Basic Elements of Quantum Computing

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Recap: Basic Boolean Algebra Concepts

- {0, 1}: The only two possible values in inputs/outputs
- Basic operators
  - AND (•) — a • b
    - returns 1 only if both a **and** b are 1s
    - otherwise returns 0
  - OR (+) — a + b
    - returns 1 if a **or** b is 1
    - returns 0 if none of them are 1s
  - NOT (') — a'
    - returns 0 if a is 1
    - returns 1 if a is 0

Recap: Truth tables

- A table sets out the functional values of logical expressions on each of their functional arguments, that is, for each combination of values taken by their logical variables

AND

Input		Output
A	B	
0	0	**0**
0	1	**0**
1	0	**0**
1	1	**1**

OR

Input		Output
A	B	
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**1**

NOT

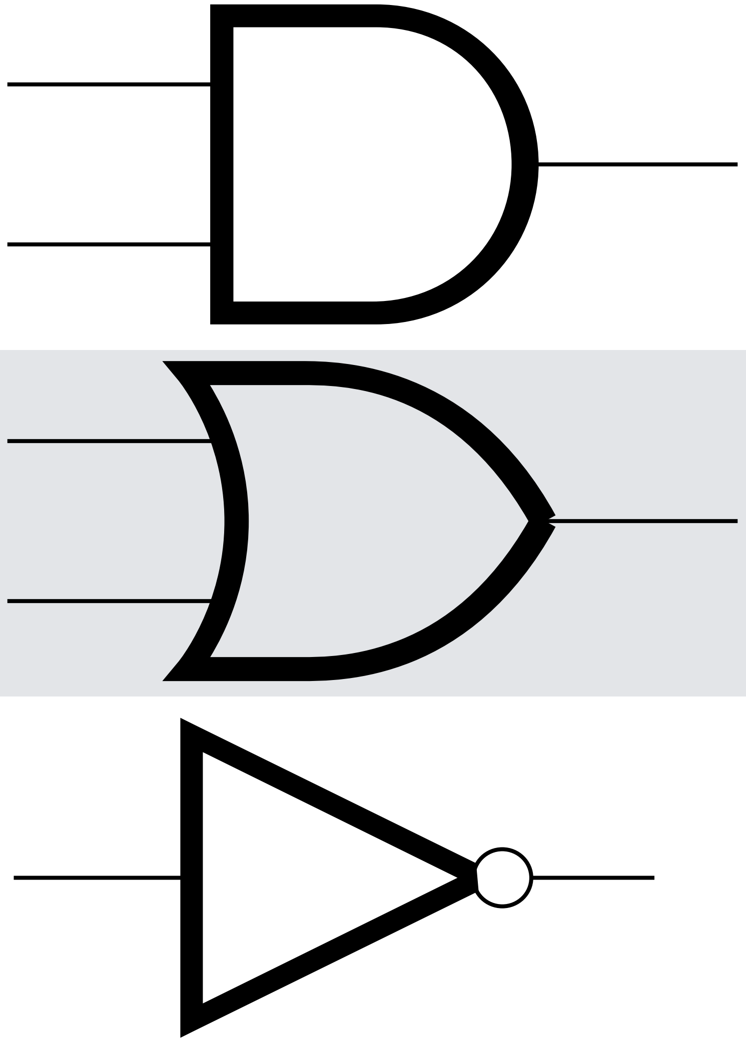
Input	Output
A	
0	1
0	1
1	0
1	0

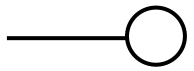
Recap: Boolean operators their circuit “gate” symbols

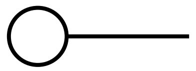
AND

OR

NOT



 represents where we take a compliment value on an input

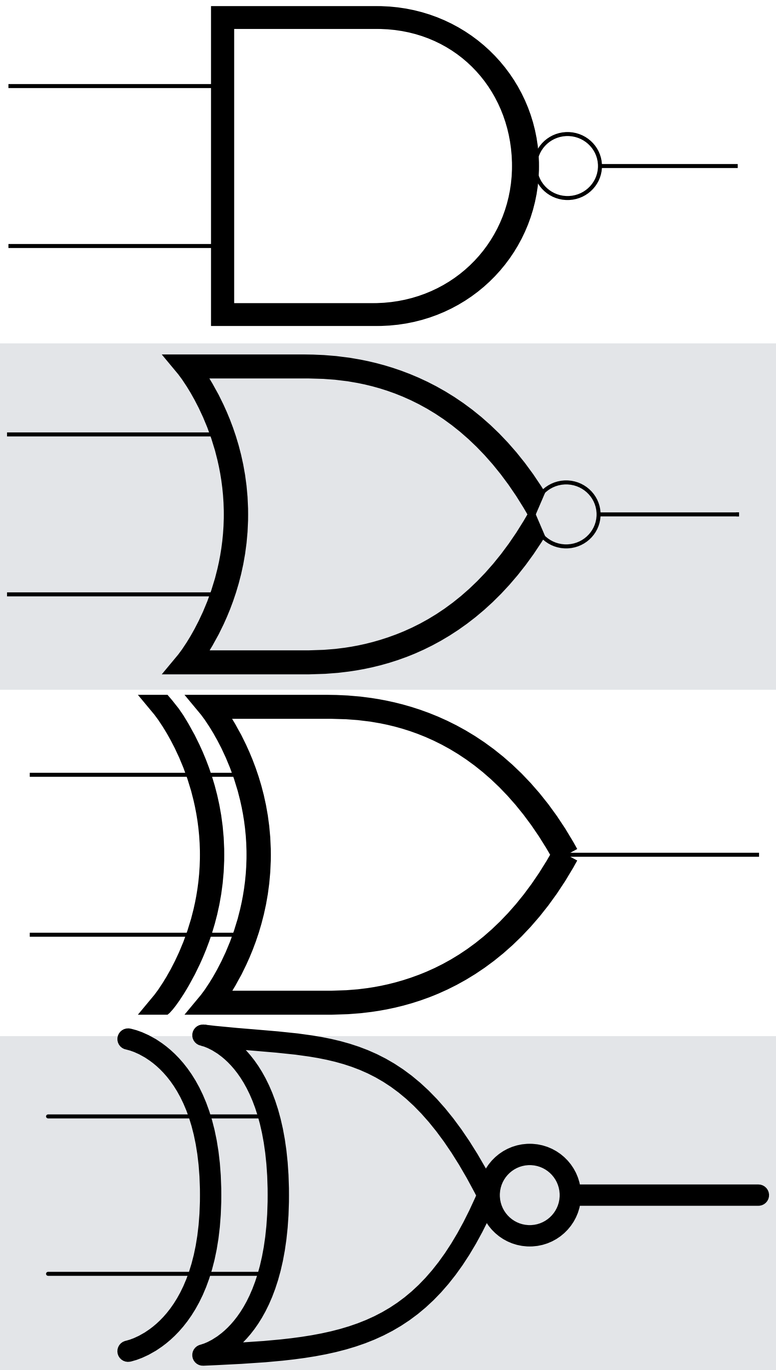
 represents where we take a compliment value on an output

