

AI1103 Assignment-5

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Download all python codes from

<https://github.com/Srivatsan-T/AI1103/tree/main/Assignment-5/codes>

and latex-tikz codes from

<https://github.com/Srivatsan-T/AI1103/blob/main/Assignment-5/Assignment-5.tex>

QUESTION (GATE-ME-2006-Q4)

A box contains 20 defective items and 80 non-defective items. If two items are selected at random without replacement, what will be the probability that both items are defective?

- 1) $\frac{1}{5}$
- 2) $\frac{1}{25}$
- 3) $\frac{20}{99}$
- 4) $\frac{19}{495}$

SOLUTION

This problem uses Hyper-Geometric distribution which involves selection of certain number of successes from a given sample without replacement

- Number of defective items = 20
- Number of non-defective items = 80

Let M be a variable representing the number of non-defective items in a selection of 2 items. M has a Hyper-Geometric probability mass function:

$$p_M(k) = \Pr(M = k) = \frac{{}^K C_k \times {}^{N-K} C_{n-k}}{{}^N C_n} \quad (0.0.1)$$

Here Success refers to selecting a defective item,

K	Total successes in population	20
N	Population size	80 + 20 = 100
k	Total observed successes	2
n	Number of draws	2

Probability that the selected set contains 2 defective items = $\Pr(M = 2)$

$$\Pr(M = 2) = \frac{{}^K C_2 \times {}^{N-K} C_{n-2}}{{}^N C_n} \quad (0.0.2)$$

$$= \frac{{}^{20} C_2 \times {}^{100-20} C_{2-2}}{{}^{100} C_2} \quad (0.0.3)$$

$$= \frac{{}^{20} C_2 \times {}^{80} C_0}}{{}^{100} C_2} \quad (0.0.4)$$

$$= \frac{20 \times 19}{100 \times 99} \quad (0.0.5)$$

$$= \frac{19}{495} \quad (0.0.6)$$

So the probability that the selected set of 2 items contain 2 defective items is $\frac{19}{495}$.

Correct Option : 4