

Assignment 3

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Download all python codes from

https://github.com/CS20BTECH11054/AI1103/blob/main/Assignment_3/codes/Assignment_3.py

and latex-tikz codes from

https://github.com/CS20BTECH11054/AI1103/blob/main/Assignment_3/Assignment_3.tex

1 PROBLEM

A continuous random variable X has a probability density function $f(x) = e^{-x}$, $0 < x < \infty$, then $P(X > 1)$ is

2 SOLUTION

Let the CDF of $f(x)$ be denoted by $F_X(x)$

We know that,

$$\begin{aligned} \frac{d(F_X(x))}{dx} &= f(x) \\ d(F_X(x)) &= f(x) dx \end{aligned} \quad (2.0.1)$$

Therefore, applying definite integral on both sides from 0 to x in (2.0.1), we get,

$$\begin{aligned} \int_0^x d(F_X(x)) &= \int_0^x f(x) dx \\ \int_0^x d(F_X(x)) &= \int_0^x e^{-x} dx \end{aligned} \quad (2.0.2)$$

We know that,

$$\begin{aligned} \int_a^b e^{-x} &= -e^{-b} - (-e^{-a}) \\ \int_a^b e^{-x} &= -e^{-a} - e^{-b} \end{aligned} \quad (2.0.3)$$

We also know that,

$$\int_a^b d(F_X(x)) = F_X(b) - F_X(a) \quad (2.0.4)$$

By using (2.0.3) and (2.0.4) in (2.0.2) we can write that,

$$\begin{aligned} F_X(x) - F_X(0) &= e^{-0} - e^{-x} \\ F_X(x) - F_X(0) &= 1 - e^{-x} \end{aligned} \quad (2.0.5)$$

We know that,

$$F_X(x) = P(X \leq x) \quad (2.0.6)$$

Substituting $x = 0$ in (2.0.6), we get

$$F_X(0) = P(X \leq 0) \quad (2.0.7)$$

But we know that there is no probability for the random variable X to satisfy $X \leq 0$, So,

