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Math 4533/5533 Numerical Methods
(Homework #4)

Note that Due is Wednesday (4/13)

1. (30points) Consider the following data:

x	-2	-1	0	1	2
y	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	0
y'	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0

We will find a piecewise cubic interpolating function $S = S_1 \cup S_2 \cup S_3 \cup S_4$ satisfying the data, where S_1, S_2, S_3, S_4 are cubic on each interval.
(1) (10points) Use the form $S_i(x) = y_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$ to find S.
(2) (10points) Check the answer in the previous question (1), using the linear system. Use the Scilab.
(3) (10points) Use Scilab to graph it.

2. (30points) We want to evaluate the Chebyshev polynomial $T_n\left(\frac{1}{2}\right)$ for $n = 2, 3, 4, 5$.
(1) (30points) Use its definition to evaluate $T_n\left(\frac{1}{2}\right)$.
(2) Use the triple recursive relation to find $T_n(x)$. Then evaluate $T_n\left(\frac{1}{2}\right)$.

3. (40points) Consider the function $f(x) = \sin x$ on $[-1, 1]$.
(1) (10points) Find the linear function $p_1(x)$ that minimizes $E(p_1; f)$.
(2) (10points) Find the uniform error of approximation of $f(x)$ by $p_1(x)$.
(3) (10points) Find the least square approximation of degree 1. The least square approximation of degree n to $f(x)$ is given by

$$l_n(x) = \sum_{j=0}^{n} \frac{(f,\, P_j)}{(P_j,\, P_j)} P_j(x),$$

where P_j is a Legendre polynomial and the inner product $(\cdot, \cdot)$ is defined by

$$(f,\, g) = \int_{-1}^{1} f(x)g(x)\, dx.$$

(4) (10points) Find the uniform error of approximation of $f(x)$ by $l_1(x)$

Sincerely,

Roth P. In

HW #4

(1)

$[-2,1] = S_1(x) = y_1 + b_1(x+2) + c_1(x+2)^2 + d_1(x+2)^3$

$[-1,0] = S_2(x) = y_2 + b_2(x+1) + c_2(x+1)^2 + d_2(x+1)^3$

$[0,1] = S_3(x) = y_3 + b_3(x+0) + c_3(x+0)^2 + d_3(x+0)^3$

$[1,2] = S_4(x) = y_4 + b_4(x-1) + c_4(x-1)^2 + d_4(x-1)^3$

$S_1(-2) = 0 = y_1$

$S_1(1) = \frac{1}{6} = y_1 + 3b_1 + 9c_1 + 27d_1$ (1)

$S_2(-1) = \frac{1}{6} = y_2$

$S_2(0)\ \frac{1}{3} = y_2 + b_2 + c_2 + d_2$ (2)

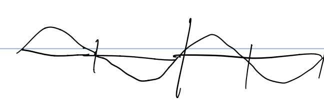
$S_3(0) = \frac{1}{3} = y_3$

$S_3(1) = \frac{1}{6} = y_3 + b_3 + c_3 + d_3$ (3)

$S_4(1) = \frac{1}{6} = y_4$

$S_4(2) = 0 = y_4 - b_4 + c_4 - d_4$ (4)

$S_1'($



(2) $\dfrac{y-y_0}{x-x_0} = \dfrac{y_1-y_0}{x_1-x_0}$

$\{-2,-1\}$ $\dfrac{y-0}{x+2} = \dfrac{\frac{1}{6}}{1} = y = \dfrac{x}{6} + \dfrac{1}{3}$

$\{-1,0\}$ $\dfrac{y-\frac{1}{6}}{x+1} = \dfrac{\frac{1}{6}}{1} = y = \frac{1}{6}x + \frac{1}{3}$

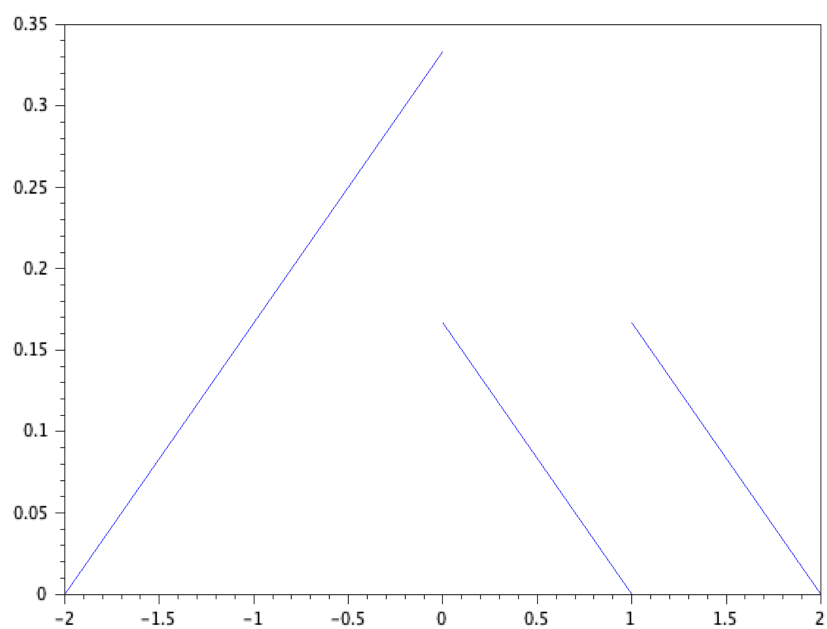
$\{0,1\}$ $\dfrac{y-\frac{1}{3}}{x-0} = \dfrac{\frac{-1}{6}}{1} = y = \dfrac{-x}{6} + \dfrac{1}{6}$

