CS-2050-All-Sections Exam 2 White (HOWARD, FAULKNER, ELLEN)

Natalie Andrea Joseph

TOTAL POINTS

95 / 105

QUESTION 1

1 MC 1 **5 / 5**

✓ **- 0 pts** *C*

- 5 pts A

- 5 pts B

- 5 pts D

- 5 pts E

- 5 pts No Answer

QUESTION 2

2 MC 2 **0 / 5**

- 0 pts B

- 5 pts A

- 5 pts C

- 5 pts D

✓ **- 5 pts** *E*

- 5 pts No Answer

QUESTION 3

3 MC 3 **5 / 5**

✓ **- 0 pts** *B*

- 5 pts A

- 5 pts C

- 5 pts D

- 5 pts E

- 5 pts No Answer

QUESTION 4

4 MC 4 **5 / 5**

✓ **- 0 pts** *D*

- 0 pts E

- 5 pts A

- 5 pts B

- 5 pts C

- 5 pts No Answer

QUESTION 5

5 MC 5 **0 / 5**

- 0 pts D

- 5 pts A

- 5 pts B

- 5 pts C

✓ **- 5 pts** *E*

- 5 pts No Answer

QUESTION 6

6 MC 6 **5 / 5**

✓ **- 0 pts** *B*

- 5 pts A

- 5 pts C

- 5 pts D

- 5 pts E

- 5 pts No Answer

QUESTION 7

Short Response 1 (Sets) 15 pts

7.1 i **5 / 5**

✓ **- 0 pts** *$$ \{ (3, \emptyset), (4, \emptyset), (\{2\}, \emptyset), (3,3), (4,3), (\{2\}, 3) \}$$*

- **- 1 pts** Used $$( )$$ instead of $$\{ \}$$ or vice-versa
- **- 1 pts** Did not have outermost $$\{ \}$$ pair

Missing/extra/incorrect elements

- **- 2.5 pts** 1 Missing/extra/incorrect elements
- **- 5 pts** 2+ Missing/extra/incorrect elements

- **- 1 pts** Minor Notation error (e,g, forgot to close brakcets)
- **- 2.5 pts** Gave answer for $$B \times A$$
- **- 4 pts** Used Set Builder Notation
- **- 5 pts** No Answer

7.2 ii **5 / 5**

✓ **- 0 pts** *$$\{ ~ \emptyset, \{ 4\}, \{ \{2\} \}, \{4,\{2\}\}\}$$*

Missing/extra/incorrect elements

- **- 2.5 pts** 1 Missing/extra/incorrect elements
- **- 5 pts** 2+ Missing/extra/incorrect elements

- **- 1 pts** Used $$( )$$ instead of $$\{ \}$$ or vice-versa
- **- 1 pts** Minor Notation error (e,g, forgot to close brakcets)
- **- 4 pts** Used Set Builder Notation
- **- 5 pts** No Answer

7.3 iii **5 / 5**

✓ **- 0 pts** *$$\{\emptyset, 3, 4, \{2\}, \{3\}\}$$*

- **- 1 pts** Used $$( )$$ instead of $$\{ \}$$ or vice-versa

Missing/extra/incorrect elements

- **- 2.5 pts** 1 Missing/extra/incorrect elements
- **- 5 pts** 2+ Missing/extra/incorrect elements

- **- 1 pts** Minor Notation error (e,g, forgot to close brakcets)
- **- 4 pts** Used Set Builder Notation
- **- 5 pts** No Answer

QUESTION 8

8 Short Response 2 (Binary Search) **7 / 7**

✓ **- 0 pts** *$$9, 13, 15, 17$$*

Missing/invalid/extra elements

- **- 2 pts** 1 missing/invalid/extra element
- **- 4 pts** 2 missing/invalid/extra element
- **- 6 pts** 3+ missing/invalid/extra element

- **- 2.5 pts** Correct intermediate steps, no final answer
- **- 5 pts** Invalid or incorrect use of inequality operators
- **- 7 pts** No Answer

