# AI1103 Assignment-8

## SRIVATSAN T - CS20BTECH11062

### QUESTION
(UGC MATH (MATHA) JUNE 2017 Q.52)

X and Y are independent random variables each having the density

$$f(t) = \frac{1}{\pi} \frac{1}{1 + t^2} \quad -\infty < t < +\infty \qquad (0.0.1)$$

Then the density function of $\dfrac{X + Y}{3}$ for $-\infty < t < +\infty$ is

1) $\dfrac{6}{\pi} \dfrac{1}{4 + 9t^2}$

2) $\dfrac{6}{\pi} \dfrac{1}{9 + 4t^2}$

3) $\dfrac{3}{\pi} \dfrac{1}{1 + 9t^2}$

4) $\dfrac{3}{\pi} \dfrac{1}{9 + t^2}$

### SOLUTION

Let us consider the random variables $\dfrac{X}{3}$ and $\dfrac{Y}{3}$.

We know that if a random variable M has a probability density $f_M(x)$, then the probability density of kM is

$$f_{kM}(x) = \frac{1}{|k|} f_M\left(\frac{x}{|k|}\right) \qquad (0.0.2)$$

Let the probability densities of $\dfrac{X}{3}$ and $\dfrac{Y}{3}$ be $f_1(x)$ and $f_2(y)$.

$$f_1(x) = 3 \times f(3x) \qquad (0.0.3)$$

$$= \frac{3}{\pi} \frac{1}{1 + 9x^2} \quad -\infty < x < +\infty \qquad (0.0.4)$$

$$f_2(y) = 3 \times f(3y) \qquad (0.0.5)$$

$$= \frac{3}{\pi} \frac{1}{1 + 9y^2} \quad -\infty < y < +\infty \qquad (0.0.6)$$

Then the probability density of $\dfrac{X}{3} + \dfrac{Y}{3}$ is the convolution of probability densities of $\dfrac{X}{3}$ and $\dfrac{Y}{3}$.

$$f_Z(z) = \int_{-\infty}^{+\infty} f_1(z - y) f_2(y) \, dy \qquad (0.0.7)$$

$$= \int_{-\infty}^{+\infty} \frac{3}{\pi} \frac{1}{1 + 9(z - y)^2} \times \frac{3}{\pi} \frac{1}{1 + 9y^2} dy \qquad (0.0.8)$$

$$= \frac{9}{\pi^2} \int_{-\infty}^{+\infty} \frac{1}{1 + 9(z - y)^2} \times \frac{1}{1 + 9y^2} dy \qquad (0.0.9)$$

$$= \frac{9}{\pi^2} \times \frac{2\pi}{3} \frac{1}{4 + 9z^2} \qquad (0.0.10)$$

$$= \frac{6}{\pi} \frac{1}{4 + 9z^2} \quad -\infty < z < +\infty \qquad (0.0.11)$$

$$f_Z(t) = \frac{6}{\pi} \frac{1}{4 + 9t^2} \quad -\infty < t < +\infty \qquad (0.0.12)$$

Correct Option : 1