NAND

NOR

XOR

NXOR



Recap: Binary

- 3 + 2 = 5

$$
\begin{array}{r}
{\color{red}1} \\
0011 \\
+\,0010 \\
\hline
0101
\end{array}
$$

- 3 + (-2) = 1

$$
\begin{array}{r}
{\color{red}1} \\
0011 \\
+\,1010 \\
\hline
1101
\end{array}
= -5 \text{ (Not 1)}
$$

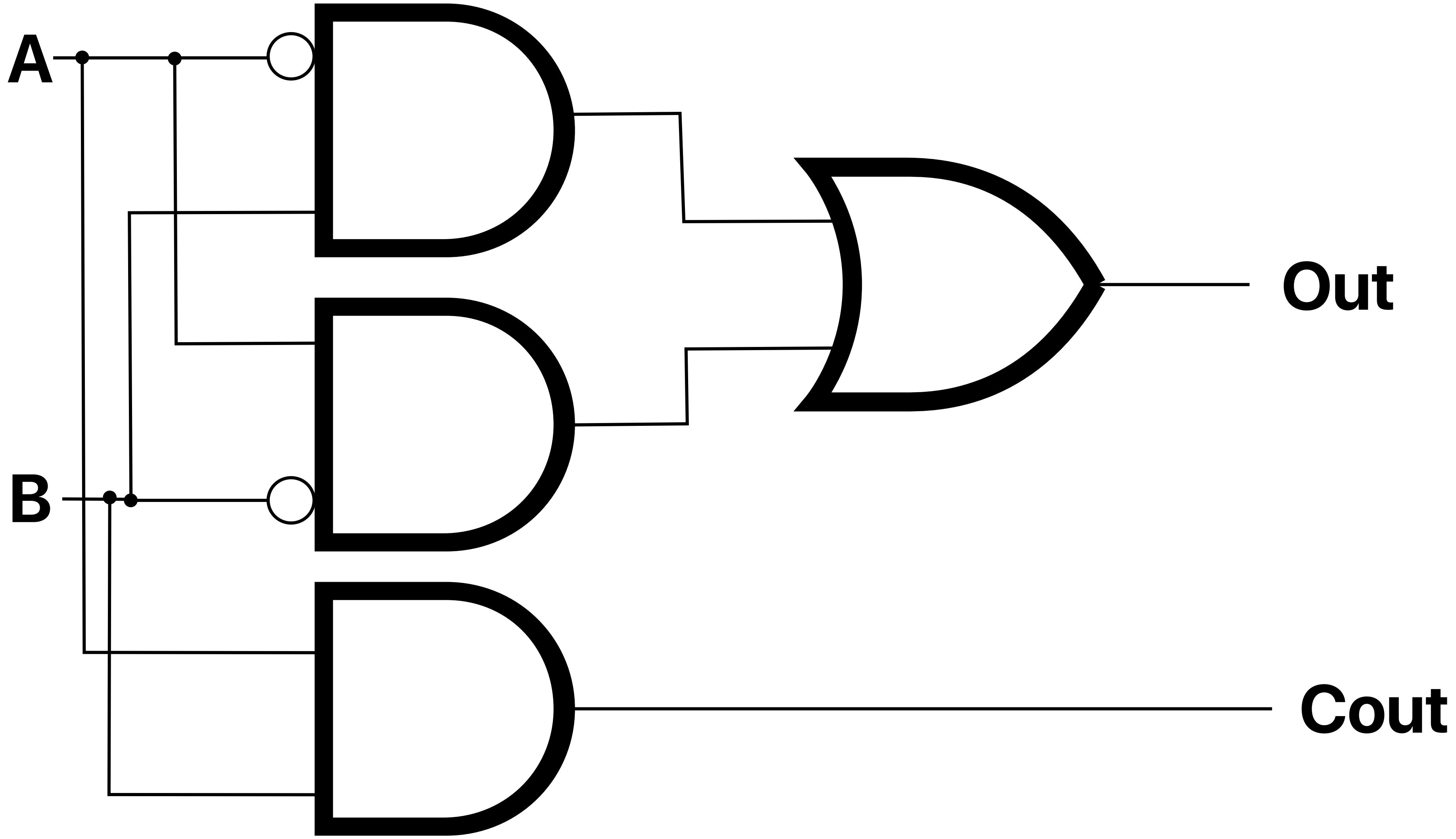
Doesn't work well and you need a separate procedure to deal with negative numbers!

Recap: Half adder

Input		Output	
A	B	Out	Cout
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Out = A′B + AB′

