Math 4533/5533 Numerical Methods (Homework #5)

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Note that Due is Thursday (4/30)

- Note that you may need to round all values to six decimal digits.

1. (30points) Approximate the integrals, using $n = 3$ Gaussian Quadrature. See the table in the lecture note. Compare with the exact value and find the error.
   (1) (10points)
   $$\int_{-1}^{1} \left(x^3 - 3x^2\right) dx$$
   (2) (10points)
   $$\int_{-1}^{1} \cos x \, dx$$
   (3) (10points)
   $$\int_{-1}^{1} e^x dx$$

2. (20points) Using Gaussian Quadrature $n = 2$, approximate the following integrals. Compare with the exact value and find the error.
   (1) (10points)
   $$\int_{-2}^{2} \frac{1}{x^2+1} dx.$$
   (2) (10points)
   $$\int_{1}^{2} \ln\left(x^2\right) dx.$$

Computer Project (50points)

1. Consider the following integral:
   $$I(f) = \int_0^{\pi/4} e^{\cos x} dx \doteq 1.939735.$$
   Then write Scilab codes.
   (1) (20points) Find $T_n(f)$, $S_n(f)$ for $n = 5$, 20.
   (2) (10points) Find their errors E_n^T and E_n^S for $n = 5$, 20.
   (3) (10points) Check the following error estimate
   $$\left|E_n^T(f)\right| \leq \frac{h^2}{12}\left(\frac{\pi}{4}\right) \max_{0\leq x\leq \pi/4} \left|f''(x)\right|.$$
   You can look up the Theorem in the Lecture note 17.
   (4) (10points) Use Gaussian Quadrature with $n = 3$ to approximate $I(f)$.

Homework #5

(#1) $\int_{-1}^{1} (x^3 - 3x^2)\,dx$ $\quad n=2 \quad a=-1 \quad b=1$

n	integral
1	0
2	-2
exact value	-2

$$\int_{-1}^{1} \left[x^3 - 3x^2\right] dx = f\left(\frac{1}{\sqrt{3}}\right) + f\left(\frac{-1}{\sqrt{3}}\right)$$

$$= \left(\frac{1}{\sqrt{3}}\right)^3 - 3\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{-1}{\sqrt{3}}\right)^3 + 3\left(\frac{-1}{\sqrt{3}}\right)^2 = -2$$

Exact: $\left[\frac{x^4}{4} - x^3\right]_{-1}^{1} = \left[\frac{1}{4} - 1\right] - \left[\frac{1}{4} + 1\right] = -2$

Error: 0

$$\int_{-1}^{1} \cos(x)\,dx$$

$n = 2$

$a = -1$

$b = 1$

n	integral
1	2
2	1.675824
exact value	1.682941969 6

$$\int_{-1}^{1} \cos(x)\,dx = \cos\left[\frac{1}{\sqrt{3}}\right] + \cos\left[\frac{-1}{\sqrt{3}}\right]$$

$$\approx 1.675823655$$

Exact: $\sin(x)\Big|_{-1}^{1} = \sin(1) - \sin(-1) \approx 1.682941969 6$

Error $= -0.007118$

$$\int_{-1}^{1} e^x \, dx$$

$n = 2$

$a = -1$

$b = 1$

n	integral
1	2
2	2.342696
exact value	$e - e^{-1} \approx$ 2.350402 3873

$$\int_{-1}^{1} e^x \, dx = e^{\left(\frac{1}{\sqrt{3}}\right)} + e^{\left(\frac{-1}{\sqrt{3}}\right)}$$

$$=$$

Exact: $e^x \Big|_{-1}^{1} = e^1 - e^{-1} \approx 2.350402 3873$

Error: -0.007706

#2

$$\int_{-2}^{2} \left\{ \frac{1}{x^2+1} \right\} dx$$

n	integral
1	2
2	1.5
exact value	$2\tan^{-1}(2) = 2.2143$

$n = 2$

$a = -2$

$b = 2$

$$\int_{-2}^{2} \left\{ \frac{1}{x^2+1} \right\} dx = \frac{1}{\left\{\frac{1}{\sqrt{3}}\right\}^2 + 1} + \frac{1}{\left\{\frac{-1}{\sqrt{3}}\right\}^2 + 1}$$

