

AI1103 Assignment-7

SRIVATSAN T - CS20BTECH11062

Download all python codes from

<https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment-7/codes>

and latex-tikz codes from

<https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment-7/Assignment-7.tex>

$$\Pr\left(U_1 \leq \frac{3}{4}\right) \times \Pr\left(U_2 \leq \frac{3}{4}\right) \dots \Pr\left(U_n \leq \frac{3}{4}\right) \quad (0.0.4)$$

$$= \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \dots \frac{3}{4} \quad (\text{n times}) \quad (0.0.5)$$

$$= \left(\frac{3}{4}\right)^n \quad (0.0.6)$$

QUESTION (CSIR UGC NET JUNE 2013 Q.59)

Let U_1, U_2, \dots, U_n be independent and identically distributed random variables each having a uniform distribution on $(0,1)$. Then,

$$\lim_{n \rightarrow \infty} \Pr\left(U_1 + U_2, \dots, U_n \leq \frac{3}{4}n\right) = \lim_{n \rightarrow \infty} \left(\frac{3}{4}\right)^n = 0 \quad (0.0.7)$$

Correct Option - 2

$$\lim_{n \rightarrow +\infty} \Pr\left(U_1 + U_2, \dots, U_n \leq \frac{3}{4}n\right)$$

- 1) does not exist
- 2) exists and equals 0
- 3) exists and equals 1
- 4) exists and equals $\frac{3}{4}$

SOLUTION

We want $U_1 + U_2, \dots, U_n \leq \frac{3}{4}n$. Arithmetic mean of $U_1, U_2, \dots, U_n \leq \frac{3}{4}$. Thus each of U_1, U_2, \dots, U_n to be lesser than or equal to $\frac{3}{4}$.

$$\Pr\left(U_1 + U_2, \dots, U_n \leq \frac{3}{4}n\right) = \Pr\left(U_1 \leq \frac{3}{4}\right) \times \Pr\left(U_2 \leq \frac{3}{4}\right) \times \Pr\left(U_3 \leq \frac{3}{4}\right) \dots \Pr\left(U_n \leq \frac{3}{4}\right). \quad (0.0.1)$$

Since $U_1, U_2, U_3, \dots, U_n$ are continuous random variables on $(0,1)$, whose probability distribution function is $\frac{1}{1-0} = 1$

$$\Pr\left(U_i \leq \frac{3}{4}\right) = \int_0^{\frac{3}{4}} 1 dx = \frac{3}{4} \quad \text{for } i \in (1, 2, 3, \dots, n) \quad (0.0.2)$$

$$(0.0.3)$$