Assignment 3

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Download all python codes from

https://github.com/CS20BTECH11054/AI1103/blob/main/Assignment_3/codes/Assignment_3.py

and latex-tikz codes from

https://github.com/CS20BTECH11054/AI1103/blob/main/Assignment_3/Assignment_3.tex

1 Problem

A continuous random variable X has a probability density function $f(x) = e^{-x}$, $0 < x < \infty$, then $P(X > 1)$ is

2 Solution

Let the CDF of $f(x)$ be denoted by $F_X(x)$
We know that,

$$\frac{d(F_X(x))}{dx} = f(x)$$
$$d(F_X(x)) = f(x)\,dx \tag{2.0.1}$$

Therefore, applying definite integral on both sides from 0 to x in (2.0.1), we get,

$$\int_0^x d(F_X(x)) = \int_0^x f(x)\,dx$$
$$\int_0^x d(F_X(x)) = \int_0^x e^{-x}\,dx \tag{2.0.2}$$

We know that,

$$\int_a^b e^{-x} = -e^{-b} - \left(-e^{-a}\right)$$
$$\int_a^b e^{-x} = -e^{-a} - e^{-b} \tag{2.0.3}$$

We also know that,

$$\int_a^b d(F_X(x)) = F_X(b) - F_x(a) \tag{2.0.4}$$

By using (2.0.3) and (2.0.4) in (2.0.2) we can write that,

$$F_X(x) - F_X(0) = e^{-0} - e^{-x}$$
$$F_X(x) - F_X(0) = 1 - e^{-x} \tag{2.0.5}$$

We know that,

$$F_X(x) = P(X \le x) \tag{2.0.6}$$

Substituting $x = 0$ in (2.0.6), we get

$$F_X(0) = P(X \le 0) \tag{2.0.7}$$

But we know that there is no probability for the random variable X to satisfy $X \le 0$,So,

$$P(X \le 0) = 0 \tag{2.0.8}$$

Substituting (2.0.8) in (2.0.6), we get

$$F_X(0) = 0 \tag{2.0.9}$$

Substituting (2.0.9) in (2.0.5), we get

$$F_X(x) = 1 - e^{-x} \tag{2.0.10}$$

From (2.0.6) and (2.0.10) we get,

$$P(X \le x) = 1 - e^{-x} \tag{2.0.11}$$

Substituting x=1 in (2.0.11) we get,

$$P(X \le 1) = 1 - e^{-1} \tag{2.0.12}$$

We know that, as total probability is equal to 1,

$$P(X > 1) + P(X \le 1) = 1 \tag{2.0.13}$$
$$P(X > 1) = 1 - P(X \le 1) \tag{2.0.14}$$

To find $P(X > 1)$, we substitute (2.0.12) in (2.0.14),

$$P(X > 1) = 1 - \left(1 - e^{-1}\right) \tag{2.0.15}$$
$$P(X > 1) = e^{-1} = \frac{1}{e} \tag{2.0.16}$$

