AI1103 Assignment-2

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Download all python codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−2/codes

and latex-tikz codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−2/Assignment−2.tex

QUESTION (GATE Prob 27)

A fair coin is tossed 10 times. What is the probability that ONLY the first 2 tosses will yield heads?

SOLUTION

Let $M \in \{0,1,2,3,4,5,6,7,8,9,10\}$ be a random variable representing number of 'heads' in 10 tosses. So M has a binomial distribution :

$$\Pr(M=k) = {}^{n}\mathrm{C}_k \times (h)^{n-k} \times (t)^k \tag{0.0.1}$$

Where

- n = Total number of tosses = 10
- h = Probability that 'head' appears in a toss = $\frac{1}{2}$
- t = Probability that 'tail' appears in a toss = $\frac{1}{2}$

So,

$$\Pr(M=k) = {}^{10}\mathrm{C}_k \times \left(\frac{1}{2}\right)^{10-k} \times \left(\frac{1}{2}\right)^{k} \tag{0.0.2}$$

n	10
Pr (2 heads)	$\Pr(X=2)$
Calculation	${}^{10}\mathrm{C}_2 \times \left(\frac{1}{2}\right)^{10}$
Value	0.043945

- Number of ways of choosing 2 positions from 10 tosses = ${}^{10}\mathrm{C}_2$
- Probability that chosen 2 'heads' are from FIRST and SECOND tosses = $\frac{1}{{}^{10}\mathrm{C}_2}$

Probability that ONLY the first 2 tosses yield heads

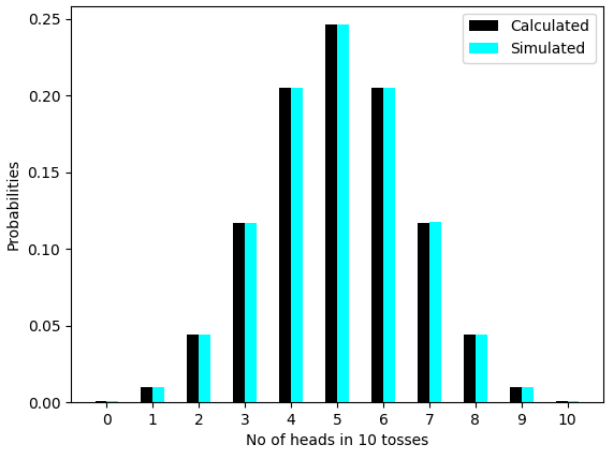
$$= \Pr(M=2) \times \frac{1}{{}^{10}\mathrm{C}_2} \tag{0.0.3}$$

$$= {}^{10}\mathrm{C}_2 \times \left(\frac{1}{2}\right)^{10} \times \frac{1}{{}^{10}\mathrm{C}_2} \tag{0.0.4}$$

$$= \left(\frac{1}{2}\right)^{10} \tag{0.0.5}$$

Probability that 'head' appears ONLY in the first two tosses is $\left(\frac{1}{2}\right)^{10}$

Correct Option : C


Calculated
Simulated
Probabilities
0.25
0.20
0.15
0.10
0.05
0.00
0
1
2
3
4
5
6
7
8
9
10
No of heads in 10 tosses