Cout = AB

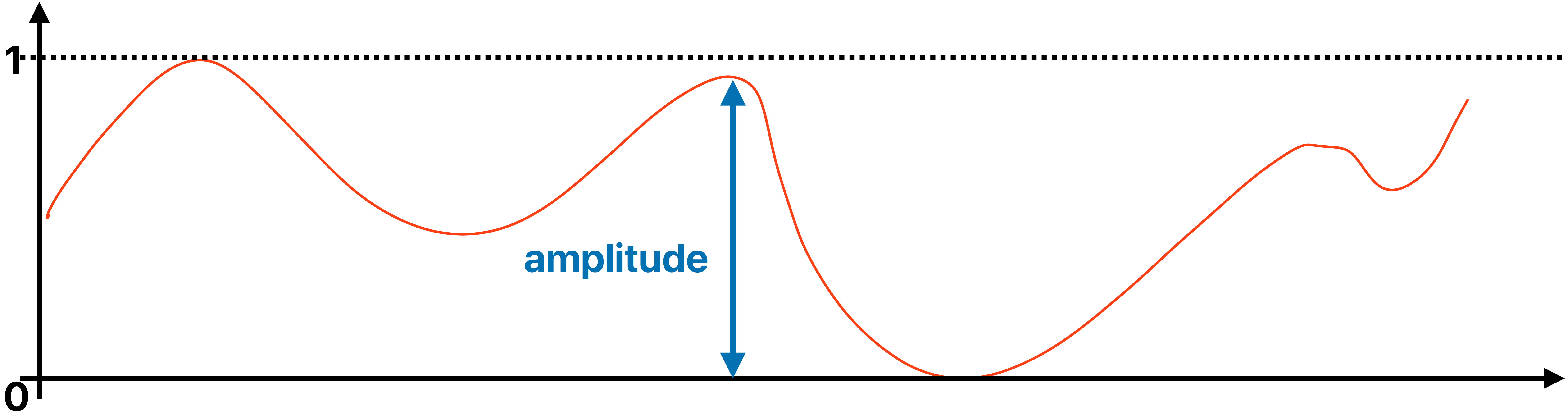


Outline

- What is a qubit?
- The logical gates/operations in quantum computing
- Designing our first quantum circuit
- Qiskit

Qubit — the basic unit of quantum computing

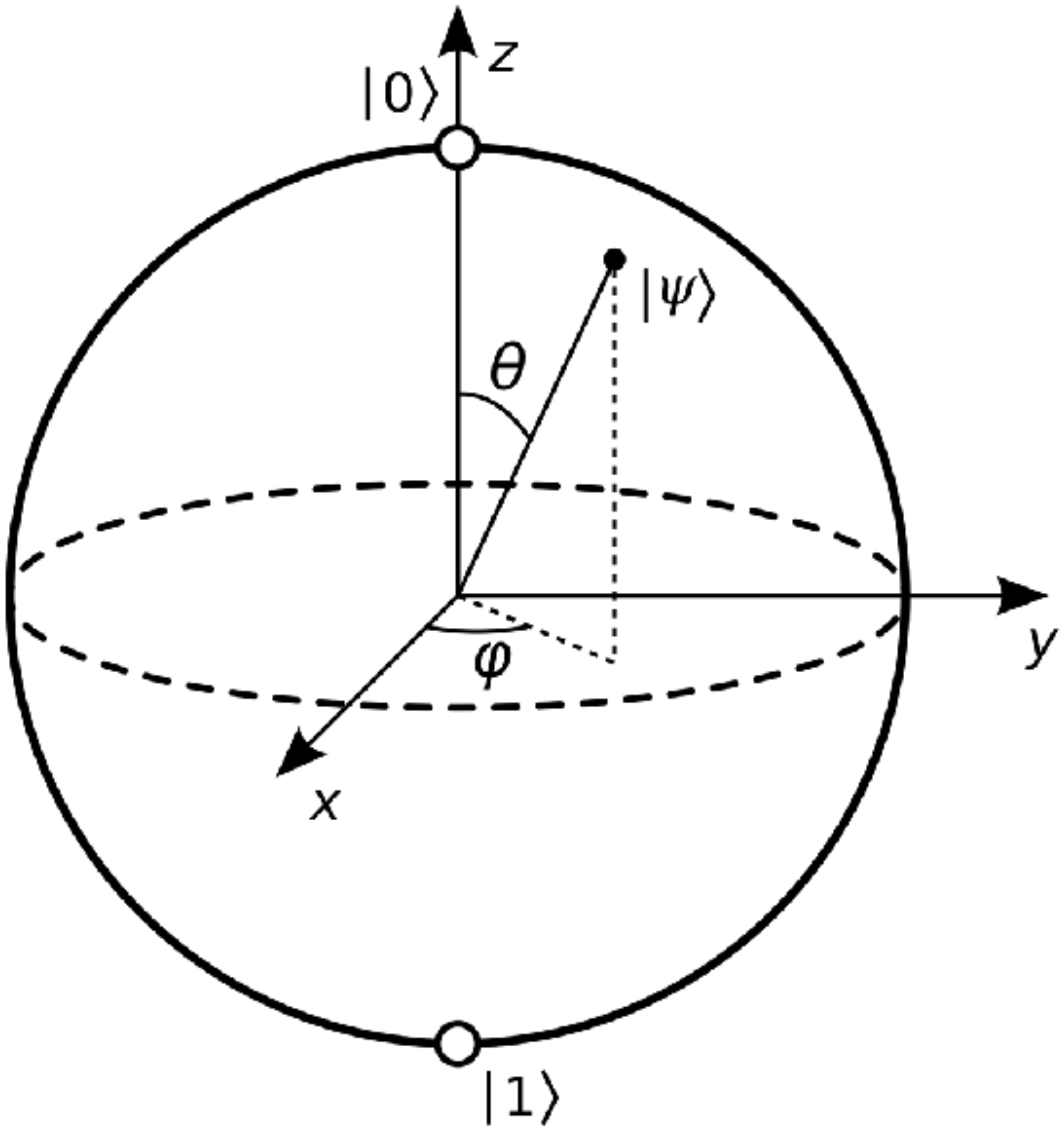
Thinking about an analog signal first



Qubit

- Similar to an analog signal, can use quantum mechanical phenomena of superposition to represent a 0, a 1, or any proportion of 0 and 1 in superposition of both states
- The **amplitude** in quantum mechanics has both **magnitude** and **direction**
  - Each possible outcome has a probability amplitude — for a qubit, the possible outcome is either 0 or 1
  - Each amplitude have a magnitude — the magnitude of that outcome's amplitude tells us how likely that outcome is to occur
- We use the complex number system to represent both **magnitude** and **direction** in qubit states
- Since the probability of all states should be 1, the magnitudes of all amplitudes must sum up to 1

Bloch sphere



$$|\psi\rangle = cos(\frac{\theta}{2})\,|\,0\rangle + e^{i\varphi}sin(\frac{\theta}{2})\,|\,1\rangle$$

Mathematical representation of a qubit

- $|\psi\rangle = a|0\rangle + b|1\rangle$
  - $|a|^2 + |b|^2 = 1$
  - $|a|^2$ is the probability that the qubit will be observed as state 0
  - $|b|^2$ is the probability that the qubit will be observed as state 1
- We can also use a tensor form to express the qubit

$$|\psi\rangle = a|0\rangle + b|1\rangle = a\underbrace{\begin{bmatrix}1\\0\end{bmatrix}} + b\underbrace{\begin{bmatrix}0\\1\end{bmatrix}}_{\text{computational basis}} = \begin{bmatrix}a\\0\end{bmatrix} + \begin{bmatrix}0\\b\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$$

