AI1103 - Assignment 5

Aman Panwar - CS20BTECH11004

Latex codes :

https://github.com/CS20BTECH11004/AI1103/blob/main/Assignment%205/Assignment%205.tex

Question: UGC/MATH (mathA_June 2017), Q.118

Suppose the random variable X has the following probability density funtion

$$f(x) = \begin{cases} \alpha\,(x-\mu)^{\alpha-1}\,e^{-(x-\mu)^{\alpha}}; & x > \mu \\ 0 & x \le \mu \end{cases}$$

where $\alpha > 0, -\infty < \mu < \infty$. Which of the following are correct? The hazard function of X is

1) an increasing function for all $\alpha > 0$
2) a decreasing function for all $\alpha > 0$
3) an increasing function for some $\alpha > 0$
4) a decreasing function for some $\alpha > 0$

Solution

For a random variable Y with P(x) as PDF and G(x) as CDF, the hazard function H(x) is defined as

$$H(x) = \frac{P(x)}{1 - G(x)}$$

For the random variable X, the CDF g(x) is

$$g(x) = \int_0^x f(y)dy \tag{0.0.1}$$

$$= \int_0^\mu 0dy + \int_\mu^x \alpha\,(y-\mu)^{\alpha-1}\,e^{-(y-\mu)^{\alpha}} \tag{0.0.2}$$

$$= 0 - e^{-(y-\mu)^{\alpha}}\Big|_\mu^x \tag{0.0.3}$$

$$= 1 - e^{-(x-\mu)^{\alpha}} \tag{0.0.4}$$

Therefore, the Hazard function of X F(x) is

$$F(x) = \begin{cases} \frac{\alpha(x-\mu)^{\alpha-1}e^{-(x-\mu)^{\alpha}}}{e^{-(x-\mu)^{\alpha}}}; & x > \mu \\ 0 & x \le \mu \end{cases}$$

$$= \begin{cases} \alpha\,(x-\mu)^{\alpha-1}; & x > \mu \\ 0 & x \le \mu \end{cases}$$

It is obvious that $F(x)$ is continuous at $x = \mu$

Differentiating F(x) w.r.t. x

$$F'(x) = \begin{cases} \alpha\,(\alpha-1)\,(x-\mu)^{\alpha-2}; & x > \mu \\ 0 & x \le \mu \end{cases}$$

When $x \le \mu$ then F'(x) is non-negative. When $x > \mu$ then $(x-\mu)^{\alpha-2}$ is positive. This implies that the sign for F'(x) for $x > \mu$ is decided by the sign of $\alpha\,(\alpha-1)$.

$$\alpha\,(1-\alpha) < 0 \implies 0 < \alpha < 1 \tag{0.0.5}$$

So, $F'(x) < 0$ for $x > \mu$ and $\alpha \in (0, 1)$ and $F(x) \ge 0$ for all other cases.

$\therefore$ For $\alpha > 0$ the Hazard function of X can be both increasing and decreasing.

$\boxed{\textbf{Solution: } \text{Options 3, 4}}$