QUESTION 9

9 Short Response 3 (Binary) **5 / 5**

✓ **- 0 pts** *False, provides correct explanation*

e.g. Binary search requires elements to be ordered

- **- 3 pts** False without any explanation
- **- 5 pts** No Answer / Incorrect

QUESTION 10

Long Response 1 (Sets) 15 pts

10.1 Proof (Set Builder) **10 / 10**

✓ **- 0 pts** *Correct*

- 6 pts Did not cite any steps

Invalid Steps

- 2 pts 1 Invalid Step

- 4 pts 2 Invalid Steps

- 6 pts 3 Invalid Steps

- 8 pts 4 Invalid Steps

- 10 pts 5+ Invalid Steps

Skipped Steps

- 2 pts 1 Skipped Step

- 4 pts 2 Skipped Steps

- 6 pts 3 Skipped Steps

- 8 pts 4 Skipped Steps

- 10 pts 5+ Skipped Steps

Uncited Steps

- 1 pts 1 Uncited Step

- 2 pts 2 Uncited Steps

- 3 pts 3 Uncited Steps

- 4 pts 4+ Uncited Steps

Miscited Steps

- 1 pts 1 Miscited Step

- 2 pts 2 Miscited Steps

- 3 pts 3 Miscited Steps

- 4 pts 4+ Miscited Steps

- 3 pts Incorrect Set Builder Notation

- 8 pts Uses a Venn Diagram for a proof

- 8 pts Uses set equivalencies.

- 8 pts Uses Set Identities

- 10 pts Disproves Statement

- 10 pts No Answer

- 8 pts Proves using examples of sets

1

need an X is an element of

10.2 Extra Credit (Venn Diagram) **5 / 5**

✓ **- 0 pts** *Correct Venn Diagram*

- 5 pts Incorrect / No Answer

QUESTION 11

11 Short Response 4 (Cashiers Algorithm) **7 / 7**

✓ **- 0 pts** *2 quarters, 1 18-cent coin, 4 pennies*

- 2 pts Minor Math Error

- 4 pts Major Math Error

- 5 pts Uses the cashier's algorithm without the 18 cent coin

- 7 pts No Answer / Incorrect

QUESTION 12

12 Proof (Cashier's Algorithm) **6 / 6**

✓ **- 0 pts** *Provided valid counterexample e.g. producing 22 cents*

- 3 pts Valid counterexample, no reasoning/proof

- 4 pts Tried to disprove but gave invalid counter example

- 4 pts Tried to prove, but provided valid reasoning

- 6 pts Tried to prove, but did not provide valid reasoning

- 6 pts No Answer

QUESTION 13

13 Proof (Witnesses) **10 / 10**

✓ **- 0 pts** *Correct witnesses and valid work*

Minor math/notational errors

- **- 2 pts** 1 Minor math/notational errors
- **- 4 pts** 2+ Minor math/notational errors

Major math/notational errors

- **- 3 pts** 1 Major math/notational errors
- **- 6 pts** 2+ Major math/notational errors

Missing/Invalid Witnesses

- **- 2 pts** 1 Missing/Invalid Witnesses
- **- 4 pts** 2 Missing/Invalid Witnesses
- **- 6 pts** 3+ Missing/Invalid Witnesses

Invalid/missing term inequalities

- **- 3 pts** 1 Invalid/missing term inequalities
- **- 6 pts** 2 Invalid/missing term inequalities
- **- 9 pts** 3+ Invalid/missing term inequalities

- 8 pts Tried to disprove statement, but provided reasonable explanation

- 10 pts Did not prove using witnesses according to Big-O definition

- 10 pts No Answer

QUESTION 14

14 Proof (Witnesses) 10 / 10

✓ **- 0 pts** *Correctly disproves using proof by contradiction*

Skipped Steps

- **- 3 pts** 1 Skipped Step
- **- 6 pts** 2 Skipped Steps
- **- 9 pts** 3+ Skipped Steps

Invalid Steps

- **- 2 pts** 1 Invalid Step
- **- 4 pts** 2 Invalid Steps
- **- 6 pts** 3 Invalid Steps
- **- 8 pts** 4+ Invalid Steps

- 7 pts Did not use proof by contradiction to disprove

- 8 pts Attempted to prove, provided valid reasoning

- 10 pts Incorrect / No Answer

CS 2050-Exam 2 White

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No notes, calculators, or other aids are allowed. Read all directions carefully and write your answers in the space provided.
Taking this exam signifies you are aware of and in accordance with the Academic Honor Code of Georgia.
Do not separate any pages from the rest of your exam.

