

# AI1103 - Assignment 5

Aman Panwar - CS20BTECH11004

Latex codes :

<https://github.com/CS20BTECH11004/AI1103/blob/main/Assignment%205/Assignment%205.tex>

QUESTION: UGC/MATH (MATHA\_JUNE 2017), Q.118  
Suppose the random variable X has the following probability density function

$$f(x) = \begin{cases} \alpha (x - \mu)^{\alpha-1} e^{-(x-\mu)^\alpha}; & x > \mu \\ 0 & x \leq \mu \end{cases}$$

where  $\alpha > 0, -\infty < \mu < \infty$ . Which of the following are correct? The hazard function of X is

- 1) an increasing function for all  $\alpha > 0$
- 2) a decreasing function for all  $\alpha > 0$
- 3) an increasing function for some  $\alpha > 0$
- 4) a decreasing function for some  $\alpha > 0$

## SOLUTION

For the random variable X, the CDF is

$$F(x) = \int_0^x f(y) dy \quad (0.0.1)$$

$$= \int_0^\mu 0 dy + \int_\mu^x \alpha (y - \mu)^{\alpha-1} e^{-(y-\mu)^\alpha} \quad (0.0.2)$$

$$= 0 - e^{-(y-\mu)^\alpha} \Big|_\mu^x \quad (0.0.3)$$

$$= 1 - e^{-(x-\mu)^\alpha} \quad (0.0.4)$$

For X, the hazard function  $H(y)$  is defined as

$$\begin{aligned} H(y) &= \frac{f(y)}{1 - F(y)} \\ \Rightarrow H(y) &= \begin{cases} \frac{\alpha(y-\mu)^{\alpha-1} e^{-(y-\mu)^\alpha}}{1 - (1 - e^{-(y-\mu)^\alpha})}; & y > \mu \\ 0 & y \leq \mu \end{cases} \\ &= \begin{cases} \alpha (y - \mu)^{\alpha-1}; & y > \mu \\ 0 & y \leq \mu \end{cases} \end{aligned}$$

Differentiating  $H(y)$  w.r.t.  $y$

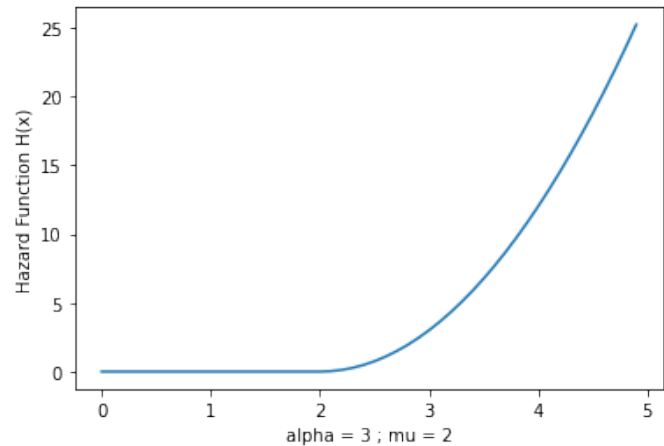
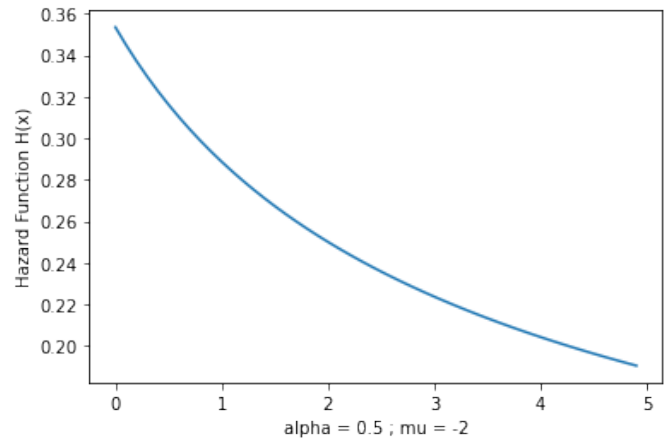
$$H'(y) = \begin{cases} \alpha(\alpha - 1)(y - \mu)^{\alpha-2}; & y > \mu \\ 0 & y \leq \mu \end{cases}$$

When  $y \leq \mu$  then  $H'(y)$  is non-negative. When  $y > \mu$  then  $(y - \mu)^{\alpha-2}$  is positive. This implies that the sign for  $F'(y)$  for  $y > \mu$  is decided by the sign of  $\alpha(\alpha - 1)$ .

$$\alpha(1 - \alpha) < 0 \Rightarrow 0 < \alpha < 1 \quad (0.0.5)$$

So,  $H'(y) < 0$  for  $y > \mu$  and  $\alpha \in (0, 1)$  and  $H'(y) \geq 0$  for all other cases.

$\therefore$  For  $\alpha > 0$ , the Hazard function of X can be both increasing and decreasing.



**Solution:** Options 3, 4