

AI1103 Assignment-3

SRIVATSAN T - CS20BTECH11062

Download all python codes from

<https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment-3/codes>

and latex-tikz codes from

<https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment-3/Assignment-3.tex>

QUESTION (GATE-MA-2014-36)

The time to failure, in months, of lights bulbs manufactured at two plants A and B obey the exponential distributions with means 6 and 2 months respectively. Plant B produces four times as many bulbs as plant A does. Bulbs from these two plants are indistinguishable. They are mixed and sold together. Given that a bulb purchased at random is working after 12 months, What is the probability that it was manufactured in plant A?

SOLUTION

This problem involves Bayes theorem and Exponential distribution

- Probability that bulb is from Plant A = $\Pr(A) = \frac{1}{5}$
- Probability that bulb is from Plant B = $\Pr(B) = \frac{4}{5}$

One can use exponential distribution to find out the probability that the bulbs work after 12 months. Let X be a variable representing the lifetime of a bulb in months.

So X has a Cumulative distribution Function:

$$F_X(x, \lambda) = \begin{cases} 1 - e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \quad (0.0.1)$$

$\frac{1}{\lambda}$	Mean of distribution
x	Time to failure (in months)
λ_A	$\frac{1}{6}$
λ_B	$\frac{1}{2}$
$\Pr(X \leq k)$	$F_X(X, \lambda)$

Let us denote that the bulbs works after 12 months with the variable W.

$$\begin{aligned} \Pr(W | A) &= 1 - \Pr(\text{Fails within 12 months} | A) \\ &= 1 - F_X(12, \lambda_A) \quad (0.0.2) \\ &= e^{-\lambda_A \times 12} \quad (0.0.3) \end{aligned}$$

$$\begin{aligned} \Pr(W | B) &= 1 - \Pr(\text{Fails within 12 months} | B) \\ &= 1 - F_X(12, \lambda_B) \quad (0.0.4) \\ &= e^{-\lambda_B \times 12} \quad (0.0.5) \end{aligned}$$

From Bayes theorem,

$$\begin{aligned} \Pr(A | W) &= \frac{\Pr(A) \times \Pr(W | A)}{\Pr(A) \times \Pr(W | A) + \Pr(B) \times \Pr(W | B)} \quad (0.0.6) \\ &= \frac{\Pr(A) \times e^{-\lambda_A \times 12}}{\Pr(A) \times e^{-\lambda_A \times 12} + \Pr(B) \times e^{-\lambda_B \times 12}} \quad (0.0.7) \end{aligned}$$

Substituting the known values, we get

$$\begin{aligned} \Pr(A | W) &= \frac{\frac{1}{5} \times e^{-2}}{\frac{1}{5} \times e^{-2} + \frac{4}{5} \times e^{-6}} \quad (0.0.8) \\ &= 0.93173845935 \quad (0.0.9) \end{aligned}$$

So the probability that the Bulb is manufactured in Plant A given that it works after a year is 0.93173845935.