Quantum Applications — image processing

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How can we represent an image using qubits?

$\theta_0, \|00\rangle$	$\theta_1, \|01\rangle$
$\theta_2, \|10\rangle$	$\theta_3, \|11\rangle$

$$
\begin{aligned}
|I\rangle = \frac{1}{2}[\quad & (\cos\theta_0\,|0\rangle + \sin\theta_0\,|1\rangle) \otimes |00\rangle \\
& + (\cos\theta_1\,|0\rangle + \sin\theta_1\,|1\rangle) \otimes |01\rangle \\
& + (\cos\theta_2\,|0\rangle + \sin\theta_2\,|1\rangle) \otimes |10\rangle \\
& + (\cos\theta_3\,|0\rangle + \sin\theta_3\,|1\rangle) \otimes |11\rangle\ ]
\end{aligned}
$$

n qubits for an $n \times n$ image

Flexible Representation of Quantum Images

- $$|I(\theta)\rangle = \frac{1}{2^n}\sum_{i=0}^{2^{2n}-1}\left(\cos\theta_i\,|0\rangle + \sin\theta_i\,|1\rangle\right)\otimes|i\rangle$$

  $$\theta_i \in \left[0,\frac{\pi}{2}\right], i = 0,1,\cdots,2^{2n}-1$$

- color information encoding: $\cos\theta_i|0\rangle + \sin\theta_i|1\rangle$
- associated pixel position encoding: $|i\rangle$

Building the FRQI State

- Initialization: $|H\rangle = \frac{1}{2^n}|0\rangle \otimes \sum_{i=0}^{2^{2n}-1} |i\rangle = \mathcal{H}\left(|0\rangle^{\otimes 2n+1}\right)$
- Transformation into FRQI state $\mathcal{R}|H\rangle = \left(\prod_{i=0}^{2^{2n}-1} R_i\right)|H\rangle = |I(\theta)\rangle$

where

$$R_i = \left(I \otimes \sum_{j=0, j\neq i}^{2^{2n}-1} |j\rangle\langle j|\right) + R_y\left(2\theta_i\right) \otimes |i\rangle\langle i| \text{ and}$$

$$R_y(2\theta_i) = \begin{pmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{pmatrix}$$

Novel Enhanced Quantum Representation

- Quadratic speedup of the time complexity to prepare the NEQR quantum image
- Optimal image compression ratio of up to 1.5×
- Accurate image retrieval after measurement, as opposed to probabilistic as FRQI
- Complex color and many other operations can be achieved

Considering an image as a bit string —

- Binary Image: 1 bit representing 0=black, and 1=white
- Grayscale Image: 8 bits representing the various shades of gray intensity values between 0 (black) and 255 (white).
- Color Image: 24 bits, are broken up into 3 groups of 8 bits, where each group of 8 bits represents the Red, Green, and Blue intensities of the pixel color.

2x2 image again

position	*binary string*	*grayscale intensity*
$\vert 00\rangle$	$\vert 00000000\rangle$	0 – *Black*
$\vert 01\rangle$	$\vert 01100100\rangle$	100 – *Darkshade*
$\vert 10\rangle$	$\vert 11001000\rangle$	200 – *Lightshade*
$\vert 11\rangle$	$\vert 11111111\rangle$	255 – *White*

$$f(Y,X) = C_{YX}^{0}, C_{YX}^{1}, \ldots C_{YX}^{q-2}, C_{YX}^{q-1} \in [0,1], f(Y,X) \in [0,2^{q-1}]$$

$$|I\rangle = \frac{1}{2^n} \sum_{Y=0}^{2^{2n-1}} \sum_{X=0}^{2^{2n-1}} |f(Y,X)\rangle |YX\rangle = \frac{1}{2^n} \sum_{Y=0}^{2^{2n-1}} \sum_{X=0}^{2^{2n-1}} | \otimes_{i=0}^{q-1} \rangle |C_{YX}^{i}\rangle |YX\rangle$$

$$f(1,0) = \overline{C_{10}^{0}}, C_{10}^{1}, C_{10}^{2}, \overline{C_{10}^{3}}, \overline{C_{10}^{4}}, C_{10}^{5}, \overline{C_{10}^{6}}, \overline{C_{10}^{7}} = 01100100 = 100$$

$$\Omega_{YX}|0\rangle^{\otimes q} = \frac{1}{\sqrt{2}}(|00000000\rangle|00\rangle + |01100100\rangle|01\rangle + |11001000\rangle|10\rangle + |11111111\rangle|11\rangle)$$

$n + log(colors)$ qubits for an $n \times n$ image

Quantum Probability Image Encoding (QPIE)

- $$|\text{Img}\rangle = \sum_{i=0}^{2^n-1} c_i |i\rangle$$

$$c_i = \frac{I_{yx}}{\sqrt{\sum I_{yx}^2}}$$

$$|\text{Img}\rangle = c_0|00\rangle + c_1|01\rangle + c_2|10\rangle + c_3|11\rangle$$

Quantum Hadamard Edge Detection (QHED)

- Hadamard gate

$$|0\rangle \rightarrow \frac{(|0\rangle + |1\rangle)}{\sqrt{2}}$$

$$|1\rangle \rightarrow \frac{(|0\rangle - |1\rangle)}{\sqrt{2}}$$

-

$$|\text{Img}\rangle = \sum_{i=0}^{N-1} c_i |i\rangle$$

$$(I_{2^{n-1}} \otimes H_0) \cdot \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_{N-2} \\ c_{N-1} \end{bmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} c_0 + c_1 \\ c_0 - c_1 \\ c_2 + c_3 \\ c_2 - c_3 \\ \vdots \\ c_{N-2} + c_{N-1} \\ c_{N-2} - c_{N-1} \end{bmatrix}$$

