

AI1103 Assignment-7

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Download all python codes from

<https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment-8/codes>

and latex-tikz codes from

<https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment-8/Assignment-8.tex>

QUESTION

(UGC MATH (MATHA) JUNE 2017 Q.52)

X and Y are independent random variables each having the density

$$f(t) = \frac{1}{\pi} \frac{1}{1+t^2} \quad -\infty < t < +\infty \quad (0.0.1)$$

Then the density function of $\frac{X+Y}{3}$ for $-\infty < t < +\infty$ is

1) $\frac{6}{\pi} \frac{1}{4+9t^2}$

2) $\frac{6}{\pi} \frac{1}{9+4t^2}$

3) $\frac{3}{\pi} \frac{1}{1+9t^2}$

4) $\frac{3}{\pi} \frac{1}{9+t^2}$

SOLUTION

Let us consider the random variables $\frac{X}{3}$ and $\frac{Y}{3}$.

We know that if a random variable M has a probability density $f_M(x)$, then the probability density of kM is

$$f_{kM}(x) = \frac{1}{|k|} f_M\left(\frac{x}{|k|}\right) \quad (0.0.2)$$

Let the probability densities of $\frac{X}{3}$ and $\frac{Y}{3}$ be $f_1(x)$ and $f_2(y)$.

$$f_1(x) = 3 \times f(3x) \quad (0.0.3)$$

$$= \frac{3}{\pi} \frac{1}{1+9x^2} \quad -\infty < x < +\infty \quad (0.0.4)$$

$$f_2(y) = 3 \times f(3y) \quad (0.0.5)$$

$$= \frac{3}{\pi} \frac{1}{1+9y^2} \quad -\infty < y < +\infty \quad (0.0.6)$$

Then the probability density of $\frac{X}{3} + \frac{Y}{3}$ is the convolution of probability densities of $\frac{X}{3}$ and $\frac{Y}{3}$.

$$f_Z(z) = \int_{-\infty}^{+\infty} f_1(z-y) f_2(y) dy \quad (0.0.7)$$

$$= \int_{-\infty}^{+\infty} \frac{3}{\pi} \frac{1}{1+9(z-y)^2} \times \frac{3}{\pi} \frac{1}{1+9y^2} dy \quad (0.0.8)$$

$$= \frac{9}{\pi^2} \int_{-\infty}^{+\infty} \frac{1}{1+9(z-y)^2} \times \frac{1}{1+9y^2} dy \quad (0.0.9)$$

$$= \frac{9}{\pi^2} \times \frac{2\pi}{3} \frac{1}{4+9z^2} \quad (0.0.10)$$

$$= \frac{6}{\pi} \frac{1}{4+9z^2} \quad -\infty < z < +\infty \quad (0.0.11)$$

Correct Option : 1