Exam 2 White

105 points

(Including 5 points for Extra Credit)

Equivalence	*Name*
$p \land \mathbf{T} \equiv p$ $p \lor \mathbf{F} \equiv p$	Identity laws
$p \lor \mathbf{T} \equiv \mathbf{T}$ $p \land \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \lor p \equiv p$ $p \land p \equiv p$	Idempotent
$\neg(\neg p) \equiv p$	Double negation law
$p \lor q \equiv q \lor p$ $p \land q \equiv q \land p$	Commutative laws
$(p \lor q) \lor r \equiv p \lor (q \lor r)$ $(p \land q) \land r \equiv p \land (q \land r)$	Associative laws
$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	Distributive laws
$p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$	Absorption laws
$p \lor \neg p \equiv \mathbf{T}$ $p \land \neg p \equiv \mathbf{F}$	Negation laws

[5] 1. Let $A = \{e, g, h\}$ and $B = \{\{1, 2\}, \emptyset\}$. What is the cardinality of $\mathcal{P}(A \cup B)$?

- ○ 2^3
- ○ 2^4
- ● 2^5
- ○ 2^6
- ○ 2^9

$|A| = 3$

$|B| = 2$

$A \cup B = \{e, g, h, \{1, 2\}, \emptyset\}$

$|A \cup B| = 5$

$|P(A \cup B)| = 2^5$

[5] 2. Let $S = \{\emptyset, 1, 2\}$ and $T = \{0, 1, 2\}$. Which of the following is true? Select only one choice.

- ○ $\{\{\emptyset, 1, 2\}\} \in \mathcal{P}(S)$ $\{S\} \subset$
- ○ $\{\{1, 2\}\} \subset \mathcal{P}(S)$ $\{1, 2\} \subset$
- ○ $\{1, 1\} \in S \cap T$
- ○ $\{1, 2\} \subset (S \cup T) - (S \cap T)$
- ● None of the above

$\{1, 3\}$

$\{\emptyset, \{1\}, \{3\}, \{1, 3\}\}$

$\{1\} \subset \{1, 3\}$

$P(S) = \{\emptyset, \{\emptyset\}, \{1\}, \{2\}, \{\emptyset, 1\}, \{\emptyset, 2\}, \{1, 2\}, \{\emptyset, 1, 2\}\}$

$S \cup T = \{\emptyset, 0, 1, 2\}$

$S \cap T = \{1, 2\}$

$(S \cup T) - (S \cap T) = \{\emptyset, 0\}$

[5] 3. Suppose $f(x)$ is $O(x^5 + x^4)$ and $g(x)$ is $O(x^3)$, where f and g are both real-valued functions. What is the smallest integer n such that $f(x) * g(x)$ is definitely $O(x^n)$?

- ○ 5
- ● 8
- ○ 9
- ○ 12
- ○ 15

$(x^3(x^5 + x^4))$

$((x^8 + x^7))$

[5] 4. Consider the relation $f(x) = \lfloor log(x) \rfloor$ where x is in $\mathbb{R}^+$ and $f(x)$ has a codomain of non-negative integers. Which of the following is true?

