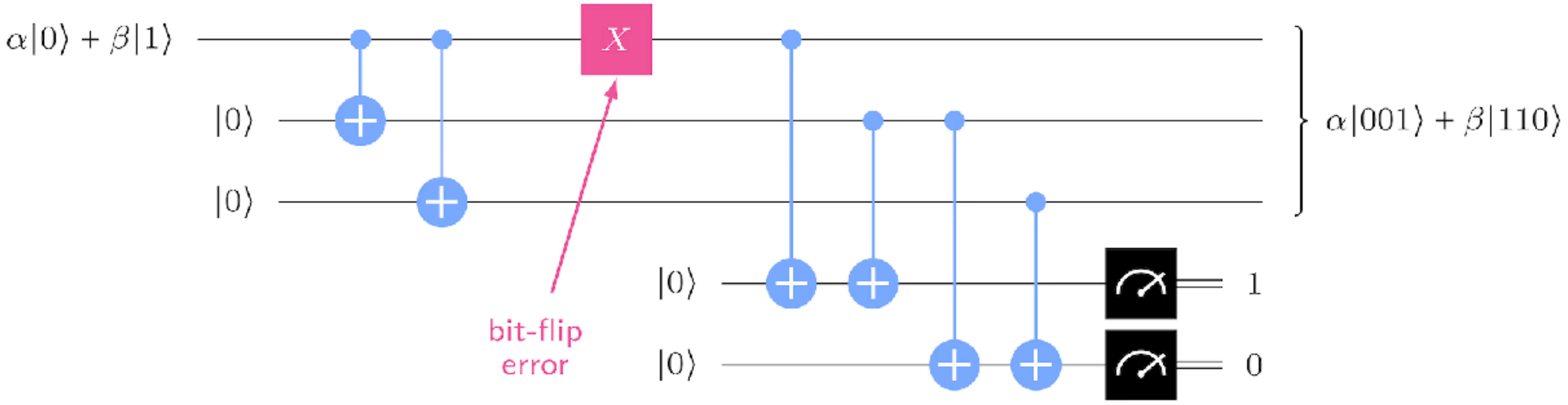
Quantum Error Correction (2)

Hung-Wei Tseng

If q_0 gets wrong

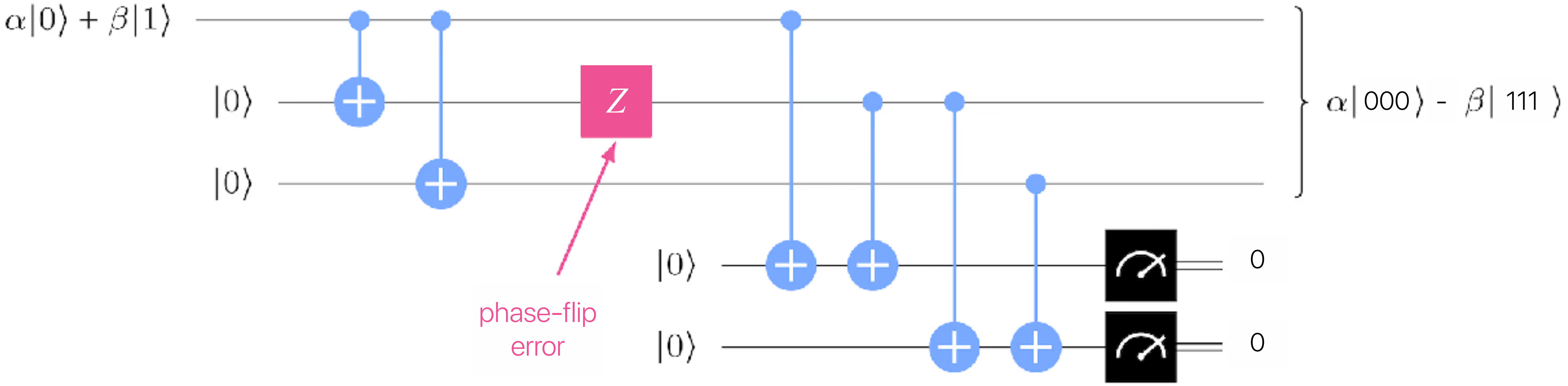


$\alpha|001\rangle+\beta|110\rangle$

01

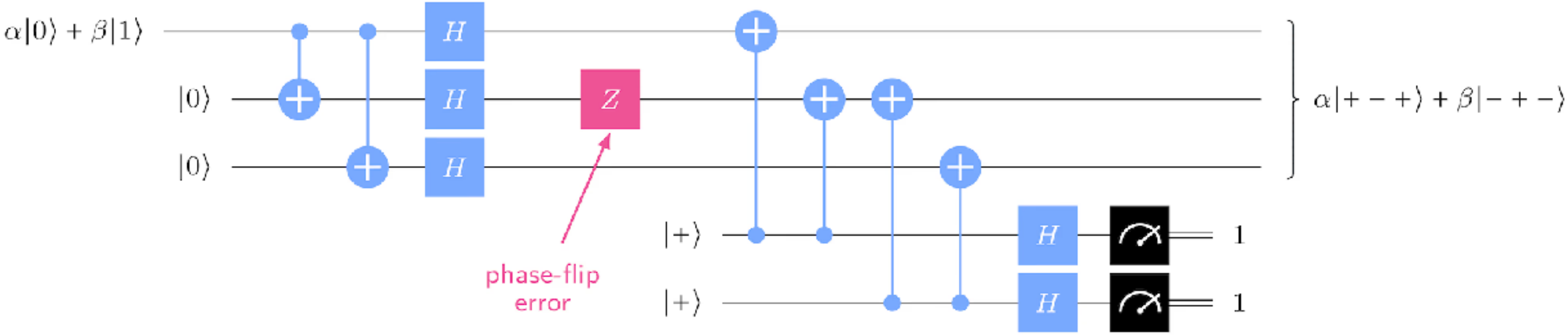
$I \otimes I \otimes X$

But, what if?

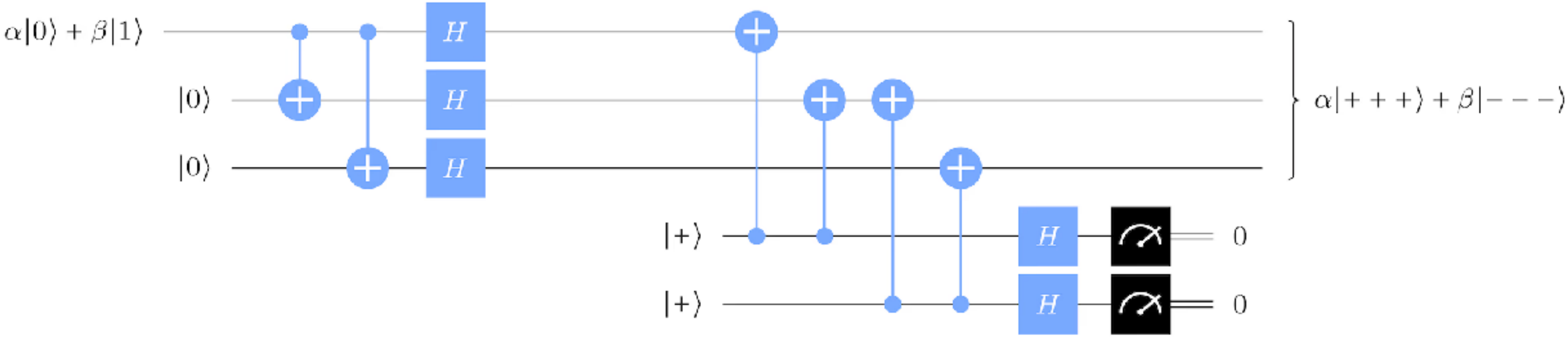


$\alpha|0\rangle + \beta|1\rangle$
$|0\rangle$
$|0\rangle$
Z
phase-flip error
$|0\rangle$
$|0\rangle$
0
0
$\alpha|000\rangle - \beta|111\rangle$

