Assignment 2

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and latex-tikz codes from

https://github.com/CS20BTECH11004/AI1103/blob/main/Assignment%202/Assignment%202.tex

1 QUESTION

(GATE MA 1999 Q1.28) Two independent events E and F are such that $P(E \cap F) = \frac{1}{6}, P(E^c \cap F^c) = \frac{1}{3}$ and $P(E) > P(F)$. Then $P(E)$ is

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) $\frac{1}{3}$

(D) $\frac{1}{4}$

2 SOLUTION

If E and F are independent, E' and F' are also independent.
So,

$$\begin{aligned}\Pr(EF) &= \Pr(E)\Pr(F)\\ &= \frac{1}{6} \qquad (2.0.1)\\ \Pr(E'F') &= \Pr(E')\Pr(F')\\ &= (1-\Pr(E))(1-\Pr(F))\\ &= \frac{1}{3} \qquad (2.0.2)\end{aligned}$$

From (2.0.1) and (2.0.2)

$$\Pr(E) + \Pr(F) = \frac{5}{6} \qquad (2.0.3)$$

From (2.0.1) and (2.0.3),

$$\begin{aligned}\Pr(E)\left(\frac{5}{6} - \Pr(E)\right) &= \frac{1}{6}\\ \equiv \Pr(E) &= \frac{1}{3} \text{or} \frac{1}{2}\end{aligned}$$

$\Pr(E) = \frac{1}{2}$ satisfies $\Pr(E) > \Pr(F)$ while $\Pr(E) = \frac{1}{3}$ does not.
$\therefore \Pr(E) = \frac{1}{2}$
Solution: Option A