Assignment 3

Vaddamani Saketh - CS20BTECH11054

Download all python codes from

https://github.com/CS20BTECH11054/AI1103/blob/main/Assignment_3/codes/Assignment_3.py

and latex-tikz codes from

https://github.com/CS20BTECH11054/AI1103/blob/main/Assignment_3/Assignment_3.tex

1 Problem

A continuous random variable X has a probability density function $f(x) = e^{-x}$, $0 < x < \infty$, then $P(X > 1)$ is

2 Solution

Let the CDF of $f(x)$ be denoted by $F_X(x)$
We know that,

$$\frac{d(F_X(x))}{dx} = f(x) \tag{2.0.1}$$

Therefore, we can find $F_X(x)$ by indefinite integral of $f(x)$ from (2.0.1), i.e.,

$$F_X(x) = \int f(x)\, dx$$

$$F_X(x) = \int e^{-x}\, dx \tag{2.0.2}$$

We know that,

$$\int e^{-x} = -e^{-x} + c \tag{2.0.3}$$

By substituting (2.0.3) in (2.0.2) we get,

$$F_X(x) = -e^{-x} + c \tag{2.0.4}$$

We know that,

$$F_X(x) = P(X \le x) \tag{2.0.5}$$

We can find c in (2.0.4) by using boundary condition that,

$$(x \to \infty) \implies (P(X \le x) \to 1) \tag{2.0.6}$$