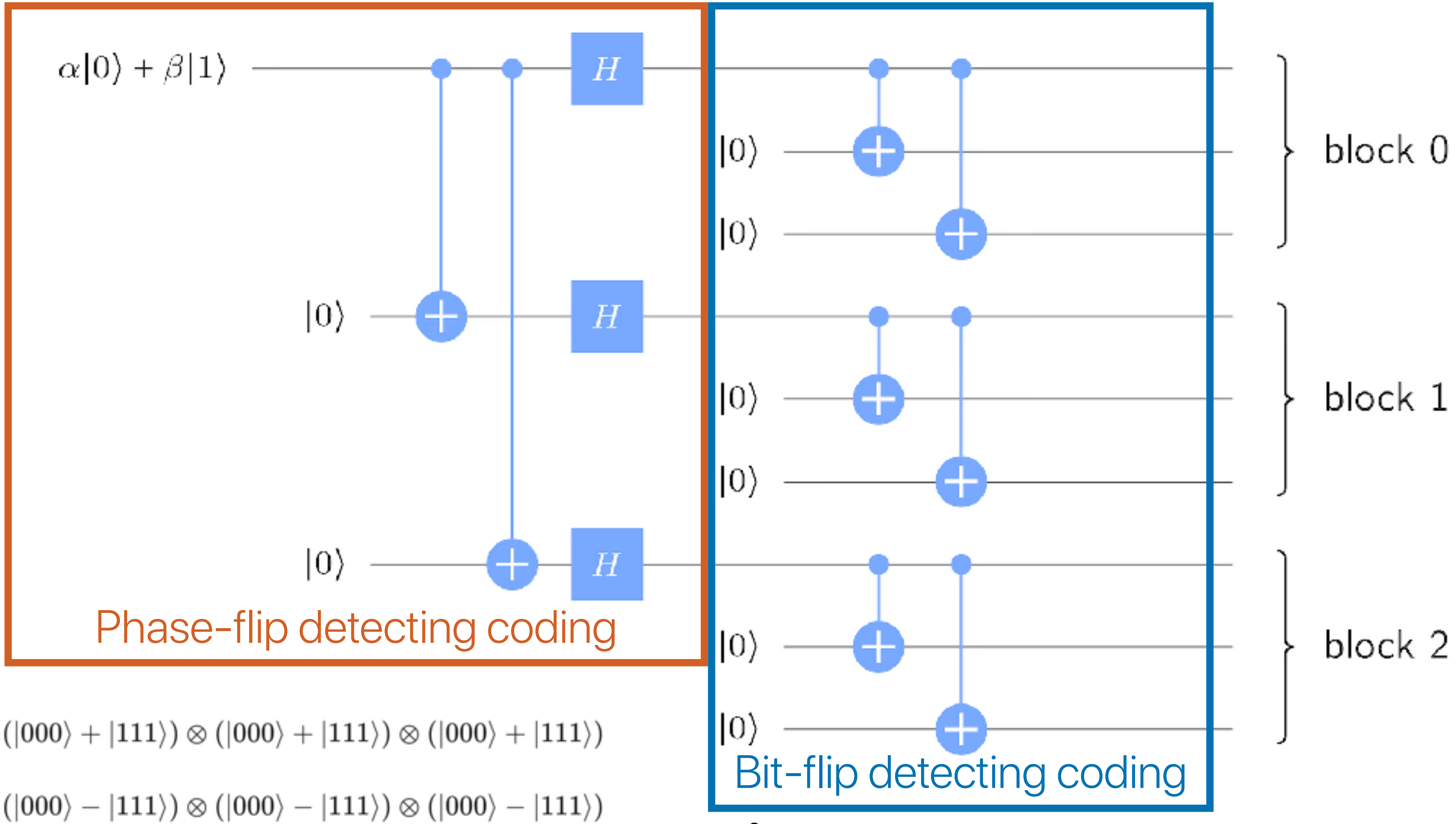
If q_1 gets wrong



Does this work for bit-flips?



