AI1103 Assignment-8

SRIVATSAN T - CS20BTECH11062

Download all python codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−8/codes

and latex-tikz codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−8/Assignment−8.tex

QUESTION

(UGC MATH (MATHA) JUNE 2017 Q.52)

X and Y are independent random variables each having the density

$$f(t) = \frac{1}{\pi}\frac{1}{1+t^2} \quad -\infty < t < +\infty \tag{0.0.1}$$

Then the density function of $\frac{X+Y}{3}$ for $-\infty <$ t $< +\infty$ is

1) $\frac{6}{\pi}\frac{1}{4+9t^2}$
2) $\frac{6}{\pi}\frac{1}{9+4t^2}$
3) $\frac{3}{\pi}\frac{1}{1+9t^2}$
4) $\frac{3}{\pi}\frac{1}{9+t^2}$

SOLUTION

Let us consider the random variables $\frac{X}{3}$ and $\frac{Y}{3}$.

We know that if a random variable M has a probability density $f_M(x)$, then the probability density of kM is

$$f_{kM}(x) = \frac{1}{|k|} f_M\left(\frac{x}{|k|}\right) \tag{0.0.2}$$

Let the probability densities of $\frac{X}{3}$ and $\frac{Y}{3}$ be $f_1(x)$ and $f_2(y)$.

$$f_1(x) = 3 \times f(3x) \tag{0.0.3}$$

$$= \frac{3}{\pi}\frac{1}{1+9x^2} \quad -\infty < x < +\infty \tag{0.0.4}$$

$$f_2(y) = 3 \times f(3y) \tag{0.0.5}$$

$$= \frac{3}{\pi}\frac{1}{1+9y^2} \quad -\infty < y < +\infty \tag{0.0.6}$$

Then the probability density of $\frac{X}{3} + \frac{Y}{3}$ is the convolution of probability densities of $\frac{X}{3}$ and $\frac{Y}{3}$.

$$f_Z(z) = \int_{-\infty}^{+\infty} f_1(z-y) f_2(y)\, dy \tag{0.0.7}$$

$$= \int_{-\infty}^{+\infty} \frac{3}{\pi}\frac{1}{1+9(z-y)^2} \times \frac{3}{\pi}\frac{1}{1+9y^2} dy \tag{0.0.8}$$

$$= \frac{9}{\pi^2}\int_{-\infty}^{+\infty} \frac{1}{1+9(z-y)^2} \times \frac{1}{1+9y^2} dy \tag{0.0.9}$$

$$= \frac{9}{\pi^2} \times \frac{2\pi}{3}\frac{1}{4+9z^2} \tag{0.0.10}$$

$$= \frac{6}{\pi}\frac{1}{4+9z^2} \quad -\infty < z < +\infty \tag{0.0.11}$$

$$f_Z(t) = \frac{6}{\pi}\frac{1}{4+9t^2} \quad -\infty < t < +\infty \tag{0.0.12}$$

Correct Option : 1

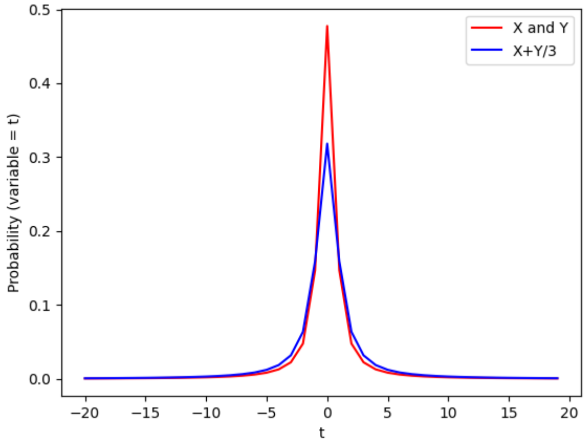

Fig. 4: Grphs showing initial and convoluted probability densities.Area under both the curves = 1