AI1103 Assignment-1

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Download all python codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−1/codes

and latex-tikz codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−1/Assignment−1.tex

QUESTION(Prob 1.4)

Suppose that 90 % of people are right handed. What is the probability that at most 6 of a random sample of 10 people are right handed?

SOLUTION

One can either be 'right' handed or 'left' handed. Given that 90 % of the population is 'right' handed and since being 'right' handed and being 'left' handed are mutually exclusive,

- Probability of 'right' handed = $\Pr(R) = \frac{9}{10}$
- Probability of 'left' handed = $\Pr(L) = \frac{1}{10}$

One can use binomial distribution to find out the probability that more that 6 people are 'right' handed.
Let M be a variable representing number of people who are 'right' handed in a given sample. Picking different number of people is an example of a Bernoulli trial.
So M has a binomial distribution :

$$\Pr(M = k) = \binom{n}{k} \times (l)^{n-k} \times (r)^k \tag{0.0.1}$$

Where

- n = Total number of people = 10
- l = Probability that a person is 'left' handed = $\frac{1}{10}$
- r = Probability that a person is 'right' handed = $\frac{9}{10}$

So,

$$\Pr(M = k) = \binom{n}{k} \times \left(\frac{1}{10}\right)^{n-k} \times \left(\frac{9}{10}\right)^k \tag{0.0.2}$$

$$\begin{aligned}\Pr(\text{at most 6 are right handed}) &= \Pr(M = 0) \\ &+ \Pr(M = 1) + \Pr(M = 2) + \Pr(M = 3) \\ &+ \Pr(M = 4) + \Pr(M = 5) + \Pr(M = 6)\end{aligned}$$

$$\implies \Pr(0 \le M \le 6) = 1 - \Pr(7 \le M \le 10) \tag{0.0.3}$$

Since

$$\sum_{k=0}^{10} \Pr(M = k) = 1 \tag{0.0.4}$$

Thus, we can write:

$$1 - \Pr(7 \le M \le 10) = 1 - \sum_{k=7}^{10} \Pr(M = k) \tag{0.0.5}$$

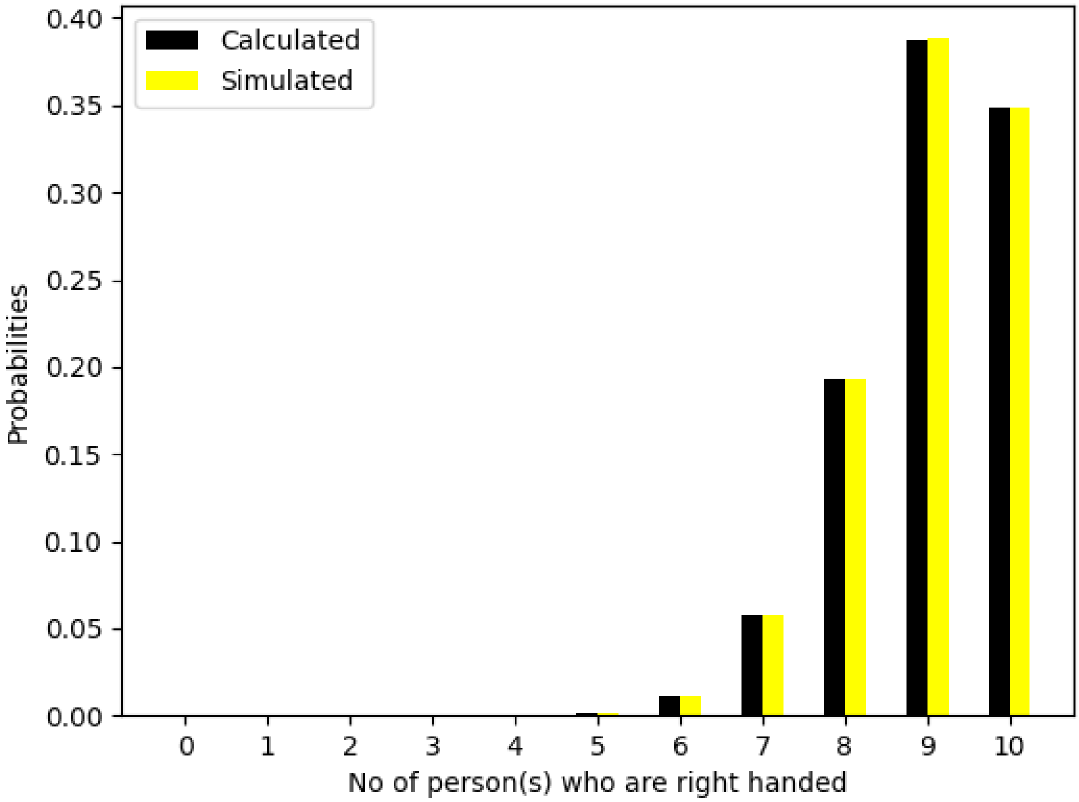
From (0.0.2) we get,

$$\implies 1 - \sum_{k=7}^{10} \binom{10}{k} \times \left(\frac{1}{10}\right)^{10-k} \times \left(\frac{9}{10}\right)^k \tag{0.0.6}$$

$$\begin{aligned}\implies 1 &- \binom{10}{7} \times \left(\frac{1}{10}\right)^3 \times \left(\frac{9}{10}\right)^7 - \binom{10}{8} \times \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^8 \\ &- \binom{10}{9} \times \left(\frac{1}{10}\right)^1 \times \left(\frac{9}{10}\right)^9 - \binom{10}{10} \times \left(\frac{1}{10}\right)^0 \times \left(\frac{9}{10}\right)^{10}\end{aligned} \tag{0.0.7}$$

$$\implies \Pr(0 \le M \le 6) = \frac{7996999}{625000000} = 0.012795198$$

Thus the probability that at most 6 people are 'right' handed out of a random sample of 10 is 0.012795198


Calculated
Simulated
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Probabilities
0
1
2
3
4
5
6
7
8
9
10
No of person(s) who are right handed