Assignment 2

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Download all python codes from

https://github.com/CS20BTECH11054/AI1103/blob/main/Assignment_2/codes/Assignment_2.py

and latex-tikz codes from

https://github.com/CS20BTECH11054/AI1103/blob/main/Assignment_2/Assignment_2.tex

1 Problem

(Prob, 5.30) Find the variance of the number obtained on a throw of an unbiased die

2 Solution

Let $X \in \{1, 2, 3, 4, 5, 6\}$, be the random variable representing outcome of the die.The probability mass function(pmf) can be expressed as

$$p_X(n) = P(X = n) = \begin{cases} \frac{1}{6}, & \text{if } 1 \le n \le 6 \\ 0, & \text{otherwise} \end{cases} \tag{2.0.1}$$

The variance (Var(X)) of this distribution can be found by definition,

$$Var(X) = E\left(X^2\right) - (E(X))^2 \tag{2.0.2}$$

where,

$$E(X) = \sum_{k=1}^{k=6} k p_X(k)$$
$$E(X) = \frac{1}{6} \sum_{k=1}^{k=6} k \tag{2.0.3}$$

We know that, sum of natural numbers from 1 to n is,

$$\sum_{k=1}^{k=n} k = \frac{n(n+1)}{2} \tag{2.0.4}$$

By substituting the formula from (2.0.4) in (2.0.3) and n=6, We get,

$$E(X) = \frac{1}{6} \times \frac{6 \times 7}{2}$$
$$E(X) = \frac{7}{2} \tag{2.0.5}$$

And,

$$E\left(X^2\right) = \sum_{k=1}^{k=6} k^2 p_X(k)$$
$$E\left(X^2\right) = \frac{1}{6} \sum_{k=1}^{k=6} k^2 \tag{2.0.6}$$

We know that, sum of squares of natural numbers from 1 to n is,

$$\sum_{k=1}^{k=n} k^2 = \frac{n(n+1)(2n+1)}{6} \tag{2.0.7}$$

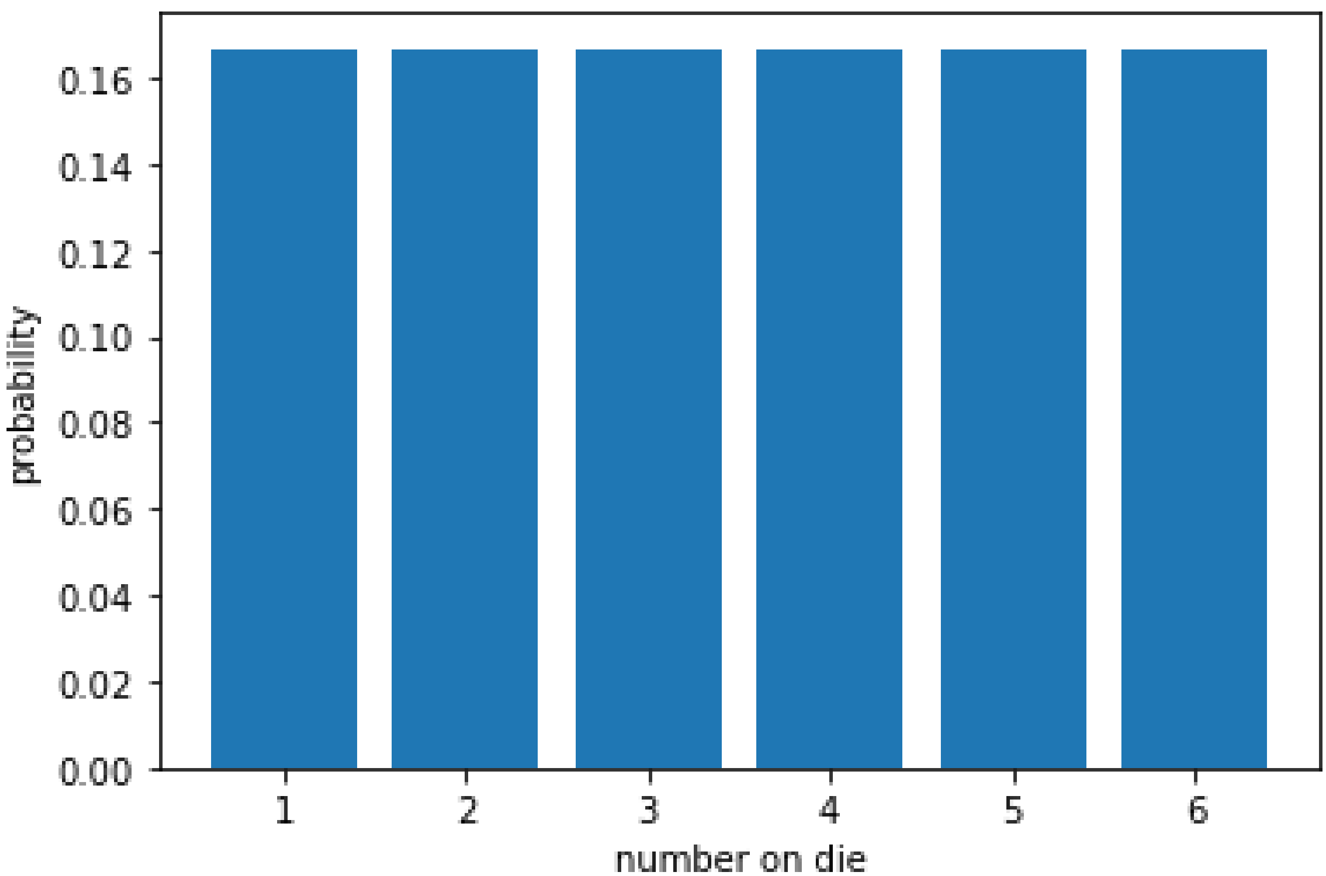
By substituting the formula from (2.0.7) in (2.0.6) and n=6, We get,

$$E\left(X^2\right) = \frac{1}{6} \times \frac{6 \times 7 \times 13}{6}$$
$$E\left(X^2\right) = \frac{91}{6} \tag{2.0.8}$$

By substituting the values from (2.0.8) and (2.0.5) in (2.0.2)

$$Var(X) = E\left(X^2\right) - (E(X))^2$$
$$Var(X) = \frac{91}{6} - \frac{49}{4}$$
$$Var(X) = \frac{70}{12}$$
$$Var(X) = 2.9167 \tag{2.0.9}$$

Therefore, the variance of the number obtained on a throw of an unbiased die is **2.9167**.


0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
probability
1
2
3
4
5
6
number on die