i < ARRAY_SIZE; i++)
{
  for(j = 0; j < ARRAY_SIZE; j++)
  {
    c[i][j] = a[i][j]+b[i][j];
  }
}
```

B

```
for(j = 0; j < ARRAY_SIZE; j++)
{
  for(i = 0; i < ARRAY_SIZE; i++)
  {
    c[i][j] = a[i][j]+b[i][j];
  }
}
```

A		B
$O(n^2)$	**Complexity**	$O(n^2)$
Same	**Instruction Count?**	**Same**
Same	**Clock Rate**	**Same**
Better	**CPI**	**Worse**

Loop optimizations

Loop interchange

A

```
for(i = 0; i < ARRAY_SIZE; i++)
{
  for(j = 0; j < ARRAY_SIZE; j++)
  {
    c[i][j] = a[i][j]+b[i][j];
  }
}
```

B

```
for(j = 0; j < ARRAY_SIZE; j++)
{
  for(i = 0; i < ARRAY_SIZE; i++)
  {
    c[i][j] = a[i][j]+b[i][j];
  }
}
```

Takeaways: Software Optimizations

- Data layout — capacity miss, conflict miss, compulsory miss
- Loop interchange — conflict/capacity miss

NVIDIA Tegra X1

- D-L1 Cache configuration of NVIDIA Tegra X1
  - Size 32KB, 4-way set associativity, 64B block, LRU policy, write-allocate, write-back, and assuming 64-bit address.

```
double a[16384], b[16384], c[16384], d[16384], e[16384];
/* a = 0x10000, b = 0x20000, c = 0x30000, d = 0x40000, e = 0x50000 */
for(i = 0; i < 8192; i++) {
    e[i] = (a[i] * b[i] + c[i])/d[i];
    //load a[i], b[i], c[i], d[i] and then store to e[i]
}
```

What's the data cache miss rate for this code?

A. 12.5%

B. 56.25%

C. 66.67%

D. 68.75%

E. $\boxed{\text{100\%}}$

Loop optimizations

Loop interchange

A

```
for(i = 0; i < ARRAY_SIZE; i++)
{
  for(j = 0; j < ARRAY_SIZE; j++)
  {
    c[i][j] = a[i][j]+b[i][j];
  }
}
```

B

```
for(j = 0; j < ARRAY_SIZE; j++)
{
  for(i = 0; i < ARRAY_SIZE; i++)
  {
    c[i][j] = a[i][j]+b[i][j];
  }
}
```

Loop fission

B

```
double a[8192], b[8192], c[8192], \
       d[8192], e[8192];
for(i = 0; i < 8192; i++) {
    e[i] = (a[i] * b[i] + c[i])/d[i];
}
```

A

```
double a[8192], b[8192], c[8192], \
       d[8192], e[8192];
for(i = 0; i < 8192; i++)
    e[i] = a[i] * b[i] + c[i];
for(i = 0; i < 8192; i++)
    e[i] /= d[i];
```

Takeaways: Software Optimizations

- Data layout — capacity miss, conflict miss, compulsory miss
- Loop interchange — conflict/capacity miss
- Loop fission — conflict miss — when $ has limited way associativity

Loop optimizations

Loop interchange

A

```
for(i = 0; i < ARRAY_SIZE; i++)
{
  for(j = 0; j < ARRAY_SIZE; j++)
  {
    c[i][j] = a[i][j]+b[i][j];
  }
}
```

B

```
for(j = 0; j < ARRAY_SIZE; j++)
{
  for(i = 0; i < ARRAY_SIZE; i++)
  {
    c[i][j] = a[i][j]+b[i][j];
  }
}
```

Loop fission

B

```
double a[8192], b[8192], c[8192], \
       d[8192], e[8192];
for(i = 0; i < 8192; i++) {
    e[i] = (a[i] * b[i] + c[i])/d[i];
}
```

A

```
double a[8192], b[8192], c[8192], \
       d[8192], e[8192];
for(i = 0; i < 8192; i++)
    e[i] = a[i] * b[i] + c[i];
for(i = 0; i < 8192; i++)
    e[i] /= d[i];
```

Loop fusion

A

```
double a[8192], b[8192], c[8192], \
       d[8192], e[8192];
for(i = 0; i < 8192; i++)
    e[i] = a[i] * b[i] + c[i];
for(i = 0; i < 8192; i++)
    e[i] /= d[i];
```

B