- ○ $f(x)$ is one-to-one only
- ○ $f(x)$ is one-to-one and onto
- ○ $f(x)$ is neither one-to-one nor onto
- ● $f(x)$ is onto only
- ○ None of the above

$\log_2(1) = 0$

$\log_2(4) = 2$

$\log_2(8) = 3$

[5] 5. Let $\mathbb{A} = \{a, b, c\}$ where a, b, c are letters (not variables) and let f and g be functions such that $f : \mathbb{A} \longrightarrow \mathbb{A}$ and $g : \mathbb{A} \longrightarrow \mathbb{Z}$, then

- ○ $g \circ f$ must be onto.
- ○ $g \circ f$ must be one to one
- ○ $g \circ g : \mathbb{Z} \longrightarrow \mathbb{Z}$
- ○ g^{-1} is not a function
- ● None of the above

$g(f(x))$

$g(g(x))$

$g(f(a)) \rightarrow g(b)$

$g(f(b)) \rightarrow g(c)$

$g(f(c)) \rightarrow g(a)$

1 2 3

neither

both

[5] 6. Determine the time complexity of the following algorithm where n is the input size. You must choose the lowest growth time complexity for this algorithm.

```
sum := 0
i := n
while i > 1 do
    sum += i
    i -= 1
end while
j := n^2
while j > n do
    j := j/n
    if sum > 4, then sum += 1
```

n

$n = 8$

j	n
64	8
8	

- ○ $O(1)$
- ● $O(n)$
- ○ $O(2^n)$
- ○ $O(n!)$
- ○ None of the above

7. Given the following sets A and B located in the same domain,

$$A = \{3, 4, \{2\}\}$$

$$B = \{\emptyset, 3\}$$

Write out the following sets in **list notation** (no credit will be given for answers written in set builder notation. As an example, list notation looks like this: $\{a, b, c\}$

[5] (i) $A \times B$

$\emptyset$ 3

3

4

$\{2\}$

$A \times B = \{(3, \emptyset), (4, \emptyset), (\{2\}, \emptyset), (3, 3), (4, 3), (\{2\}, 3)\}$

[5] (ii) $\mathcal{P}(A - B)$

$2^2 = 4$

$A - B = \{4, \{2\}\}$

$P(A - B) = \{\emptyset, \{4\}, \{\{2\}\}, \{4, \{2\}\}\}$

[5] (iii) $(A \cup B) \cup \mathcal{P}(A \cap B)$

$A \cup B = \{3, 4, \{2\}, \emptyset\}$

$A \cap B = \{3\}$

$P(A \cap B) = \{\emptyset, \{3\}\}$

$(A \cup B) \cup P(A \cap B) = \{3, 4, \{2\}, \emptyset, \{3\}\}$

[7] 8. List all the numbers you would compare with 17 while searching for the number 17 in the sequence $(1, 3, 7, 9, 12, 13, 15, 17)$ using the binary search algorithm outlined in class. Make sure to write every value compare against in the order the comparisons occur. This includes all values in the "inequality" comparisons as well as the final "equality" check. You must use the algorithm as taught in class.

1 3 7 9 | 12 13 15 17   17 > 9 ? ✓

12 13 | 15 17   17 > 13 ? ✓

15 | 17   17 > 15 ? ✓

17   17 = 17 ? ✓

{9, 13, 15, 17}

[5] 9. Determine whether the following statement is True or False and explain why:
"A binary search does not require an ordered list."

False: A binary search does require an ordered list. If the list is not ordered, then the sublist that contains the number could be discarded after a comparison check. For instance, in the sequen (1, 5, 3, 4), in searching for 4, 4 would be compared to 5 first. 4 is not greater than 5 so the right sublist is discarded, which has 4. Therefore, the list must be ordered.

$A = \{1, 2, 3\}$ $\quad p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$C = \{2, 4, 5\}$ $\quad A - C = \{1, 3\}$

[10] 10. (i) Prove or disprove that if A and B are sets such that $B \cup (A - C) = (B \cup A) \cap (B \cup \overline{C})$. You cannot use set identities in this proof.

I proceed directly with set builder notation.