Therefore, P$(X > 1) = \frac{1}{e}$

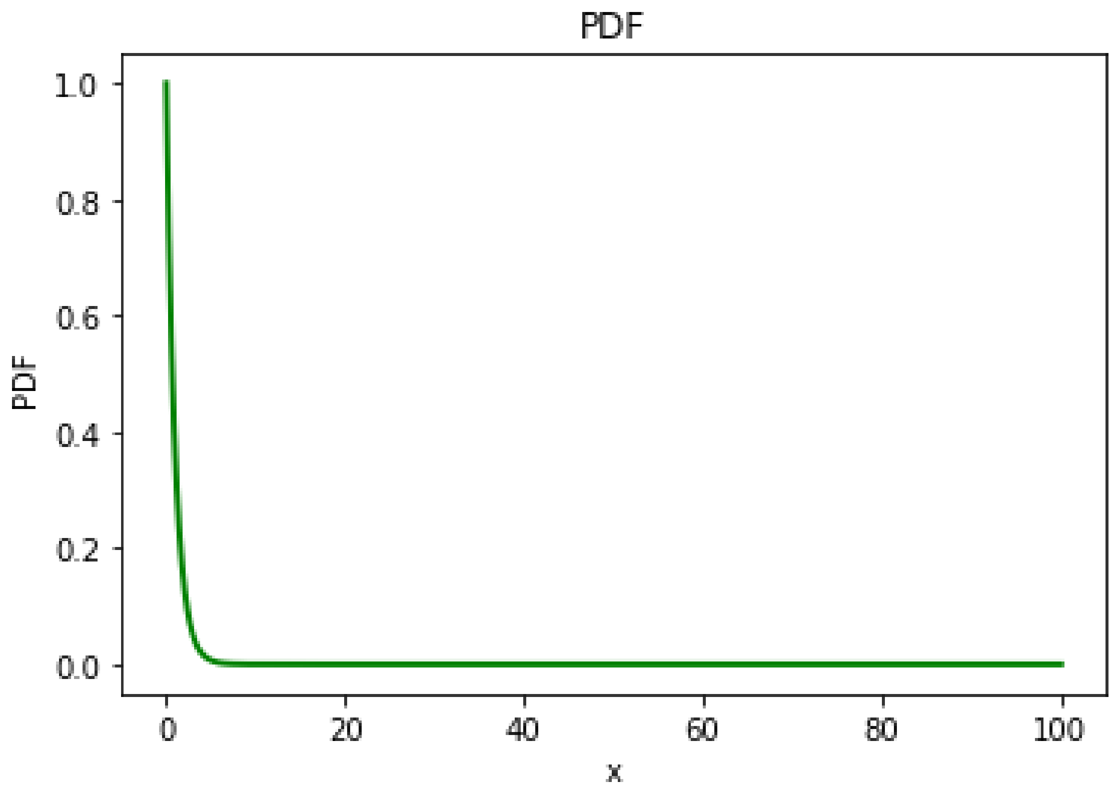

Fig. 0: **PDF**

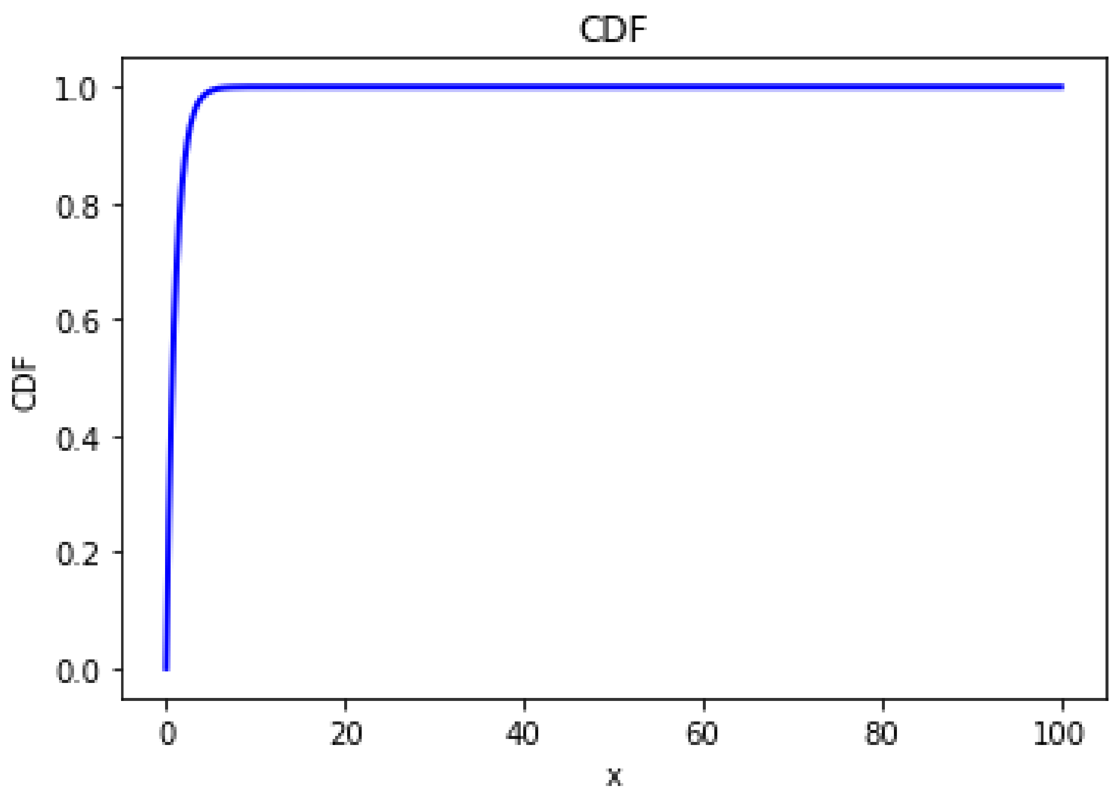

Fig. 0: **CDF**