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AI1103 Assignment-1

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Download all python codes from

https://github.com/CS20BTECH11062/AI1103/tree/ main/Assignment-1/codes

and latex-tikz codes from

https://github.com/CS20BTECH11062/AI1103/tree/ main/Assignment-1/Assignment-1.tex

QUESTION(Prob 1.4)

Suppose that 90 % of people are right handed. What is the probability that at most 6 of a random sample of 10 people are right handed?

SOLUTION

One can either be 'right' handed or 'left' handed. Given that 90 % of the population is 'right' handed and since being 'right' handed and being 'left' handed are mutually exclusive,

- Probability of 'right' handed = $Pr(R) = \frac{9}{10}$ Probability of 'left' handed = $Pr(L) = \frac{1}{10}$

One can use binomial distribution to find out the probability that more that 6 people are 'right' handed.

Let M be a variable representing number of people who are 'right' handed in a given sample. Picking different number of people is an example of a Bernoulli trial.

So M has a binomial distribution:

$$\Pr\left(M = k\right) = \binom{n}{k} \times (l)^{n-k} \times (r)^{k} \tag{0.0.1}$$

Where

- n = Total number of people = 10
- 1 = Probability that a person is 'left' handed =
- r = Probability that a person is 'right' handed

So,

$$\Pr\left(M = k\right) = \binom{n}{k} \times \left(\frac{1}{10}\right)^{n-k} \times \left(\frac{9}{10}\right)^{k} \qquad (0.0.2)$$

 $Pr(at most 6 are right handed) = Pr(M \le 6)$ \implies 1 - Pr $(M \ge 7)$ Since

$$\sum_{k=1}^{10} Pr(M=k) = 1 \tag{0.0.3}$$

Thus, we can write:

$$1 - \Pr(M \ge 7) = 1 - \sum_{k=7}^{10} \Pr(M = k) \qquad (0.0.4)$$

$$\implies 1 - \sum_{k=7}^{10} {10 \choose k} \times \left(\frac{1}{10}\right)^{10-k} \times \left(\frac{9}{10}\right)^k \qquad (0.0.5)$$

$$\implies 1 - \binom{10}{7} \times \left(\frac{1}{10}\right)^{3} \times \left(\frac{9}{10}\right)^{7} - \binom{10}{8} \times \left(\frac{1}{10}\right)^{2} \times \left(\frac{9}{10}\right)^{8} \\ - \binom{10}{9} \times \left(\frac{1}{10}\right)^{1} \times \left(\frac{9}{10}\right)^{9} - \binom{10}{10} \times \left(\frac{1}{10}\right)^{0} \times \left(\frac{9}{10}\right)^{10}$$
(0.0.6)

$$\implies \Pr(M \le 6) = \frac{7996999}{625000000} \tag{0.0.7}$$

Thus the probability that at most 6 people are 'right' handed out of a random sample of 10 is 0.012795198

