AI1103 - Assignment 5

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Latex codes :

https://github.com/CS20BTECH11004/AI1103/blob/main/Assignment%205/Assignment%205.tex

QUESTION: UGC/MATH (mathA_June 2017), Q.118

Suppose the random variable X has the following probability density funtion

$$f(x) = \begin{cases} \alpha\,(x-\mu)^{\alpha-1}\,e^{-(x-\mu)^{\alpha}}; & x > \mu \\ 0 & x \le \mu \end{cases}$$

where $\alpha > 0, -\infty < \mu < \infty$. Which of the following are correct? The hazard function of X is

1) an increasing function for all $\alpha > 0$
2) a decreasing function for all $\alpha > 0$
3) an increasing function for some $\alpha > 0$
4) a decreasing function for some $\alpha > 0$

Solution

For a random variable Y with P(x) as PDF and G(x) as CDF, the hazard function H(x) is defined as

$$H(x) = \frac{P(x)}{1 - G(x)}$$

For the random variable X, the CDF g(x) is

$$g(x) = \int_{0}^{x} f(y)dy \tag{0.0.1}$$

$$= \int_{0}^{\mu} 0dy + \int_{\mu}^{x} \alpha\,(y-\mu)^{\alpha-1}\,e^{-(y-\mu)^{\alpha}} \tag{0.0.2}$$

$$= 0 - e^{-(y-\mu)^{\alpha}}\Big|_{\mu}^{x} \tag{0.0.3}$$

$$= 1 - e^{-(x-\mu)^{\alpha}} \tag{0.0.4}$$

Therefore, the Hazard function of X F(x) is

$$F(x) = \begin{cases} \frac{\alpha(x-\mu)^{\alpha-1}e^{-(x-\mu)^{\alpha}}}{e^{-(x-\mu)^{\alpha}}}; & x > \mu \\ 0 & x \le \mu \end{cases}$$

$$= \begin{cases} \alpha\,(x-\mu)^{\alpha-1}; & x > \mu \\ 0 & x \le \mu \end{cases}$$

Differentiating F(x) wrt x

$$F'(x) = \begin{cases} \alpha\,(\alpha-1)\,(x-\mu)^{\alpha-2}; & x > \mu \\ 0 & x \le \mu \end{cases}$$

When $x \le \mu$ then F'(x) is non-negative. When $x > \mu$ then $(x-\mu)^{\alpha-2}$ is positive. This implies that the sign for F'(x) for $x > \mu$ is decided by the sign of $\alpha\,(\alpha-1)$. As, $\alpha > 0$, $F'(x) < 0$ for $x > \mu$ and $\alpha \in (0, 1)$

$\therefore$ For $\alpha > 0$ the Hazard function of X can be both increasing and decreasing.

Solution: Options 3, 4