AI1103 Assignment-2

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Download all python codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−2/codes

and latex-tikz codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−2/Assignment−2.tex

QUESTION (GATE Prob 27)

A fair coin is tossed 10 times. What is the probability that ONLY the first 2 tosses will yield heads?

1) $\left(\frac{1}{2}\right)^2$
2) $\left(\frac{1}{2}\right)^{10}$
3) ${}^{10}C_2 \times \left(\frac{1}{2}\right)^2$
4) ${}^{10}C_2 \times \left(\frac{1}{2}\right)^{10}$

SOLUTION

Let $M \sim B(n,h)$ be a random variable representing number of 'heads' in 10 tosses.
So M has a binomial distribution :

$$\Pr(M=k) = {}^{n}C_k \times (h)^{n-k} \times (t)^k \tag{0.0.1}$$

Where

- n = Total number of tosses = 10
- h = Probability that 'head' appears in a toss = $\frac{1}{2}$
- t = Probability that 'tail' appears in a toss = $\frac{1}{2}$

So,

$$\Pr(M=k) = {}^{10}C_k \times \left(\frac{1}{2}\right)^{10-k} \times \left(\frac{1}{2}\right)^k \tag{0.0.2}$$

- Number of ways of choosing 2 positions from 10 tosses = ${}^{10}C_2$
- Number of favourable outcome = 1 (Choosing FIRST and SECOND tosses as heads)

n	10
$\Pr(M=2)$	${}^{10}C_2 \times \left(\frac{1}{2}\right)^{10-2} \times \left(\frac{1}{2}\right)^2$
Calculation	${}^{10}C_2 \times \left(\frac{1}{2}\right)^{10}$
Value	0.043945

- Probability that chosen 2 'heads' are from FIRST and SECOND tosses = $\frac{1}{{}^{10}C_2}$

Probability that ONLY the first 2 tosses yield heads

$$= \Pr(M=2) \times \frac{1}{{}^{10}C_2} \tag{0.0.3}$$

$$= {}^{10}C_2 \times \left(\frac{1}{2}\right)^{10} \times \frac{1}{{}^{10}C_2} \tag{0.0.4}$$

$$= \left(\frac{1}{2}\right)^{10} \tag{0.0.5}$$

Probability that 'head' appears ONLY in the first two tosses is $\left(\frac{1}{2}\right)^{10}$

Correct Option : 2