

AI1103 Assignment-2

SRIVATSAN T - CS20BTECH11062

Download all python codes from

<https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment-2/codes>

and latex-tikz codes from

<https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment-2/Assignment-2.tex>

QUESTION (GATE PROB 27)

A fair coin is tossed 10 times. What is the probability that ONLY the first 2 tosses will yield heads?

SOLUTION

Let M be a random variable representing number of 'heads' in 10 tosses.

So M has a binomial distribution :

$$\Pr(M = k) = {}^nC_k \times (h)^{n-k} \times (t)^k \quad (0.0.1)$$

Where

- n = Total number of tosses = 10
- h = Probability that 'head' appears in a toss = $\frac{1}{2}$
- t = Probability that 'tail' appears in a toss = $\frac{1}{2}$

So,

$$\Pr(M = k) = {}^{10}C_k \times \left(\frac{1}{2}\right)^{10-k} \times \left(\frac{1}{2}\right)^k \quad (0.0.2)$$

$\Pr(\text{'head' appears twice in 10 tosses}) = \Pr(M = 2)$

$$\begin{aligned} \Pr(M = 2) &= {}^{10}C_2 \times \left(\frac{1}{2}\right)^{10-2} \times \left(\frac{1}{2}\right)^2 \quad (0.0.3) \\ &= {}^{10}C_2 \times \left(\frac{1}{2}\right)^{10} \end{aligned}$$

\Rightarrow Probability that 'head' appears 2 times in 10 tosses is 0.0439453125

Now, these 2 heads can occur at any position in 10 tosses.

- Number of ways of choosing 2 positions from 10 tosses = ${}^{10}C_2$
- Probability that chosen 2 'heads' are from FIRST and SECOND tosses = $\frac{1}{{}^{10}C_2}$

Probability that ONLY the first 2 tosses yield heads = $\Pr(M = 2) \times$ Probability that chosen 2 'heads' are from FIRST and SECOND tosses.

$$\Rightarrow \binom{10}{2} \times \left(\frac{1}{2}\right)^{10} \times \frac{1}{\binom{10}{2}} = \left(\frac{1}{2}\right)^{10}$$

Probability that 'head' appears ONLY in the first two tosses is $\left(\frac{1}{2}\right)^{10}$.

Correct Option : C