Amplitude and probability

- Which of the following is a valid amplitude but invalid probability?
  A. $\boxed{-1}$
  B. $\frac{1}{3}$
  C. 1.01
  D. $\sqrt{-2}$

Multi-qubit system

- If we have an n-qubit system, we can have 2^n potential outcomes —
$|\psi\rangle = \sum_{i=0}^{2^{n-1}} c_i|x_i\rangle$
$$\sum_{i=0}^{2^{n-1}} |c_i|^2 = 1$$
- We can also express the states with a vector with 2^n elements
- For a 2-qubit system —
$|\psi\rangle = c_0|00\rangle + c_1|01\rangle + c_2|10\rangle + c_3|11\rangle$
$$= c_0 \begin{bmatrix}1\\0\\0\\0\end{bmatrix} + c_1 \begin{bmatrix}0\\1\\0\\0\end{bmatrix} + c_2 \begin{bmatrix}0\\0\\1\\0\end{bmatrix} + c_3 \begin{bmatrix}0\\0\\0\\1\end{bmatrix} = \begin{bmatrix}a\\b\\c\\d\end{bmatrix}$$

computational basis

Amplitude of a 2-qubit system

- Given the state vector —

$$\begin{bmatrix} \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} \\ 0 \\ 0 \end{bmatrix}$$

what is the amplitude of the outcome of 01?

A. 1

B. $\boxed{\sqrt{\frac{1}{2}}}$

C. $\frac{1}{2}$

D. 0

Probability of a 2-qubit system

- Given the state vector —
  $$\begin{bmatrix} \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} \\ 0 \\ 0 \end{bmatrix}$$
  how likely are we going to measure the outcome as 00?
  A. 1
  B. $\sqrt{\frac{1}{2}}$
  C. $\frac{1}{2}$
  D. 0

Valid state

- Which of the following is a valid quantum state?

A. $\boxed{\sqrt{\frac{1}{3}}\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}}$

B. $\sqrt{\frac{1}{2}}\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$

C. $\frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

D. $\sqrt{\frac{1}{2}}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

Superposition

- If the state has more than one possible outcome
- For the state

$$|x\rangle = \begin{bmatrix} \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} \\ 0 \\ 0 \end{bmatrix} = \sqrt{\frac{1}{2}}(|00\rangle + |01\rangle)$$

We call $|x\rangle$ the superposition of $|00\rangle$ and $|01\rangle$

We can also use other basis

- Another popular basis
  - $|+\rangle = \sqrt{\frac{1}{2}}\begin{bmatrix}1\\1\end{bmatrix}$
  - $|-\rangle = \sqrt{\frac{1}{2}}\begin{bmatrix}1\\-1\end{bmatrix}$

Try other basis!

- Find the values of α, β such that the following equation is true
  - $\alpha|+\rangle + \beta|-\rangle = |0\rangle$

$$\alpha\sqrt{\frac{1}{2}}\begin{bmatrix}1\\1\end{bmatrix} + \beta\sqrt{\frac{1}{2}}\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$$

$$\sqrt{\frac{1}{2}}\alpha + \sqrt{\frac{1}{2}}\beta = 1$$

$$\sqrt{\frac{1}{2}}\alpha - \sqrt{\frac{1}{2}}\beta = 0$$

$$2\sqrt{\frac{1}{2}}\alpha = \sqrt{2}\alpha = 1$$

$$\alpha = \frac{1}{\sqrt{2}} \quad \beta = \frac{1}{\sqrt{2}}$$

Try other basis!!

- Find the values of γ, δ such that the following equation is true
  - $\gamma|+\rangle + \delta|-\rangle = |1\rangle$

$$\gamma\sqrt{\frac{1}{2}}\begin{bmatrix}1\\1\end{bmatrix} + \delta\sqrt{\frac{1}{2}}\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}$$

$$\sqrt{\frac{1}{2}}\gamma + \sqrt{\frac{1}{2}}\delta = 0$$

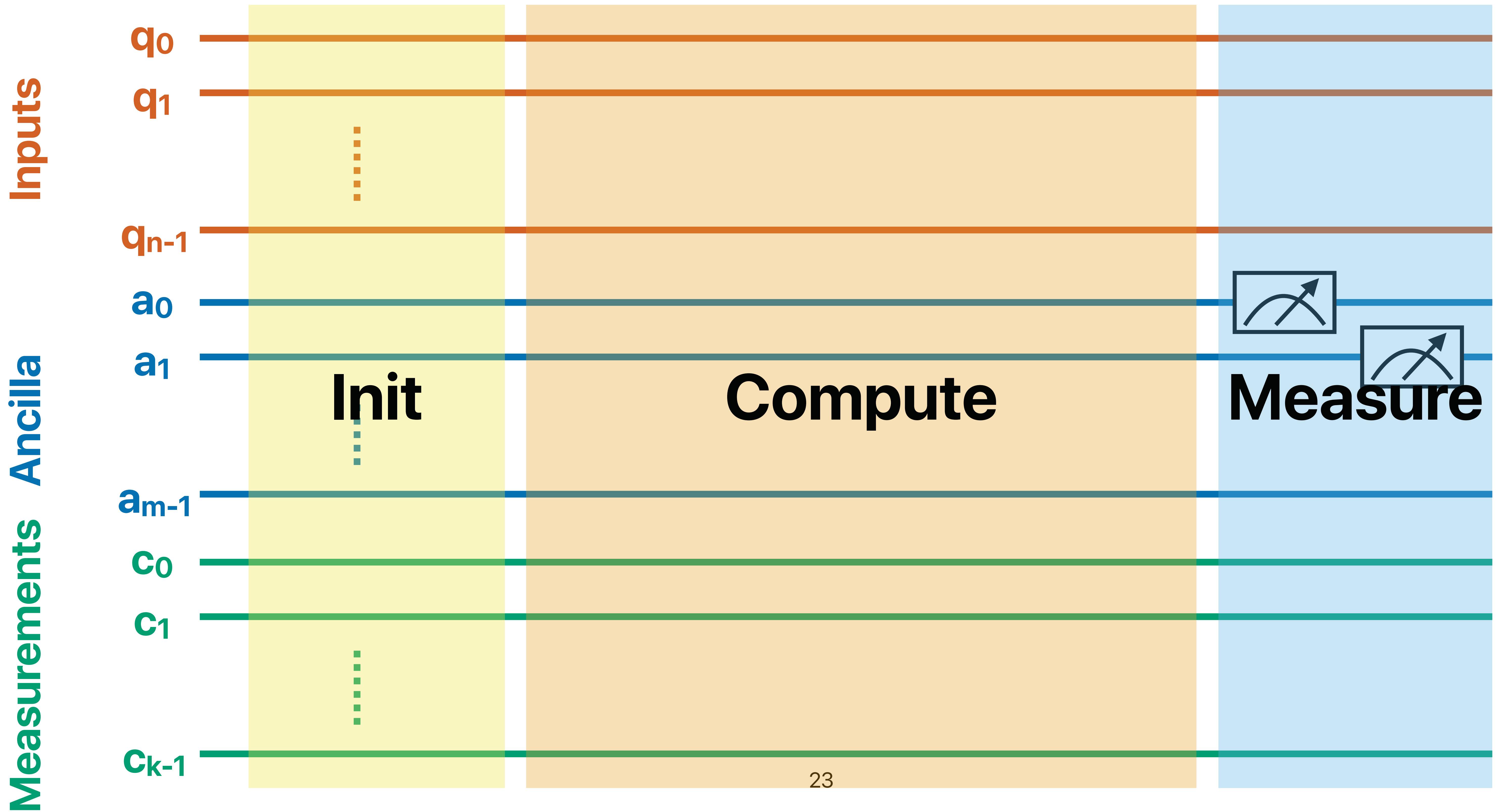
$$\sqrt{\frac{1}{2}}\gamma - \sqrt{\frac{1}{2}}\delta = 1$$

