AI1103 Assignment-1

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Download all python codes from

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https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−3/codes
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and latex-tikz codes from

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https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−3/Assignment−3.tex
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QUESTION (GATE-MA-2014-36)

The time to failure, in months, of lights bulbs manufactured at two plants A and B obey the exponential distributions with means 6 and 2 months respectively. Plant B produces four times as many bulbs as plant A does. Bulbs from these two plants are indistinguishable. They are mixed and sold together. Given that a bulb purchased at random is working after 12 months, What is the probability that it was manufactured in plant A?

SOLUTION

Given that 90 % of the population is 'right' handed and since being 'right' handed and being 'left' handed are mutually exclusive,

- Probability of 'right' handed = $\Pr(R) = \frac{9}{10}$
- Probability of 'left' handed = $\Pr(L) = \frac{1}{10}$

One can use binomial distribution to find out the probability that more that 6 people are 'right' handed.
Let M be a variable representing number of people who are 'right' handed in a given sample.
So M has a binomial distribution :

$$\Pr(M = k) = \binom{n}{k} \times (l)^{n-k} \times (r)^k \tag{0.0.1}$$

Where

- n = Total number of people = 10
- l = Probability that a person is 'left' handed = $\frac{1}{10}$
- r = Probability that a person is 'right' handed = $\frac{9}{10}$

So,

$$\Pr(M = k) = \binom{n}{k} \times \left(\frac{1}{10}\right)^{n-k} \times \left(\frac{9}{10}\right)^k \tag{0.0.2}$$

$$\begin{aligned} \Pr(\text{at most 6 are right handed}) &= \Pr(M = 0) \\ &+ \Pr(M = 1) + \Pr(M = 2) + \Pr(M = 3) \\ &+ \Pr(M = 4) + \Pr(M = 5) + \Pr(M = 6) \end{aligned}$$

$$\implies \Pr(0 \le M \le 6) = 1 - \Pr(7 \le M \le 10) \tag{0.0.3}$$

$$\left(\text{Since } \textstyle\sum_{k=0}^{10} \Pr(M = k) = 1\right)$$

$$\implies 1 - \Pr(7 \le M \le 10) = 1 - \sum_{k=7}^{10} \Pr(M = k) \tag{0.0.4}$$

From (0.0.2) we get,

$$\implies 1 - \sum_{k=7}^{10} \binom{10}{k} \times \left(\frac{1}{10}\right)^{10-k} \times \left(\frac{9}{10}\right)^k \tag{0.0.5}$$

$$\begin{aligned} \implies 1 &- \binom{10}{7} \times \left(\frac{1}{10}\right)^3 \times \left(\frac{9}{10}\right)^7 - \binom{10}{8} \times \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^8 \\ &- \binom{10}{9} \times \left(\frac{1}{10}\right)^1 \times \left(\frac{9}{10}\right)^9 - \binom{10}{10} \times \left(\frac{1}{10}\right)^0 \times \left(\frac{9}{10}\right)^{10} \end{aligned} \tag{0.0.6}$$

$$\implies \Pr(0 \le M \le 6) = \frac{7996999}{625000000} = 0.012795198$$

Thus the probability that at most 6 people are 'right' handed is 0.012795198