

AI1103 Assignment-6

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Download all python codes from

<https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment-6/codes>

and latex-tikz codes from

<https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment-6/Assignment-6.tex>

QUESTION (CSIR UGC NET JUNE 2013 Q.59)

The joint probability density function of (X,Y) is

$$f(x, y) = \begin{cases} 6(1-x) & \text{if } 0 < y < x, 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.1)$$

Which among the following are correct?

1) X and Y are not independent

2) $f_Y(y) = \begin{cases} 3(y-1)^2 & \text{if } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$

3) X and Y are independent

4) $f_Y(y) = \begin{cases} 3\left(y - \frac{1}{2}y^2\right) & \text{if } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$

SOLUTION

Given joint probability density function of X and Y, marginal probability density functions are as follows:

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy \quad (0.0.2)$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx \quad (0.0.3)$$

Calculating $f_X(x)$

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy \quad (0.0.4)$$

$$= \int_0^x 6(1-x) dy \quad (0.0.5)$$

$$f_X(x) = \begin{cases} 6x(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.6)$$

Calculating $f_Y(y)$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx \quad (0.0.7)$$

$$= \int_y^1 6(1-x) dx \quad (0.0.8)$$

$$= 6x - 3x^2 \Big|_y^1 \quad (0.0.9)$$

$$= 3 - 6y + 3y^2 \quad (0.0.10)$$

$$= 3(y-1)^2 \quad (0.0.11)$$

$$f_Y(y) = \begin{cases} 3(y-1)^2 & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.12)$$

To check whether X and Y are independent, we calculate $f_X(x) \times f_Y(y)$. From (0.0.6) and (0.0.12)

$$f_X(x) \times f_Y(y) = \begin{cases} 18x(1-x)(y-1)^2 & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.13)$$

$$\neq f(x, y) \quad (0.0.14)$$

Since $f(x, y)$ and $f_X(x) \times f_Y(y)$ are different, random variables X and Y are not independent.

Options 1 and 2 are correct