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AI1103 Assignment-5

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Download all python codes from

https://github.com/Srivatsan-T/AI1103/tree/main/Assignment-5/codes

and latex-tikz codes from

https://github.com/Srivatsan-T/AI1103/blob/main/Assignment-5/Assignment-5.tex

QUESTION (GATE-ME-2006-Q4)

A box contains 20 defective items and 80 non-defective items. If two items are selected at random without replacement, what will be the probability that both items are defective?

- 1) $\frac{1}{5}$
- 2) $\frac{1}{25}$
- 3) $\frac{20}{99}$
- 4) $\frac{19}{495}$

SOLUTION

This problem uses Hyper-Geometric distribution which involves selection of certain number of successes from a given sample without replacement

- Number of defective items = 20
- Number of non-defective items = 80

Let M be a variable representing the number of nondefective items in a selection of 2 items. M has a Hyper-Geometric probability mass function:

$$p_M(k) = \Pr(M = k) = \frac{{}^{K}C_k \times {}^{N-K}C_{n-k}}{{}^{N}C_n}$$
 (0.0.1)

Here Success refers to selecting a defective item,

K	Total successes in population	20
N	Population size	80 + 20 = 100
k	Total observed successes	2
n	Number of draws	2

Probability that the selected set contains 2 defective items = Pr(M = 2)

$$\Pr(M=2) = \frac{{}^{K}C_{2} \times {}^{N-K}C_{n-2}}{{}^{N}C_{n}}$$
 (0.0.2)

$$= \frac{{}^{20}\text{C}_2 \times {}^{100-20}\text{C}_{2-2}}{{}^{100}\text{C}_2} \tag{0.0.3}$$

$$= \frac{^{20}\text{C}_2 \times ^{80}\text{C}_0}{^{100}\text{C}_2} \tag{0.0.4}$$

$$= \frac{20 \times 19}{100 \times 99} \tag{0.0.5}$$

$$=\frac{19}{495}\tag{0.0.6}$$

So the probability that the selected set of 2 items contain 2 defective items is $\frac{19}{495}$.

Correct Option: 4