AI1103 Assignment-8

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Download all python codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−8/codes

and latex-tikz codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−8/Assignment−8.tex

QUESTION

(UGC MATH (MATHA) JUNE 2017 Q.52)

X and Y are independent random variables each having the density

$$f(t) = \frac{1}{\pi}\frac{1}{1+t^2} - \infty < t < +\infty \quad (0.0.1)$$

Then the density function of $\frac{X+Y}{3}$ for $-\infty <$ t $< +\infty$ is

1) $\frac{6}{\pi}\frac{1}{4+9t^2}$
2) $\frac{6}{\pi}\frac{1}{9+4t^2}$
3) $\frac{3}{\pi}\frac{1}{1+9t^2}$
4) $\frac{3}{\pi}\frac{1}{9+t^2}$

SOLUTION

Let us consider the random variables X and Y. The Characteristic function of the probability density $f(t)$ is

$$g(w) = \int_{-\infty}^{\infty} f(t)e^{iwt}dt \quad (0.0.2)$$

$$= \int_{-\infty}^{\infty} \frac{1}{\pi}\frac{1}{1+t^2}e^{iwt}dt \quad (0.0.3)$$

$$= e^{-|w|} \ , -\infty < w < \infty \quad (0.0.4)$$

The product of the Characteristic function of probability density of X and Y is

$$h(w) = g_1(w) \times g_2(w) = e^{-2|w|} \quad (0.0.5)$$

To get the probability density of X+Y, we find the inverse characteristic function of h(w). But since there is a one to one correspondence between a function and its fourier transform and $h(w) = g(2w)$

$$F_{X+Y}(t) = \frac{1}{2}f(\frac{t}{2}) \quad (0.0.6)$$

$$= \frac{1}{2\pi}\frac{4}{4+t^2} \ , -\infty < t < \infty \quad (0.0.7)$$

We know that if a random variable M has a probability density $f_M(x)$, then the probability density of random variable kM is

$$f_{kM}(x) = \frac{1}{|k|}f_M\left(\frac{x}{|k|}\right) \quad (0.0.8)$$

Probability density of $Z = \frac{X+Y}{3}$ given $F_{X+Y}(t)$ is

$$F_Z(t) = 3 \times f_{X+Y}(3t) \quad (0.0.9)$$

$$= \frac{6}{\pi}\frac{1}{4+9t^2} \quad (0.0.10)$$

Correct Option : 1

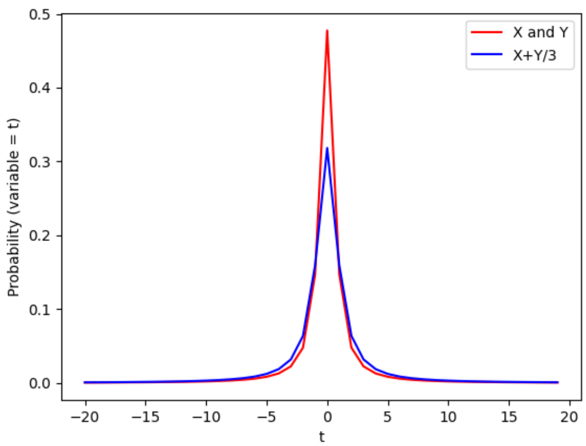

Fig. 4: Graph showing X,Y and $\frac{X+Y}{3}$ probability densities.Area under both the curves = 1