- We now have access to the gradient between the pixel intensities of neighboring pixels in the form of $(c_i - c_{i+1})$
- Detection of horizontal boundaries between the even-pixels-pairs

QHED (with an auxiliary qubit)

- Adding an additional qubit

$$|\text{Img}\rangle \otimes \frac{(|0\rangle + |1\rangle)}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{bmatrix} c_0 \\ c_0 \\ c_1 \\ c_1 \\ c_2 \\ c_2 \\ \vdots \\ c_{N-2} \\ c_{N-2} \\ c_{N-1} \\ c_{N-1} \end{bmatrix}$$

- Amplitude permutation unitary to transform the amplitudes into a structure which will make it easier to calculate the image gradients further ahead

$$D_{2^{n+1}} = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ 1 & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}$$

QHED (with an auxiliary qubit) (cont.)

- $$(I_{2^n} \otimes H) \cdot \begin{bmatrix} c_0 \\ c_1 \\ c_1 \\ c_2 \\ c_2 \\ c_3 \\ \vdots \\ c_{N-2} \\ c_{N-1} \\ c_{N-1} \\ c_0 \end{bmatrix} \to \begin{bmatrix} c_0 + c_1 \\ c_0 - c_1 \\ c_1 + c_2 \\ c_1 - c_2 \\ c_2 + c_3 \\ c_2 - c_3 \\ \vdots \\ c_{N-2} + c_{N-1} \\ c_{N-2} - c_{N-1} \\ c_{N-1} + c_0 \\ c_{N-1} - c_0 \end{bmatrix}$$
- measuring this state conditioned on the auxiliary qubit being in state $|1\rangle$, we will get the resultant horizontal gradient values $(c_i - c_{i+1})$ for all possible pairs of adjacent qubits.

Assignment Q1

- Question 1

  Assume we apply the **CNOT** gate to states ψ. So the transformation can be expressed as $\text{CNOT}\,|\psi\rangle = x\,|\psi\rangle$
- Please write a qiskit program in the cell below and evaluate the eigenvalue.

Assignment Q2

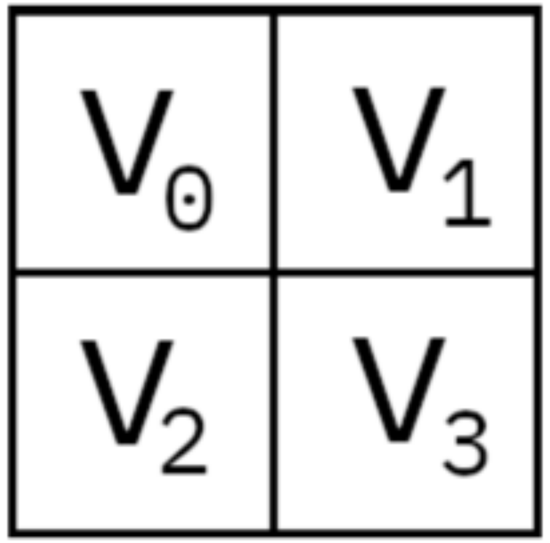
- We want to create a quantum circuit that solves a 2×2 binary sudoku. The 2×2 binary sudoku problem has two simple rules:
  - No column may contain the same value twice
  - No row may contain the same value twice
- If we assign each square in our sudoku to a variable like so:

V_0	V_1
V_2	V_3

we want our circuit to output a solution to this sudoku.

Thinking process of Q2

- We need to check the followings
  - v0 ≠ v1   # check along top row
  - v2 ≠ v3   # check along bottom row
  - v0 ≠ v2   # check down left column
  - v1 ≠ v3   # check down right column
- How to check? — XOR gate in digital circuit — CNOTs in QC!
- All must be 1 if the initial assignments passed!



Assignment Q3

- Implementing a Bernstein-Vazirani Algorithm that contains \`11101101\` as the secret string.

Paper presentations

- Assignment #1 due this evening
- 3/11/2025 Haotian Lu — Hanrui Wang, Zirui Li, Jiaqi Gu, Yongshan Ding, David Z. Pan, and Song Han. OC: quantum on-chip training with parameter shift and gradient pruning. In the 59th ACM/IEEE Design Automation Conference (DAC '22)
- 3/11/2025
- 3/13/2025
- 3/13/2025
- Topics
  - Quantum optimization algorithms
  - Quantum compilers
  - Quantum architectures/memory

Candidates to be claimed

- Wang, H., Liu, P., Tan, B., Liu, Y., Gu, J., Pan, D. Z., ... & Han, S. Atomique: A Quantum Compiler for Reconfigurable Neutral Atom Arraysy. ISCA 2024
- S. Xu, C. T. Hann, B. Foxman, S. M. Girvin, and Y. Ding. Systems Architecture for Quantum Random Access Memory. MICRO 2023
- P. Li n, J. Liu , A. Gonzales , Z. Saleem, H. Zhou n & P. Hovland. QuTracer: Mitigating Quantum Gate and Measurement Errors by Tracing Subsets of Qubits. ISCA 2024
- Cerezo, M., Verdon, G., Huang, HY. et al. Challenges and opportunities in quantum machine learning. Nat Comput Sci 2, 567–576 (2022). https://doi.org/10.1038/s43588-022-00311-3
- A. Seif, H. Liao, V. Tripathi, K. Krsulich, P. Jurcevic, M. Malekakhlagh, A. Javadi-Abhari. Suppressing Correlated Noise in Quantum Computers via Context-Aware Compiling. ISCA 2024