$$= \frac{3}{2}$$

Exact: $\int_{-2}^{2} \frac{1}{x^2+1} = 2\tan^{-1}(2) \approx 2.2143$

Error: -0.7143

$$\int_{-2}^{2} \ln(x^2)\,dx$$

$n = 2$

n	integral
1	undefined
2	-2.19722457
exact value	$\ln(16)-2 \approx 0.77259$

$a = 1$

$b = +2$

$$\int_{1}^{2} \ln(x^2)\,dx = \ln\left[\left(\frac{1}{\sqrt{3}}\right)^2\right] + \ln\left[\left(\frac{-1}{\sqrt{3}}\right)^2\right]$$

$$= -2.19722457$$

Exact: $\int_{1}^{2} \ln(x^2)\,dx = \ln(16)-2 \approx 0.77259$

Error: -2.96725

Computer Project:

#1

hw5.sce (/Users/rothin1553/Desktop/NM/hw5/hw5.sce) - SciNotes

hw5.sce

```
1
1  function value = testFunc(x)
2      value = exp(cos(x));
3  endfunction
5
1  function approx = tapez(a,b,n,func)
2      h = (b-a)/n;
3      sum = 0;
4      for i = 1:n
5          sum = sum + func(a+(i-1)*h) + func(a+i*h);
6      end
7      approx = sum * h / 2;
8  endfunction
14
1  function approx = simpson(a,b,n,func)
2      h = (b-a)/n;
3      sum = 0;
4      for i = 1:n/2
5          sum = sum + func(a+2*(i-1)*h)+(4*func(a+(2*(i-1)+1)*h)) + func(a
   +(2*(i-1)+2)*h);
6      end
7      approx = sum*h/3;
8  endfunction
23
24 //test trap for n = 5, 20
25 disp(tapez(0,%pi/4,5,testFunc));
26 disp(tapez(0,%pi/4,20,testFunc));
27
28 // test simpson for n = 5, 20
29 disp(simpson(0,%pi/4,5,testFunc));
30 disp(simpson(0,%pi/4,20,testFunc));
31
```

Line 19, Column 88. Function 'simpson' at Line 5.

```
--> exec('/Users/rothin1553/Desktop/NM/hw5/hw5.sce', -1)
   1.9367829
   1.9395505
   1.6038499
   1.9397349
-->
```

$T_5(f)$

$T_{20}(f)$

$S_5(f)$

$S_{20}(f)$

#2 Error:

$$E_5^T = -0.002952$$

$$E_{20}^T = -0.000185$$

$$E_5^S = -0.3358851$$

$$E_{20}^S = -0.0000001$$

#3

$$\left|E_5^1(4)\right| = \frac{\left(\frac{\pi}{20}\right)^2}{12}\left(\frac{\pi}{4}\right)\left|e^{\cos(0)}\left(\sin^2(0) - \cos(0)\right)\right|$$

$$\approx 0.004390$$

$$\left|E_{20}^1(4)\right| = \frac{\left(\frac{\pi}{80}\right)^2}{12}\left(\frac{\pi}{4}\right)\left|e^{\cos(0)}\left(\sin^2(0) - \cos(0)\right)\right|$$

$$\approx 0.000274$$

#4

$$f(x) = \int_0^{\pi/4} e^{\cos(x)}\,dx \qquad n = 3 \quad a = 0 \quad b = \pi/4$$

$$\int_0^{\frac{\pi}{4}} e^{\cos(x)} \approx \frac{5}{9} e^{\cos\left(\sqrt{\frac{3}{5}}\right)} + \frac{5}{9} e^{\cos\left(\sqrt{\frac{3}{5}}\right)} + \frac{8}{9} e^{\cos(0)}$$

$$\approx 5.621506$$