```
double a[8192], b[8192], c[8192], \
       d[8192], e[8192];
for(i = 0; i < 8192; i++) {
    e[i] = (a[i] * b[i] + c[i])/d[i];
}
```

Takeaways: Software Optimizations

- Data layout — capacity miss, conflict miss, compulsory miss
- Loop interchange — conflict/capacity miss
- Loop fission — conflict miss — when $ has limited way associativity
- Loop fusion — capacity miss — when $ has enough way associativity

Tiling/Blocking Algorithm

What is an M by N “2-D” array in C?

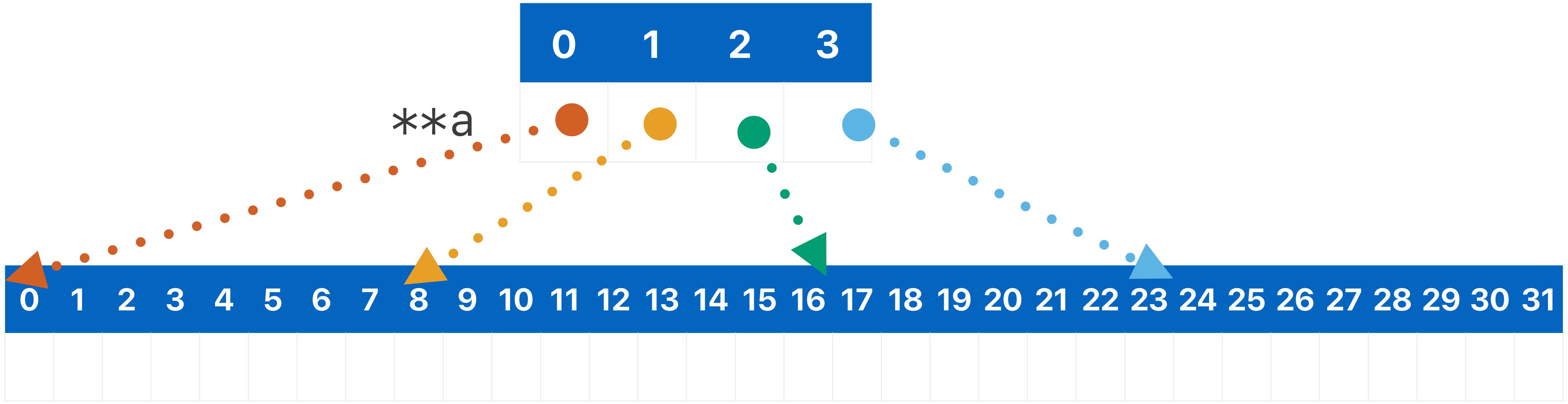
```c
a = (double **)malloc(M*sizeof(double *));
for(i = 0; i < N; i++)
{
   a[i] = (double *)malloc(N*sizeof(double));
}
```

`a[i][j]` is essentially `a[i*N+j]`

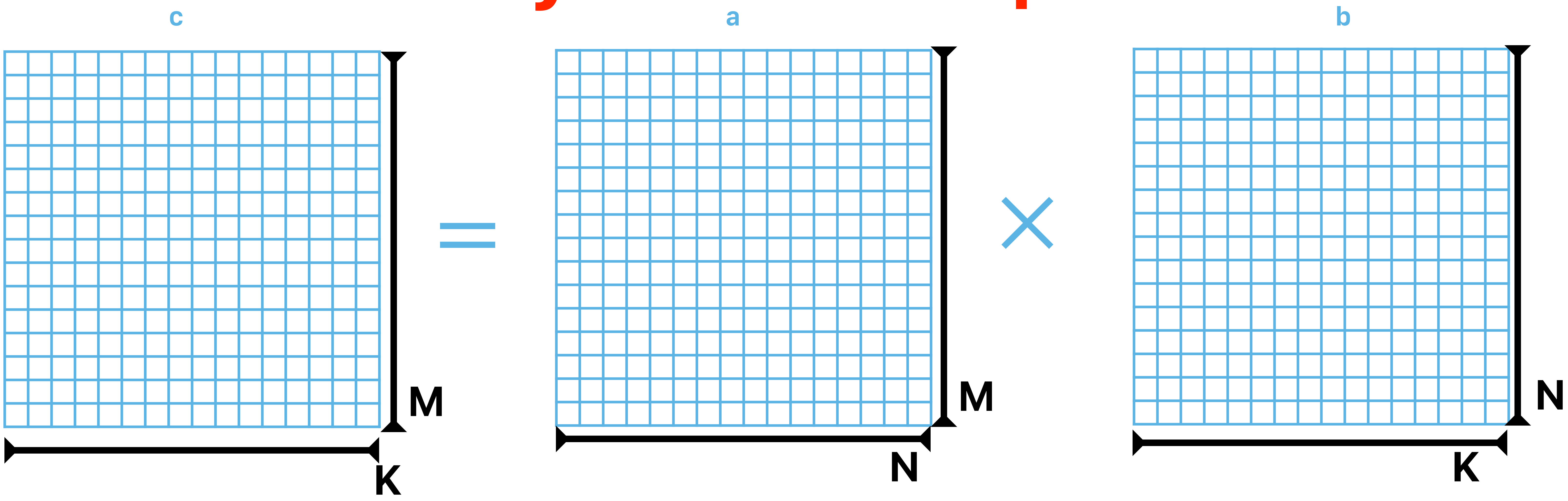
abstraction

	0	1	2	3	4	5	6	7
0								
1								
2								
3								

physical implementation



Case Study: Matrix Multiplications



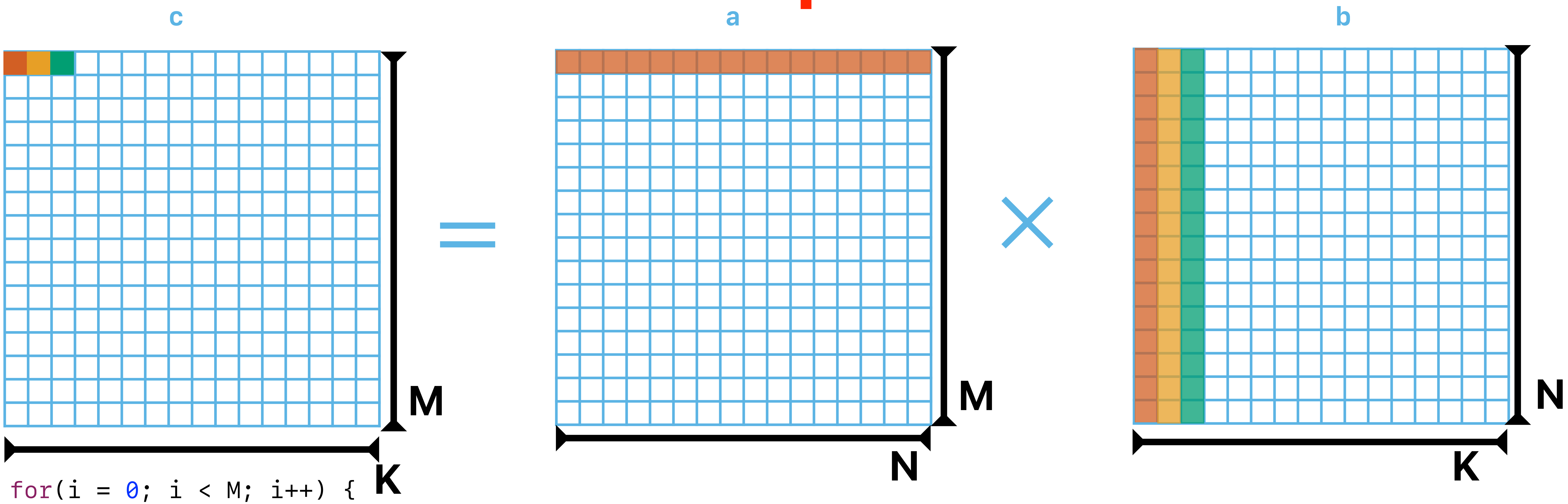