$$P(X \leq 0) = 0 \quad (2.0.8)$$

Substituting (2.0.8) in (2.0.6), we get

$$F_X(0) = 0 \quad (2.0.9)$$

Substituting (2.0.9) in (2.0.5), we get

$$F_X(x) = 1 - e^{-x} \quad (2.0.10)$$

From (2.0.6) and (2.0.10) we get,

$$P(X \leq x) = 1 - e^{-x} \quad (2.0.11)$$

Substituting $x=1$ in (2.0.11) we get,

$$P(X \leq 1) = 1 - e^{-1} \quad (2.0.12)$$

We know that, as total probability is equal to 1,

$$P(X > 1) + P(X \leq 1) = 1 \quad (2.0.13)$$

$$P(X > 1) = 1 - P(X \leq 1) \quad (2.0.14)$$

To find $P(X > 1)$, we substitute (2.0.12) in (2.0.14),

$$P(X > 1) = 1 - (1 - e^{-1}) \quad (2.0.15)$$

$$P(X > 1) = e^{-1} = \frac{1}{e} \quad (2.0.16)$$

Therefore, $P(X > 1) = \frac{1}{e}$

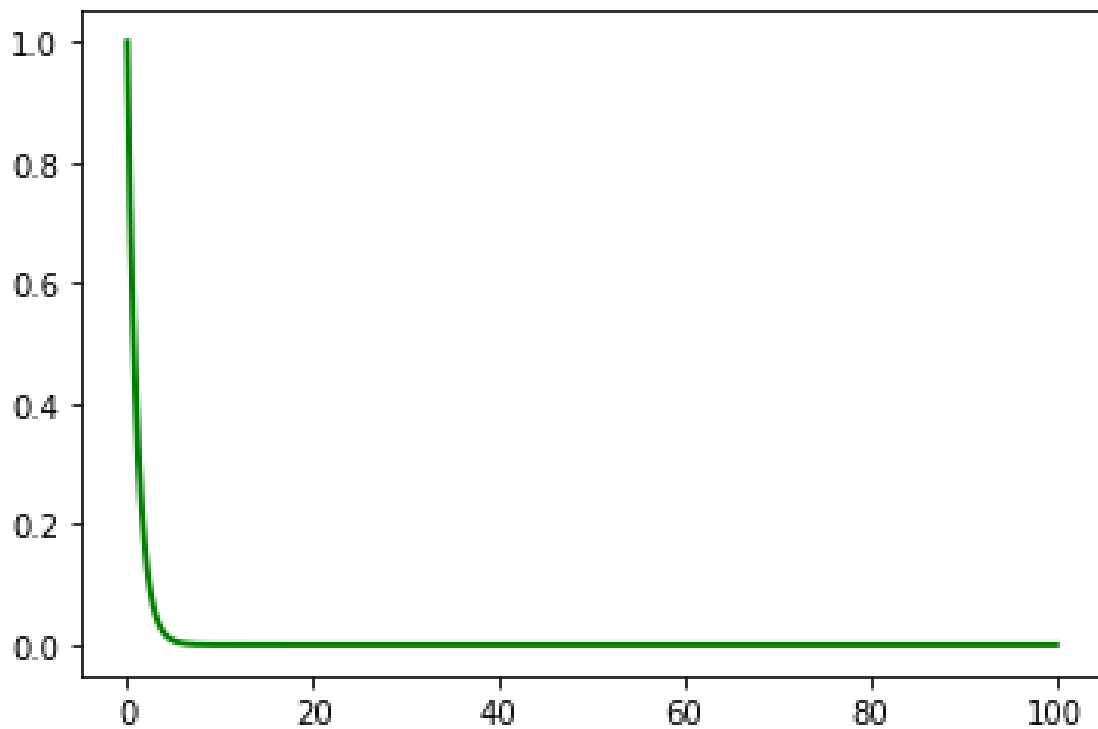


Fig. 0: **PDF**

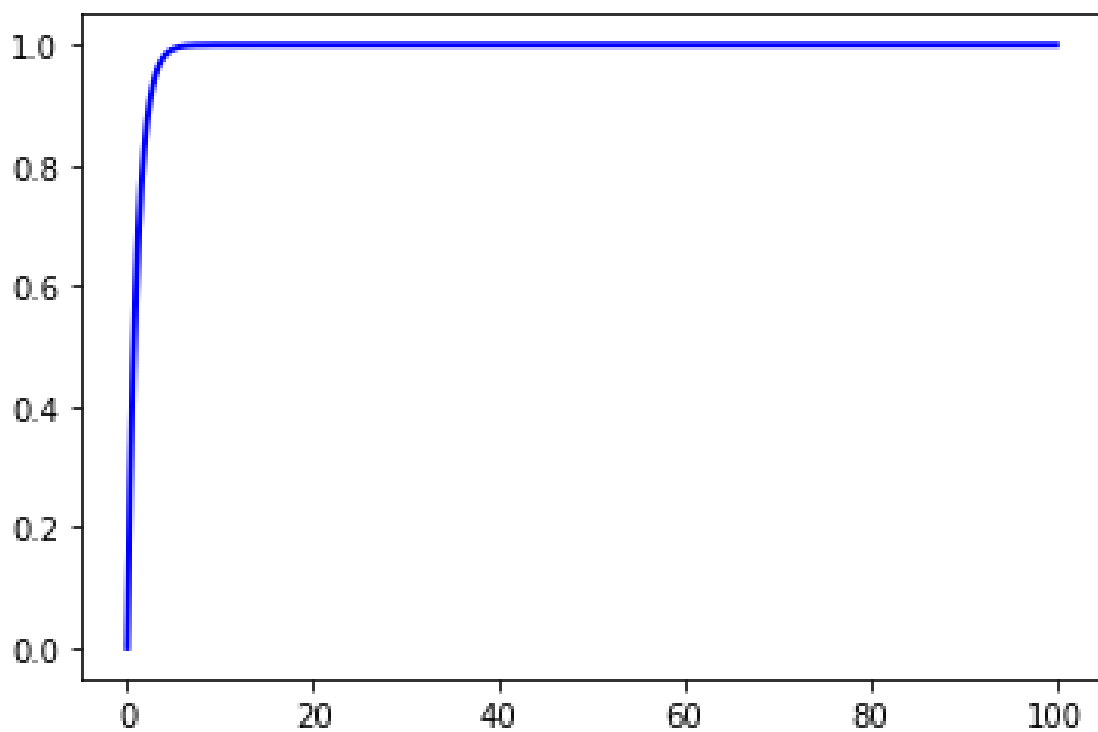


Fig. 0: **CDF**