AI1103 Assignment-2

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Download all python codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−2/codes

and latex-tikz codes from

https://github.com/CS20BTECH11062/AI1103/tree/main/Assignment−2/Assignment−2.tex

QUESTION (GATE Prob 27)

A fair coin is tossed 10 times. What is the probability that ONLY the first 2 tosses will yield heads?

SOLUTION

Since the given coin is fair,

- Probability of getting a 'head' = $\Pr(H) = \frac{1}{2}$
- Probability of getting a 'tail' = $\Pr(T) = \frac{1}{2}$

One can use binomial distribution to find out the probability that 'head' appears twice in 10 tosses.
Let M be a random variable representing number of 'heads' in 10 tosses.
So M has a binomial distribution :

$$\Pr(M = k) = \binom{n}{k} \times (h)^{n-k} \times (t)^k \tag{0.0.1}$$

Where

- n = Total number of tosses = 10
- h = Probability that 'head' appears in a toss = $\frac{1}{2}$
- t = Probability that 'tail' appears in a toss = $\frac{1}{2}$

So,

$$\Pr(M = k) = \binom{10}{k} \times \left(\frac{1}{2}\right)^{10-k} \times \left(\frac{1}{2}\right)^k \tag{0.0.2}$$

Pr('head' appears twice in 10 tosses) = $\Pr(M = 2)$

$$\begin{aligned} \Pr(M = 2) &= \binom{10}{2} \times \left(\frac{1}{2}\right)^{10-2} \times \left(\frac{1}{2}\right)^2 \\ &= \binom{10}{2} \times \left(\frac{1}{2}\right)^{10} \end{aligned} \tag{0.0.3}$$

$\implies$ Probability that 'head' appears 2 times in 10 tosses is 0.0439453125

Now, these 2 heads can occur at any position in 10 tosses. To find the probability that these 2 heads occur ONLY in first 2 tosses:

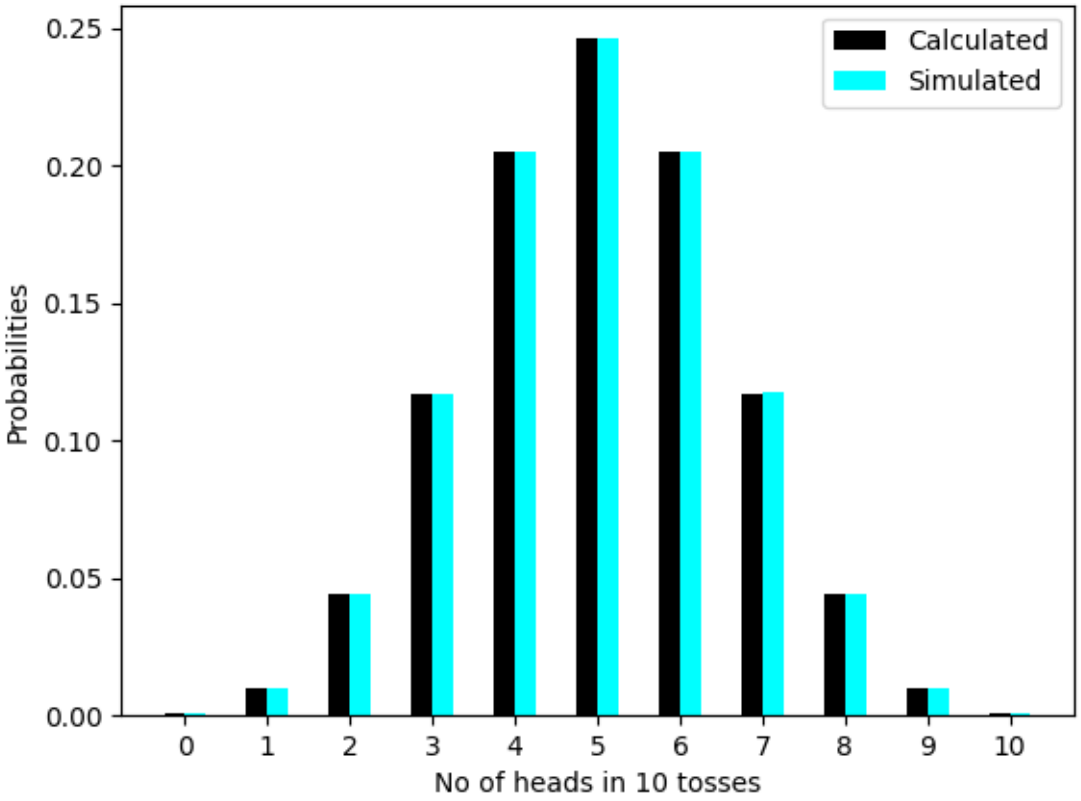
- Number of ways of choosing position of 2 'heads' from 10 tosses = $\binom{10}{2}$
- Number of favourable outcomes of choosing 2 'heads' = 1 (choosing FIRST and SECOND toss's outcome as 'heads')
- Probability that chosen 2 'heads' are from FIRST and SECOND tosses = $\frac{1}{\binom{10}{2}}$

So probability that ONLY the first 2 tosses yield heads = $\Pr(M = 2) \times$ Probability that chosen 2 'heads' are from FIRST and SECOND tosses.

$$\implies \binom{10}{2} \times \left(\frac{1}{2}\right)^{10} \times \frac{1}{\binom{10}{2}} = \left(\frac{1}{2}\right)^{10}$$

Probability that 'head' appears ONLY in the first two tosses is $\left(\frac{1}{2}\right)^{10}$.

Correct Option : C


Calculated
Simulated
Probabilities
0.25
0.20
0.15
0.10
0.05
0.00
0
1
2
3
4
5
6
7
8
9
10
No of heads in 10 tosses