```
for(i = 0; i < M; i++) {
  for(j = 0; j < K; j++) {
    for(k = 0; k < N; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
  }
}
```

Algorithm class tells you it's $O(n^3)$

If M=N=K=1024, it takes about 2 sec

How long is it take when M=N=K=2048?

Matrix Multiplications




```
for(i = 0; i < M; i++) {
  for(j = 0; j < K; j++) {
    for(k = 0; k < N; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
  }
}
```

Matrix Multiplication — let's consider "b"

```
for(i = 0; i < M; i++) {
  for(j = 0; j < K; j++) {
    for(k = 0; k < N; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
  }
}
```

	Address	Tag	Index
b[0][0]	0x20000	0x20	0x0
b[1][0]	0x24000	0x24	0x0
b[2][0]	0x28000	0x28	0x0
b[3][0]	0x2C000	0x2C	0x0
b[4][0]	0x30000	0x30	0x0
b[5][0]	0x34000	0x34	0x0
b[6][0]	0x38000	0x38	0x0
b[7][0]	0x3C000	0x3C	0x0
b[8][0]	0x40000	0x40	0x0
b[9][0]	0x44000	0x44	0x0
b[10][0]	0x48000	0x48	0x0
b[11][0]	0x4C000	0x4C	0x0
b[12][0]	0x50000	0x50	0x0
b[13][0]	0x54000	0x54	0x0
b[14][0]	0x58000	0x58	0x0
b[15][0]	0x5C000	0x5C	0x0
b[16][0]	0x60000	0x60	0x0

- If the row dimension (N) of your matrix is 2048, each row element with the same column index is

  $$2048 \times 8 = 16384 = 0x4000$$

  away from each other

Each set can store only 12 blocks! So we will start to kick out b[0][0-7], b[1][0-7] …

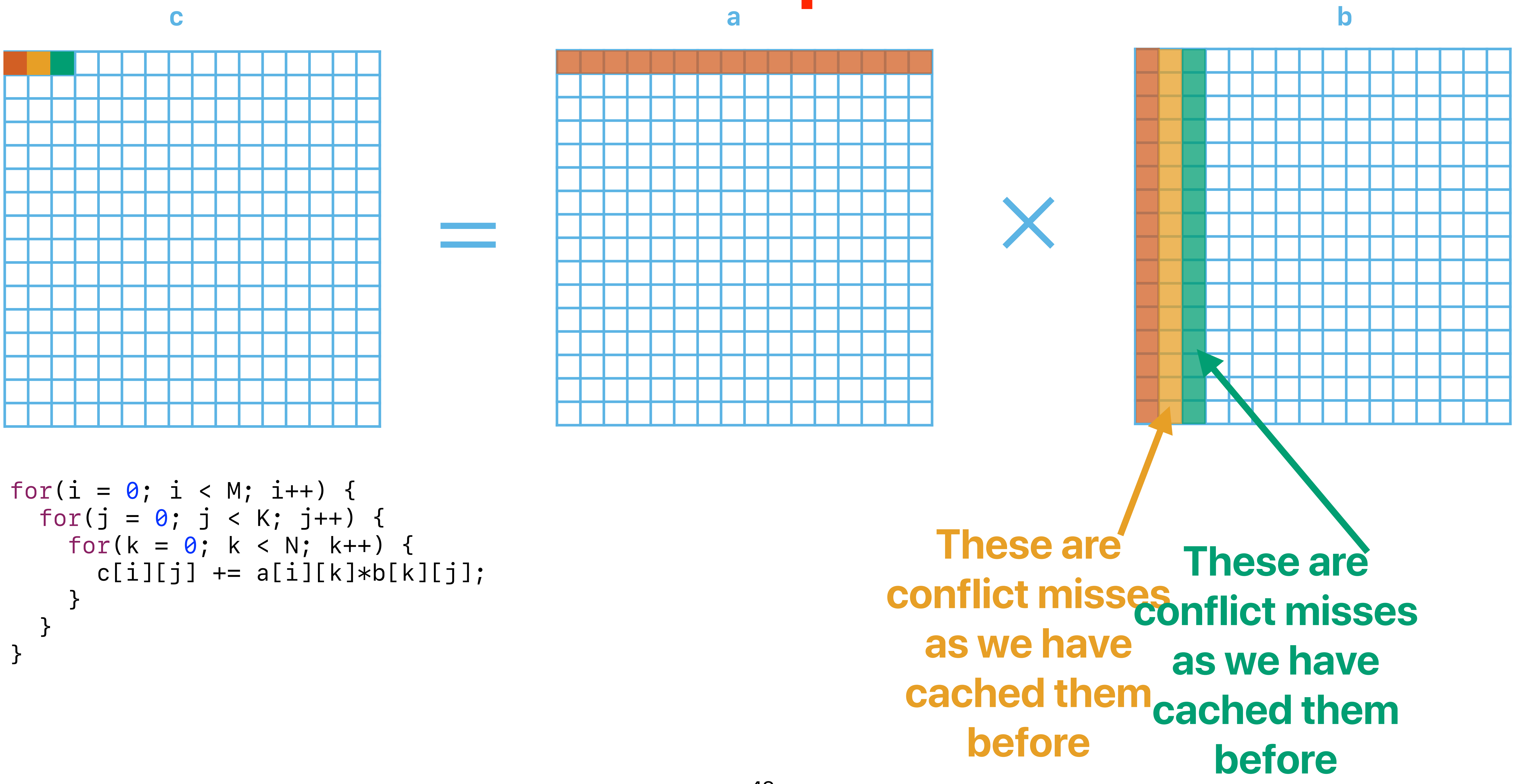