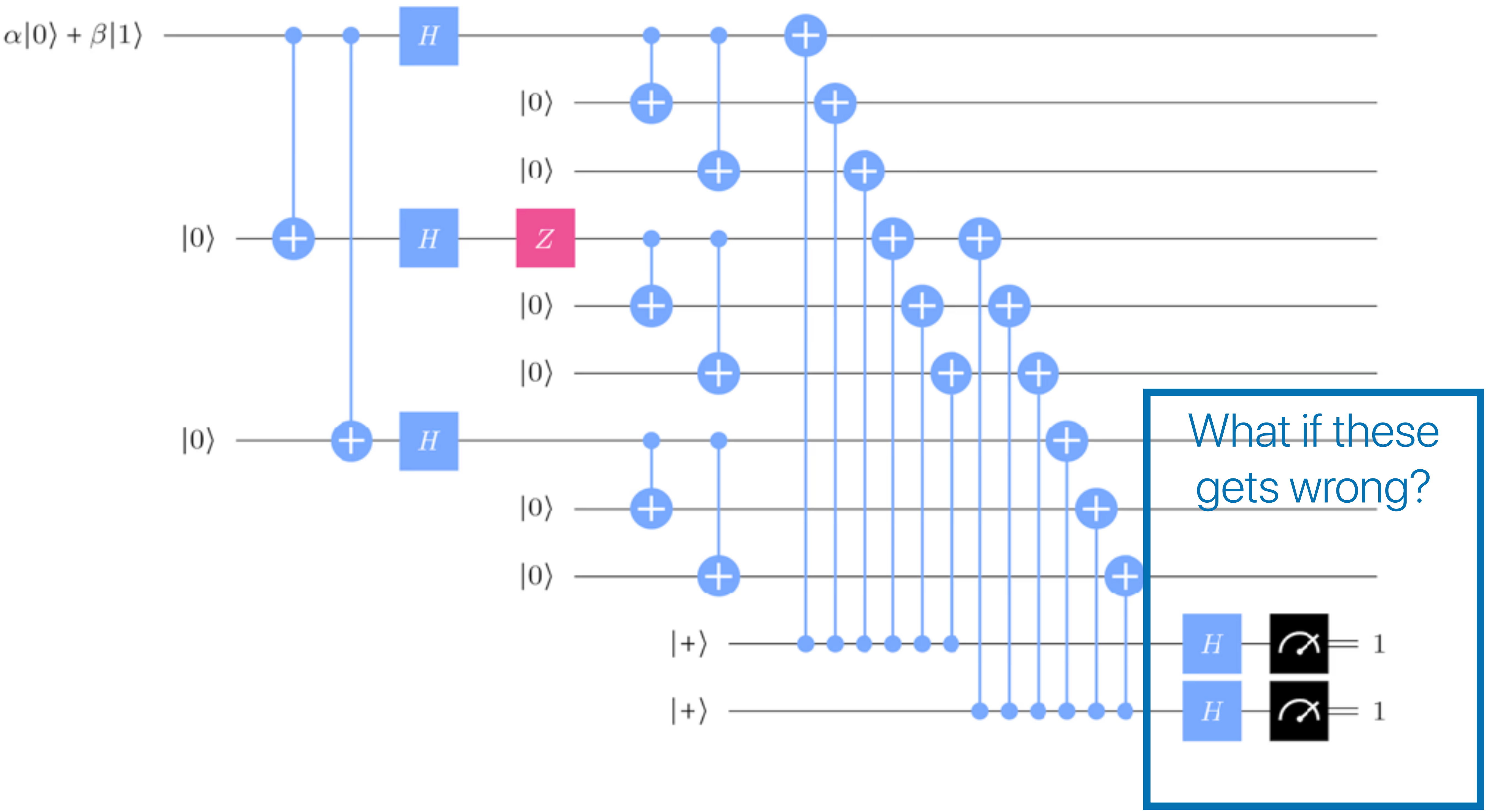
9-qubit Shor code



$$|0\rangle \mapsto \frac{1}{2\sqrt{2}}(|000\rangle + |111\rangle) \otimes (|000\rangle + |111\rangle) \otimes (|000\rangle + |111\rangle)$$

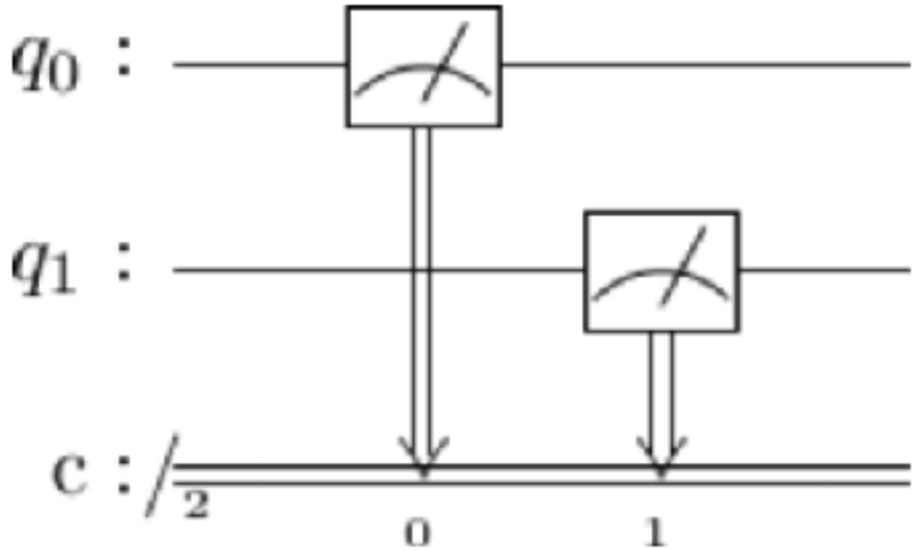
$$|1\rangle \mapsto \frac{1}{2\sqrt{2}}(|000\rangle - |111\rangle) \otimes (|000\rangle - |111\rangle) \otimes (|000\rangle - |111\rangle)$$

Shor coding with error corrections



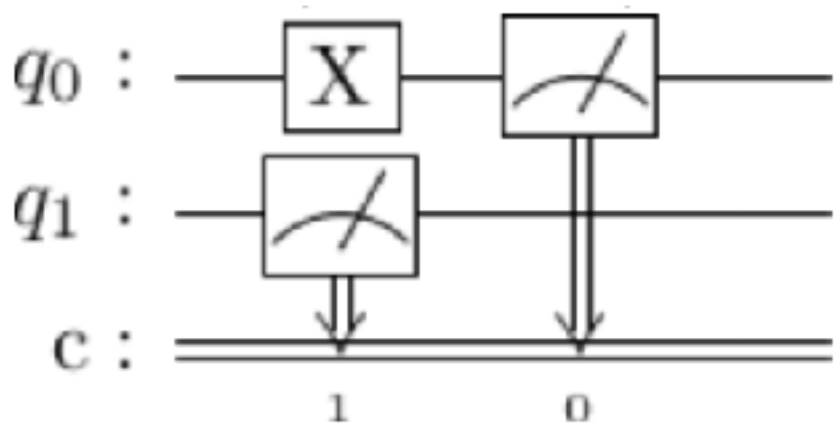
Correct measurement errors

Not only gates will have errors, measurements too!



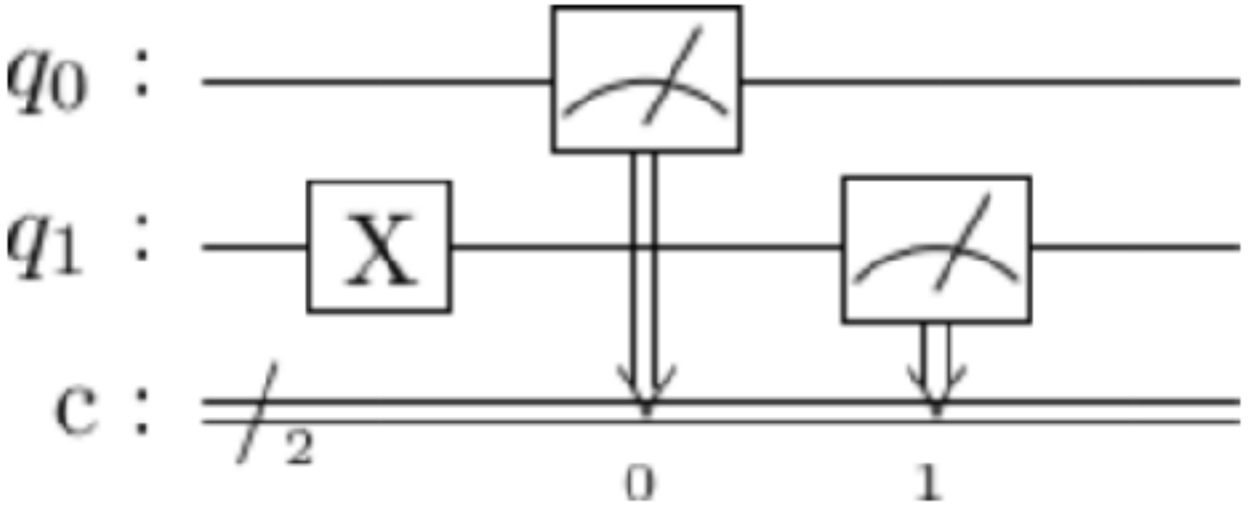
{'11': 1, '01': 94, '10': 96, '00': 9809}

