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AI1103 Assignment-2

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Download all python codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment-2/codes

and latex-tikz codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment-2/Assignment-2.tex

QUESTION (GATE Prob 27)

A fair coin is tossed 10 times. What is the probability that ONLY the first 2 tosses will yield heads?

SOLUTION

Let M be a random variable representing number of 'heads' in 10 tosses.

So M has a binomial distribution:

$$\Pr(M = k) = {}^{n}C_{k} \times (h)^{n-k} \times (t)^{k}$$
 (0.0.1)

Where

- n = Total number of tosses = 10
- h = Probability that 'head' appears in a toss = $\frac{1}{2}$
- $t = Probability that 'tail' appears in a toss = \frac{1}{2}$

So.

$$\Pr(M = k) = {}^{10}C_k \times \left(\frac{1}{2}\right)^{10-k} \times \left(\frac{1}{2}\right)^k$$
 (0.0.2)

n	10
Pr (2heads)	$\Pr\left(X=2\right)$
Calculation	10 C ₂ × $\left(\frac{1}{2}\right)^{10-2}$ × $\left(\frac{1}{2}\right)^2$
Value	0.043945

- Now, these 2 heads can occur at any position in 10 tosses.
- Number of ways of choosing 2 positions from $10 \text{ tosses} = {}^{10}\text{C}_2$

• Probability that chosen 2 'heads' are from FIRST and SECOND tosses = $\frac{1}{^{10}\text{C}_2}$

Probability that ONLY the first 2 tosses yield heads = $Pr(M = 2) \times Probability$ that chosen 2 'heads' are from FIRST and SECOND tosses.

$$\implies {}^{10}\mathrm{C}_2 \times \left(\frac{1}{2}\right)^{10} \times \frac{1}{{}^{10}\mathrm{C}_2} = \left(\frac{1}{2}\right)^{10}$$

Probability that 'head' appears ONLY in the first two tosses is $\left(\frac{1}{2}\right)^{10}$

Correct Option: C