Now, when we work on c[0][1]

	Address	Tag	Index
b[0][0]	0x20000	0x20	0x0
b[1][0]	0x24000	0x24	0x0
b[2][0]	0x28000	0x28	0x0
b[3][0]	0x2C000	0x2C	0x0
b[4][0]	0x30000	0x30	0x0
b[5][0]	0x34000	0x34	0x0
b[6][0]	0x38000	0x38	0x0
b[7][0]	0x3C000	0x3C	0x0
b[8][0]	0x40000	0x40	0x0
b[9][0]	0x44000	0x44	0x0
b[10][0]	0x48000	0x48	0x0
b[11][0]	0x4C000	0x4C	0x0
b[12][0]	0x50000	0x50	0x0
b[13][0]	0x54000	0x54	0x0
b[14][0]	0x58000	0x58	0x0
b[15][0]	0x5C000	0x5C	0x0
b[16][0]	0x60000	0x60	0x0

	Address	Tag	Index	
b[0][1]	0x20008	0x20	0x0	Conflict Miss
b[1][1]	0x24008	0x24	0x0	Conflict Miss
b[2][1]	0x28008	0x28	0x0	Conflict Miss
b[3][1]	0x2C008	0x2C	0x0	Conflict Miss
b[4][1]	0x30008	0x30	0x0	Conflict Miss
b[5][1]	0x34008	0x34	0x0	Conflict Miss
b[6][1]	0x38008	0x38	0x0	Conflict Miss
b[7][1]	0x3C008	0x3C	0x0	Conflict Miss
b[8][1]	0x40008	0x40	0x0	Conflict Miss
b[9][1]	0x44008	0x44	0x0	Conflict Miss
b[10][1]	0x48008	0x48	0x0	Conflict Miss
b[11][1]	0x4C008	0x4C	0x0	Conflict Miss
b[12][1]	0x50008	0x50	0x0	Conflict Miss
b[13][1]	0x54008	0x54	0x0	Conflict Miss
b[14][1]	0x58008	0x58	0x0	Conflict Miss
b[15][1]	0x5C008	0x5C	0x0	Conflict Miss
b[16][1]	0x60008	0x60	0x0	Conflict Miss

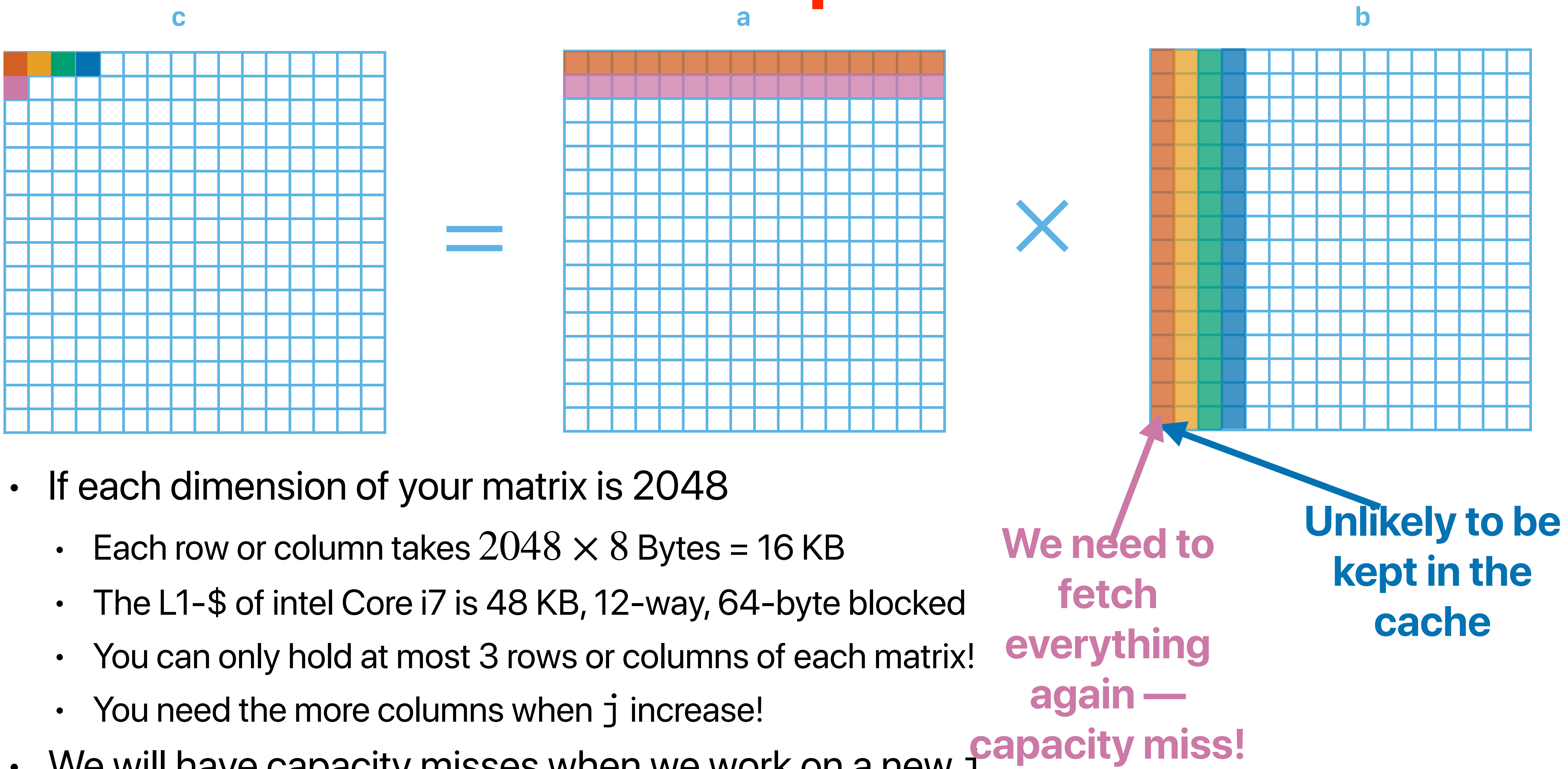
Each set can store only 12 blocks! So we will start to kick out b[0][0-7], b[1][0-7] …

Matrix Multiplications




