

Math 4533/5533 Numerical Methods (Homework #5)

Note that Due is Thursday (4/30)

- Note that you may need to round all values to six decimal digits.
- 1. (30points) Approximate the integrals, using n=3 Gaussian Quadrature. See the table in the lecture note. Compare with the exact value and find the error.
 - (1) (10points)

$$\int_{-1}^{1} \left(x^3 - 3x^2 \right) dx$$

(2) (10points)

$$\int_{-1}^{1} \cos x \, dx$$

(3) (10points)

$$\int_{-1}^{1} e^x dx$$

- 2. (20points) Using Gaussian Quadrature n=2, approximate the following integrals. Compare with the exact value and find the error.
 - (1) (10points)

$$\int_{-2}^{2} \frac{1}{x^2 + 1} dx.$$

(2) (10points)

$$\int_{1}^{2} \ln\left(x^{2}\right) dx.$$

Computer Project (50points)

1. Consider the following integral:

$$I(f) = \int_0^{\pi/4} e^{\cos x} dx \doteq 1.939735.$$

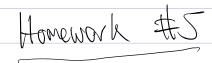
Then write Scilab codes.

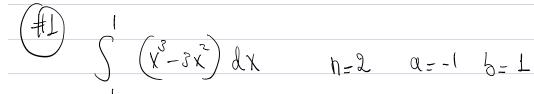
- (1) (20points) Find $T_n(f)$, $S_n(f)$ for n = 5, 20.
- (2) (10points) Find their errors E_n^T and E_n^S for $n=5,\,20$. (3) (10points) Check the following error estimate

$$\left| E_n^T(f) \right| \le \frac{h^2}{12} \left(\frac{\pi}{4} \right) \max_{0 \le x \le \pi/4} \left| f''(x) \right|.$$

You can look up the Theorem in the Lecture note 17.

(4) (10points) Use Gaussian Quadrature with n=3 to approximate I(f).





U	integral
1	0
2	-2
exact value	- 2

$$\int_{-1}^{1} \left(x^{2} - 3x^{2} \right) dx = f\left(\frac{1}{13} \right) + f\left(\frac{1}{13} \right) + f\left(\frac{1}{13} \right) + f\left(\frac{1}{13} \right) = -2$$

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$$\int_{-1}^{1} \left(x^{2} - 3x^{2} \right) dx = f\left(\frac{1}{13} \right) + f\left($$

Ener: 0

	cos(x) dx					
		l ~ t	n=2			
	N N	integral	d=-1 b=1			
	2	1.675824	051			
	exact value	1.6829419696				
			,			
$\int \cos(x) dx = \cos\left(\frac{1}{\sqrt{3}}\right) + \cos\left(\frac{-1}{\sqrt{3}}\right)$						
~ 1.67582365S						
Exort: $570(4) = 510(1) - 500(4) \approx 1.6829419696$						
Ever = -0.007/18						

ex dx N=2 integral \mathcal{U} 2.342696 exact value e-e ≈ 2.3 504023873 Choef: et | - e'-e' ≈ 2.3504623873 Ever: -0.007706



$$\int_{-2}^{2} \left(\frac{1}{x^2 + 1} \right) dx$$

U	integral	n=2
1	2	Q = -2
2	1,5	h-2
exact value	2ton-1(2) = 2.2143	

$$\int_{-\infty}^{\infty} \left(\frac{1}{x^2} \right) dx = \frac{1}{(13)^2 + 1} \left(\frac{1}{13} \right) + 1$$

Exold:
$$\int_{-2}^{2} \frac{1}{\chi^{2}+1} = 2+\cos^{-1}(2) \approx 2.2143$$

$$\int_{-2}^{2} \ln(x^{2}) dx$$

n=2

Q= \

6-+2

U	integral	
1	undernal	
2	-2.197224577	
exoct value	(n(16)-2 = 0.77259	

 $\int_{1}^{2} \ln(y^{2}) dy = \ln\left(\frac{1}{\sqrt{3}}\right) + \ln\left(\frac{1}{\sqrt{3}}\right)$

= -2.197224577

Exact: $\int_{1}^{7} \ln(x^2) dx = \ln(16) - 2 \approx 0.77259$

Evror: -2.969725

Computer Project:

```
hw5.sce (/Users/rothin1553/Desktop/NM/hw5/hw5.sce) – SciNotes
                                 hw5.sce 🗶
    1
    1 function value = testFunc(x)
    2 - - - value = exp(cos(x));
    3 endfunction
    5
    1 function approx = tapez(a,b,n,func)
       a - b - h = a (b-a)/n;
    2
    3
       ---sum-=-0:
      ---for-i-=-1:n
    4
      -----sum = -sum + -func(a+(i-1)*h) + -func(a+i*h);
    5
    6
         - approx - = - sum - * - h - / - 2;
    7
    8 endfunction
   14
    1 function approx = simpson(a,b,n,func)
         h = - (b-a)/n;
    2
         sum - = - 0;
    3
         for - i - = - 1: n/2
    4
             +(2*(i-1)+2)*h);
    6
         approx = = sum*h/3;
    8 endfunction
   23
   24 //test-trap-for-n-=-5,-20
   25 disp(tapez(0,%pi/4,5,testFunc));
   26 disp(tapez(0,%pi/4,20,testFunc));
   27
   28 //-test-simpson-for-n-=-5,-20
   29 disp(simpson(0,%pi/4,5,testFunc));
   30 disp(simpson(0,%pi/4,20,testFunc));
   31
 Line 19, Column 88. Function 'simpson' at Line 5.
  -> exec('/users/rotninibb3/Desktop/NM/nwb/nwb.sce', -1)
    1.9367829
    1.9395505
    1.6038499
    1.9397349
```

$$E_{s}(4) = \frac{1}{20}(T_{4}) = \frac{(20)(5m^{2}(0) - (090))}{12}$$