00 → '10': 96, '11': 1, '01': 94, '00': 9809



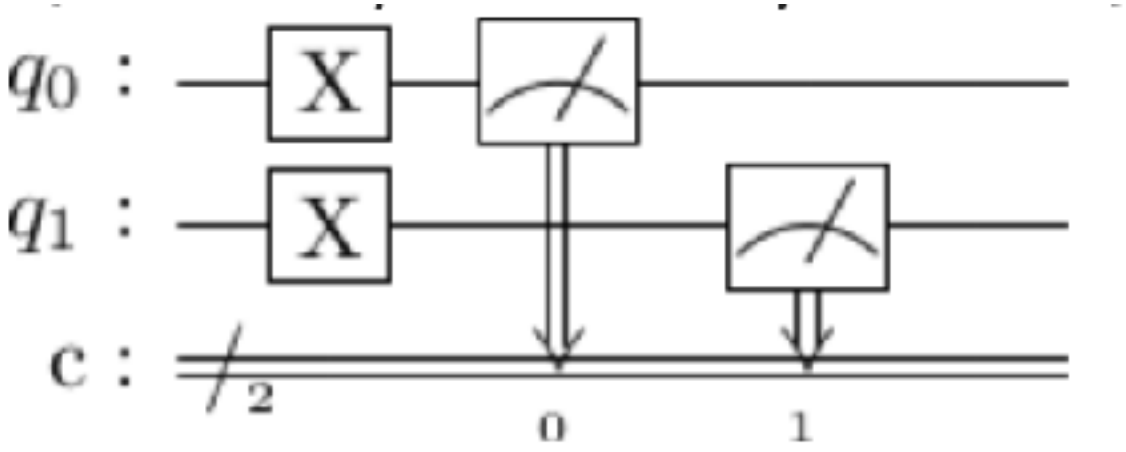
{'11': 106, '00': 110, '01': 9784}

01 → '10': 0, '11': 106, '01': 9784, '00': 110



{'00': 123, '10': 9778, '11': 99}

10 → '10': 9778, '11': 99, '01': 0, '00': 123



{'01': 87, '10': 82, '11': 9831}

11 → '10': 82, '11': 9831, '01': 87, '00': 0

Error mitigation in with linear algebra

$00 \rightarrow$ '10': 96, '11': 1, '01': 94, '00': 9809

$01 \rightarrow$ '10': 0, '11': 106, '01': 9784, '00': 110

$10 \rightarrow$ '10': 9778, '11': 99, '01': 0, '00': 123

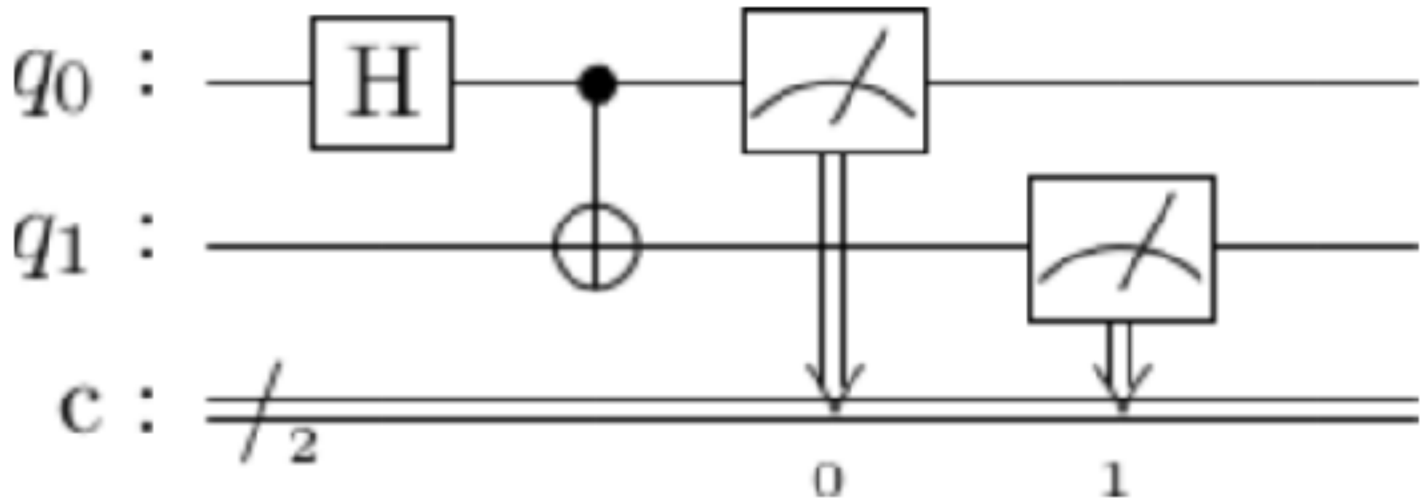
$11 \rightarrow$ '10': 82, '11': 9831, '01': 87, '00': 0

$$M = \begin{pmatrix} 0.9809 & 0.0110 & 0.0123 & 0.0000 \\ 0.0094 & 0.9784 & 0.0000 & 0.0087 \\ 0.0096 & 0.0000 & 0.9778 & 0.0082 \\ 0.0001 & 0.0106 & 0.0099 & 0.9831 \end{pmatrix}$$

$$C_{\text{noisy}} = M\ C_{\text{ideal}}$$

Consider the state of $|+\rangle$

$$\begin{pmatrix} 0.9809 & 0.0110 & 0.0123 & 0.0000 \\ 0.0094 & 0.9784 & 0.0000 & 0.0087 \\ 0.0096 & 0.0000 & 0.9778 & 0.0082 \\ 0.0001 & 0.0106 & 0.0099 & 0.9831 \end{pmatrix} \begin{pmatrix} 5000 \\ 0 \\ 0 \\ 5000 \end{pmatrix} = \begin{pmatrix} 4905. \\ 100.5 \\ 91.5 \\ 4903 \end{pmatrix}$$




```
{'01': 99, '11': 4870, '10': 92, '00': 4939}
```

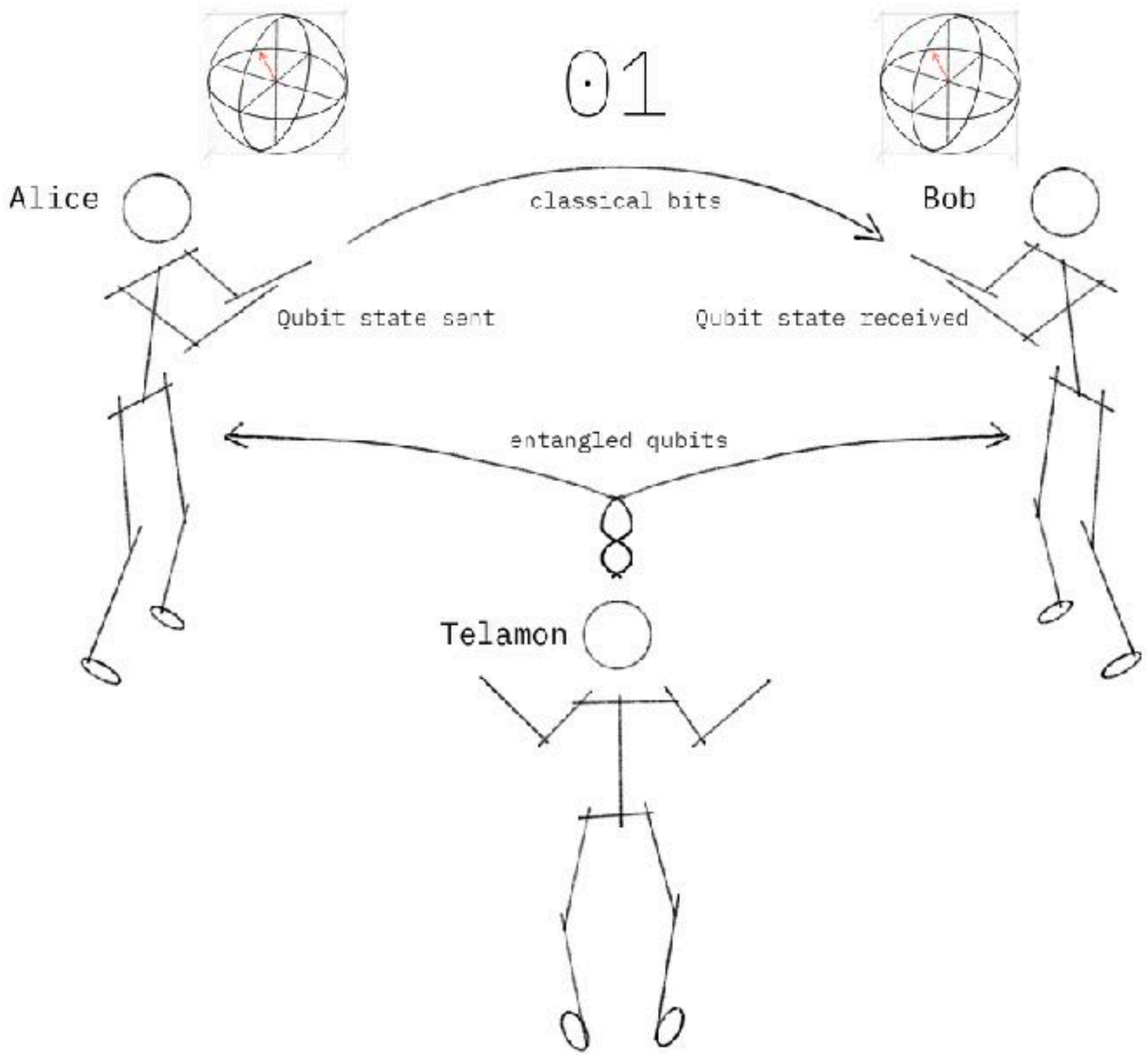
$$\begin{pmatrix} 4939 \\ 99 \\ 92 \\ 4870 \end{pmatrix}$$