$$2\sqrt{\frac{1}{2}}\gamma = \sqrt{2}\gamma = 1$$

$$\gamma = \frac{1}{\sqrt{2}} \qquad \delta = -\frac{1}{\sqrt{2}}$$

High-level view of quantum circuits

Quantum circuits



Quantum Logical Gates

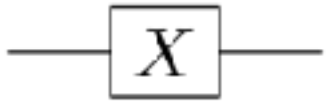
Quantum Logical Gates

- Transition the amplitudes of the original n-qubit inputs to another set of amplitudes
- $2^n \rightarrow 2^n$ transitions
- Can be expressed as an $2^n \times 2^n$ matrix

$$\begin{bmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{bmatrix} \rightarrow \begin{bmatrix} b_{00} \\ b_{01} \\ b_{10} \\ b_{11} \end{bmatrix} \qquad \begin{bmatrix} a_{00} \rightarrow b_{00} & a_{01} \rightarrow b_{00} & a_{10} \rightarrow b_{00} & a_{11} \rightarrow b_{00} \\ a_{00} \rightarrow b_{01} & a_{01} \rightarrow b_{01} & a_{10} \rightarrow b_{01} & a_{11} \rightarrow b_{01} \\ a_{00} \rightarrow b_{10} & a_{01} \rightarrow b_{10} & a_{10} \rightarrow b_{10} & a_{11} \rightarrow b_{10} \\ a_{00} \rightarrow b_{11} & a_{01} \rightarrow b_{11} & a_{10} \rightarrow b_{11} & a_{11} \rightarrow b_{11} \end{bmatrix}$$

Pauli gates

- Rotate the qubit around the x, y, or z axis

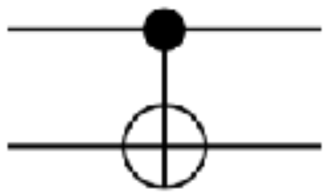
Pauli-X / NOT / Bit-flip	1-bit		$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli Y	1-bit		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli Z / Phase flip	1-bit		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} i \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Controlled gates — CNOT

Controlled NOT 2-bit 

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$|x\rangle = \boxed{|00\rangle}$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \boxed{|01\rangle}$$

$|x\rangle = \boxed{|10\rangle}$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \boxed{|11\rangle}$$

$|x\rangle = \boxed{|01\rangle}$

Nothing changes if the first qubit is 0

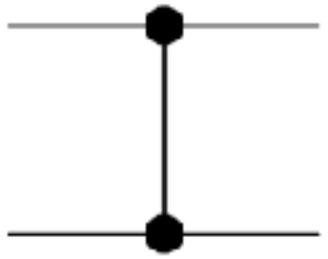
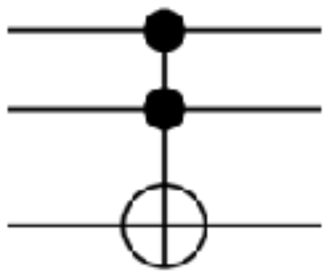
$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \boxed{|01\rangle}$$

$|x\rangle = \boxed{|11\rangle}$

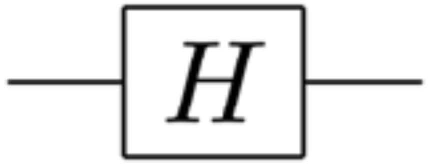
2nd bit is flipped if the 1st bit is 1

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \boxed{|10\rangle}$$

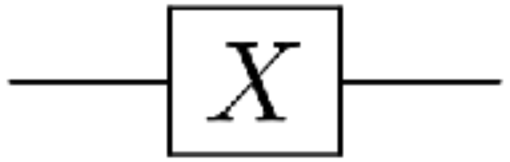
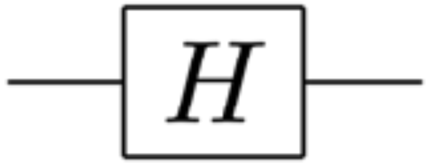
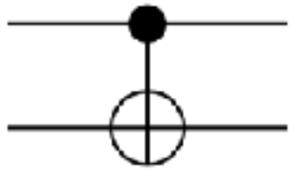
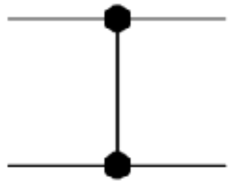
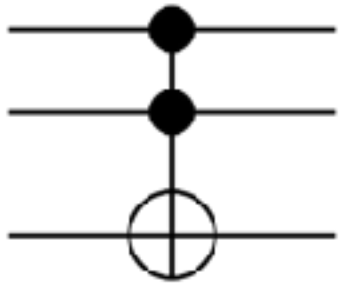
Other controlled gates

