

Assignment 4

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Download all python codes from

https://github.com/CS20BTECH11054/AI1103/blob/main/Assignment_4/codes/Assignment_4.py

and latex-tikz codes from

https://github.com/CS20BTECH11054/AI1103/blob/main/Assignment_4/Assignment_4.tex

1 PROBLEM

Let a random variable X follow exponential distribution with mean 2. Define $Y = [X - 2 | X > 2]$. The value of $\Pr(Y \geq t)$ is

2 SOLUTION

Given that, $Y = [X - 2 | X > 2]$

$$\Pr(Y \geq t) = \frac{\Pr(X - 2 \geq t, X > 2)}{\Pr(X > 2)} \quad (2.0.1)$$

Let the PDF, CDF, and mean for the distribution be $f(x)$, $F_X(x)$ and $E(x)$ such that

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } 0 < x < \infty \\ 0, & \text{otherwise} \end{cases} \quad (2.0.2)$$

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & \text{if } 0 < x < \infty \\ 0, & \text{otherwise} \end{cases} \quad (2.0.3)$$

$$E(x) = \frac{1}{\lambda} \quad (2.0.4)$$

Given, the mean or expected value of the distribution is 2, So, from (2.0.4), we get

$$\begin{aligned} \frac{1}{\lambda} &= 2 \\ \lambda &= \frac{1}{2} \end{aligned} \quad (2.0.5)$$

$\Pr(X \geq t + 2)$ can be found by

$$\Pr(X \geq t + 2) = \Pr(X > t + 2) + \Pr(X = t + 2) \quad (2.0.6)$$

As this probability distribution is continuous, at every random variable x , $\Pr(X = x)$ should be zero. So, using this in (2.0.6), we get

$$\Pr(X \geq t + 2) = \Pr(X > t + 2) = 1 - F_X(t + 2) = e^{-\lambda(t+2)} \quad (2.0.7)$$

$\Pr(X > 2)$ can be found by

$$\Pr(X > 2) = 1 - F_X(2) = e^{-2\lambda} \quad (2.0.8)$$

Substituting (2.0.7) and (2.0.8) in (2.0.1), we get

$$\Pr(Y \geq t) = \frac{e^{-\lambda(t+2)}}{e^{-2\lambda}} = e^{-\lambda t} = e^{-\frac{t}{2}} \quad (2.0.9)$$

Therefore, $\Pr(Y \geq t) = e^{-\frac{t}{2}}$

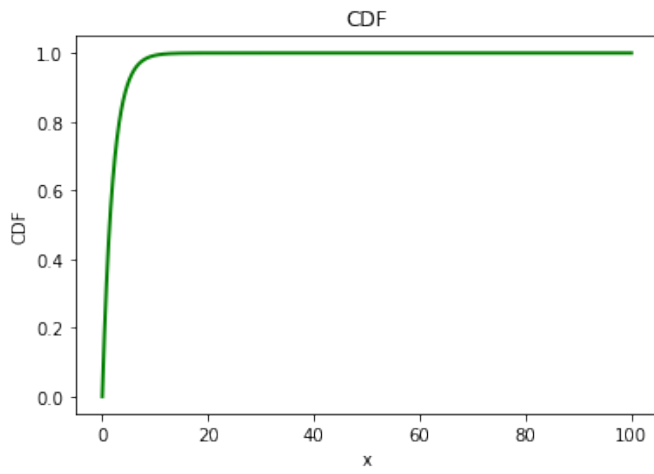


Fig. 0: **CDF**

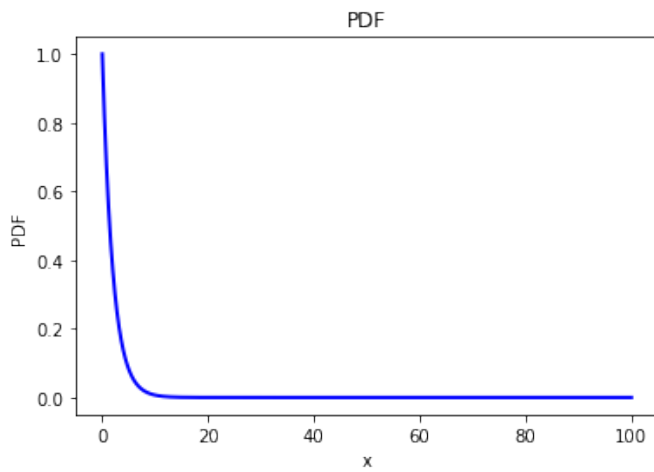


Fig. 0: **PDF**