AI1103 Assignment-2

SRIVATSAN T - CS20BTECH11062

Download all python codes from

https://github.com/CS20BTECH11062/AI1103/tree/ main/Assignment-2/codes

and latex-tikz codes from

https://github.com/CS20BTECH11062/AI1103/tree/ main/Assignment-2/Assignment-2.tex

QUESTION (GATE Prob 27)

A fair coin is tossed 10 times. What is the probability that ONLY the first 2 tosses will yield heads?

- 1) $\left(\frac{1}{2}\right)^2$ 2) $\left(\frac{1}{2}\right)^{10}$
- 3) ${}^{10}C_2 \times \left(\frac{1}{2}\right)^2$ 4) ${}^{10}C_2 \times \left(\frac{1}{2}\right)^{10}$

SOLUTION

Let $M \sim B(n, h)$ be a random variable representing number of 'heads' in 10 tosses.

So M has a binomial distribution:

$$\Pr(M = k) = {}^{n}C_{k} \times (h)^{n-k} \times (t)^{k}$$
 (0.0.1)

Where

- n = Total number of tosses = 10
- h = Probability that 'head' appears in a toss =
- $t = \text{Probability that 'tail' appears in a toss} = \frac{1}{2}$

So,

$$\Pr(M = k) = {}^{10}C_k \times \left(\frac{1}{2}\right)^{10-k} \times \left(\frac{1}{2}\right)^k$$
 (0.0.2)

n	10
$\Pr\left(M=2\right)$	10 C ₂ × $\left(\frac{1}{2}\right)^{10-2}$ × $\left(\frac{1}{2}\right)^2$
Calculation	$^{10}\mathrm{C}_2 \times \left(\frac{1}{2}\right)^{10}$
Value	0.043945

- Number of ways of choosing 2 positions from 10 tosses = ${}^{10}C_2$
- Number of favourable outcome = 1 (Choosing FIRST and SECOND tosses as heads)
- Probability that chosen 2 'heads' are from FIRST and SECOND tosses = $\frac{1}{10}$ C₂

Probability that ONLY the first 2 tosses yield heads

$$= \Pr(M = 2) \times \frac{1}{{}^{10}\text{C}_2}$$
 (0.0.3)

$$={}^{10}C_2 \times \left(\frac{1}{2}\right)^{10} \times \frac{1}{{}^{10}C_2} \tag{0.0.4}$$

$$= \left(\frac{1}{2}\right)^{10} \tag{0.0.5}$$

Probability that 'head' appears ONLY in the first two tosses is $\left(\frac{1}{2}\right)$

Correct Option: 2

