AI1103-Challenging Problem 1

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Latex codes :

QUESTION: CHALLENGING PROBLEM 1

Let X be a random variable such that $E(X) = E(X^2) = 1$. Then $E(X^{100}) = ?$

(A) 0
(B) 1
(C) 2^{100}
(D) $2^{100} + 1$

SOLUTION

$$\rho(X)^2 = E[X^2] - E[X]^2 \qquad (0.0.1)$$
$$= 1 - 1^2 \qquad (0.0.2)$$
$$= 0 \qquad (0.0.3)$$

We also know

$$\rho(X)^2 = E[(X - E[X])^2] \qquad (0.0.4)$$
$$= \frac{\sum(X - E[X])^2}{N} \qquad (0.0.5)$$

Where N is the number of terms of summation.
As $\rho(X)^2 = 0$ and each term summation in (0.0.5) ≥ 0, we can conclude that each term of summation is 0.
Or we can write

$$X = E[X] \qquad (0.0.6)$$
$$= 1 \qquad (0.0.7)$$

∴ X takes only one value which is 1. Alternatively we can write

$$\Pr(X = 1) = 1 \qquad (0.0.8)$$

So, finally we can say that

$$E[X^{100}] = \sum X^{100} \Pr(X) \qquad (0.0.9)$$
$$= 1^{100} \Pr(X = 1) \qquad (0.0.10)$$
$$= 1 \qquad (0.0.11)$$

Solution: Option B