```
for(i = 0; i < M; i++) {
  for(j = 0; j < K; j++) {
    for(k = 0; k < N; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
  }
}
```

Matrix Multiplications



- If each dimension of your matrix is 2048
  - Each row or column takes 2048×8 Bytes = 16 KB
  - The L1-$ of intel Core i7 is 48 KB, 12-way, 64-byte blocked
  - You can only hold at most 3 rows or columns of each matrix!
  - You need the more columns when `j` increase!
- We will have capacity misses when we work on a new `i`

Ideas regarding reducing misses in matrix multiplications

- Reducing conflict misses — we need to reduce the length of a column that we visit within a period of time
- Reducing capacity misses — we need to reduce the length of a row that we visit within a period of time

Mathematical view of MM

$$c_{i,j} = \sum_{k=0}^{k=N-1} a_{i,k} \times b_{k,j} = \sum_{k=0}^{k=\frac{N}{2}-1} a_{i,k} \times b_{k,j} + \sum_{k=\frac{N}{2}}^{k=N-1} a_{i,k} \times b_{k,j}$$

$$= \sum_{k=0}^{k=\frac{N}{4}-1} a_{i,k} \times b_{k,j} + \sum_{k=\frac{N}{4}}^{k=\frac{N}{2}-1} a_{i,k} \times b_{k,j} + \sum_{k=\frac{N}{2}}^{k=\frac{3N}{4}-1} a_{i,k} \times b_{k,j} + \sum_{k=3N4-1}^{k=N-1} a_{i,k} \times b_{k,j}$$

$$\vdots$$

Let's break up the multiplications and accumulations into something fits in the cache well

Matrix Multiplication — let's consider "b"

