AI1103 Assignment-5

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QUESTION (GATE-ME-2006-Q4)

A box contains 20 defective items and 80 non-defective items. If two items are selected at random without replacement, what will be the probability that both items are defective?

A. $\dfrac{1}{5}$ B. $\dfrac{1}{25}$ C. $\dfrac{20}{99}$ D. $\dfrac{19}{495}$

SOLUTION

This problem involves Hyper-Geometric distribution
- Number of defective items = 20
- Number of non-defective items = 80

One can use Hyper-Geometric distribution to find out the probability that the selected set contains 2 defective items.

Let M be a variable representing the number of non-defective items in a selection of 2 items.

M has a probability mass Function:

$$p_M(k) = \Pr(M = k) = \frac{{}^K C_k \times {}^{N-K} C_{n-k}}{{}^N C_n} \tag{0.0.1}$$

Here Success refers to selecting a defective item,

K	Total successes in population	20
N	Population size	80 + 20 = 100
k	Total observed successes	2
n	Number of draws	2

Probability that the selected set contains 2 defective items = $\Pr(M = 2)$

$$\Pr(M = 2) = \frac{{}^K C_2 \times {}^{N-K} C_{n-2}}{{}^N C_n} \tag{0.0.2}$$

$$= \frac{{}^{20} C_2 \times {}^{100-20} C_{2-2}}{{}^{100} C_2} \tag{0.0.3}$$

$$= \frac{{}^{20} C_2 \times {}^{80} C_0}{{}^{100} C_2} \tag{0.0.4}$$

$$= \frac{20 \times 19}{100 \times 99} \tag{0.0.5}$$

$$= \frac{19}{495} \tag{0.0.6}$$

So the probability that the selected set of 2 items contain 2 defective items is $\dfrac{19}{495}$.

Correct Option : D