Controlled Z	2-bit		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
Toffoli	3-bit		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

Hadamard gate

Hadamard	1-bit	 	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$

Important quantum logical gates

Pauli-X / NOT / Bit-flip	1-bit		$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli Y	1-bit		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli Z / Phase flip	1-bit		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard	1-bit		$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Controlled NOT	2-bit		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z	2-bit		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
Toffoli	3-bit		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

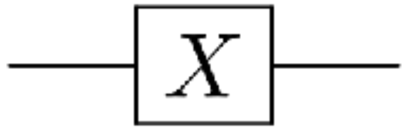
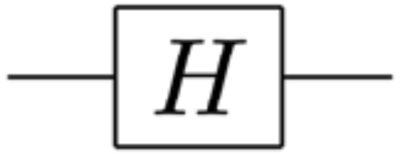
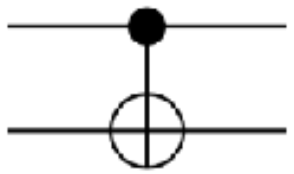
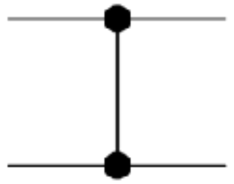
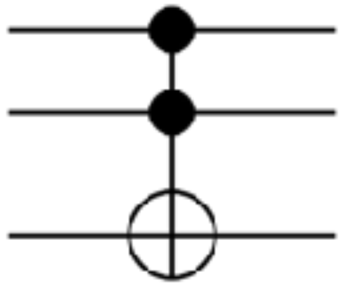
Conversion between classical and quantum gates

XOR

Input		Output
A	**B**	
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**0**

$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |01\rangle$$
$$|10\rangle \rightarrow |11\rangle$$
$$|11\rangle \rightarrow |10\rangle$$

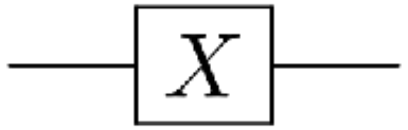
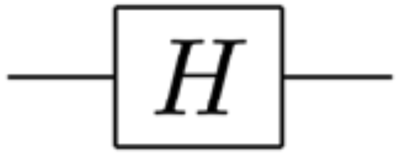
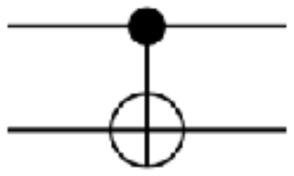
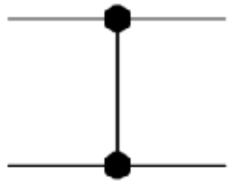
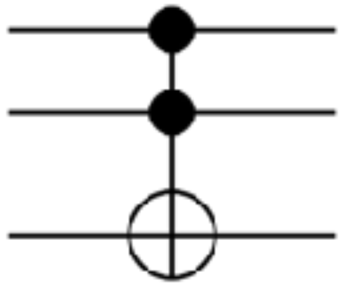
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Pauli-X / NOT / Bit-flip		$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli Y		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli Z / Phase flip		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard		$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Controlled NOT		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
Toffoli		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

AND

Input		Output
A	B	
0	0	**0**
0	1	**0**
1	0	**0**
1	1	**1**

$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |00\rangle$$
$$|10\rangle \rightarrow |10\rangle$$
$$|11\rangle \rightarrow |11\rangle$$

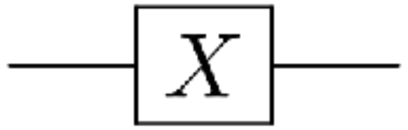
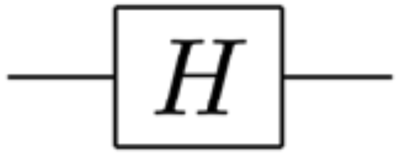
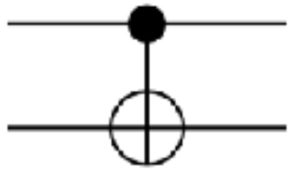
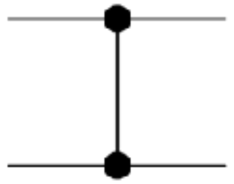
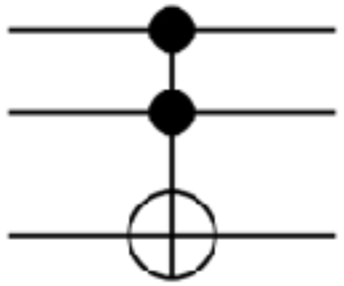
Pauli-X / NOT / Bit-flip		$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli Y		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli Z / Phase flip		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard		$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Controlled NOT		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
Toffoli		$\begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \end{bmatrix}$

AND

Input			Output
A	B		
0	0	**0**	**0**
0	1	**0**	**0**
1	0	**0**	**0**
1	1	**0**	**1**

$$
\begin{aligned}
&|000 \rightarrow |000\\
&\cancel{|001 \rightarrow |001}\\
&|010 \rightarrow |010\\
&\cancel{|011 \rightarrow |011}\\
&|100 \rightarrow |100\\
&\cancel{|101 \rightarrow |101}\\
&|110 \rightarrow |111\\
&\cancel{|111 \rightarrow |110}
\end{aligned}
$$

$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}
$$

Pauli-X / NOT / Bit-flip		$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli Y		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli Z / Phase flip		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard		$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Controlled NOT		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
Toffoli		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

