AI1103 Assignment-4

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QUESTION (GATE-ME-2012-45)

A box contains 4 red balls and 6 black balls. Three balls are selected randomly from the box one after another, without replacement. What is the probability that the selected set contains one red ball and two black balls?

1) $\dfrac{1}{20}$

2) $\dfrac{1}{12}$

3) $\dfrac{3}{10}$

4) $\dfrac{1}{2}$

SOLUTION

This problem uses Hyper-Geometric distribution which involves selection of certain number of successes from a given sample without replacement.

- Number of Red balls = 4
- Number of Black balls = 6

Let M be a variable representing the number of black balls in a selection of 3 balls. M has a Hyper-Geometric probability mass function:

$$p_M(k) = \Pr(M = k) = \frac{{}^{K}C_k \times {}^{N-K}C_{n-k}}{{}^{N}C_n} \qquad (0.0.1)$$

Here Success refers to selecting a black ball,

K	Total successes in population	6
N	Population size	6 + 4 = 10
k	Total observed successes	2
n	Number of draws	3

Probability that the selected set contains 2 black balls and 1 red ball = $\Pr(M = 2)$

$$\Pr(M = 2) = \frac{{}^{K}C_2 \times {}^{N-K}C_{n-2}}{{}^{N}C_n} \qquad (0.0.2)$$

$$= \frac{{}^{6}C_2 \times {}^{10-6}C_{3-2}}{{}^{10}C_3} \qquad (0.0.3)$$

$$= \frac{{}^{6}C_2 \times {}^{4}C_1}{{}^{10}C_3} \qquad (0.0.4)$$

$$= \frac{15 \times 4}{120} \qquad (0.0.5)$$

$$= \frac{1}{2} \qquad (0.0.6)$$

So the probability that the selected set of 3 balls contain 2 black balls and 1 red ball is $\dfrac{1}{2}$.

Correct Option : 4