$\{1,2\}$ $\dfrac{y-\frac{1}{6}}{x-1} = \dfrac{-1}{6} = y = \dfrac{-x}{6} + \dfrac{1}{3}$

hw4.sce (/Users/rothin1553/Desktop/NM/hw4/hw4.sce) - SciNotes

hw4.sce

```
x1= -2:0.1:-1;

x2=-1:0.1:0;

x3=0:0.1:1;

x4=1:0.1:2;

y1= (x1/6) + (1/3)
y2= (x2/6) + (1/3)
y3= (-x3/6) + (1/6)
y4= (-x4./6) + (1/3)

plot(x1,y1)
plot(x2,y2)
plot(x3,y3)
plot(x4,y4)
```



② (1) $T_0(0.5) = \cos(0) = 1$

$T_1(0.5) = \cos(0.5) = 0.5$

$T_2(0.5) = \cos(2\theta) = 2(0.5)^2 - 1 = \boxed{-0.5}$

$T_3(0.5) = \cos(3\theta) = 4(0.5)^3 - 3(0.5) = \boxed{-1}$

$T_4(0.5) = \cos(4\theta) = 8(0.5)^4 - 8(0.5)^2 + 1 = \boxed{-0.5}$

$T_5(0.5) = \cos(5\theta) = 16(0.5)^5 - 20(0.5)^3 + 5(0.5) = \boxed{0.5}$

(2).

$$T_2(0.5) = 2(0.5)^2 - 1 = \boxed{-0.5}$$

$$T_3(0.5) = 4(0.5)^3 - 3(0.5) = \boxed{-1}$$

$$T_4(0.5) = 8(0.5)^4 - 8(0.5)^2 + 1 = \boxed{-0.5}$$

$$T_5(x) = 2(x)\,T_4(x) - T_3(x)$$

$$= 16x^5 - 16x^3 + 2x - (4x^3 - 3x)$$

$$= 16x^5 - 20x^3 + 5x$$

$$T_5(0.5) = 16(0.5)^5 - 20(0.5)^3 + 5(0.5) = \boxed{0.5}$$

③ ① $P_1(x) = \sin(0) + \cos(0)\,x$

$= x$

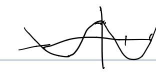
② $\sin(x) - P_1(x) \approx \frac{(x-x_0)(x-x_1)}{2} - \sin(x_0)$

$\approx -\frac{(x+1)(x-1)}{2}\sin(-1)$

≈ 0

③ $p(x) = x_0 + x_1 x$

$g(x_0, x_1) = \int_{-1}^{1} (\sin x - x_0 - x_1 x)^2 \, dx$



$\frac{\partial g}{\partial x_0} = 2\int_{-1}^{1} (\sin x - x_0 - x_1 x)(-1)\,dx = -2\left[-x_0 x\right]\Big|_{-1}^{1}$

$= -2\left[-x_0 - x_0\right] = 2(1 - x_0)$

$\frac{\partial g}{\partial x_1} = 2\int_{-1}^{1} (\sin x - x_0 - x_1 x)(-x)\,dx = 2\left[-\cos x - \frac{x_1 x^2}{2}\right]\Big|_{-1}^{1}$

$= 2\left\{1 - \frac{1}{2}x_1 - \left(1 - \frac{1}{2}x_1\right)\right\}$

$= 2(\cos 1 - \sin 1) + \frac{2x_1}{3}$

$x_0 = 1 \quad x_1 = 3(\sin 1 - \cos 1)$

$p(x) = 1 + 3(\sin 1 - \cos 1)\,x$

(4) $$\max_{-1 \le x \le x} \left| \sin(x) - P(x) \right|$$

$$= \max_{-1 \le x \le x} \left| \sin(x) - 1 - 3(\sin 1 - \cos 1)x \right|$$