How to recover?

$$C_{\text{noisy}} = M\ C_{\text{ideal}}$$

$$\rightarrow C_{\text{noisy}} M^{-1} = C_{\text{ideal}}$$

Recap on Quantum Teleportation


01
Alice
classical bits
Bob
Qubit state sent
Qubit state received
entangled qubits
Telamon

How to make a good quantum computer

What affects computing power?

- Qubit count — Number of qubits (more is better)
- Readout errors — Gate and readout errors (less is better)
- Connectivity — Qubit-Qubit connectivity (more is better)
- Gate set — Gate set (larger / more powerful is better)
- Software stack — Compilers and software stack (more intelligent is better)

Qubit Technologies

Trapped Ion Qubits

- Optical Qubits
- Hyperfine Qubits

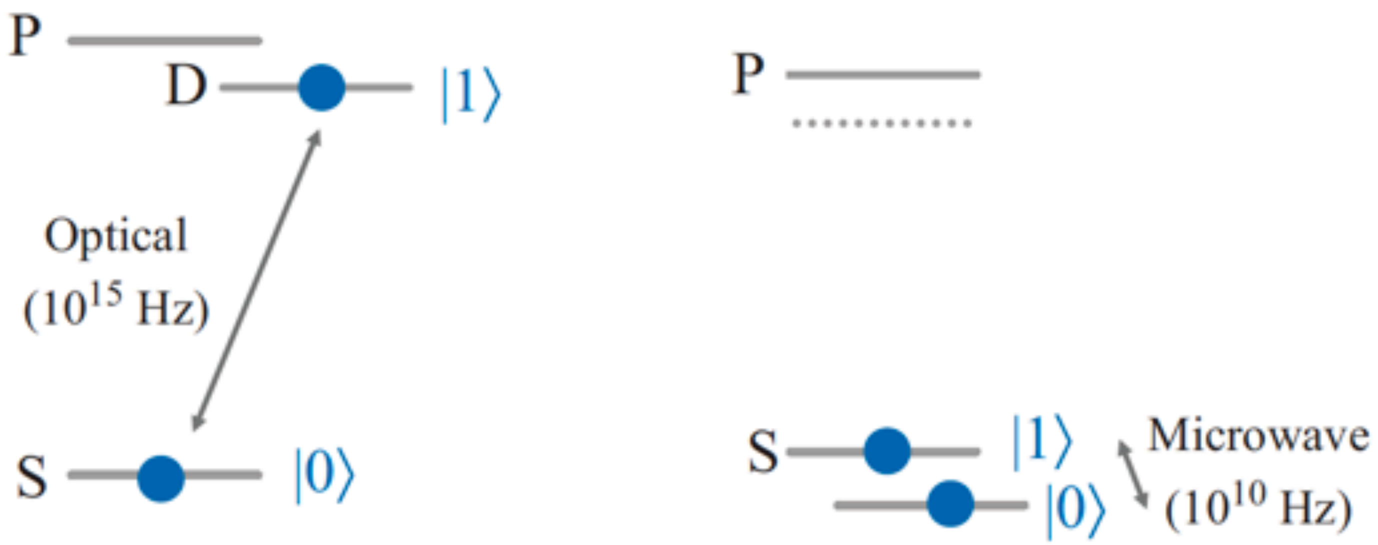


Figure 2.6: State transitions for two common types of trapped ion qubits: the optical qubit and the hyperfine qubit.

Measuring Qubits

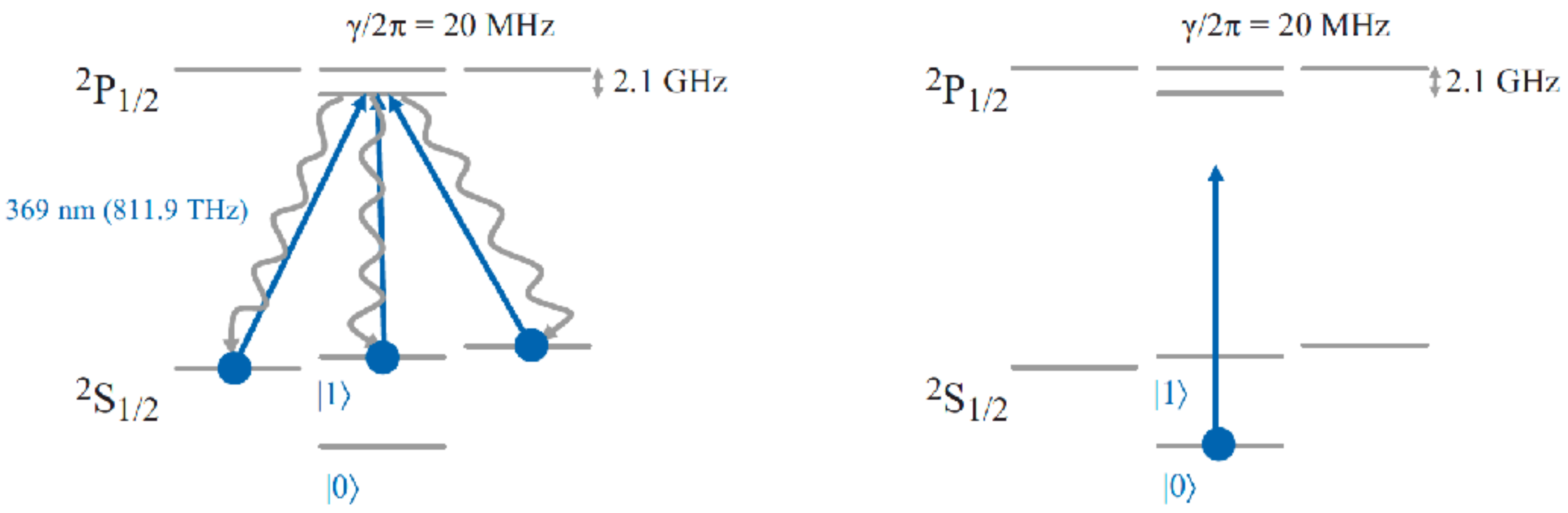


Figure 2.7: Measurement outcome is observed by state-dependent flourescence.