```
for(i = 0; i < M; i++) {
  for(j = 0; j < K; j++) {
    for(k = 0; k < N; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
  }
}
```

	Address	Tag	Index
b[0][0]	0x20000	0x20	0x0
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b[3][0]	0x2C000	0x2C	0x0
b[4][0]	0x30000	0x30	0x0
b[5][0]	0x34000	0x34	0x0
b[6][0]	0x38000	0x38	0x0
b[7][0]	0x3C000	0x3C	0x0
b[8][0]	0x40000	0x40	0x0
b[9][0]	0x44000	0x44	0x0
b[10][0]	0x48000	0x48	0x0
b[11][0]	0x4C000	0x4C	0x0
b[12][0]	0x50000	0x50	0x0
b[13][0]	0x54000	0x54	0x0
b[14][0]	0x58000	0x58	0x0
b[15][0]	0x5C000	0x5C	0x0
b[16][0]	0x60000	0x60	0x0

- If the row dimension of your matrix is 2048, each row element with the same column index is

$$2048 \times 8 = 16384 = 0x4000$$

away from each other

Each set can store only 12 blocks! So we will start to kick out b[0][0-7], b[1][0-7] ...

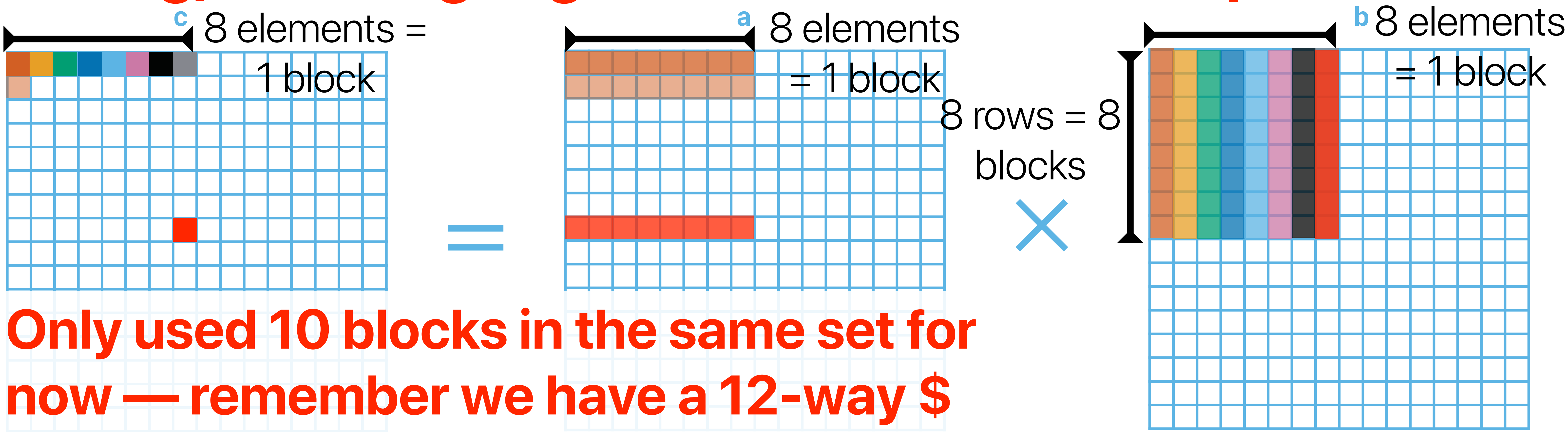
If we stop at somewhere before 12 blocks, we should be fine!

Since each block has 8 elements, let's break up in 8 for now

— 8 elements from a[i]

— 8 columns each covers 8 rows

Tiling/Blocking Algorithm for Matrix Multiplications



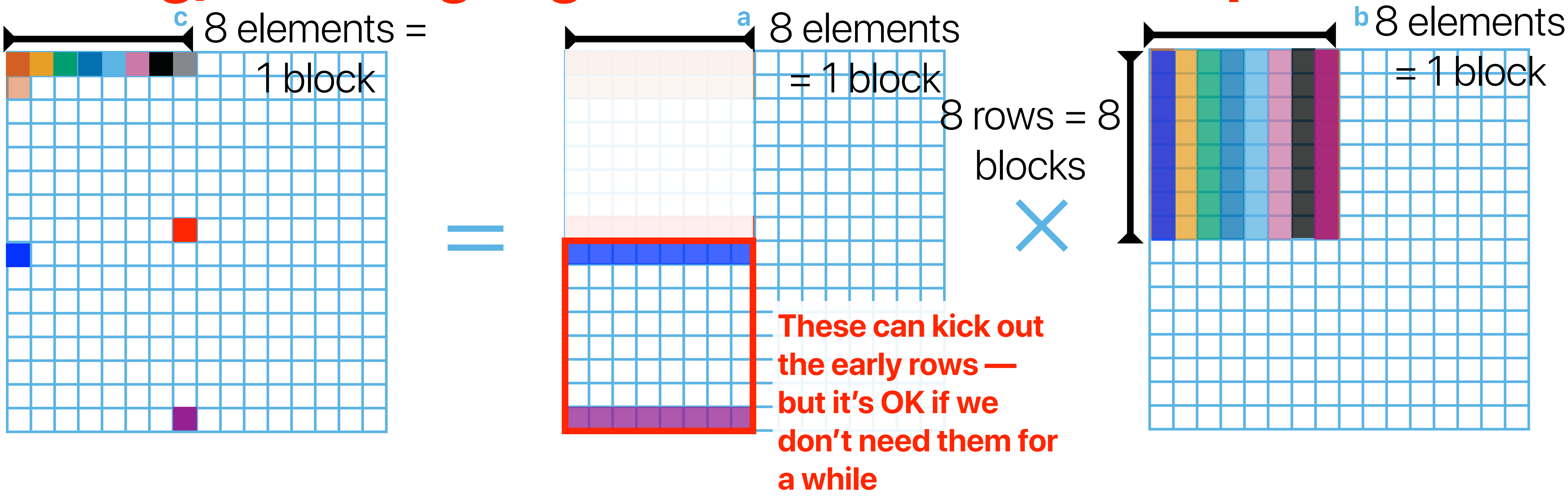
Only used 10 blocks in the same set for now — remember we have a 12-way $

These are still around when we move to the next row in the "tile"

Only compulsory misses —

$$miss\_rate = \frac{total\ misses}{total\ accesses} = \frac{8+8+8}{3 \times 8 \times 8 \times 8} = 0.015625$$

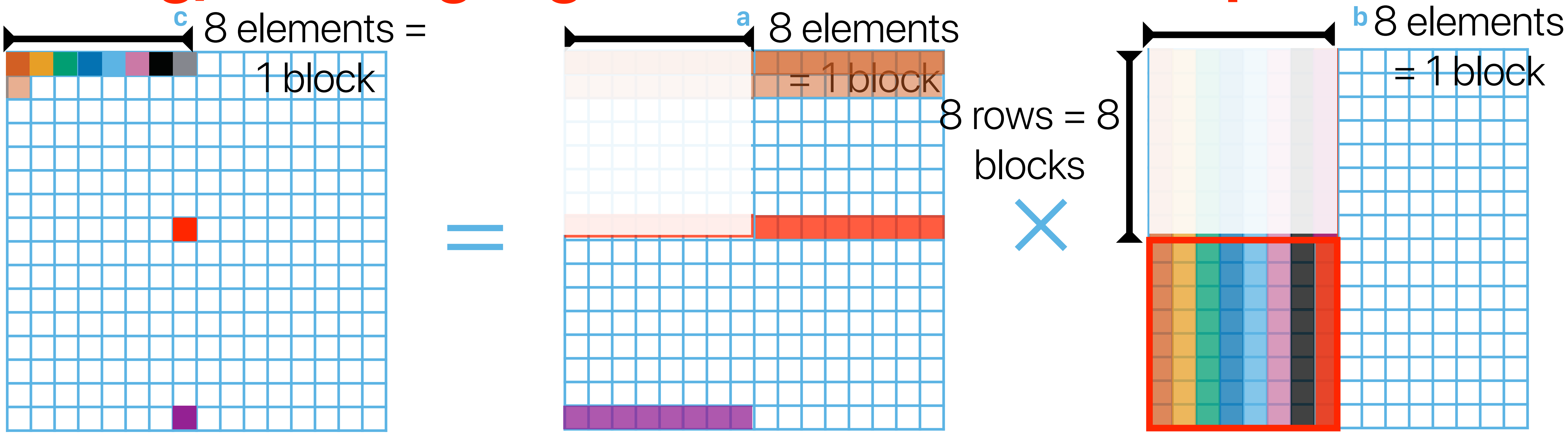
Tiling/Blocking Algorithm for Matrix Multiplications



Bringing miss rate even further lower now —

$$miss\_rate = \frac{total\ misses}{total\ accesses} = \frac{8+8+8}{3\times8\times8+3\times8\times8} = 0.042$$

Tiling/Blocking Algorithm for Matrix Multiplications




