Assignment 4

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Download all python codes from

https://github.com/CS20BTECH11054/AI1103/blob/main/Assignment_4/codes/Assignment_4.py

and latex-tikz codes from

https://github.com/CS20BTECH11054/AI1103/blob/main/Assignment_4/Assignment_4.tex

1 Problem

Let a random variable X follow exponential distribution with mean 2. Define $Y = [X-2|X>2]$. The value of $\Pr(Y \geq t)$ is

2 Solution

Given that, $Y = [X-2|X>2]$

$$\Pr(Y \geq t) = \frac{\Pr(X-2 \geq t, X>2)}{\Pr(X>2)} \tag{2.0.1}$$

Let the PDF,CDF, and mean for the distribution be $f(x)$, $F_X(x)$ and $E(x)$ such that

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } 0 < x < \infty \\ 0, & \text{otherwise} \end{cases} \tag{2.0.2}$$

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & \text{if } 0 < x < \infty \\ 0, & \text{otherwise} \end{cases} \tag{2.0.3}$$

$$E(x) = \frac{1}{\lambda} \tag{2.0.4}$$

Given, the mean or expected value of the distribution is 2, So, from (2.0.4), we get

$$\frac{1}{\lambda} = 2$$
$$\lambda = \frac{1}{2} \tag{2.0.5}$$

$\Pr(X>2)$ can be found by

$$\Pr(X>2) = 1 - F_X(2) = e^{-2\lambda} \tag{2.0.6}$$

There are two cases possible depending on the value of t, They are,

$$(i)\, t > 0$$
$$(ii)\, t \leq 0$$

$\underline{Case-(i):}$ $t>0$

$$\Pr(X \geq t+2, X>2) = \Pr(X \geq t+2) \tag{2.0.7}$$

(2.0.7) can be explained by the fact that if $t>0$ and $X>2$, then

$$\{X : X > t+2\} \supset \{X : X > 2\}$$

$$\begin{aligned}\Pr(X \geq t+2) &= \Pr(X > t+2) \\ &= 1 - F_X(t+2) = e^{-\lambda(t+2)}\end{aligned} \tag{2.0.8}$$

Substituting (2.0.7), (2.0.6) and (2.0.10) in (2.0.1), we get

$$\Pr(Y \geq t) = \frac{e^{-\lambda(t+2)}}{e^{-2\lambda}} = e^{-\lambda t} = e^{-\frac{t}{2}} \tag{2.0.9}$$

$\underline{Case-(ii):}$ $t \leq 0$

$$\Pr(X \geq t+2, X>2) = \Pr(X>2) \tag{2.0.10}$$

(2.0.10) can be explained by the fact that if $t \leq 0$ and $X \geq t+2$, then

$$\{X : X > 2\} \supset \{X : X \geq t+2\}$$

Substituting (2.0.10) in (2.0.1), we get

$$\Pr(Y \geq t) = \frac{\Pr(X>2)}{\Pr(X>2)} = 1 \tag{2.0.11}$$

Therefore,

$$\Pr(Y \geq t) = \begin{cases} e^{-\frac{t}{2}}, & \text{if } t > 0 \\ 1, & \text{otherwise} \end{cases} \tag{2.0.12}$$

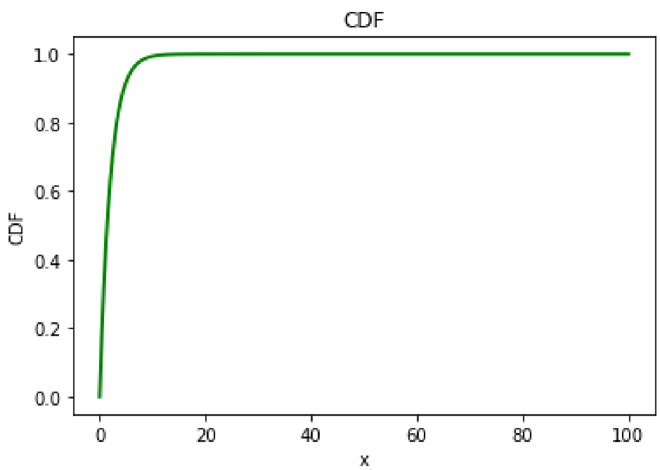

Fig. 0: **CDF**

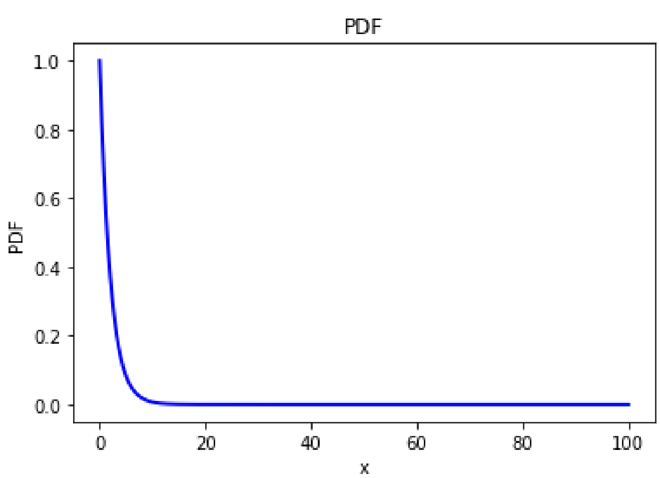

Fig. 0: **PDF**