Single-Qubit Gate: Raman or Microwave Transition

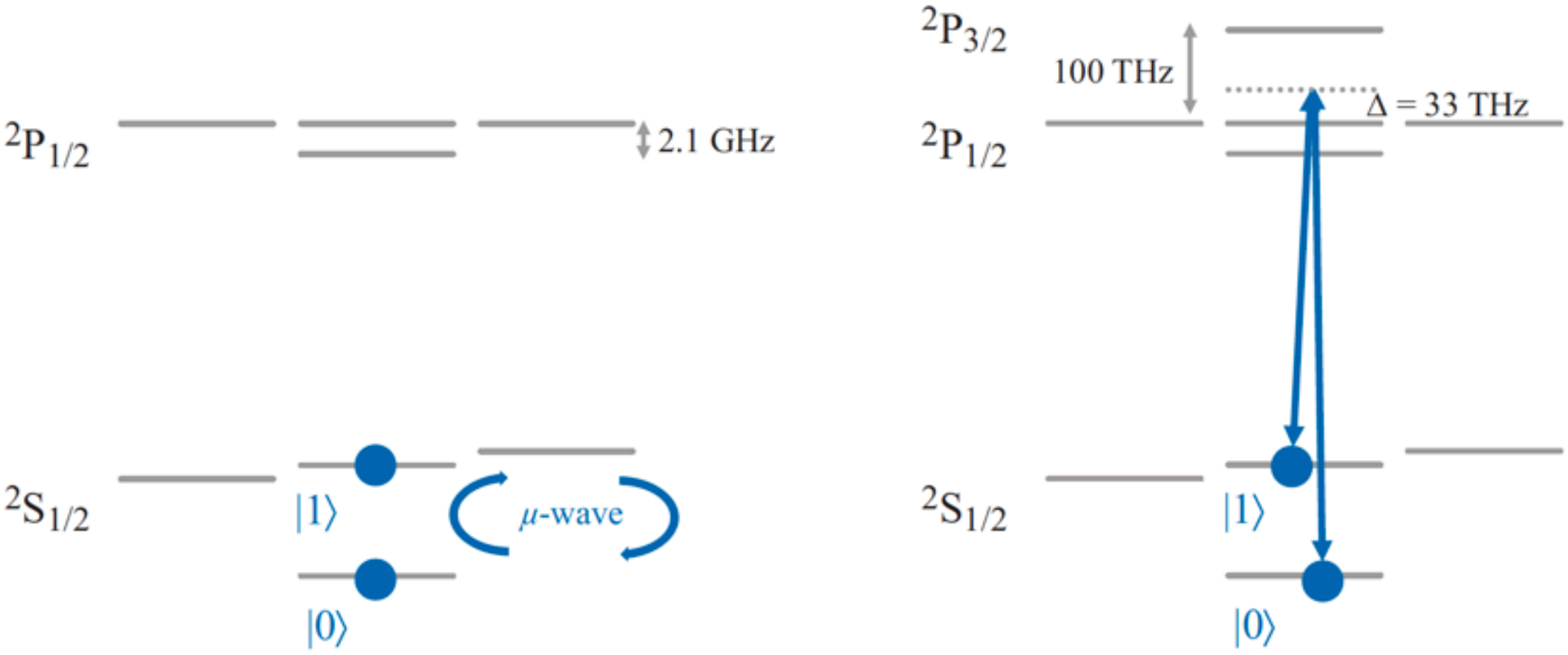


Figure 2.8: Single qubit gates via Raman transition or microwave transition.

Loading Qubits

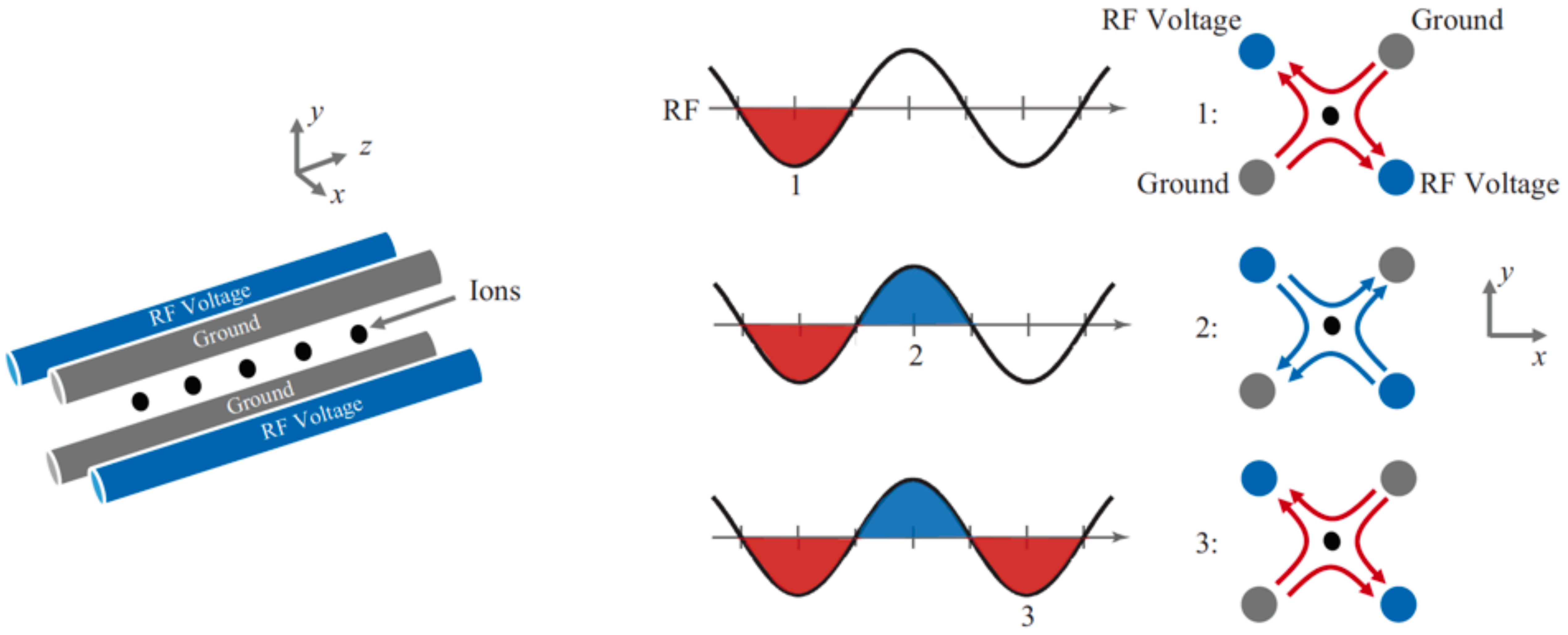


Figure 2.9: Schematics for a RF Paul trap.

Trapped Ion Qubits

- Take about 1 ms to transport across a 30 mm chip (big enough to host 10-100 qubits)
- Entangling gates can be executed in 50-500 us
- With coherence times of about 60 s - typical for atomic clock states - that's about 0.002% error on top of the entangling gate.
- 2019 estimates for Shor factoring of 2048-bit integers with superconducting qubits give an 8-hour run-time with physical error rates of 0.1% and a surface code cycle time of 1 us.