```
for(i = 0; i < M; i+=tile_size)
    for(j = 0; j < K; j+=tile_size)
        for(k = 0; k < N; k+=tile_size)
            for(ii = i; ii < i+tile_size; ii++)
                for(jj = j; jj < j+tile_size; jj++)
                    for(kk = k; kk < k+tile_size; kk++)
                        c[ii][jj] += a[ii][kk]*b[kk][jj];
```

These can kick out the upper portion of the columns — but it's OK if we don't need them for a while

Why is “8” not the best performing?

size	tile_size	IC	Cycles	CPI	CT_ns	ET_s	DL1_miss_rate	DL1_accesses
2048	4	97765686275	24149510064	0.247014	0.193189	4.665430	0.015102	43641501149
2048	8	80996985555	21043742544	0.259809	0.193444	4.070776	0.010135	38128531445
2048	16	74473114435	19369857501	0.260092	0.193204	3.742332	0.071790	36105122733
2048	32	71543334296	27812871208	0.388756	0.193112	5.371009	0.217011	35214370198

More instructions due to more loop control overhead!

“8” indeed has the best miss rate

Takeaways: Software Optimizations

- Data layout — capacity miss, conflict miss, compulsory miss
- Loop interchange — conflict/capacity miss
- Loop fission — conflict miss — when $ has limited way associativity
- Loop fusion — capacity miss — when $ has enough way associativity
- Blocking/tiling — capacity miss, conflict miss

Matrix Transpose

```
for(i = 0; i < M; i+=tile_size) {
  for(j = 0; j < K; j+=tile_size) {
    for(k = 0; k < N; k+=tile_size) {
      for(ii = i; ii < i+tile_size; ii++)
        for(jj = j; jj < j+tile_size; jj++)
          for(kk = k; kk < k+tile_size; kk++)
            c[ii][jj] += a[ii][kk]*b[kk][jj];
    }
  }
}
```

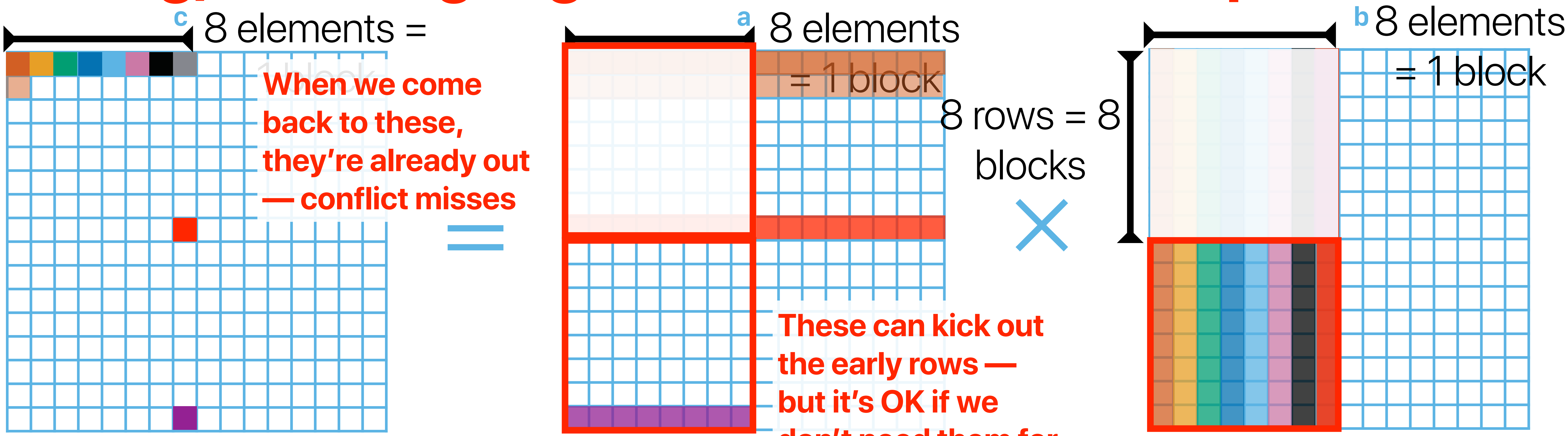
```
// Transpose matrix b into b_t
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {
  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {
      b_t[i][j] += b[j][i];
  }
}