Design the first quantum circuit

1-bit half adder

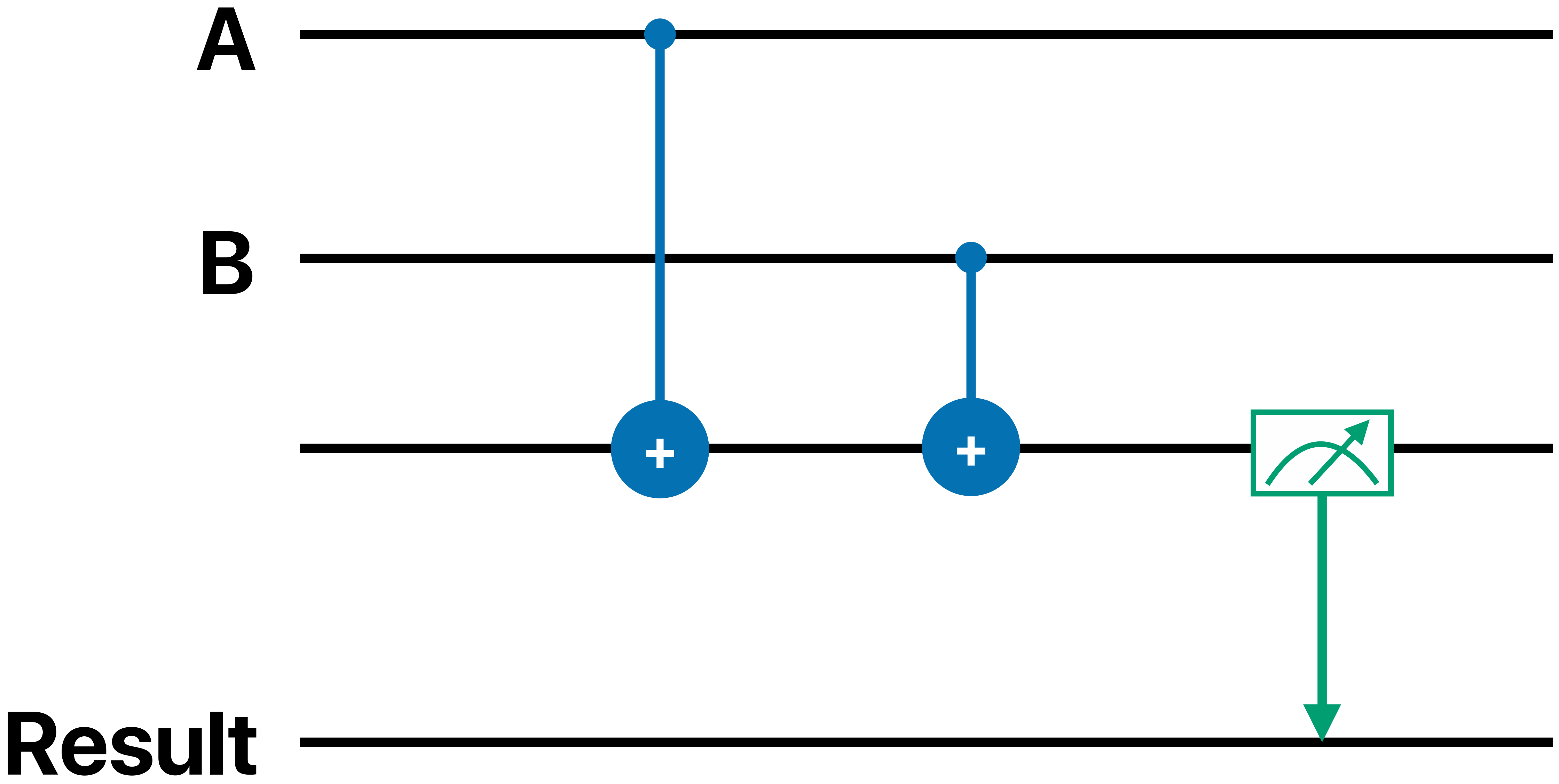
A	B	carry	result
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Which gate do you have in mind?

Result is the same as b if a is 0

Result is the flipped of b if a is 1

1-bit half adder



1-bit half adder

A	B	carry	result
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Which gate do you have in mind?

