Assignment 4

Vaddamani Saketh - CS20BTECH11054

Download all python codes from

```
https://github.com/CS20BTECH11054/AI1103/blob
    /main/Assignment_4/codes/Assignment_4.py
```

and latex-tikz codes from

```
https://github.com/CS20BTECH11054/AI1103/blob
    /main/Assignment_4/Assignment_4.tex
```

1 Problem

Let a random variable X follow exponential distribution with mean 2. Define $Y = [X - 2|X > 2]$. The value of $\Pr(Y \geq t)$ is

2 Solution

Given that, $Y = [X - 2|X > 2]$

$$\Pr(Y \geq t) = \frac{\Pr(X - 2 \geq t, X > 2)}{\Pr(X > 2)} \tag{2.0.1}$$

Let the PDF,CDF, and mean for the distribution be $f(x)$, $F_X(x)$ and $E(x)$ such that

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } 0 < x < \infty \\ 0, & \text{otherwise} \end{cases} \tag{2.0.2}$$

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & \text{if } 0 < x < \infty \\ 0, & \text{otherwise} \end{cases} \tag{2.0.3}$$

$$E(x) = \frac{1}{\lambda} \tag{2.0.4}$$

Given, the mean or expected value of the distribution is 2, So, from (2.0.4), we get

$$\frac{1}{\lambda} = 2$$

$$\lambda = \frac{1}{2} \tag{2.0.5}$$

$\Pr(X > 2)$ can be found by

$$\Pr(X > 2) = 1 - F_X(2) = e^{-2\lambda} \tag{2.0.6}$$

There are two cases possible depending on the value of t, They are,

$$(i)\, t > 0$$

$$(ii)\, t \leq 0$$

$\underline{Case - (i):}$ $t > 0$

$$\Pr(X \geq t + 2, X > 2) = \Pr(X \geq t + 2) \tag{2.0.7}$$

(2.0.7) can be explained by the fact that if $t > 0$ and $X > 2$, then $X > t + 2$ is a superset of $X > 2$.

$$\Pr(X \geq t + 2) = \Pr(X > t + 2) = 1 - F_X(t + 2) = e^{-\lambda(t+2)} \tag{2.0.8}$$

Substituting (2.0.7), (2.0.6) and (2.0.10) in (2.0.1), we get

$$\Pr(Y \geq t) = \frac{e^{-\lambda(t+2)}}{e^{-2\lambda}} = e^{-\lambda t} = e^{-\frac{t}{2}} \tag{2.0.9}$$

$\underline{Case - (ii):}$ $t \leq 0$

$$\Pr(X \geq t + 2, X > 2) = \Pr(X > 2) \tag{2.0.10}$$

(2.0.10) can be explained by the fact that if $t \leq 0$ and $X \geq t + 2$, then $X > 2$ is a superset of $X \geq t + 2$. Substituting (2.0.10) in (2.0.1), we get

$$\Pr(Y \geq t) = \frac{\Pr(X > 2)}{\Pr(X > 2)} = 1 \tag{2.0.11}$$

Therefore,

$$\Pr(Y \geq t) = \begin{cases} e^{-\frac{t}{2}}, & \text{if } t > 0 \\ 1, & \text{otherwise} \end{cases} \tag{2.0.12}$$

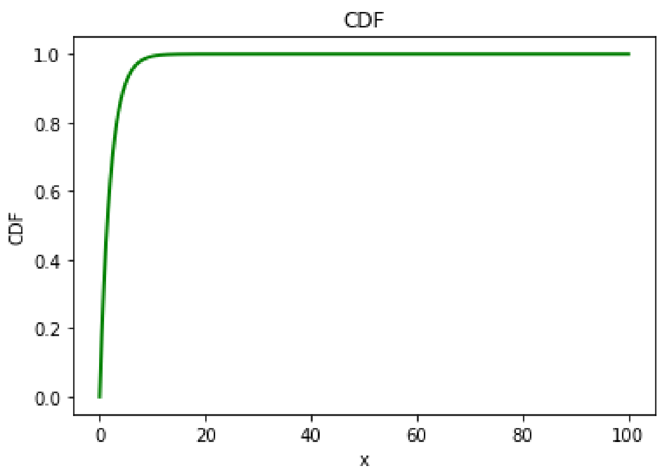

Fig. 0: **CDF**

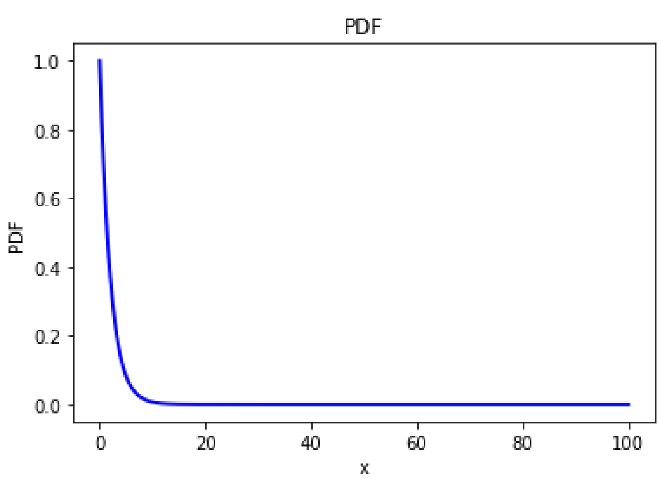

Fig. 0: **PDF**