 for(i = 0; i < M; i+=tile_size) {
   for(j = 0; j < K; j+=tile_size) {
     for(k = 0; k < N; k+=tile_size) {
       for(ii = i; ii < i+tile_size; ii++)
         for(jj = j; jj < j+tile_size; jj++)
           for(kk = k; kk < k+tile_size; kk++)
             // Compute on b_t
             c[ii][jj] += a[ii][kk]*b_t[jj][kk];
     }
   }
 }
```

The effect of transposition

size	tile_size	IC	Cycles	CPI	CT_ns	ET_s	DL1_miss_rate	DL1_accesses
2048	4	97765686275	24149510064	0.247014	0.193189	4.665430	0.015102	43641501149
2048	8	80996985555	21043742544	0.259809	0.193444	4.070776	0.010135	38128531445
2048	16	74473114435	19369857501	0.260092	0.193204	3.742332	$\boxed{0.071790}$	36105122733
2048	32	71543334296	27812871208	0.388756	0.193112	5.371009	0.217011	35214370198
2048	16	64810073062	15221864848	0.234869	0.193117	2.939604	$\boxed{0.024597}$	27045326530

Transpose improves the miss rate!

Tiling/Blocking Algorithm for Matrix Multiplications

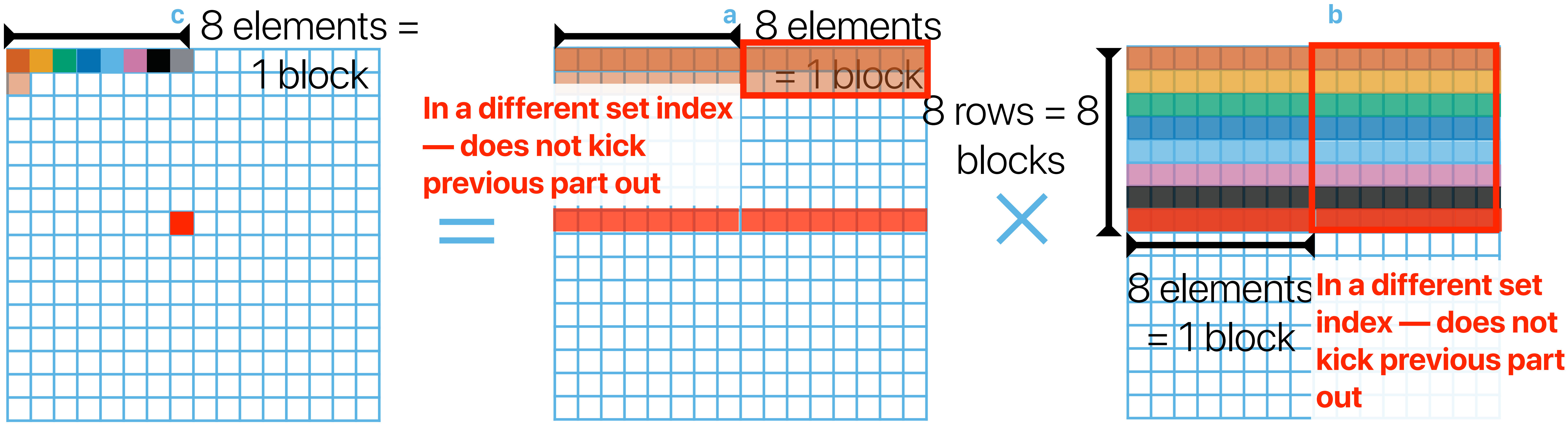


These can kick out the early rows — but it's OK if we don't need them for a while

These can kick out the upper portion of the columns — but it's OK if we don't need them for a while

```
for(i = 0; i < M; i+=tile_size)
    for(j = 0; j < K; j+=tile_size)
        for(k = 0; k < N; k+=tile_size)
            for(ii = i; ii < i+tile_size; ii++)
                for(jj = j; jj < j+tile_size; jj++)
                    for(kk = k; kk < k+tile_size; kk++)
                        c[ii][jj] += a[ii][kk]*b[kk][jj];
```

Tiling/Blocking Algorithm for Transposed Matrix Multiplications



We can make the "tile_size" larger without interfacing conflict misses

```
for(i = 0; i < M; i+=tile_size)
    for(j = 0; j < K; j+=tile_size)
        for(k = 0; k < N; k+=tile_size)
            for(ii = i; ii < i+tile_size; ii++)
                for(jj = j; jj < j+tile_size; jj++)
                    for(kk = k; kk < k+tile_size; kk++)
                        c[ii][jj] += a[ii][kk]*b_t[jj][kk];
```

Takeaways: Software Optimizations

- Data layout — capacity miss, conflict miss, compulsory miss
- Loop interchange — conflict/capacity miss
- Loop fission — conflict miss — when $ has limited way associativity
- Loop fusion — capacity miss — when $ has enough way associativity
- Blocking/tiling — capacity miss, conflict miss
- Matrix transpose (a technique changes layout) — conflict misses

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