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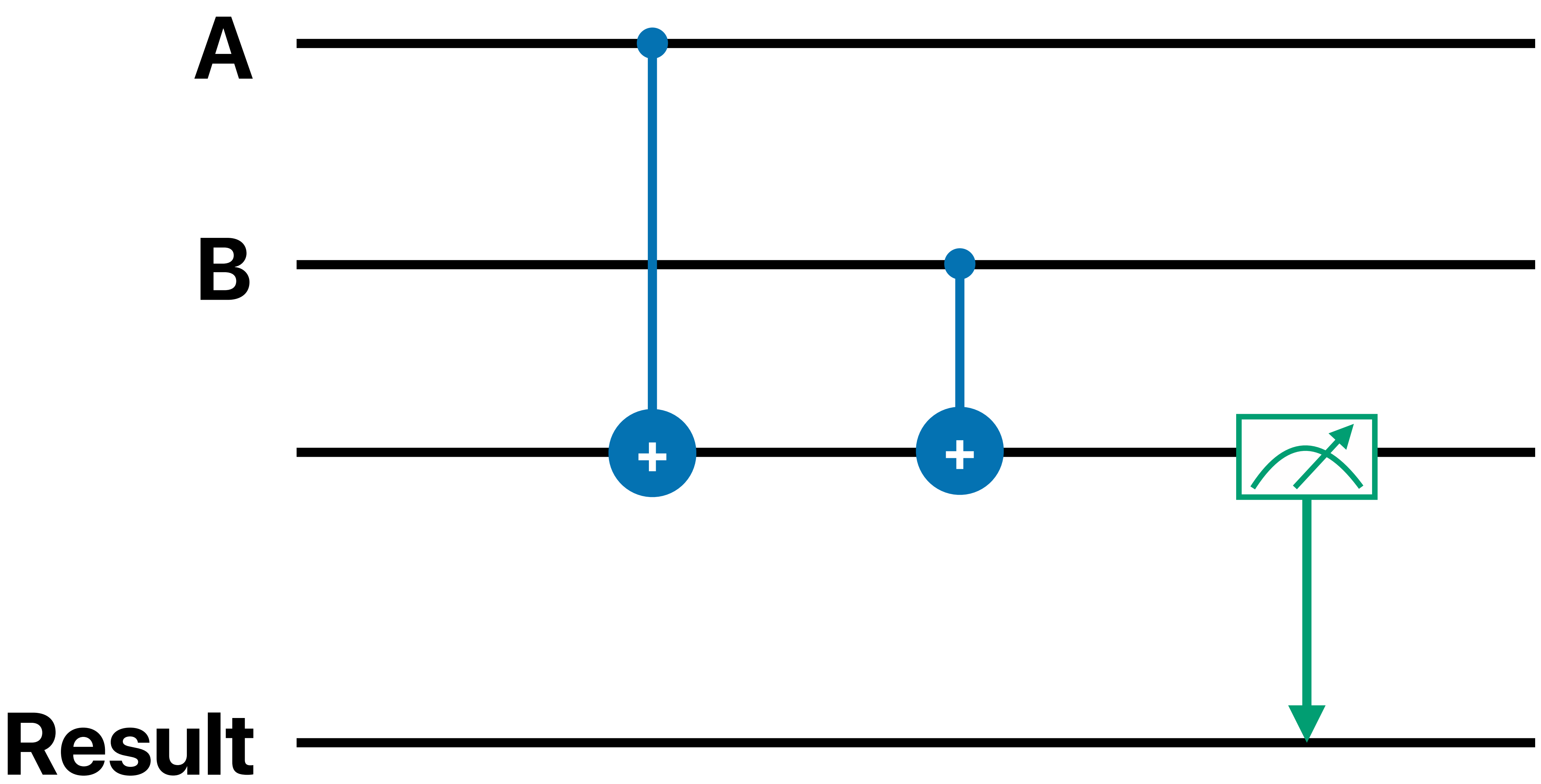
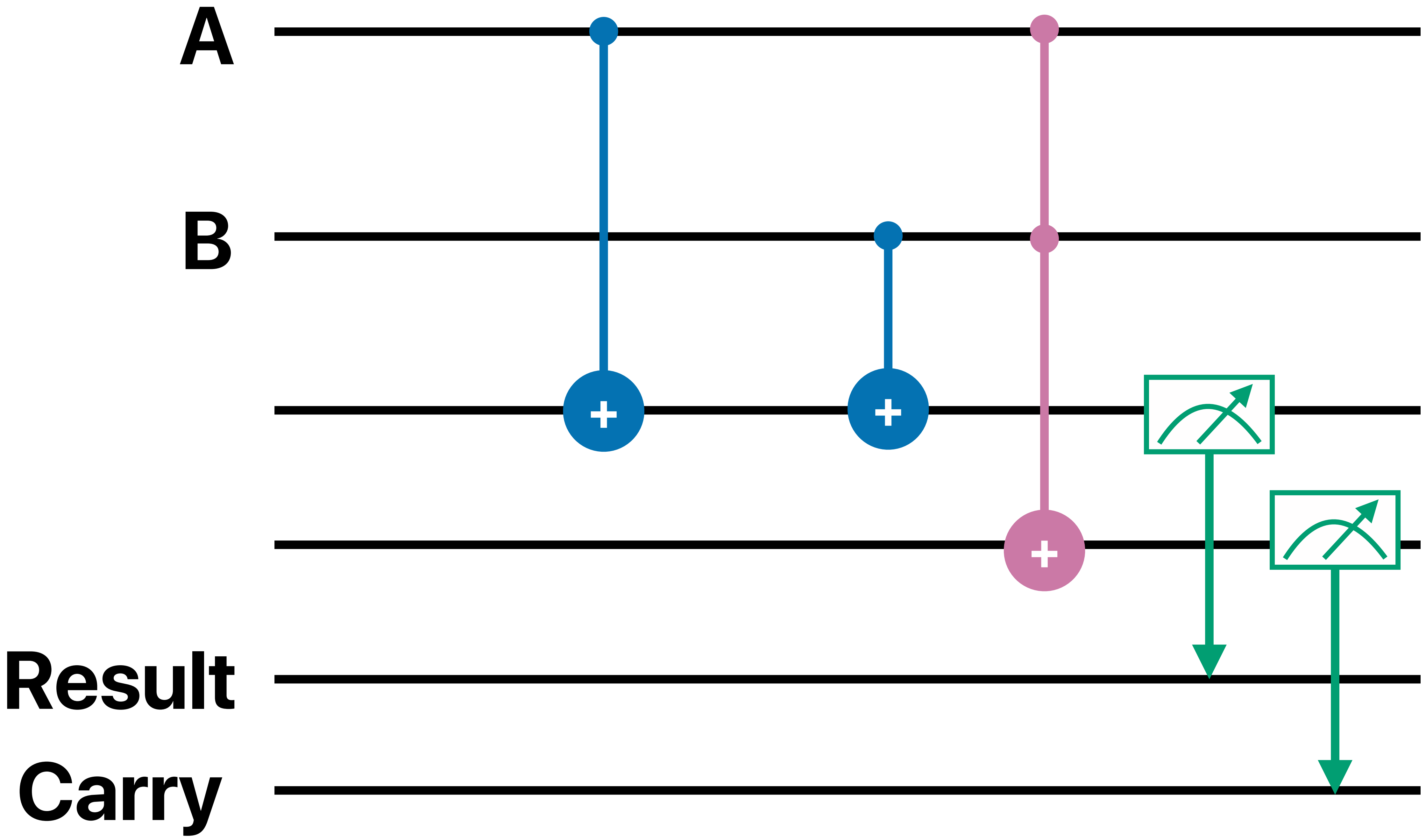
1-bit half adder

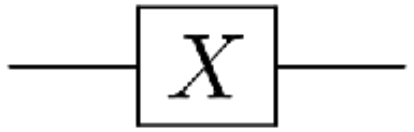
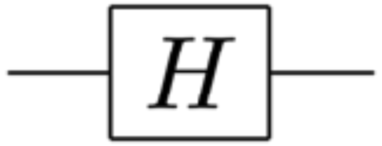
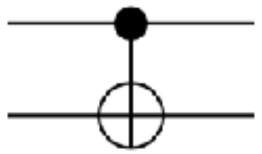
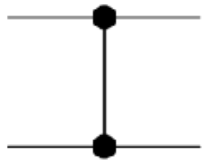
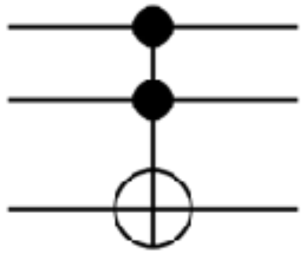


1-bit half adder



Using Qiskit

Qiskit & quantum gates

Gate	Input/Output bits	Symbol	Transition Matrix	Qiskit Method
Pauli-X / NOT / Bit-flip	1-bit		$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	QuantumCircuit.x
Pauli Y	1-bit		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	QuantumCircuit.y
Pauli Z / Phase flip	1-bit		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	QuantumCircuit.z
Hadamard	1-bit		$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	QuantumCircuit.h
Controlled NOT	2-bit		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	QuantumCircuit.cx
Controlled Z	2-bit		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$	QuantumCircuit.cz
Toffoli	3-bit		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$	QuantumCircuit.ccx

Using qiskit on datahub

- Go to datahub.escalab.org
- Login with your UCR gmail
- Select EE214 container to start
- Open a terminal from the user interface and
  `git clone git@github.com:CS203UCR/ee214_25wi.git`