therefore, from (2.0.5) and (2.0.6) we get,

$$(x \to \infty) \implies (F_X(x) \to 1) \tag{2.0.7}$$

Substituting (2.0.4) in (2.0.7), we get

$$(x \to \infty) \implies ((-e^{-x} + c) \to 1) \tag{2.0.8}$$

We know that,

$$(x \to \infty) \implies (e^{-x} \to 0) \tag{2.0.9}$$

Substituting (2.0.9) in (2.0.8), we get

$$c = 1 \tag{2.0.10}$$

Substituting (2.0.10) in (2.0.4), we get

$$F_X(x) = 1 - e^{-x} \tag{2.0.11}$$

From (2.0.5) and (2.0.11) we get,

$$P(X \le x) = 1 - e^{-x} \tag{2.0.12}$$

Substituting x=1 in (2.0.12) we get,

$$P(X \le 1) = 1 - e^{-1} \tag{2.0.13}$$

We know that, as total probability is equal to 1,

$$P(X > 1) + P(X \le 1) = 1 \tag{2.0.14}$$

$$P(X > 1) = 1 - P(X \le 1) \tag{2.0.15}$$

To find $P(X > 1)$, we substitute (2.0.13) in (2.0.15),

$$P(X > 1) = 1 - \left(1 - e^{-1}\right) \tag{2.0.16}$$

$$P(X > 1) = e^{-1} = \frac{1}{e} \tag{2.0.17}$$

Therefore, P$(X > 1)$=$\frac{1}{e}$

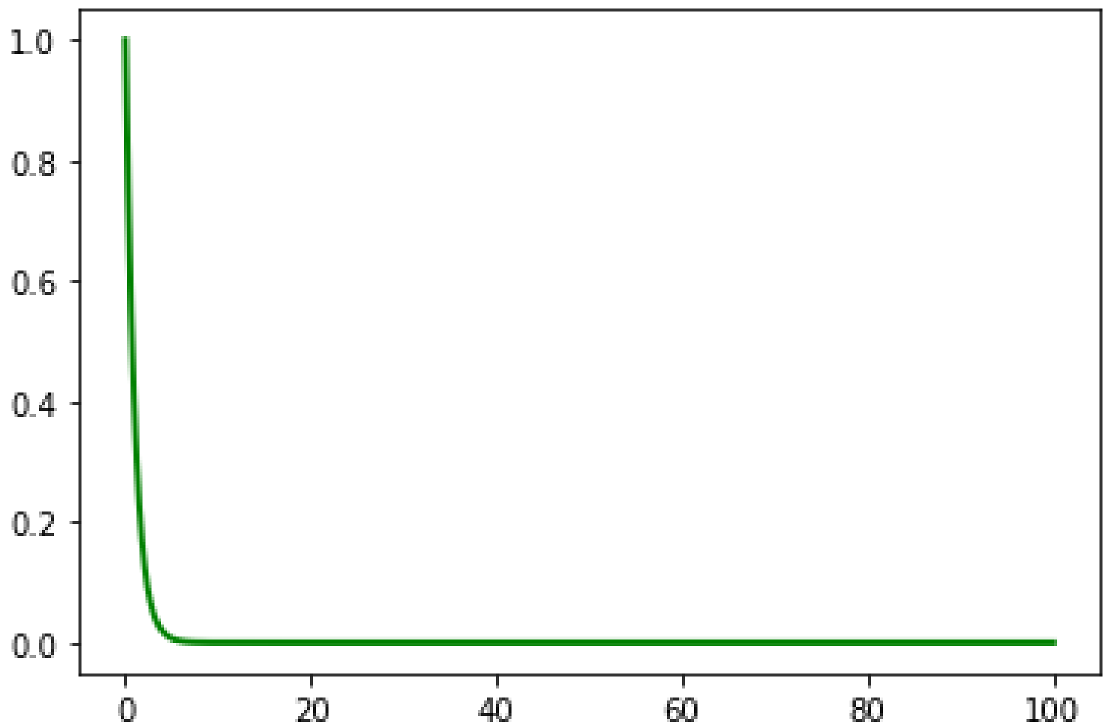

Fig. 0: **PDF**

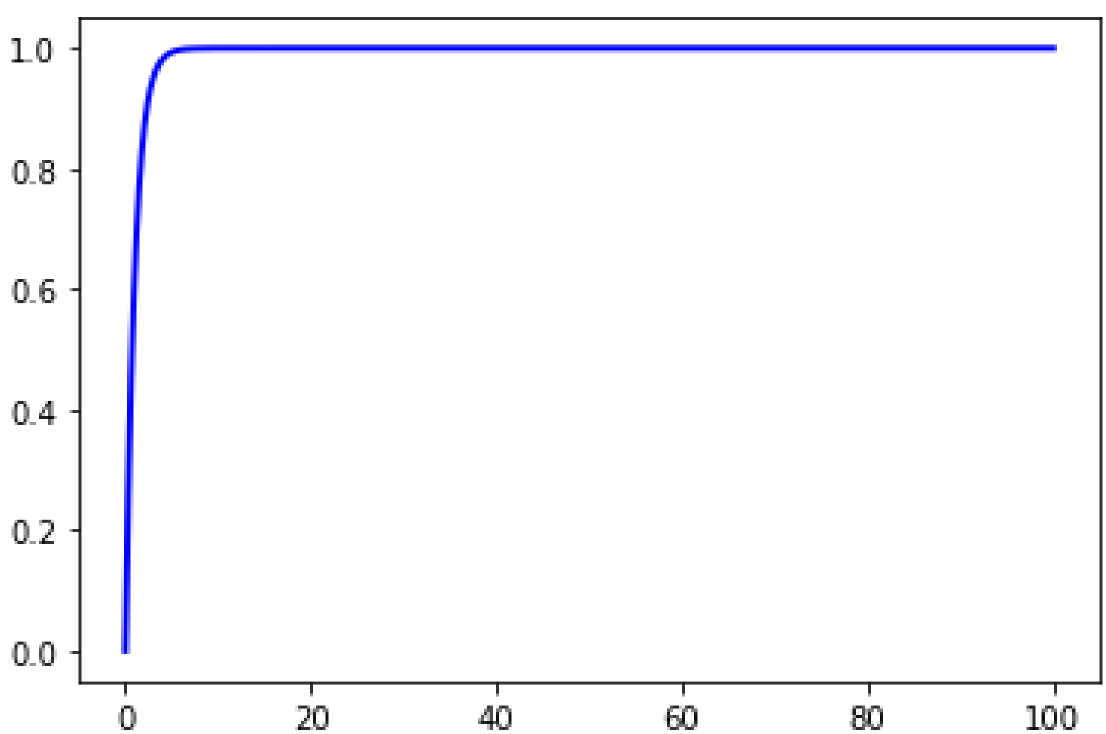

Fig. 0: **CDF**