AI1103 Assignment-3

SRIVATSAN T - CS20BTECH11062

Download all python codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−3/codes

and latex-tikz codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−3/Assignment−3.tex

QUESTION (GATE-MA-2014-36)

The time to failure, in months, of lights bulbs manufactured at two plants A and B obey the exponential distributions with means 6 and 2 months respectively. Plant B produces four times as many bulbs as plant A does. Bulbs from these two plants are indistinguishable. They are mixed and sold together. Given that a bulb purchased at random is working after 12 months, What is the probability that it was manufactured in plant A?

SOLUTION

This problem involves Baye's theorem and Exponential distribution

- Probability that bulb is from Plant A = $\Pr(A) = \frac{1}{5}$
- Probability that bulb is from Plant B = $\Pr(B) = \frac{4}{5}$

One can use exponential distribution to find out the probability that the bulbs work after 12 months

Let X be a variable representing the lifetime of a bulb in months.

So X has a Cumulative distribution Function:

$$F(x,\lambda) = \begin{cases} 1 - e^{-\lambda x} & if \quad x \geq 0 \\ 0 & if \quad x < 0 \end{cases} \tag{0.0.1}$$

Where

- $\frac{1}{\lambda}$ = Mean of distribution
- x = Time to failure (in months)

So,

$$\Pr(M \leq k) = F(M,\lambda) \tag{0.0.2}$$

Pr(Works after 12 months | Plant A) =
1 - Pr(Fails within 12 months | Plant A)

$$\implies \Pr(0 \leq M \leq 6) = 1 - \Pr(7 \leq M \leq 10) \tag{0.0.3}$$

Pr(Works after 12 months | Plant A) =
1 - Pr(Fails within 12 months | Plant A)

$$\implies 1 - \Pr(7 \leq M \leq 10) = 1 - \sum_{k=7}^{10} \Pr(M = k) \tag{0.0.4}$$

From (0.0.2) we get,

$$\implies 1 - \sum_{k=7}^{10} \binom{10}{k} \times \left(\frac{1}{10}\right)^{10-k} \times \left(\frac{9}{10}\right)^{k} \tag{0.0.5}$$

$$\implies 1 - \binom{10}{7} \times \left(\frac{1}{10}\right)^{3} \times \left(\frac{9}{10}\right)^{7} - \binom{10}{8} \times \left(\frac{1}{10}\right)^{2} \times \left(\frac{9}{10}\right)^{8} - \binom{10}{9} \times \left(\frac{1}{10}\right)^{1} \times \left(\frac{9}{10}\right)^{9} - \binom{10}{10} \times \left(\frac{1}{10}\right)^{0} \times \left(\frac{9}{10}\right)^{10} \tag{0.0.6}$$

$$\implies \Pr(0 \leq M \leq 6) = \frac{7996999}{625000000} = 0.012795198$$

Thus the probability that at most 6 people are 'right' handed is 0.012795198