$B \cup (A - C) = \{x \mid x \in B \cup (A - C)\}$ set builder notation

" " $= \{x \mid x \in B \vee x \in (A - C)\}$ def. of union

" " $= \{x \mid x \in B \vee (x \in A \wedge x \in \overline{C})\}$ def. of set subtraction → A − C, where C is a complement with respect to A

" " $= \{x \mid (x \in B \vee x \in A) \wedge (x \in B \vee x \in \overline{C})\}$ distributive law for propositions

" " $= \{x \mid (x \in B \vee x \in A) \cap (x \in B \vee x \in \overline{C})\}$ def. of intersection

" " $= \{x \mid (x \in B \cup A) \cap (x \in B \vee x \in \overline{C})\}$ def. of union

" " $= \{x \mid x \in (B \cup A) \cap (B \cup \overline{C})\}$ def. of union

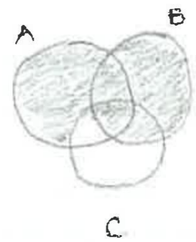
" " $= (B \cup A) \cap (B \cup \overline{C})$ set builder notation reversed

Therefore, $B \cup (A - C) = (B \cup A) \cap (B \cup \overline{C})$.

□

$B \cup (A - C) = (B \cup A) \cap (B \cup \overline{C})$

[5] (ii) *Extra Credit*: Show an example of the set equality in part (i) of this question using a Venn Diagram.



[7] 11. Use the cashier's algorithm to make change for 72 cents using pennies, nickels, dimes, quarters, and a newly introduced 18 cent coin that has been added to the existing currency system.

$$
\begin{array}{r}
.72 \\
-.25 \\
\hline
.47 \\
-.25 \\
\hline
.22 \\
-.18 \\
\hline
.04 \\
-.04 \\
\hline
.00
\end{array}
$$

q = ||

18 = |

p = ||||

Two quarters, one 18 cent coin, four pennies (7 coins)

[6] 12. Prove or disprove the statement: "The cashier's algorithm using quarters, dimes, nickels, pennies, and a 18 cent coin will always produce change using the fewest coins possible when compared to making change using any other methods of making change with those denominations."

False: the cashier's algorithm using the original US currency system is most optimal. For instance, 72¢ would be two quarters, two dimes, and two pennies, which is 6 coins. However, with the 18¢ coin, 72¢ required 7 coins.

[10] 13. Let $f(x) = 2x^2 + 3 * 2^x + 25$, where $x \in \mathbb{R}$. Prove or disprove that $f(x)$ is $O(2^x)$ using witnesses. You must use witnesses to receive credit.

I proceed directly.

$2x^2 \leq (2)2^x \quad \forall x > 4$

$3 \cdot 2^x \leq (3) 2^x \quad \forall x > 1$

$25 \leq (1) 2^x \quad \forall x > 5$

Therefore, $2x^2 + 3 \cdot 2^x + 25 \leq 6 \cdot 2^x \quad \forall x > 5$. With $C = 6$ and $k = 5$ as witnesses, $f(x) = 2x^2 + 3 \cdot 2^x + 25$ is $O(2^x)$ by def. of big-O.

[10] 14. Let $f(x) = 4x^2 + x^5$, where $x \in \mathbb{R}$. Prove or disprove that $f(x)$ is $O(x^2)$ using witnesses. You must use witnesses to receive credit.

I proceed with a proof by contradiction and assume $f(x) = 4x^2 + x^5$ is $O(x^2)$. Then by defintion of big-O, there exists constants C and k such that $4x^2 + x^5 \leq C|x^2|$ $\forall x > k$. Assume $x > 0$, then $C|x^2| = Cx^2$.

Then:

$$4x^2 + x^5 \leq Cx^2 \quad \forall x > k$$

$$4 + x^3 \leq C \quad \forall x > k \quad \text{(divide both sides by } x^2\text{)}$$

$4 + x^3$ will grow infinitely past C because C is a constant. Therefore, $4x^2 + x^5$ is $O(x^2)$ is a contradiction. Thus, $4x^2 + x^5$ is not $O(x^2)$.

$\square$

This page provides extra space if needed. Clearly mark any question that has its answer here.