Superconducting Qubits

- Charge qubits
- Flux qubits

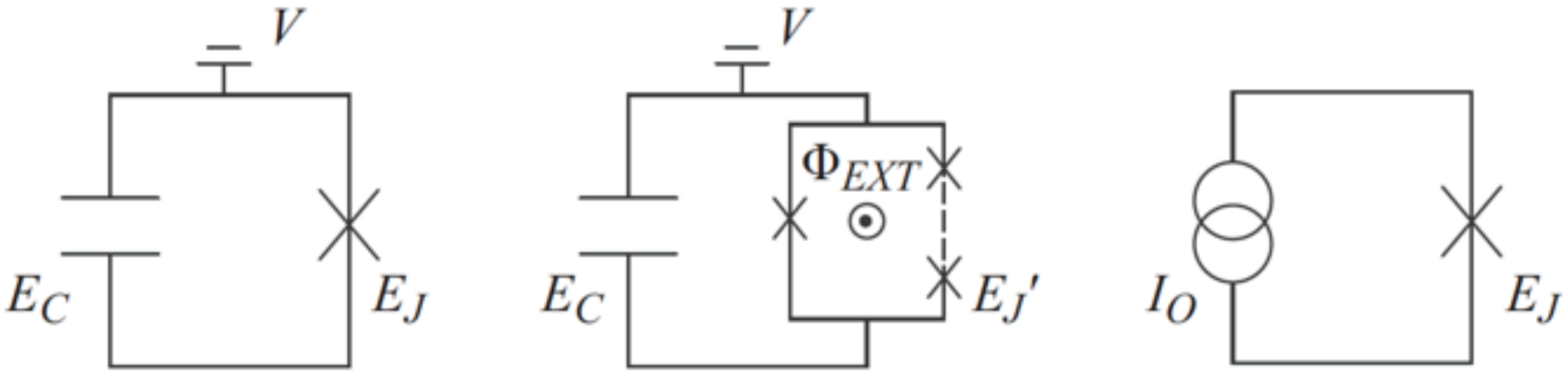


Figure 2.11: Types of superconducting qubits. **Left:** Circuit diagram for charge qubits (when $E_J \leq E_C$) and transmon qubit (when $E_J \gg E_C$), consisting of capacitor C and Josephson junction J. **Center:** Circuit diagram for a c-shunted flux qubit, where a junction is shunted with a number of junctions. **Right:** Circuit diagram for a phase qubit with current bias I_0.

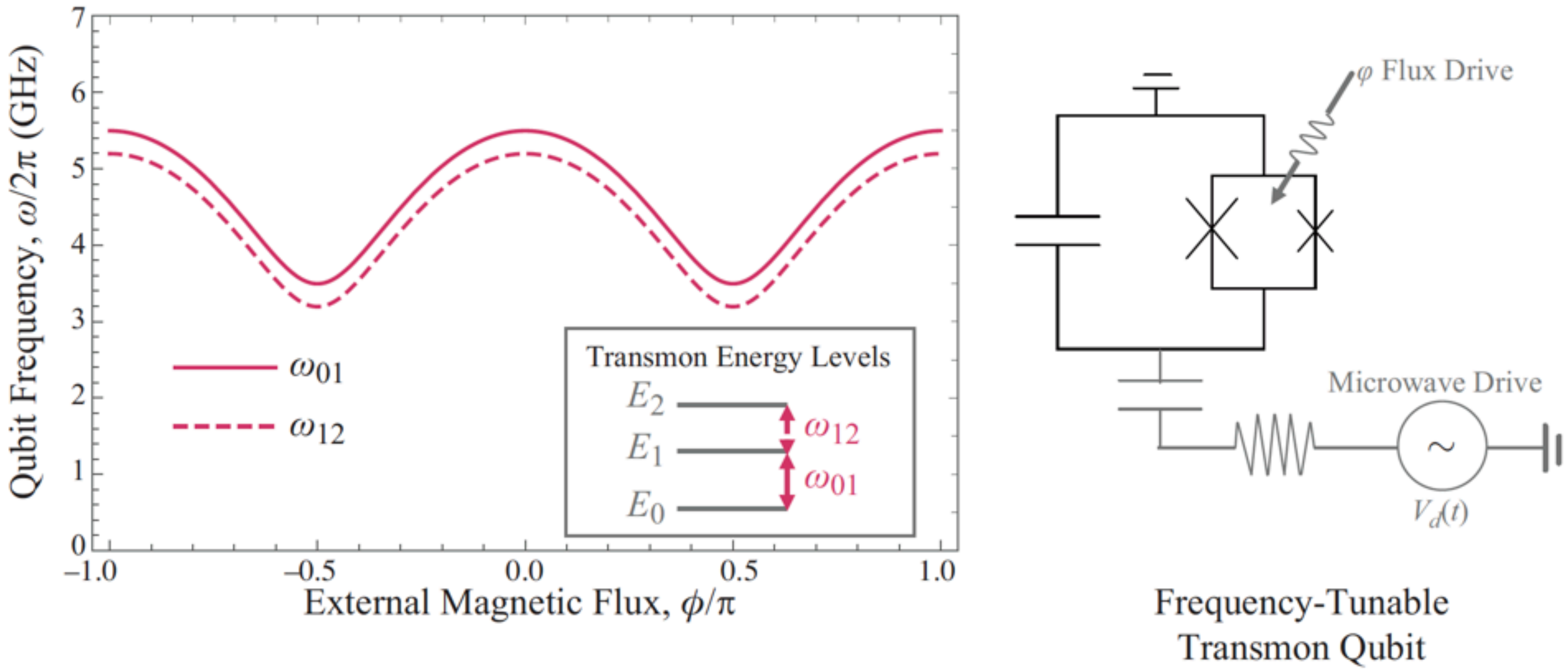


Figure 2.12: **Left:** Qubit frequencies as a function of external magnetic flux. The first three levels of the transmon, ω_{01} and ω_{12}, are plotted. **Right:** Circuit diagram for a frequency-tunable (asymmetric) transmon qubit (highlighted in black), consisting of a capacitor and two asymmetric Josephson junctions. Highlighted in gray are two control lines: the external magnetic flux control φ and microwave voltage drive line $V_d(t)$ for each transmon qubit.

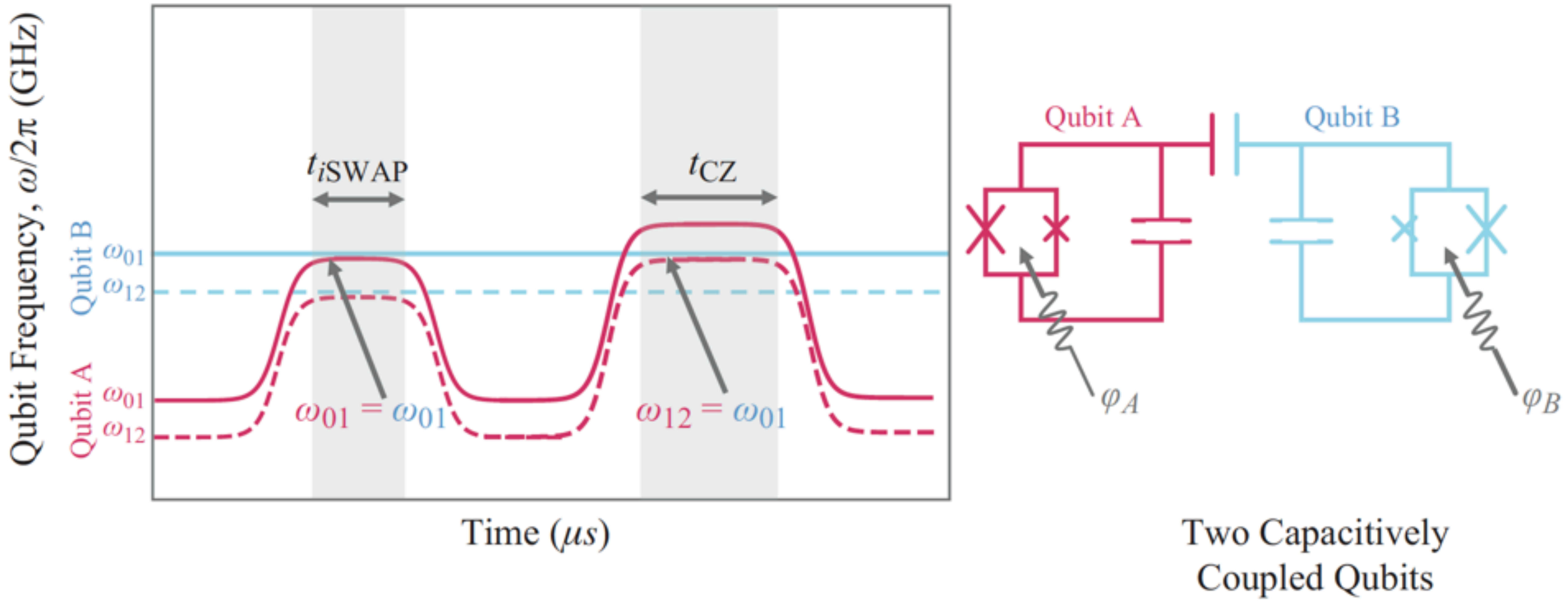
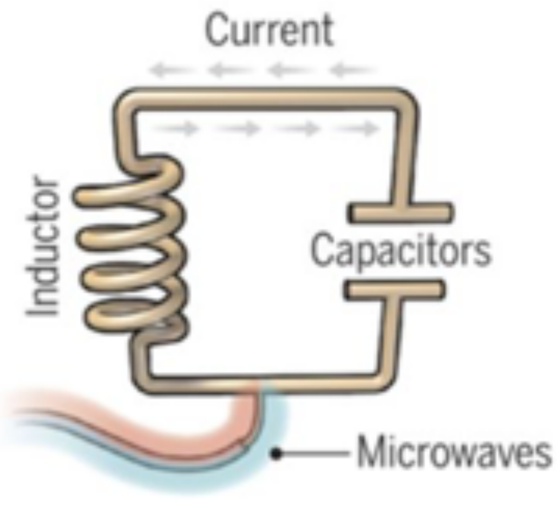
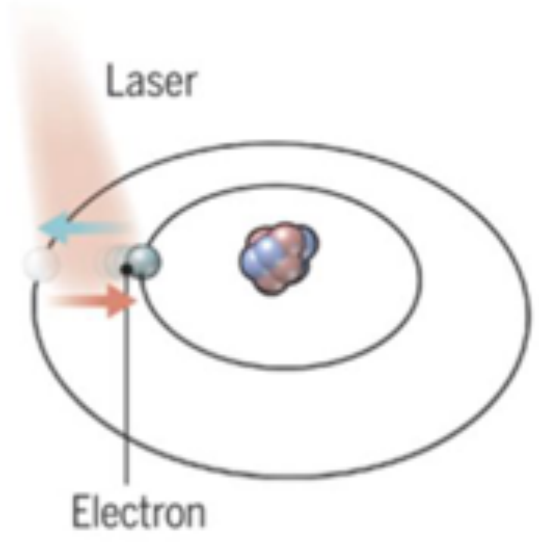
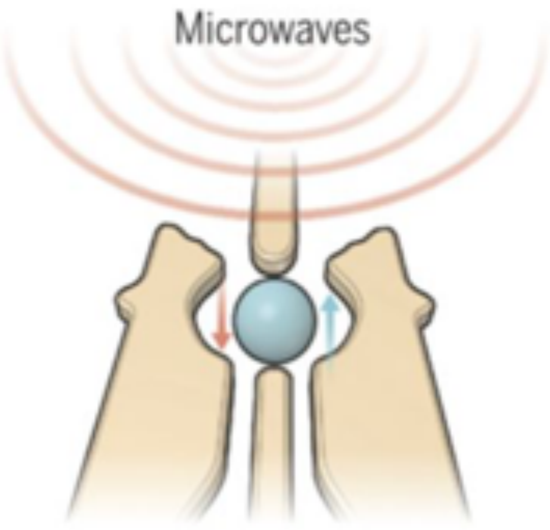


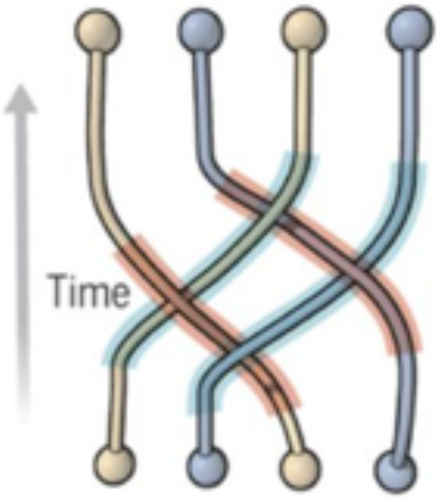
Figure 2.13: Two-qubit interactions for two capacitively coupled transmons. **Left:** Two-qubit gates are implemented with resonance of qubit frequencies. Shown here are how qubit frequencies are tuned for i`SWAP` gate and `CZ` gate. **Right:** Circuit diagram of two capacitively coupled transmon qubits.

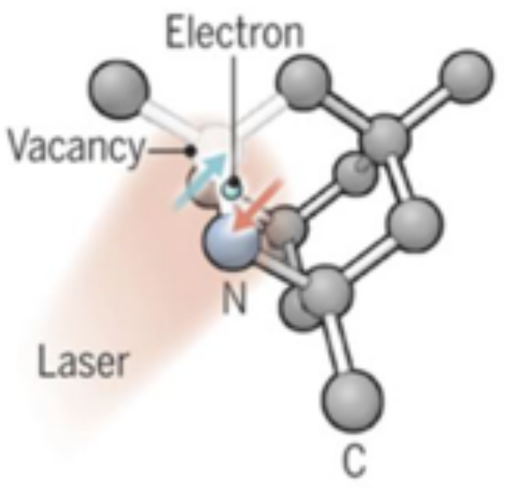
Qubit technologies











Superconducting loops
A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into super-position states.

Trapped ions
Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in superposition states.

Silicon quantum dots
These "artificial atoms" are made by adding an electron to a small piece of pure silicon. Microwaves control the electron's quantum state.

Topological qubits
Quasiparticles can be seen in the behavior of electrons channeled through semi-conductor structures.Their braided paths can encode quantum information.

Diamond vacancies
A nitrogen atom and a vacancy add an electron to a diamond lattice. Its quantum spin state, along with those of nearby carbon nuclei, can be controlled with light.

	Superconducting loops	Trapped ions	Silicon quantum dots	Topological qubits	Diamond vacancies
Longevity (seconds)	0.00005	>1000	0.03	N/A	10
Logic success rate	99.4%	99.9%	~99%	N/A	99.2%

Announcement

- Assignment #1 available on website. Please submit through gradescope